

Exercice 617 (Martin)

$$A, B \in \mathcal{S}_m^+(\mathbb{R})$$

th spectral $A = P D P^T$
 $D \in \mathcal{D}_m^+(\mathbb{R})$

$\lambda_1, \dots, \lambda_m$ valeurs propres de A

$$\begin{aligned} \text{tr}(AB) &= \text{tr} P D P^T B \\ &= \text{tr} (D \underbrace{P^T B P}_{= B'}) \end{aligned}$$

$$= \sum \underbrace{\lambda_i}_{\geq 0} \underbrace{b'_{ii}}_{\geq 0}$$

$$\leq \underbrace{\sum \lambda_i}_{\text{tr } D = \text{tr}(A)} \underbrace{\sum b'_{ii}}_{\text{tr } B' = \text{tr } B}$$

$$\langle \underbrace{B E_{ii}, E_{ii}}_{\geq 0} \rangle = b_{ii}$$

$$\langle B P E_{ii}, P E_{ii} \rangle \geq 0$$

Exercice 634 (Amcis)

$$a \in \mathbb{R}_+^* \quad \left[\begin{array}{cc} 1 & \frac{a}{n} \\ -\frac{a}{n} & 1 \end{array} \right]^n \xrightarrow{n \rightarrow \infty} ?$$

$$A = \begin{bmatrix} 1 & a/n \\ -a/n & 1 \end{bmatrix}$$

$$\chi_A = x^2 - 2x + 1 + \frac{a^2}{n^2}$$

$$\Delta = -\frac{a^2}{n^2} < 0$$

A est diagonalisable dans $\mathbb{C}$

$$\chi_A = (X - (1 + i\frac{a}{n}))(X - (1 - i\frac{a}{n}))$$

$$A = P \begin{bmatrix} \overbrace{1 + i\frac{a}{n}}^{\lambda_1} & 0 \\ 0 & \underbrace{1 - i\frac{a}{n}}_{\lambda_2} \end{bmatrix} P^T$$

$$E_{\lambda_1}(A) = \text{Ker}(A - \lambda_1 I_n)$$

$$= \text{Ker} \begin{bmatrix} -i\frac{a}{n} & \frac{a}{n} \\ -\frac{a}{n} & -i\frac{a}{n} \end{bmatrix}$$

$$= \text{Vect} \left(\begin{pmatrix} 1 \\ i \end{pmatrix} \right) \quad (C_1 + iC_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix})$$

$$E_{\lambda_2}(A) = \text{Vect} \left(\begin{pmatrix} 1 \\ -i \end{pmatrix} \right)$$

$$P = \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$$

$$A^n = P D^n P^{-1} = P \begin{bmatrix} (1 + i\frac{a}{n})^n & 0 \\ 0 & (1 - i\frac{a}{n})^n \end{bmatrix} P^{-1}$$

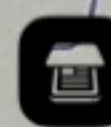
$$z = \rho e^{i\theta}$$

$$1 + i\frac{a}{n} = \sqrt{1 + \frac{a^2}{n^2}} e^{i \arctan(\frac{a}{n})}$$

$$1 - i\frac{a}{n} = \sqrt{1 + \frac{a^2}{n^2}} e^{-i \arctan(\frac{a}{n})}$$

$$D^n = \begin{bmatrix} e^{\frac{n}{2} \ln(1 + \frac{a^2}{n^2})} & i \arctan(\frac{a}{n}) \\ 0 & 0 \end{bmatrix}$$

$$e^{\frac{n}{2} \ln(1 + \frac{a^2}{n^2})} e^{i \arctan(\frac{a}{n})}$$



Exercice 634 (suite)

$$D^n \xrightarrow{n \rightarrow \infty} \begin{bmatrix} e^{ia} & 0 \\ 0 & e^{-ia} \end{bmatrix}$$

$$A^n \xrightarrow{n \rightarrow \infty} \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$$

Exercice 658 (William)

$$\text{Sur } \mathbb{R}^n, \|x\|_1 = \sum_{k=1}^n |x_k|$$

$$\text{Mq Soient } (x, y) \in \mathbb{R}^n, \|x+y\|_1 + \|x-y\|_1 = 2(\|x\|_1 + \|y\|_1)$$

$$\Leftrightarrow \forall k \in [1, n], x_k y_k = 0$$

$$\Rightarrow \sum_{k=1}^n |x_k + y_k| + \sum_{k=1}^n |x_k - y_k| = 2 \sum_{k=1}^n |x_k| + |y_k|$$

$$\text{Or } \forall k \in [1, n], |x_k + y_k| + |x_k - y_k| \leq 2(|x_k| + |y_k|)$$

$$\sum_{k=1}^n \underbrace{2(|x_k| + |y_k|) - |x_k + y_k| - |x_k - y_k|}_{\geq 0} = 0$$

$$\Rightarrow \forall k \in [1, n], 2(|x_k| + |y_k|) = |x_k + y_k| + |x_k - y_k|$$

$$\Rightarrow x_k, y_k, -y_k \text{ de même signe } \forall k \in [1, n].$$

$$\Rightarrow x_k y_k = 0 \forall k \in [1, n]$$