Exercice 617 (Martin)

$A, B \in \mathcal{S}_n^+(\mathbb{R})$

th spectral $A = P \underbrace{D}_{\in D_n^+(\mathbb{R})} P^T$

$\lambda_1, — \lambda_n$ valeurs propres de A

$$\begin{aligned} tr(AB) &= tr\, PDP^T B \\ &= tr(D\underbrace{P^T B P}_{= B'}) \\ &= \sum \underbrace{\lambda_i}_{\geq 0} \underbrace{b'_{ii}}_{\geq 0} \\ &\leq \underbrace{\sum \lambda_i}_{\substack{tr\, D \\ = tr(A)}} \underbrace{\sum b'_{ii}}_{\substack{tr\, B' \\ = tr\, B}} \end{aligned}$$

$$\underbrace{\langle BE_{ii}, E_{ii} \rangle}_{\geq 0} = b_{ii}$$

$$\langle BPE_{ii}, PE_{ii} \rangle \geq 0$$

Exercice 634 (Anaïs)

$a \in \mathbb{R}_+^*$ $\quad \begin{bmatrix} 1 & \frac{a}{n} \\ -\frac{a}{n} & 1 \end{bmatrix}^n \xrightarrow[n\infty]{} \; ?$

$$A = \begin{bmatrix} 1 & a/n \\ -a/n & 1 \end{bmatrix}$$

$$\chi_A = x^2 - 2x + 1 + \frac{a^2}{n^2}$$

$$\Delta = -\frac{a^2}{n^2} < 0$$

A est diagonalisable dans $\mathbb{C}$