

$$\text{dnc } \text{sp}(P) = \{f(\lambda), \lambda \in \text{sp}(S)\}$$

$$f(\lambda) = (\lambda - a)(\lambda - b) \geq 0.$$

Exercice 436:

Cas 1: Si $A = \lambda I_2$ alors $C(A) = M_2(\mathbb{R})$
dnc $\dim C(A) = 4.$

Cas 2: Si $A \neq \lambda I_2$
 (A, I_2) libre dans $\mathcal{C}(A)$ dnc $\dim C(A) \geq 2.$

Supposons $\dim \mathcal{C}(A) = 3$

Si A diagonalisable, $A = PDP^{-1}$

Soit $\Pi \in \mathcal{C}(A)$, $\Pi A = A\Pi$ dnc $\Pi P D P^{-1} = P D P^{-1} \Pi$
dnc $P^{-1} \Pi P \in \mathcal{C}(D)$

dnc $\dim C(A) = \dim [C(D)]$ et $D \neq \lambda I_2$ par hyp
Et $\Pi D = D\Pi$ dnc $\Pi \in \mathcal{D}_2(\mathbb{C})$

Dnc $\dim C(A) = \dim (\mathcal{D}_2(\mathbb{C})) = 2$ $\varphi \in A$

$A \in M_2(\mathbb{C})$ dnc A diagonalisable $A = PTP^{-1}$

dnc $\dim C(A) = \dim [C(T)]$

$$\Pi T = T\Pi \Leftrightarrow \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{\Pi} \begin{pmatrix} \alpha & \beta \\ 0 & \alpha \end{pmatrix} = \begin{pmatrix} \alpha a & a\beta + b\alpha \\ \alpha c & c\beta + d\alpha \end{pmatrix}$$

$$T = \alpha I_2 + \begin{pmatrix} 0 & \beta \\ 0 & 0 \end{pmatrix}$$

Dnc $\mathcal{C}(T) = \mathcal{C}(E_{12})$ dnc $\dim \mathcal{C}(A) = 2$ $\varphi \in A$

Ccl: $\dim \mathcal{C}(A) \in \{2, 4\}$