$$\forall n, \begin{cases} X_{n+1} = (1-x_n)^2 = X_n^2 \\ Y_{n+1} = (1-y_n)^2 = Y_n^2 \end{cases}$$

$$\forall\, n \in \mathbb{N}, \begin{cases} X_n = X_0^{2^n} \\ Y_n = Y_0^{2^n} \end{cases}$$

$$\Rightarrow x_n = 1 - X_n = 1 - X_0^{2^n}$$
$$y_n = 1 - Y_0^{2^n}$$

Soit $n \in \mathbb{N}$.

$$A_n = P \begin{pmatrix} 1-X_0^{2^n} & X \\ 0 & 1-Y_0^{2^n} \end{pmatrix} P^{-1}$$

<u>Pour $n \in \mathbb{N}$</u>, $\det A_n = (1-X_0^{2^n})(1-Y_0^{2^n})$

$$1-X_0^{2^n} = 1-\left(1-1+\frac{\sqrt{5}}{4}\right)^{2^n}$$
$$= 1-\left(\frac{\sqrt{5}}{4}\right)^{2^n}$$

On a $0 < \frac{\sqrt{5}}{4} < 1 \Rightarrow 1-X_0^{2^n} \xrightarrow[+\infty]{} 1$

Idem : $1-Y_0^{2^n} = 1-\left(1-1-\frac{\sqrt{5}}{4}\right)^{2^n} = 1-\left(\frac{\sqrt{5}}{4}\right)^{2^n} \xrightarrow[+\infty]{} 1$

donc $\det(A_n) \xrightarrow[+\infty]{} 1$

Exercice 374 : William

$P \in \mathbb{C}[X]$, $P(\mathbb{Q}) \subset \mathbb{Q}$

$$P = \sum_{k=0}^{n} a_k X^k \qquad (a_0, \dots, a_n) \in \mathbb{C}^n$$

$P(1) \in \mathbb{Q}$ et $P(1) = \sum_{k=0}^{n} a_k$

De même, $P(2) \in \mathbb{Q}$ donc $\sum_{k=0}^{n} a_k 2^k \in \mathbb{Q}$

etc... $P(n+1) = \sum_{k=0}^{n} a_k (n+1)^k \in \mathbb{Q}$

$$\begin{pmatrix} 1 & \cdots & \cdots & 1 \\ \vdots & 2 & \cdots & 2^n \\ \vdots & \vdots & & \\ 1 & n+1 & \cdots & (n+1)^n \end{pmatrix} \begin{pmatrix} a_0 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} P(1) \\ \vdots \\ P(n+1) \end{pmatrix} \in \mathbb{Q}^n$$

$$\begin{pmatrix} a_0 \\ \vdots \\ a_n \end{pmatrix} = A^{-1} \begin{pmatrix} P(1) \\ \vdots \\ P(n+1) \end{pmatrix} \in \mathbb{Q}^{n+1}$$

Exercice 351 : Ulysse

$$M = \begin{pmatrix} a & c & b \\ c & b & a \\ b & a & c \end{pmatrix}$$

$$M^2 = \begin{pmatrix} a & c & b \\ c & b & a \\ b & a & c \end{pmatrix} \begin{pmatrix} a & c & b \\ c & b & a \\ b & a & c \end{pmatrix} = \begin{pmatrix} 1 & \alpha & \beta \\ \alpha & 1 & \gamma \\ \beta & \gamma & 1 \end{pmatrix}$$

$$\det M^2 = 1(1-\gamma^2) - \alpha(\alpha - \beta\gamma) + \beta(\alpha\gamma - \beta)$$

$$= 1 - \gamma^2 - \beta^2 - \alpha^2$$