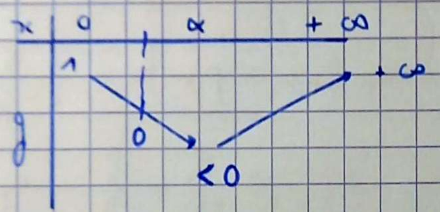
Exercice 69 (Quentin)

On pose : $\begin{cases} u_{n+1} = \sqrt{2}^{\,u_n} \\ u_0 = \sqrt{2} \end{cases}$

Soit $f : x \longmapsto \sqrt{2}^{\,x}$ et $g : x \longmapsto f(x) - x$

$$g'(x) = \frac{\ln(2)}{2}\sqrt{2}^{\,x} - 1$$

On a : $g'(x) = 0 \iff x = 2\Big[1 - \underbrace{\frac{\ln(\ln(2))}{\ln(2)}}_{<0}\Big] > 2$
$= \alpha$



Récurrence : $1 \leq u_n \leq \alpha$ $\qquad \mathcal{I} = [1, 2] \subset f(\mathcal{I})$

$\forall x \in [1,2],\ f(x) \geq x \Longrightarrow u_{n+1} - u_n$

TLM $(u_n)_n \nearrow$ & majorée. $u_n \longrightarrow l$.

$u_n \longrightarrow 2$ seul point fixe de f sur $[0, 2]$

Donc $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}\cdots}}} = 2$.

Exercice 58 (Lucas)

$\forall n,\quad u_{n+1} = |u_n - n| = \begin{cases} u_n - n & \text{si } u_n \geq n \\ n - u_n & \text{si } u_n \leq n \end{cases}$

$u_1 = |u_0|$