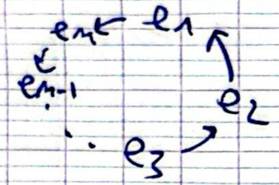
$$= \frac{V(b_1,\dots,b_n)\,V(a_1,\dots,a_n)}{\prod_{i\neq j}(a_i+b_j)}$$

Exercice 352 : Ulysse

$$H = \begin{pmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & & 1 \\ 1 & & & 0 \end{pmatrix} \qquad M = P(H) \text{ où } P(X) = a_0 + \dots + a_{n-1}X^{n}$$

Soit u canoniquement associé à H



$\forall i \in [\![1,n]\!],\ u^n(e_i) = e_i$

$q(X) = X^n - 1$ annule u

donc A

$$q(X) = \prod_{k=0}^{n-1}(X-\omega^k)$$

donc H diagonalisable.

$\underline{Sp(H) \subset \mathbb{U}_n}$

Soit $\lambda \in Sp(H)$, $\lambda I_n - H = \begin{pmatrix} \lambda & -1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & -1 \\ -1 & & & \lambda \end{pmatrix}$

$$rg(\lambda I_n - H) \geq n-1$$

th du rg $\Downarrow$

$$\dim(\ker(\lambda I_n - H)) \leq 1$$

Or, H diagonalisable : $\bigoplus_{\lambda \in Sp(H)} E_\lambda = E$

donc $card(Sp(H)) = n$

donc $Sp(H) = \mathbb{U}_n$

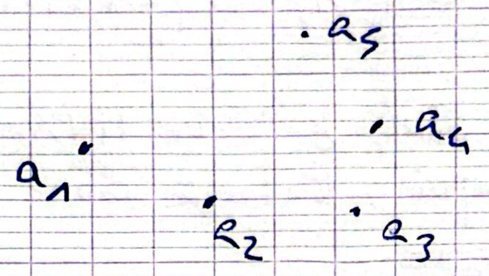
$$H = P^{-1}\begin{pmatrix} \omega^0 & & \\ & \ddots & \\ & & \omega^{n-1} \end{pmatrix}P$$

$$P(M) = P^{-1} \begin{pmatrix} P(\omega^0) & & 0 \\ & \ddots & \\ 0 & & P(\omega^{n-1}) \end{pmatrix} P$$

donc $\boxed{\det M = \prod_{k=0}^{n-1} P(\omega^k)}$

Exercice 375 : William + M. Fagebaum + Diane

Soient $a_1, \dots, a_n$ des points du plan



Soit $M_1, \dots, M_n$ un n-gone tq
$\forall i \in [\![1,n]\!], \ \overrightarrow{M_iM_{i+1}} = 2\overrightarrow{M_ia_i}$

Notons (x_i, y_i) les coordonnées de a_i
et (m_i, p_i) celles de M_i.

Donc $\forall i \in [\![1,n]\!]$, $\begin{cases} m_{i+1} - m_i = 2(x_i - m_i) \\ p_{i+1} - p_i = 2(y_i - p_i) \end{cases}$

$\iff \begin{cases} x_i = \frac{1}{2}(m_i + m_{i+1}) \\ y_i = \frac{1}{2}(p_i + p_{i+1}) \end{cases}$

$$a_i = \frac{1}{2}(z_i + z_{i+1})$$

$\left| \begin{array}{l} a_1 = \frac{1}{2}(z_1 + z_2) \\ \vdots \\ a_n = \frac{1}{2}(z_n + z_1) \end{array} \right.$ $\iff \begin{cases} 2a_1 = z_1 + z_2 \\ \vdots \\ 2a_n = z_1 + z_n \end{cases}$

$$\iff \underbrace{\begin{bmatrix} 1 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 1 & & & 1 \end{bmatrix}}_{= A_n} \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} = \begin{bmatrix} 2a_1 \\ \vdots \\ 2a_n \end{bmatrix}$$