4.13 $E = \mathbb{R}^{\mathbb{N}}$ Soit $T \left| \begin{array}{l} \mathbb{R}^{\mathbb{N}} \longrightarrow \mathbb{R}^{\mathbb{N}} \\ (u_n) \longrightarrow (v_n) \end{array} \right.$

où $v_n = \frac{1}{n+1} \sum_{k=0}^{n} u_k$

$\forall n \in \mathbb{N}$

Éléments propres de T.

T injective donc $0 \notin Sp(T)$

Soit $\lambda \in Sp(T)$,

alors $\forall n \in \mathbb{N}$ $v_n = \lambda u_n$

$$\Longleftrightarrow \sum_{k=0}^{n} u_k = (n+1) u_n$$

<u>$n=0$</u> $u_0 = \lambda u_0$

<u>cas 1</u> $u_0 \neq 0 \Rightarrow \lambda = 1$

$u_1 + u_0 = 2u_1 \Rightarrow u_1 = u_0$

$u_2 + u_1 + u_0 = 3u_2$

$\Rightarrow u_1 + u_0 = 2u_2$

$\Rightarrow u_2 = u_0$

on trouve $E_1(T) = \{\text{suite } c^{te}\}$

<u>cas 2</u> $u_0 = 0$

$u_0 + u_1 = 2\lambda u_1$

donc $u_1 = 2\lambda u_1$

Si $u_1 \neq 0$, $\lambda = \frac{1}{2}$; $\rightarrow u_n = u_1$ $E_{\frac{1}{2}}(T)$

$$\lambda = \frac{1}{i}$$

$$\sum_{k=0}^{n} u_k = \frac{n+1}{i} u_n$$

$$u_{i-1} = u_{i-1}$$

$$u_i + u_{i-1} = \frac{i+1}{i} u_i$$

$$u_{i-1} = \frac{1}{i} u_i$$

$$\boxed{u_i = i u_{i-1}}$$

$$u_{i-1} + u_i + u_{i+1} = \frac{i+2}{i} u_{i+1}$$

$$(i+1) u_{i-1} = \frac{2}{i} u_{i+1}$$

$$\boxed{u_{i+1} = \frac{i(i+1)}{2} u_{i-1}}$$

$$u_{i-1} + u_i + u_{i+1} + u_{i+2} = \frac{i+3}{i} u_{i+2}$$

$$(i+1) u_{i-1} + \frac{i(i+1)}{2} u_{i-1} = u_{i+2} \left(\frac{3}{i}\right)$$

$$\frac{2(i+1) + i(i+1)}{2} u_{i-1} = \frac{3}{i} u_{i+2}$$

$$\frac{(i+1)(i+2)}{2} = \frac{3}{i} u_{i+2}$$

$$\boxed{u_{i+2} = \frac{i(i+1)(i+2)}{6}}$$

$$u_{i-1+l} = \frac{\sum_{k=0}^{l} g(i+k)}{l!} u_{i-1}$$

$$u_{i-1+l} = g(i) u_{i-1}$$

$$u_{i+l} -$$

$$u_{i-1} + u_i + u_{i+1} + u_{i+2} + u_{i+3} = \frac{i+4}{i} u_{i+3}$$

$$\frac{(i+1)(2+i)}{2} + \frac{i(i+1)(i+2)}{6} = \frac{4}{i} u_{i+3}$$

$$\frac{(i+1)(i+2)(i+3)}{6} u_{i-1} = \frac{4}{i} u_{i+3}$$

$$u_{i+3} = \frac{i(i+1)(i+2)(i+3)}{24}$$

$$u_{i+3} = n$$

$$u_{i+l} = \frac{(i+l)!}{(l+1)!(i-1)!}$$

$$= \frac{(i+l)!}{(l+1)!(i-1)!} u_{i-1}$$

$$\boxed{E_{1/i}(T) = \left\{ u \in \mathbb{R}^{\mathbb{N}} ;\ \forall n \geq i-1,\ u_n = \frac{n!}{(n-i+1)!(i-1)!} u_{i-1} \right\}}$$

$$n = l + i$$

$$l + 1 = n - i + 1$$