supposons $H(k)$.

On a : $u_i \circ g^{k+1} = u_i \circ g^k \circ g$
$$= f^k \circ \underbrace{u_i \circ g}_{= f \circ u_i}$$
$$= f^{k+1} \circ u_i$$

d'où $H(k+1)$.

<u>CCL</u> : $\forall k \in \mathbb{N}$, $u_i \circ g^k = f^k \circ u_i$

par linéarité, $\forall P \in \mathbb{K}[X]$, on a : $u_i \circ P(g) = P(f) \circ u_i$.

f diagonalisable donc $\exists P$ SARS tq $P(f) = 0_{\mathcal{L}(E)}$

d'où $\forall i \in [\![1;n]\!]$, $u_i \circ P(g) = \underbrace{P(f)}_{=0_{\mathcal{L}(E)}} \circ u_i = 0_{\mathcal{L}(E)}$

$\forall i \in [\![1;n]\!]$, $u_i \circ P(g) = 0_{\mathcal{L}(E)}$

$\forall i \in [\![1;n]\!]$, $\mathrm{Im}(P(g)) \subset \mathrm{Ker}\, u_i$

d'où : $\{0\} \subset \mathrm{Im}\, P(g) \subset \bigcap_{i=1}^{n} \mathrm{Ker}(u_i) = \{0\}$

donc $P(g) = 0_{\mathcal{L}(E)}$ avec P SARS donc g diagonalisable.

<u>Exercice 525</u> (Ulysse).

1. Soit $(v_1, \dots, v_p) \in (\mathbb{R}^n)^p$

tq $\forall (i,j) \in [\![1;p]\!]^2$ tq $i \neq j$, $\langle v_i, v_j \rangle < 0$.

$$\|x\|^2 = \left\langle \sum_{i=1}^{p} x_i v_i , \sum_{j=1}^{p} x_j v_j \right\rangle$$
$$= \sum_{i=1}^{p} x_i^2 \|v_i\|^2 + 2 \sum_{1 \leq i < j \leq p} x_i x_j \langle v_i, v_j \rangle$$

$$\|x\|^2 - \|y\|^2 = 2 \sum_{1 \leq i < j \leq p} \underbrace{(x_i x_j - |x_i x_j|)}_{\leq 0} \underbrace{\langle v_i, v_j \rangle}_{< 0} \geq 0$$

d'où : $\|x\| \geq \|y\|$

2. On suppose $\|x\| = 0 \Rightarrow \|y\| = 0$

Soit $i_0 \in [\![1 ; p]\!]$ tq $x_{i_0} = 0$

$\langle y, v_{i_0} \rangle = \sum_{i \neq i_0} \underbrace{|x_i|}_{\geq 0} \underbrace{\langle v_i, v_{i_0} \rangle}_{< 0} + 0$

donc $\forall i \neq i_0$, $x_i = 0$.

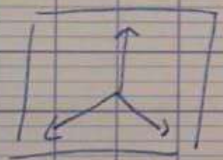
3. Soit $(\lambda_1, \dots, \lambda_{p-1}) \in \mathbb{R}^{p-1}$ tq :

$$x = \sum_{i=1}^{p-1} \lambda_i v_i = 0$$

$\Rightarrow \forall i \in [\![1 ; p-1]\!]$, $\lambda_i = 0$

d'où $(v_1, \dots, v_{p-1})$ libre

$n \geq p-1 \Leftrightarrow p \leq n+1$

5.   passage 2D à 3D.

Soit $n \in \mathbb{N}$ et $(v_1, \dots, v_{n+1})$ obtusangles.

$v_{n+2} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$, $v_i' = v_i - \alpha v_{n+2}$

Pour $i \in [\![1 ; n+1]\!]$, $\langle v_{n+2}, v_i' \rangle = \langle v_{n+2}, v_i \rangle - \alpha \|v_{n+2}\|^2$

$= -\alpha$

Soit $(i,j) \in [\![1 ; n+1]\!]^2$

$\langle v_i', v_j' \rangle = \langle v_i - \alpha v_{n+2}, v_j - \alpha v_{n+2} \rangle$

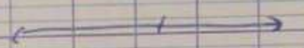
$= \langle v_i, v_j \rangle + \alpha^2 \underbrace{\|v_{n+2}\|^2}_{=1}$

< 0

donc $\alpha^2 < -\langle v_i, v_j \rangle$

$$\alpha = -\frac{1}{2} \min_{(i,j) \in [\![1; n+1]\!]^2} \langle v_i, v_j \rangle$$

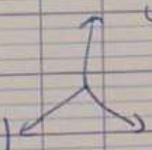
$\mathbb{R}^1$:



$v_2 = (-1)$ $\quad$ $v_1 = (1)$

$\mathbb{R}^2$

$v_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$v_2 = \begin{pmatrix} -1 \\ -\frac{1}{2} \end{pmatrix}$ $\quad$ $v_1 = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix}$

$$\left(\begin{pmatrix} 1 \\ -\frac{1}{2} \\ \vdots \\ -\frac{1}{2^{n+1}} \end{pmatrix}, \begin{pmatrix} -1 \\ -\frac{1}{2} \\ \vdots \\ -\frac{1}{2^{n}} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -\frac{1}{2} \\ \vdots \\ -\frac{1}{2^{n-1}} \end{pmatrix}, \dots, \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \right)$$