

Exercice 144 (William)

$$\text{Calculer } \sum_{z \in \mathbb{U}_n} \frac{1}{2-z}.$$

$$P(X) = X^n - 1.$$

$$= \prod_{k=0}^{n-1} (X - z_k) \quad \text{car } z_k = e^{\frac{2\pi i k}{n}} \quad \forall k \in \{0, \dots, n-1\}.$$

$$P(X) = \sum_{k=0}^{n-1} \prod_{i \neq k} (X - z_i)$$

$$= \sum_{k=0}^{n-1} \frac{P(X)}{X - z_k}$$

$$\text{Donc } P(X) = P(X) \sum_{z \in \mathbb{U}_n} \frac{1}{X - z}$$

$$\frac{P(2)}{P(2)} = \sum_{z \in \mathbb{U}_n} \frac{1}{2-z}$$

$$P(1) = n \cdot 2^{n-2}$$

$$\text{Donc : } \boxed{\sum_{z \in \mathbb{U}_n} \frac{1}{2-z} = \frac{n 2^{n-2}}{2^n - 1}}.$$