De même, $P(2) \in \mathbb{Q}$ donc $\sum_{k=0}^{n} a_k 2^k \in \mathbb{Q}$

etc... $P(n+1) = \sum_{k=0}^{n} a_k (n+1)^k \in \mathbb{Q}$

$$\begin{pmatrix} 1 & \cdots & \cdots & 1 \\ \vdots & 2 & \cdots & 2^n \\ \vdots & \vdots & & \\ 1 & n+1 & \cdots & (n+1)^n \end{pmatrix} \begin{pmatrix} a_0 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} P(1) \\ \vdots \\ P(n+1) \end{pmatrix} \in \mathbb{Q}^n$$

$$\begin{pmatrix} a_0 \\ \vdots \\ a_n \end{pmatrix} = A^{-1} \begin{pmatrix} P(1) \\ \vdots \\ P(n+1) \end{pmatrix} \in \mathbb{Q}^{n+1}$$

Exercice 351 : Ulysse

$$M = \begin{pmatrix} a & c & b \\ c & b & a \\ b & a & c \end{pmatrix}$$

$$M^2 = \begin{pmatrix} a & c & b \\ c & b & a \\ b & a & c \end{pmatrix} \begin{pmatrix} a & c & b \\ c & b & a \\ b & a & c \end{pmatrix} = \begin{pmatrix} 1 & \alpha & \beta \\ \alpha & 1 & \gamma \\ \beta & \gamma & 1 \end{pmatrix}$$

$$\det M^2 = 1(1-\gamma^2) - \alpha(\alpha - \beta\gamma) + \beta(\alpha\gamma - \beta)$$

$$= 1 - \gamma^2 - \beta^2 - \alpha^2$$