Exercice 165 (Léon)

$P \in \mathbb{R}_n[X]$ tq $\forall x \in \mathbb{R}$, $P(x) \geq 0$

$$Q = \sum_{k=0}^{n} P^{(k)}$$

Mq $\forall x \in \mathbb{R}$, $Q(x) \geq 0$.

$$Q' = P' + \dots + P^{(n)} + \underbrace{P^{(n+1)}}_{0}$$

Ainsi, $Q = P + Q' \iff P = Q - Q'$, d'où $\deg P = \deg Q$.

si $\deg P \equiv 1 [2]$: Absurde $\forall cd(P)$, $P < 0$ en $+\infty$ ou $-\infty$

Donc $\deg P \equiv 0 [2]$

$$P(x) \xrightarrow[\substack{+\infty \\ -\infty}]{} +\infty$$

$$\Rightarrow Q(x) \xrightarrow[\substack{+\infty \\ -\infty}]{} +\infty$$

$\Rightarrow$ Q admet un minimum en $m \in \mathbb{R}$

$$P(m) = Q(m) - \underbrace{Q'(m)}_{=0} \geq 0$$

$$\boxed{\forall x \in \mathbb{R},\ Q(x) \geq Q(m) \geq 0.}$$