(4.11) (Mcaun) $\begin{cases} A = (a_{ij})_{1 \le i,j \le n} \\ B = (b_{ij})_{1 \le i,j \le n} \end{cases}$

$$A \otimes B = \begin{pmatrix} \boxed{a_{11} B} & \cdots & \boxed{a_{1n} B} \\ \vdots & \ddots & \\ & & \boxed{a_{nn} B} \end{pmatrix}$$

$$(A \otimes B)(C \otimes D) = \begin{pmatrix} \boxed{a_{11} B} & \cdots & \boxed{a_{1n} B} \\ \vdots & \ddots & \\ \boxed{a_{n1} D} & & \boxed{a_{nn} B} \end{pmatrix} \begin{pmatrix} \boxed{c_{1} D} & \cdots & \boxed{c_{1n} D} \\ \vdots & \ddots & \\ \boxed{c_{n1} D} & & \boxed{c_{nn} D} \end{pmatrix}$$

$$= \begin{pmatrix} \boxed{\sum_{i=1}^{n} a_{1,i} c_{i,1} BD} & \cdots & \boxed{\sum_{i=1}^{n} a_{1,i} c_{i,n} BD} \\ \vdots & & \vdots \\ \boxed{\sum_{i=1}^{n} a_{n,i} c_{i,1} BD} & \cdots & \boxed{\sum_{i=1}^{n} a_{n,i} c_{i,n} BD} \end{pmatrix}$$

$$= \begin{pmatrix} (AC)_{1,1} BD & \cdots & (AC)_{1,n} BD \\ & & \vdots \\ (AC)_{n,1} BD & & (AC)_{n,n} BD \end{pmatrix}$$

d'où $(A \otimes B)(C \otimes D) = AC \otimes BD$ $(*)$

d'où $(A \otimes B)(A^{-1} \otimes B^{-1}) = (AA^{-1}) \otimes (BB^{-1})$

si $A, B \in GL_n(\mathbb{K})$

$= I_n \otimes I_n$

$= I_n$

2) Hyp : A, B diagonalisable

$\exists P$: $A = PDP^{-1}$

$\exists Q$: $B = QD'Q^{-1}$

alors $A \otimes B = \underbrace{(P \otimes Q)}_{M} (D \otimes D') \underbrace{(P^{-1} \otimes Q^{-1})}_{M^{-1}}$ $(*)$

d'où $\underline{A \otimes B \text{ diagonalisable}}$

173 (Dan) $M = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

$\oplus$ sev stables par u, ECA de M

$\chi_A(x) = (x-2)((x-3)(x-1)-1)$

$= (x-2)(x^2-4x+2)$

$= (x-2)(x-(2+\sqrt{2}))(x-(2-\sqrt{2}))$

χ_A SARS, donc A diagonalisable.

$\Longrightarrow$ on cherche X_i tq $MX_i = \lambda_i X_i$, $\lambda_i = \{2, 2+\sqrt{2}, 2-\sqrt{2}\}$