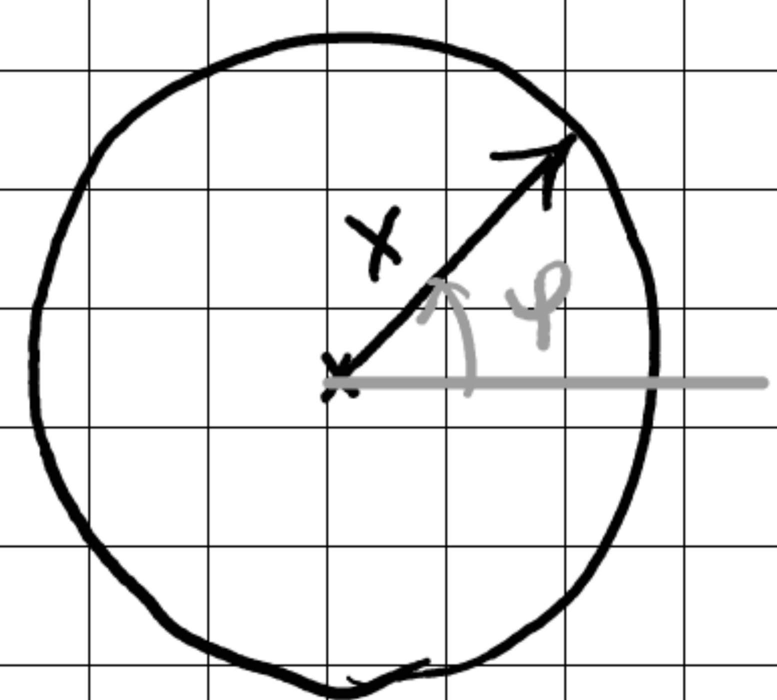
Ex681 - Ulysse (Fogebaume)

a)



Soit $X \in \mathbb{R}^2$. $\|X\| = 1$

$$X = \begin{pmatrix} \cos 2\pi\varphi \\ \sin 2\pi\varphi \end{pmatrix}$$

On cherche M sous la forme d'une matrice rotation.

$$M_\alpha = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

<u>$\alpha = 1$</u> $\quad G = 2\pi\mathbb{Z} + \mathbb{Z}$ sous groupe additif de $\mathbb{R}$ => soit discret soit dense

si $G = a\mathbb{Z} \Rightarrow \begin{cases} 1 = ka \\ 2\pi = k'a \end{cases} \Rightarrow \pi \in \mathbb{Q}$
ABSURDE

Donc G dense, donc <u>$\mathbb{Z} + 2\pi\mathbb{Z} + \varphi$ aussi dense</u>

$$\overline{\mathbb{Z} + 2\pi\mathbb{Z}} = \mathbb{R}$$

$$\overline{\cos(\mathbb{Z} + 2\pi\mathbb{Z})} = [-1, 1]$$

$$\overline{\sin(\mathbb{Z} + 2\pi\mathbb{Z})} = [-1, 1]$$

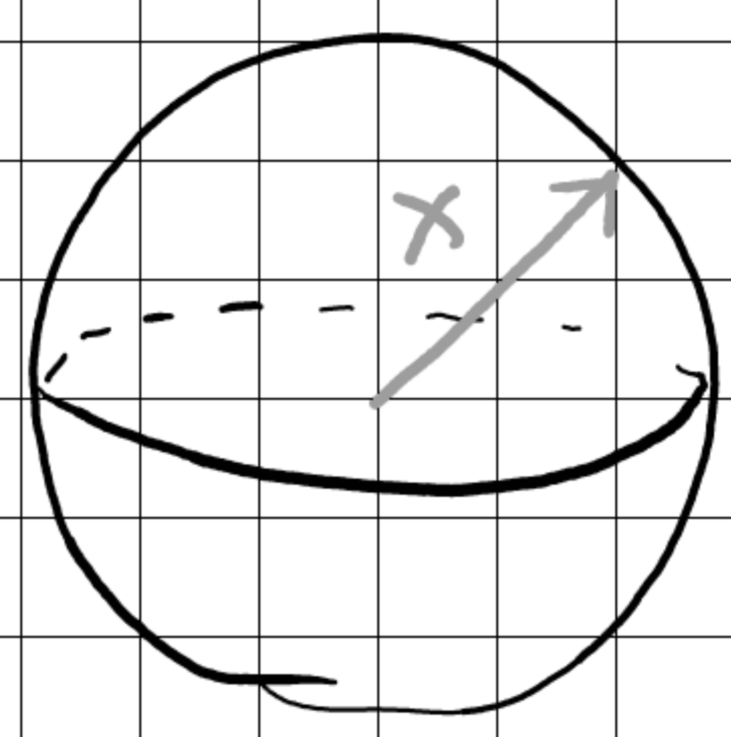
$$\overline{\exp(i(\mathbb{Z} + 2\pi\mathbb{Z}))} = \mathbb{U}$$

$$e^{i\theta} = \lim_{n \to +\infty} \exp(i z_n) \qquad z_n = \underbrace{a_n}_{\in \mathbb{Z}} + 2\pi \underbrace{b_n}_{\in \mathbb{Z}} + \varphi$$

$(M^n X)_{n\in\mathbb{Z}}$ dense dans S^1

$$M^{a_n} X = e^{i(\varphi + a_n)} = e^{i(\varphi + a_n + 2\pi b_n)} \xrightarrow[n\infty]{} e^{i\theta}$$

b)



Soit $M \in O_3$

$$\exists \theta \in \mathbb{R},\ M = P \begin{pmatrix} 1 & 0 \\ 0 & R(\theta) \end{pmatrix} P^{-1}$$