CHAPTER 1 NUMBERS

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Chapter 1.1 Quiz for Consolidating Knowledge

Chapter 1.2 Adding and Subtracting Negative Integers

Adding Negative Integers

- **When adding two negative integers**: You can think of it like moving farther away from zero in the negative direction.
  - Example: **−5 + −3** = **−8** (Move 5 units to the left, then 3 more units to the left).

Adding Positive and Negative Integers

- **When adding a positive and a negative integer**, subtract the smaller absolute value and keep the sign of the integer with the larger absolute value.
  - Example: **−7 + 4** = **−3** (Move 7 units to the left, then 4 units to the right, leaving you at −3).
  - Example: **5 + −3** = **2** (Move 3 units to the left, leaving you at 2).

Subtracting Negative Integers

- **Subtracting a negative number** is the same as **adding** its positive counterpart.
  - Example: **8 − (−5)** = **8 + 5 = 13** (Subtracting −5 is the same as adding 5).

Practice Questions

Basic Questions

1. $-3 + (-5) =$

2. $7 - (-2) =$

3. $-8 + 4 =$

4. $-12 - (-6) =$

5. $5 - 9 =$

Intermediate Questions

6. $-15 + (-10) =$

7. $20 - (-7) =$

8. $-25 + 13 =$

9. $-9 - 6 =$

10. $10 - (-3) + (-8) =$

Advanced Questions

11. $-30 + 18 - (-12) =$

12. $-45 - (-20) + (-5) =$

13. $-100 + (-50) - (-25) =$

14. $-7 + (-3) + (-10) =$

15. $50 - 60 + (-15) - (-5) =$

16. $-80 - (-25) + 40 =$

17. $-22 + (-11) - (-6) =$

18. $-14 - 9 + (-5) =$

19. $-100 + 75 - (-30) =$

20. $-55 + (-45) + (-10) =$

Word Problems

1. The temperature in a city was −5℃ in the morning. By noon, it dropped by 7℃, but in the evening, it rose by 4℃. What was the temperature in the evening?

2. A deep-sea diver starts at −20 meters below sea level. He ascends 15 meters but then descends 10 meters. What is his final depth?

3. A football team gains 12 yards on one play but loses 18 yards on the next. What is the total yardage change?

4. A business had a loss of $5000 in January, a gain of $2000 in February, and a loss of $3000 in March. What is the net profit or loss after three months?

5. A hiker descends 30 meters down a mountain, then climbs up 25 meters, and later descends 15 meters. Where is the hiker now relative to the starting point?

6. The stock market dropped 45 points in the morning but recovered 20 points in the afternoon. By the evening, it dropped another 10 points. What is the overall change?

7. A bank account has a balance of −$250. A deposit of $100 is made, followed by a withdrawal of $50. What is the final balance?

8. A submarine is at −120 meters below sea level. It rises by 40 meters, then sinks 30 meters. What is its new depth?

9. The temperature was $-18°C$ in the early morning. By midday, it increased by $9°C$, but by night, it dropped by $15°C$. What is the final temperature?

10. A cyclist moves 10 meters backward, then moves 20 meters forward, and finally moves 5 meters backward. What is the final position relative to the starting point?

Chapter 1.3 Multiplying and Dividing Negative Integers

Multiplying Negative Integers

The rule is simple:

- Negative × Negative = Positive
- Positive × Negative = Negative
- Negative × Positive = Negative

Example:

- $-3 \times 4 = -12$ (A positive times a negative gives a negative product).
- $-2 \times -6 = 12$ (A negative times a negative gives a positive product).

Dividing Negative Integers

The rule for division is similar to multiplication:

- Negative ÷ Positive = Negative
- Positive ÷ Negative = Negative
- Negative ÷ Negative = Positive

Example:

- $-12 \div 3 = -4$ (A negative divided by a positive gives a negative quotient).
- $-20 \div -4 = 5$ (A negative divided by a negative gives a positive quotient).

Practice Questions

Basic Questions

1. $-5 \times 3 =$

2. $-8 \times (-2) =$

3. $12 \div (-4) =$

4. $-16 \div (-2) =$

5. $-7 \times (-6) =$

Intermediate Questions

6. $-9 \times 4 =$

7. $-36 \div 6 =$

8. $-10 \times (-5) =$

9. $24 \div (-8) =$

10. $-50 \div (-10) =$

Advanced Questions

11. $-12 \times (-3) \times 2 =$

12. $-48 \div 8 \times (-3) =$

13. $-6 \times 4 + (-2) =$

14. $-(30 + 5) \times (-2) =$

15. $-100 + (-10) \times 5 =$

16. $-7 \times (-2) \times (-3) =$

17. $(-64 + 8) \times (-2) =$

18. $-9 \times 5 + (-3) =$

19. $-45 + 5 \times (-2) =$

20. $-72 + (-6) \times 4 =$

Word Problems

1. A submarine descends 5 meters per minute. How far will it have descended after 8 minutes?

2. A business made a $200 loss per day for 6 days. What is the total loss?

3. A factory decreases production by 7 units per day. How many units are lost in 10 days?

4. A diver descends at −4 meters per second. How deep is the diver after 12 seconds?

5. A glacier moves -3 meters per year. How much will it have moved in 9 years?

6. The temperature drops $2°C$ per hour. What is the total change in temperature after 24 hours?

7. A stock loses $5 per share for 15 days. How much value is lost?

8. A scientist studying bacteria finds that a population decreases by −8% per day. What is the total change in 5 days?

9. A person earns −$10 per hour in debt for working unpaid. How much debt is accumulated in 20 hours?

10. A hiker descends −12 meters every hour. How far will they have descended in 5 hours?

Chapter 1.4 Order of Operations and Substitution

When solving mathematical expressions, we follow the **BODMAS** rule to ensure accurate calculations. **BODMAS** stands for:

- **B**rackets ( ) [ ] { } – Solve anything inside brackets first.
- **O**rders – Solve exponents or powers (e.g., ^{2}, ^{3}) and roots (√).
- **D**ivision and **M**ultiplication – Work from left to right.
- **A**ddition and **S**ubtraction – Work from left to right.

Example:

Solve **8 + 3 × (6 − 2) ÷ 2**

1. Brackets first: **8 + 3 × 4 ÷ 2**
2. Division and multiplication from left to right: **8 + 12 ÷ 2** → **8 + 6**
3. Addition last: **14**

Substitution

Substitution means replacing variables with their values in an expression before solving it using **BODMAS**.

Example: If **x = 4** and **y = −2**, evaluate **2x + 3y**

1. Substitute values: **2(4) + 3(−2)** → **8 − 6**
2. Solve: **2**

Practice Questions

Basic Questions

1. $(6 + 4) \div 2 =$
2. $8 + 3 \times (6 - 2) =$
3. $(12 + 4) \times (5 + 7) - 3 =$
4. $15 - 3 \times 2 + 4 + 2 =$
5. $[10 + (6 \div 2)] \times 3 =$

Intermediate Questions

6. $(20 - 5 \times 3) + 5 + 7 =$

7. $(-8 + 4) \times 3 - 6 + 2 =$

8. $[5 \times (2 + 3)] - (4 + 2) =$

9. $(18 + 3) + (5 \times 2) - 7 =$

10. $\{[10 + 5 \times (2 - 1)] + 5\} \times 4 =$

Advanced Questions

11. $(30 - 12) + 6 \times 5 + 4 =$

12. $\{(50 + 5) + (4 \times 3)\} - 8 =$

13. $(-6 \times 3) + (4 + 2) =$

14. $(10 + 2 \times 3) + (5 - 3) =$

15. $(40 + 5) \times (3 + 2) - 8 =$

16. If $x = 4$ and $y = -2$, evaluate: $2x + 3y$

17. If $a = -5$ and $b = 3$, find: $a^2 - b^2$

18. Given $m = 7$ and $n = -3$, solve:
$m \times n + 2n$

19. If $p = 2$ and $q = -6$, evaluate:
$(p + q)^2 - 4p$

20. If $x = -3$ and $y = 5$, find the value of:
$xy - (x + y)$

Word Problems

1. A rectangle has a length of $(5 + 3)$ meters and a width of 4 meters. Find its area.

2. A shop offers a 20% discount on an item originally priced at \$50. After the discount, an additional \$5 is deducted. What is the final price?

3. A person drives at $(60 \div 2)$ km per hour for 3 hours. How far does the person travel?

4. A class has 5 groups, each with $(3 + 2)$ students. How many students are in the class?

5. The cost of 3 shirts is $20 each, and a jacket cost $50. What is the total cost?

6. A machine produces (8 × 4) widgets per hour. How many widgets are produced in 6 hours?

7. A person earns \$15 per hour and works for (5 × 2) hours. How much does the person earn?

8. If a rectangle has a length of (10 ÷ 2) meters and a width of (6 × 3) meters, what is its area?

9. A cyclist rides at a speed of $(48 \div 6)$ km per hour for 5 hours. How far does the cyclist travel?

10. A store sells apples in packs of $(4 + 2)$. If you buy 5 packs, how many apples do you get?

CHAPTER 2 GEOMETRIES AND ANGLES

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Chapter 2.1 Quiz for Consolidating Knowledge

Naming Angles

- An angle can be named using three points, with the middle letter representing the vertex (e.g., **∠ABC** or **∠CBA**).
- It can also be named using just the vertex (**∠B**) or a special notation (**ABˆC**).
- The size of the angle is represented as **b°**.

Types of Angles

- **Acute Angle**: Between **0° and 90°**
- **Right Angle**: Exactly **90°**
- **Obtuse Angle**: Between **90° and 180°**
- **Straight Angle**: Exactly **180°**
- **Reflex Angle**: Between **180° and 360°**
- **Revolution**: Exactly **360°**

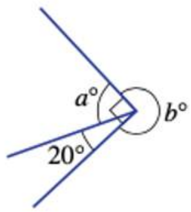
Special Angle Pairs

- **Complementary Angles**: Two angles that sum to **90°** (**a + b = 90°**)
- **Supplementary Angles**: Two angles that sum to **180°** (**a + b = 180°**)
- **Vertically Opposite Angles**: Formed when two lines intersect, and opposite angles are equal (**a = b** and **c = d**).

Practice Questions

1. Determine the value of the pronumerals in these diagrams.

a.



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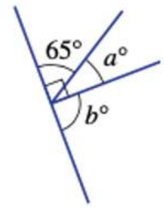

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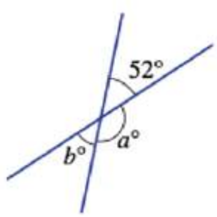

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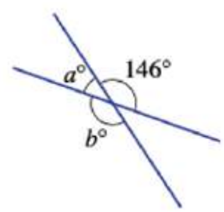

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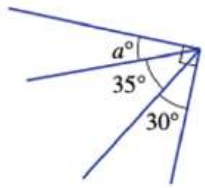

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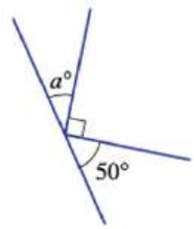

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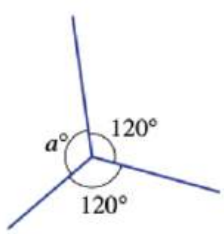

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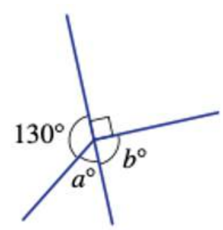

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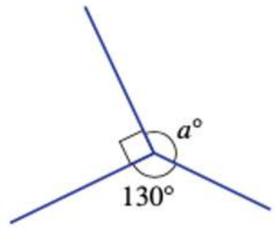

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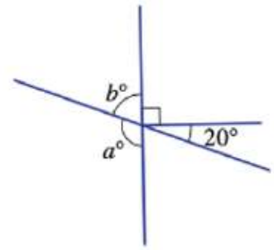

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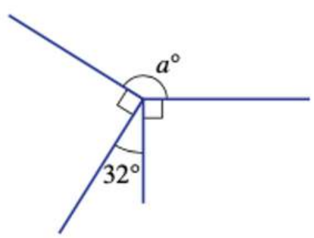

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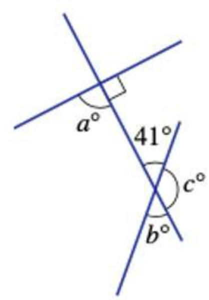

2. Find the value of a in these diagrams.

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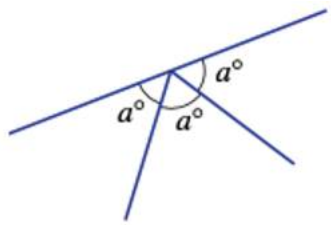

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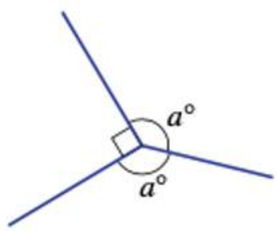

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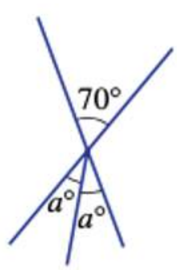

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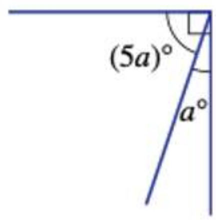

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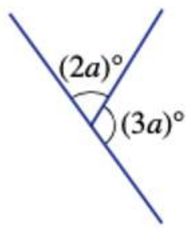

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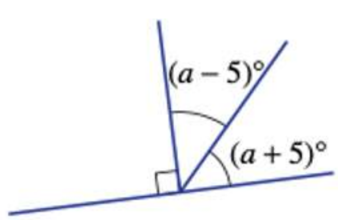

3. Equations can be helpful in solving geometric problems in which more complex expressions are involved. Find the value of a in these diagrams.

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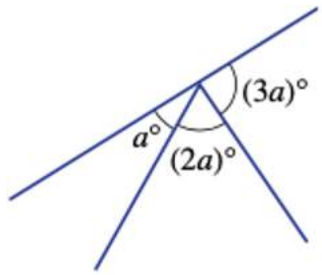

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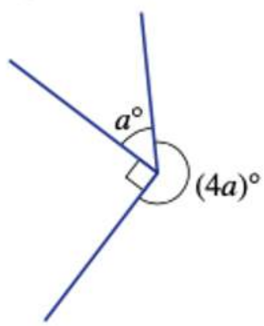

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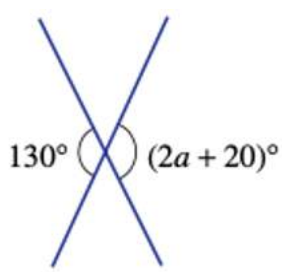

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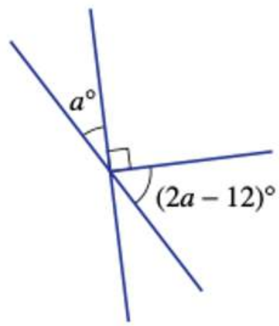

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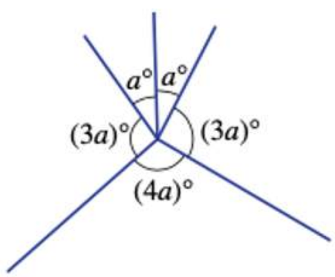

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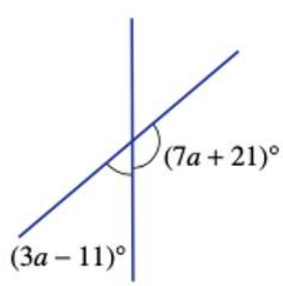


Chapter 2.2 Line Geometry

What is a Transversal?

- A **transversal** is a line that cuts across at least **two other lines**.
- It creates different types of **angle pairs**.

Angle Pairs Formed by a Transversal

- **Corresponding Angles**: Found in matching positions; they are **equal**.
- **Alternate Angles**: On **opposite sides** of the transversal, inside the two lines; they are **equal**.
- **Co-Interior Angles**: On the **same side** of the transversal, inside the two lines; they **sum to 180°**.

Parallel Lines

- **Parallel lines** do not intersect.
- They are marked with the **same number of arrows**.

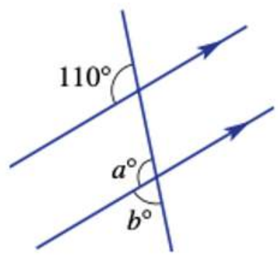
When a Transversal Cuts Two Parallel Lines

- **Corresponding angles** are **equal** (4 pairs).
- **Alternate angles** are **equal** (2 pairs).
- **Co-interior angles** add up to **180°**.

Practice Questions

1. Find the value of the pronumerals in these diagrams, starting reasons.

a.



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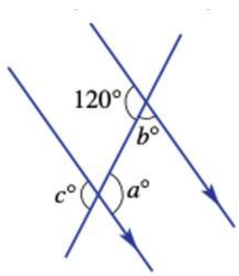

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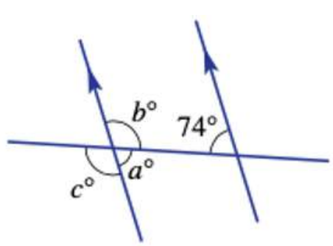

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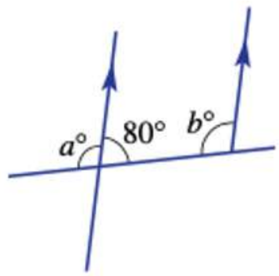

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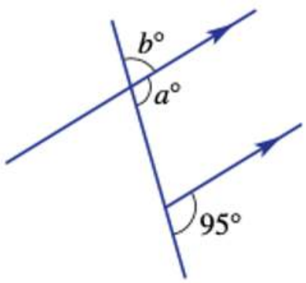

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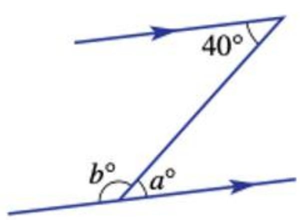

2. Decide if the following diagrams include a pair of parallel lines. Given a reason for each answer.

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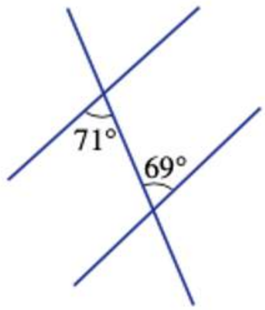

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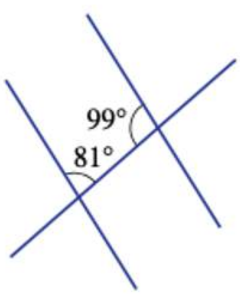

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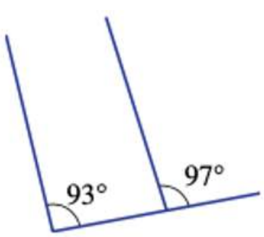
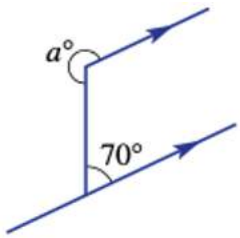


3. Find the value of a in these diagrams.

a.



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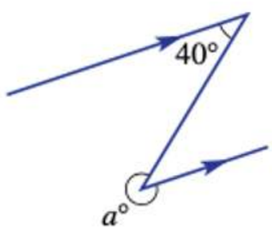

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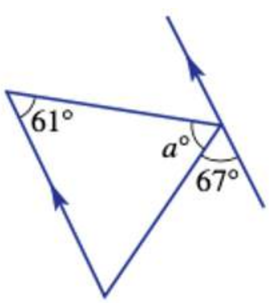

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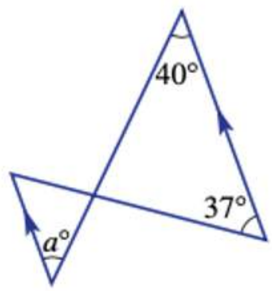

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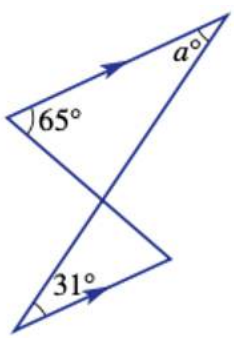

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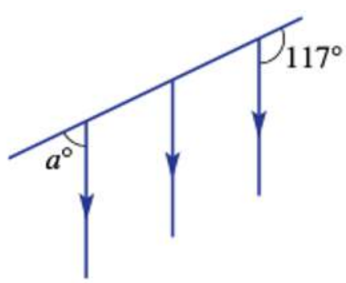

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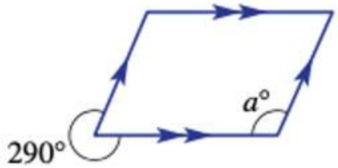

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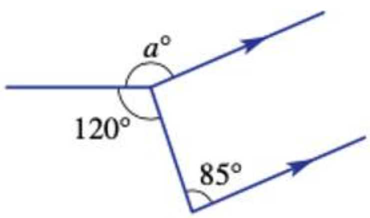

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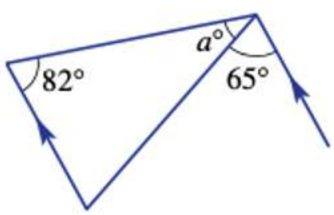


Chapter 2.3 Triangles and Quadrilaterals

Basic Properties of a Triangle

- A triangle has **3 sides**, **3 vertices**, and **3 interior angles**.
- The sum of the interior angles in a triangle is **always 180°** (**a + b + c = 180°**).

Types of Triangles by Side Lengths

- **Scalene Triangle**: All sides and angles are different.
- **Isosceles Triangle**: Two sides are equal, and the base angles are also equal.
- **Equilateral Triangle**: All three sides and angles are equal (**each angle is 60°**).

Types of Triangles by Interior Angles

- **Acute Triangle**: All three angles are **less than 90°**.
- **Right Triangle**: Contains **one right angle (90°)**.
- **Obtuse Triangle**: Contains **one obtuse angle (greater than 90°)**.

Types of Quadrilaterals

- **Convex Quadrilaterals**:
  - All vertices point outward.
  - All interior angles are **less than 180°**.
  - Both diagonals lie **inside** the figure.
- **Non-Convex (Concave) Quadrilaterals**:
  - At least one vertex points inward.
  - Has **one reflex interior angle** (greater than 180°).
  - One diagonal lies **outside** the figure.

Parallelograms

A **parallelogram** is a quadrilateral with **two pairs of parallel sides**. Special types include:

- **Rectangle**: Parallelogram with **all angles 90°**.
- **Rhombus**: Parallelogram with **all sides equal**.
- **Square**: A **rhombus** with **all angles 90°** (or a **rectangle** with **all sides equal**).

Other Special Quadrilaterals

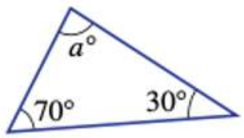
- **Kite**: A quadrilateral with **two pairs of adjacent equal sides**.
- **Trapezium (Trapezoid)**: A quadrilateral with **at least one pair of parallel sides**.

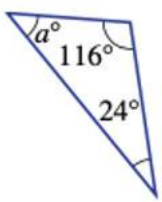
Angle Sum of a Quadrilateral

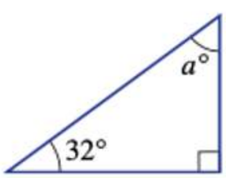
- The sum of all interior angles in a quadrilateral is always **360°**.
  - **a + b + c + d = 360°**

Practice Questions

1. Use the angle sum of a triangle to help find the value of a in these triangles.

a. 

b. 

c. 

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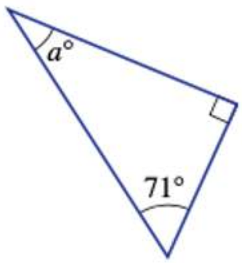
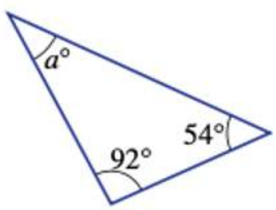
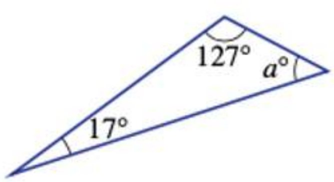


e.

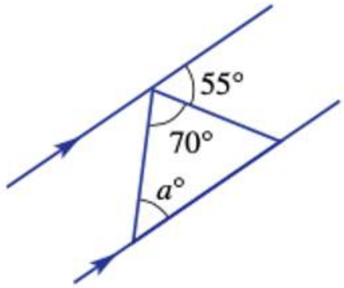


f.



2. Use your knowledge of parallel lines and triangles to find the unknown angle a.

a.



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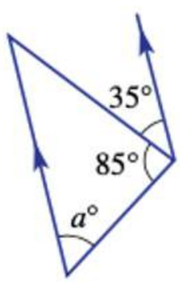
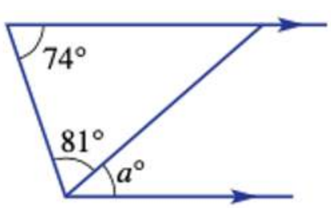
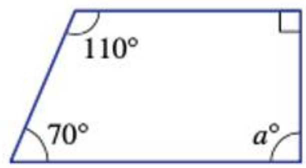


c.

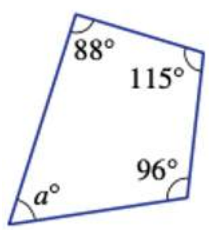


3. Use the quadrilateral angle sum to find the value of a in these quadrilaterals.

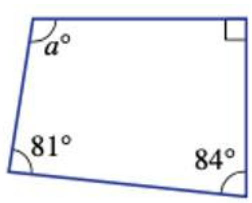
a.



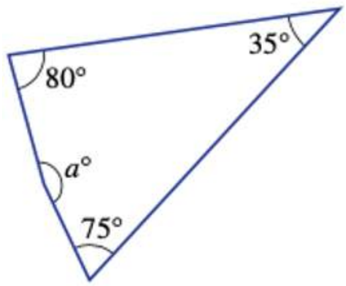
b.



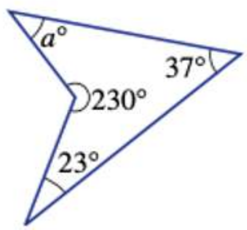
c.



d.



e.



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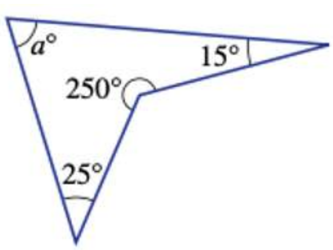
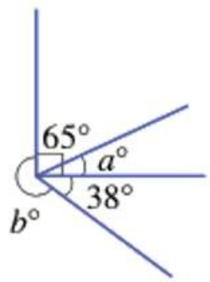


4. Determine the value of the pronumerals in these diagrams.

a.



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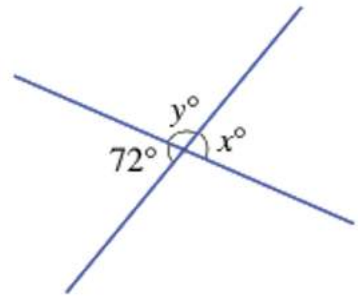

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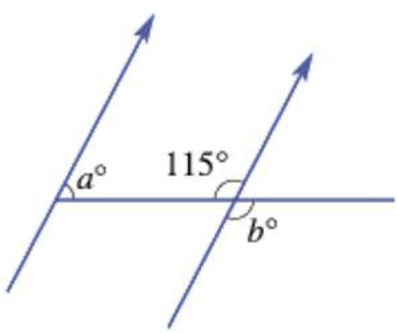
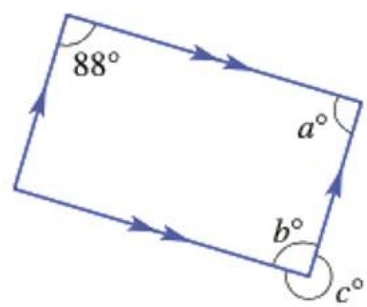
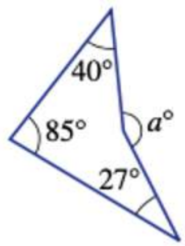


d.



5. Use your knowledge of geometry from the previous sections to find the values of a.

a.



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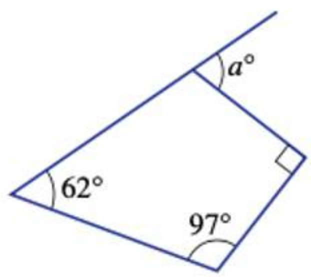
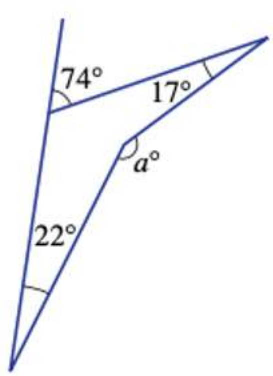
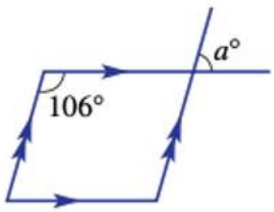


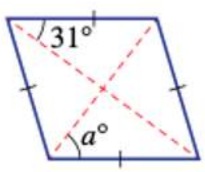
c.



d.



e.



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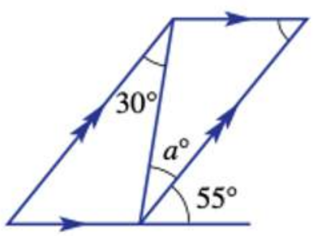


Word Problems

<u>Triangles:</u>

1. In a triangle, if two angles are 45° and 65°, what is the measure of the third angle?

2. The angles of a triangle are in the ratio 2:3:4. What are the actual measures of the angles?

3. In an equilateral triangle, if one side is 8 cm, what is the perimeter of the triangle?

4. Find the area of a triangle with a base of 10 cm and a height of 6 cm.

5. In an isosceles triangle, the base angles are both 70°. What is the measure of the third angle?

6. In a right triangle, the lengths of the two legs are 5 cm and 12 cm. Find the length of the hypotenuse.

7. The interior angles of a triangle are x, x +10 x +10 x +10, and $2x$. Find the value of x.

8. The perimeter of a triangle is 36 cm. Two sides are 10 cm and 12 cm. What is the length of the third side?

9. In an acute triangle, one angle measures 35° and another measures 85°. What is the third angle?

10. A triangle has sides 7 cm, 24 cm, and 25 cm. Prove whether it is a right triangle.

Quadrilaterals:

1. The angles of a quadrilateral are in the ratio 2:3:4:5. What are the actual measures of the angles?

2. If a parallelogram, one angle measures 50°. What is the measure of the opposite angle?

3. Find the area of a rectangle if its length is 12 cm and its width is 8 cm.

4. In a rhombus, one of the diagonals is 10 cm and the other is 12 cm. Find the area of the rhombus.

5. In a square, the length of each side is 6 cm. What is the perimeter of the square?

6. A trapezium has one pair of parallel sides measuring 10 cm and 15 cm, and the height is 8 cm. Calculate its area.

7. In a kite, the lengths of the diagonals are 8 cm and 6 cm. What is the area of the kite?

8. In a parallelogram, the base is 14 cm, and the height is 9 cm. What is the area of the parallelogram?

9. A quadrilateral has interior angles measuring 90°, 110°, 85°, and 75°. Verify that this is a valid quadrilateral.

10. In a trapezoid, the lengths of the parallel sides are 8 cm and 14 cm. If the height is 5 cm, calculate the area of the trapezoid.

Chapter 2.4 Polygons

What is a Polygon?

- A **polygon** is a closed shape with **straight sides**.
- Polygons can be **convex** or **non-convex (concave)**.

Types of Polygons

- **Convex Polygon**:
  - All interior angles are **less than 180°**.
  - All vertices point **outward**.
- **Non-Convex (Concave) Polygon**:
  - At least **one reflex angle** (**greater than 180°**).
  - At least **one vertex points inward**.

Naming Polygons by Number of Sides

Number of Sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

Angle Sum of a Polygon

- The sum of the interior angles of a polygon with **n sides** is:

  $$S = (n - 2) \times 180^\circ$$

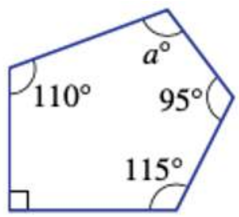
Regular Polygons

- A **regular polygon** has **all sides and all angles equal**.

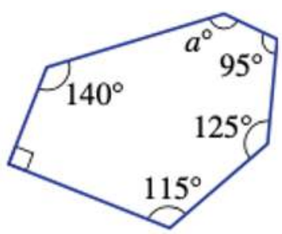
Practice Questions

1. Find the value of a in these polygons.

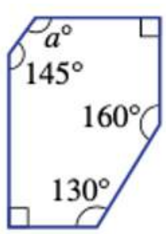
a.



b.



c.



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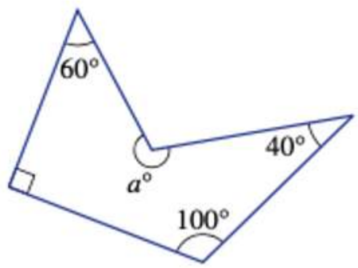
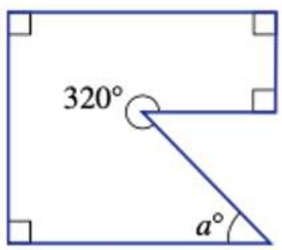
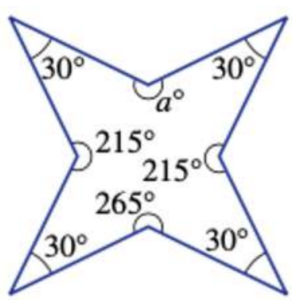


e.

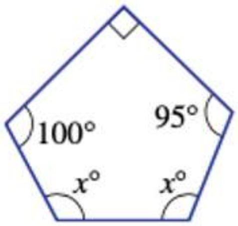


f.



2. Find the value of x in these diagrams.

a.



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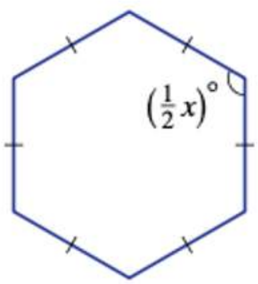
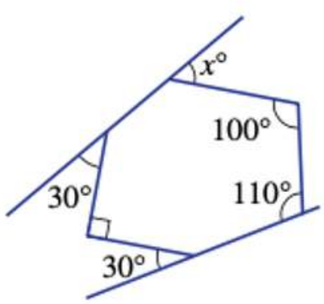


c.



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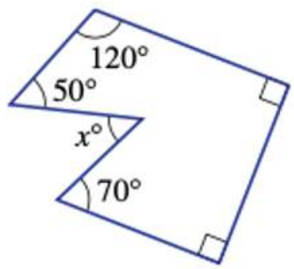

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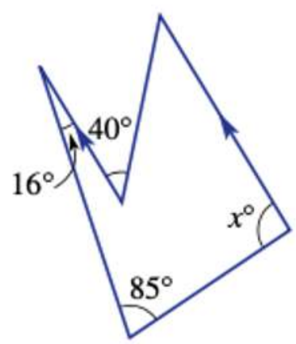

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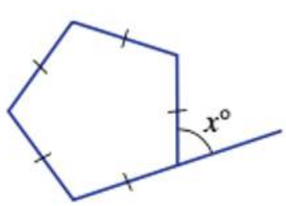
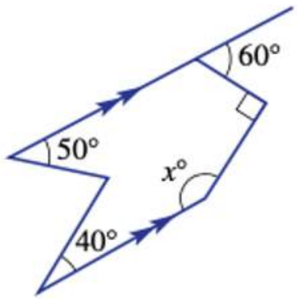


3. With the limited information provided, find the value of x in these diagrams.

a.



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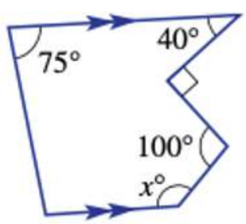
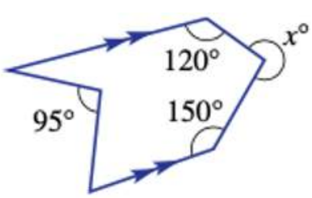


c.



Word Problems

1. What is the sum of the interior angles of triangle?

2. How many sides does a heptagon have?

3. Calculate the sum of the interior angles of a hexagon.

4. Is a rectangle a convex polygon or a concave polygon?

5. How many sides does a nonagon have?

6. What is the sum of the interior angles of a quadrilateral?

7. If a polygon has 10 sides, what is the sum of its interior angles?

8. A regular polygon has 14 sides. Calculate the sum of its interior angle.

9. The sum of the interior angles of a polygon is 2520°. How many sides does the polygon have?

10. In a regular decagon (10 sides), what is the measure of each interior angle?

Chapter 2.5 3D Shapes

What is a Polyhedron?

- A **polyhedron** is a closed **3D solid** with **flat faces, vertices, and edges**.
- Polyhedra are named by their **number of faces**, for example:
    - **Tetrahedron** (4 faces)
    - **Pentahedron** (5 faces)
    - **Hexahedron** (6 faces, e.g., a cube)

Euler's Formula for Polyhedra

For a polyhedron with **F** faces, **V** vertices, and **E** edges:

$$E = F + V - 2$$

Types of Polyhedra

Prisms

- A **prism** has **two identical (congruent) parallel bases**.
- The bases define the **cross-section** and **name** of the prism.
- The **other faces are parallelograms**.
- If the parallelogram faces are **rectangles**, it is called a **right prism**.
    - Example: **Hexagonal prism**

Pyramids

- A **pyramid** has a **base** and **triangular faces** that meet at a single **apex**.
- The **base shape determines the name** of the pyramid.
    - Example: **Square pyramid**

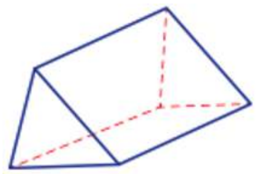
Solids with Curved Surfaces

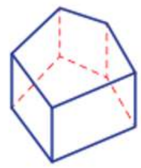
Some **3D solids** do not have flat faces:

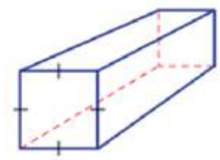
- **Cylinder** (two circular bases, curved surface)
- **Sphere** (completely curved, no edges or vertices)
- **Cone** (one circular base, curved surface, and an apex)

Practice Questions

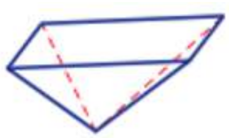
1. Name these prisms.

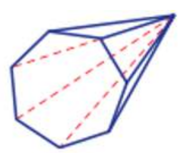
a. 

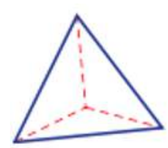
b. 

c. 

2. Name these pyramids.

3. 

4. 

5. 

3. Use Euler's formula to calculate the missing numbers in this table.

Faces (F)	**Vertices (V)**	**Edges (E)**
6	8	
	5	8
5		8
7		12
	4	6
11	11	

Word Problems

1. What is the number of edges on a cube?

2. How many vertices does a tetrahedron have?

3. Using Euler's formula, if a polyhedron has 12 edges and 6 faces, how many vertices does it have?

4. How many faces does a rectangular prism have?

5. Calculate the number of edges on a triangular prism.

6. A square pyramid has 5 faces and 8 edges. How many vertices does it have?

7. If a pyramid has a hexagonal base, how many faces does it have?

8. A triangular pyramid has 4 faces, 6 edges, and 4 vertices. Verify Euler's formula for this pyramid.

9. How many edges does a pentagonal prism have?

10. How many faces does a hexagonal prism have?

Chapter 2.6 3D Shapes Coordinates

In a 3D coordinate system:

- **Axes**: There are three axes: x, y, and z, all at right angles to each other.
- **Origin**: The origin is at $(0,0,0)$, where all axes intersect.
- **Coordinates**: A point in 3D space is represented as (x, y, z), indicating its position relative to the axes.
- **Example**: A point P with coordinates $(1,2,4)$ is 1 unit along the x-axis, 2 units along the y-axis, and 4 units along the z-axis.

This system helps describe positions in three-dimensional space.

Practice Questions

1. What are the three axes in a 3D coordinate system?

2. What are the coordinates of the origin in a 3D space?

3. How do you represent a point in 3D space?

4. If a point is located at $(3,4,5)$, how many units is it from the origin along each axis?

5. What is the difference between the x-axis, y-axis, and z-axis in a 3D coordinate system?

6. Given the point $(1,2,3)$, identify the position along each axis.

7. How would you describe the position of a point at $(0,0,5)$?

8. If a point has coordinates $(2,0,4)$, what is its location in terms of distance from the origin on the x-axis, y-axis, and z-axis?

9. Describe the location of a point at $(5,3,0)$.

10. How can you find the distance between two points in 3D space, for example, from $(1,1,1)$ to $(4,5,6)$?

11. What would the coordinates be for a point 3 units along the x-axis, 2 units along the y-axis, and 6 units along the z-axis?

12. How does the position of a point change if you increase its z-coordinate while keeping x and y the same?

13. In the diagram of 3D space, what do you understand by the concept of a cubic unit of space?

14. If the coordinates of a point are $(0,3,2)$, what is the point's distance from the origin along each axis?

15. What is the coordinate of the point located 4 units along the x-axis, 5 units along the y-axis, and 7 units along the z-axis?

16. Given the points $A(2,3,1)$ and $B(4,2,5)$, how would you describe their relative position?

17. How can you represent a cube in 3D space using coordinates for its vertices?

18. What happens to the coordinates of a point when you reflect it across the x-axis?

19. How do you plot the point $(-3,4,2)$ on a 3D coordinate grid?

20. Can you describe the location of a point at $(-1,-2,3)$? What does the negative value represent?

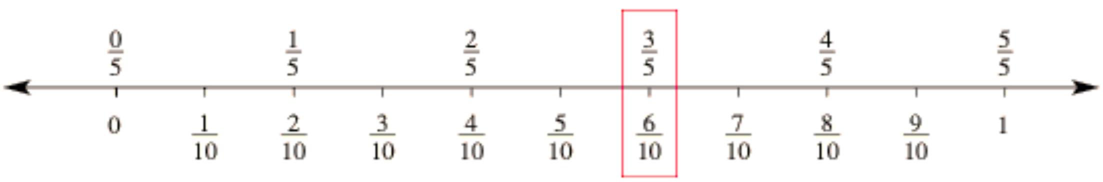
CHAPTER 3 FRACTIONS DECIMALS AND PERCENTAGES

Table of Contents

Chapter 3.1 Equivalent and Simplifying Fractions

Equivalent fractions are fractions that represent the same value or point on a number line. They can be created by either multiplying or dividing the numerator and denominator of a fraction by the same number.

Simplifying fractions to their simplest form involves dividing both the numerator and denominator by their highest common factor.



Practice Questions

1. Find the missing value to make the statement true.

a. $\frac{2}{5} = \frac{\square}{15}$

b. $\frac{7}{9} = \frac{14}{\square}$

c. $\frac{7}{14} = \frac{1}{\square}$

d. $\frac{21}{30} = \frac{\square}{10}$

e. $\frac{4}{3} = \frac{\square}{21}$

f. $\frac{8}{5} = \frac{80}{\square}$

g. $\frac{3}{12} = \frac{\square}{60}$

h. $\frac{7}{11} = \frac{28}{\square}$

Word Problems

1. Simplify the fraction $\frac{48}{64}$ to its simplest form.

2. Are the fractions $\frac{5}{10}$ and $\frac{1}{2}$ equivalent? Justify your answer.

3. Multiply both the numerator and denominator of $\frac{7}{11}$ by 4. What is the equivalent fraction?

4. Simplify the fraction $\frac{36}{54}$ by dividing both the numerator and denominator by their greatest common factor.

5. What is the highest common factor (HCF) of 42 and 56?

6. A container holds $\frac{3}{5}$ of a litre of water. If you need to fill the container in $\frac{1}{5}$ litre increments, how many increments are needed to fill the entire container?

7. Find two equivalent fractions for $\frac{7}{8}$ by multiplying the numerator and denominator by different whole numbers.

8. Simplify the fraction $\frac{98}{126}$ by dividing both the numerator and denominator by their highest common factor.

9. Are the fractions $\frac{9}{15}$ and $\frac{3}{5}$ equivalent? Explain how you know.

10. If a map is drawn with a scale of $\frac{1}{1000}$, how would you express this scale as an equivalent fraction with a denominator of 100?

Chapter 3.2 Review Operations with Fractions

Practice Questions

1. $\frac{3}{4}+\frac{1}{4}$

2. $\frac{7}{8}-\frac{3}{8}$

3. $\frac{5}{6}+\frac{1}{3}$

4. $\frac{9}{10}-\frac{4}{5}$

5. $\frac{2}{3}\times\frac{4}{5}$

6. $\frac{7}{9}\times\frac{3}{7}$

7. $\frac{5}{8}\div\frac{1}{2}$

8. $\frac{3}{4}\div\frac{5}{6}$

9. $\frac{2}{5}+\frac{3}{10}$

10. $\frac{5}{12}-\frac{1}{4}$

11. $\frac{3}{8}\times\frac{5}{12}$

12. $\frac{6}{7}+\frac{8}{7}$

13. $\frac{9}{11}\times\frac{2}{3}$

14. $\frac{7}{12}\div\frac{3}{4}$

15. $\frac{5}{9}+\frac{2}{3}$

16. $\frac{3}{5}-\frac{2}{5}$

17. $\frac{4}{5}+\frac{1}{5}$

18. $\frac{2}{5}\times\frac{3}{8}$

19. $\frac{11}{14}\div\frac{1}{2}$

20. $\frac{4}{9}+\frac{2}{9}$

21. $\frac{7}{9}+\frac{5}{12}$

22. $\frac{8}{15}-\frac{4}{9}$

23. $\frac{11}{14}+\frac{7}{8}$

24. $\frac{3}{5}-\frac{2}{7}$

25. $\frac{4}{7} \times \frac{5}{9}$

26. $\frac{13}{18} \times \frac{7}{12}$

27. $\frac{5}{6} \div \frac{2}{3}$

28. $\frac{15}{20} \div \frac{5}{8}$

29. $\frac{7}{12} + \frac{5}{16}$

30. $\frac{9}{10} - \frac{7}{18}$

31. $\frac{5}{8} \times \frac{12}{19}$

32. $\frac{19}{24} + \frac{13}{32}$

33. $\frac{15}{28} \times \frac{8}{13}$

34. $\frac{23}{30} \div \frac{11}{15}$

35. $\frac{7}{9} + \frac{5}{18}$

36. $\frac{11}{14} - \frac{2}{5}$

37. $\frac{6}{13} \times \frac{9}{11}$

38. $\frac{8}{9} \div \frac{4}{5}$

39. $\frac{12}{17} + \frac{19}{51}$

40. $\frac{5}{12} - \frac{3}{8}$

41. $3\frac{1}{4} + \frac{5}{12}$

42. $4\frac{2}{5} - \frac{3}{7}$

43. $2\frac{3}{8} + \frac{7}{9}$

44. $5\frac{1}{6} - \frac{2}{9}$

45. $3\frac{2}{3} \times \frac{4}{7}$

46. $7\frac{1}{4} \times \frac{5}{8}$

47. $4\frac{3}{5} \div \frac{2}{3}$

48. $6\frac{2}{3} \div \frac{5}{8}$

49. $1\frac{3}{4}+2\frac{1}{6}$

50. $5\frac{1}{2}-3\frac{3}{4}$

51. $3\frac{2}{5}\times 4\frac{1}{2}$

52. $7\frac{3}{8}+\frac{5}{16}$

53. $6\frac{1}{3}\times 2\frac{2}{5}$

54. $9\frac{1}{4}\div 4\frac{1}{2}$

55. $3\frac{5}{9}+4\frac{2}{3}$

56. $8\frac{1}{2}-3\frac{7}{8}$

57. $2\frac{1}{3} \times \frac{5}{6}$

58. $5\frac{1}{4} \div \frac{7}{9}$

59. $4\frac{2}{5} + 2\frac{1}{10}$

60. $6\frac{3}{4} - 2\frac{5}{8}$

61. $\frac{-3}{4} + \frac{2}{5}$

62. $\frac{7}{8} - \frac{-3}{8}$

63. $\frac{-5}{6} + \frac{-2}{3}$

64. $\frac{-4}{9} - \frac{1}{6}$

65. $\frac{-7}{10} \times \frac{3}{5}$

66. $\frac{-6}{7} \times \frac{-4}{9}$

67. $\frac{-5}{12} \div \frac{1}{3}$

68. $\frac{-3}{8} \div \frac{-2}{5}$

69. $\frac{-9}{14} + \frac{4}{7}$

70. $\frac{-2}{3} + \frac{5}{6}$

71. $\frac{-7}{8} - \frac{-3}{4}$

72. $\frac{-5}{9} \times \frac{7}{8}$

73. $\frac{-12}{17} \div \frac{2}{3}$

74. $\frac{-8}{11} + \frac{5}{6}$

75. $\frac{4}{9} - \frac{-7}{12}$

76. $\frac{-5}{6} \times \frac{-3}{4}$

77. $\frac{-7}{15} \div \frac{3}{5}$

78. $\frac{-6}{8} + \frac{5}{6}$

79. $\frac{-11}{20} - \frac{-5}{12}$

80. $\frac{3}{7} + \frac{-4}{9}$

Word Problems

1. Sarah is baking a cake and needs $2\frac{1}{2}$ cups of flour. She already has $1\frac{3}{4}$ cups of flour in the bowl. How much more flour does she need to add?

2. A class activity lasts $3\frac{1}{4}$ hours. If the class has already spent $1\frac{3}{5}$ hours on the activity, how much more time do they need to finish?

3. A pizza was divided into 8 equal slices. Sarah ate $2\frac{1}{2}$ slices, and her friend ate $3\frac{1}{4}$ slices. How many slices of pizza are left?

4. Michael has $3\frac{2}{3}$ meters of garden space to plant flowers. He has already planted flowers in $2\frac{1}{4}$ meters of the space. How much more space does he have left for planting?

5. A car travels $5\frac{3}{4}$ miles before stopping for gas. After refueling, the car travels another $4\frac{1}{2}$ miles. How far did the car travel in total?

Chapter 3.3 Decimals

Converting Decimals to Fractions:

1. **Count the decimal places**: The number of decimal places determines how many zeros you'll place in the denominator.
2. **Convert**: Write the decimal as a fraction with the corresponding denominator.
3. **Simplify**: If possible, reduce the fraction to its simplest form.

For example:

- $0.75 = \frac{75}{100} = \frac{3}{4}$
- $0.125 = \frac{125}{1000} = \frac{1}{8}$

Converting Fractions to Decimals:

- **If the denominator is a power of 10** (like 10, 100, 1000), convert the fraction directly into a decimal by using its place value.
  For example:
  - $\frac{56}{100} = 0.56$
  - $\frac{72}{1000} = 0.072$
- **If the denominator is not a power of 10**, find an equivalent fraction where the denominator is a power of 10, then convert it to a decimal.
  For example:
  - $\frac{7}{25} = \frac{28}{100} = 0.28$
  - $\frac{5}{8} = \frac{62.5}{100} = 0.625$

Practice Questions

1. Convert 0.4 to a fraction.
2. Convert 0.25 to a fraction.
3. Convert 0.75 to a fraction.
4. Convert 0.5 to a fraction.
5. Convert 0.125 to a fraction.
6. Convert 0.6 to a fraction.
7. Convert 0.2 to a fraction.
8. Convert 0.375 to a fraction.
9. Convert 0.9 to a fraction.
10. Convert 0.8 to a fraction.

11. Convert 0.06 to a fraction.

12. Convert 0.45 to a fraction.

13. Convert 0.12 to a fraction.

14. Convert 0.04 to a fraction.

15. Convert 0.8 to a fraction.

16. Convert 0.03 to a fraction.

17. Convert 0.125 to a fraction.

18. Convert 0.9 to a fraction.

19. Convert 0.625 to a fraction.

20. Convert 0.33 to a fraction.

21. Convert $\frac{3}{4}$ to a decimal.

22. Convert $\frac{7}{10}$ to a decimal.

23. Convert $\frac{5}{8}$ to a decimal.

24. Convert $\frac{9}{20}$ to a decimal.

25. Convert $\frac{2}{5}$ to a decimal.

26. Convert $\frac{3}{25}$ to a decimal.

27. Convert $\frac{1}{2}$ to a decimal.

28. Convert $\frac{7}{8}$ to a decimal.

29. Convert $\frac{11}{50}$ to a decimal.

30. Convert $\frac{5}{6}$ to a decimal.

31. Convert $\frac{3}{16}$ to a decimal.

32. Convert $\frac{13}{25}$ to a decimal.

33. Convert $\frac{1}{8}$ to a decimal.

34. Convert $\frac{9}{40}$ to a decimal.

35. Convert $\frac{17}{100}$ to a decimal.

36. Convert $\frac{19}{50}$ to a decimal.

37. Convert $\frac{7}{16}$ to a decimal.

38. Convert $\frac{4}{9}$ to a decimal.

39. Convert $\frac{5}{12}$ to a decimal.

40. Convert $\frac{17}{8}$ to a decimal.

41. $3.24 + 2.56$

42. $7.85 + 4.23$

43. $5.67 + 8.21$

44. $9.8 + 12.6$

45. $0.75 + 3.14$

46. $13.42 + 7.58$

47. $2.99 + 5.01$

48. $6.76 + 3.34$

49. $14.5 + 6.25$

50. $4.9 + 1.6$

51. $8.95 - 3.22$

52. $7.8 - 5.4$

53. $15.67 - 6.8$

54. $5.2 - 2.9$

55. $9.99 - 4.56$

56. $12.3 - 8.14$

57. $20.8 - 11.5$

58. $3.45 - 1.25$

59. $8.4 - 2.5$

60. 6.76×3.2

61. 5.5×3.6

62. 7.2×4.5

63. 9.8×2.4

64. 6.4×2.5

65. 5.2×4.1

66. 3.3×1.8

67. 7.5×0.6

68. 3.2×2.2

69. $5.6 \div 2.8$

70. $12.6 \div 3.6$

71. $6.3 \div 1.5$

72. $9.6 \div 3.2$

73. $4.8 \div 1.2$

74. $8.4 \div 2$

75. $15.6 \div 2.4$

76. $6.75 \div 0.25$

77. $12.5 \div 2.5$

78. $7.2 \div 4$

79. $4.9 \div 1.7$

80. $3.6 \div 0.6$

81. Convert $\frac{3}{4}$ to a decimal.

82. Convert 0.75 to a fraction.

83. Convert 0.85 to a percentage.

84. Convert $\frac{1}{2}$ to a percentage.

85. Convert 0.45 to a percentage.

86. Convert $\frac{5}{8}$ to a percentage.

87. Convert $\frac{7}{10}$ to a decimal.

88. Convert 0.6 to a fraction.

89. Convert $\frac{2}{3}$ to a percentage.

90. Convert 0.3 to a percentage.

91. Convert $\frac{9}{20}$ to a decimal.

92. Convert 0.125 to a percentage.

93. Convert $\frac{3}{5}$ to a percentage.

94. Convert 0.8 to a fraction.

95. Convert $\frac{7}{10}$ to a percentage.

96. Convert 0.4 to a fraction.

97. Convert 0.25 to a fraction.

98. Convert $\frac{1}{5}$ to a decimal.

99. Convert $\frac{5}{8}$ to a decimal.

100. Convert 0.6 to a percentage.

Word Problems

1. Emily bought a notebook for $4.75 and a pen for $2.50. How much did she spend in total?

2. A car's fuel tank was filled with 12.6 liters of petrol. If it already had 5.8 liters, how many more liters were added?

3. John ran 3.75 kilometers in the morning and 4.5 kilometers in the afternoon. How far did he run in total?

4. Sarah works 8 hours a day. She earns $12.50 per hour. How much does she earn in a day?

5. Amy bought 2 cans of soup for $3.25 each and 1 loaf of bread for $2.80. How much did she spend on groceries?

6. A piece of cloth measures 5.6 meters in length. If 2.75 meters of cloth are used, how much cloth is left?

7. A bottle contains 1.25 liters of juice. If 0.75 liters is poured out, how much juice remains in the bottle?

8. A gardener plants 4.8 meters of flowers in the morning and 2.3 meters in the afternoon. How many meters of flowers did the gardener plant?

9. A recipe calls for 0.75 cups of sugar, but you only have 0.5 cups. How much more sugar do you need to complete the recipe?

10. Lisa buys a bag of flour for \$3.80 and a bag of sugar for \$2.65. How much did she pay for both items?

11. Sarah invested $\frac{3}{8}$ of her savings in stocks, $\frac{1}{4}$ in bonds, and the rest in real estate. If her total savings were \$12,000, how much did she invest in real estate? Express your answer as a percentage of her total savings.

12. A store offers a 15% discount on a jacket originally priced at \$120. After applying the discount, a sales tax of 8% is added to the reduced price. What is the final price of the jacket after both the discount and the tax are applied?

13. The population of a city increased by 12.5% over the last year. If the current population is 450,000, what was the population a year ago? Convert your final answer into a fraction, decimal, and percentage.

14. The radius of a circular garden is 7.5 meters. Calculate the area of the garden. Express your answer as a decimal and then convert the result into a fraction and a percentage of 100 square meters.

15. A car travels 2.5 kilometers every minute. If the car maintains this speed for 3 hours and 15 minutes, how far does the car travel? Convert the total distance into a fraction, decimal, and percentage of 1,000 kilometers.

Chapter 3.4 Calculating Percentages

To express one quantity as a percentage of another:

1. Write a fraction where the part is the numerator and the whole is the denominator.
2. Multiply the fraction by 100 to get the percentage.

For example, to express a score of 18 out of 25 as a percentage, the fraction would be $\frac{18}{25}$, and multiplying by 100 gives the percentage.

To find a certain percentage of a quantity:

1. Convert the percentage into a fraction.
2. Replace the word "of" with a multiplication sign.
3. Express the number as a fraction.
4. Multiply the fractions following the rules for fraction multiplication.

Practice Questions

1. Find:

   a. 50% of 36
   b. 20% of 45
   c. 25% of 68

   d. 5% of 60
   e. 2% of 150
   f. 14% of 40

   g. 15% of 880
   h. 45% of 88
   i. 80% of 56

2. Find:

a. 130% of 10

b. 200% of 40

c. 400% of 25

d. 125% of 54

e. 320% of 16

f. 105% of 35

Word Problems

1. Express 15 out of 25 as a percentage.

2. Express 30 out of 50 as a percentage.

3. Express 8 out of 12 as a percentage.

4. Express 45 out of 60 as a percentage.

5. Express 7 out of 20 as a percentage.

6. Express 12 out of 40 as a percentage.

7. Express 18 out of 25 as a percentage.

8. Express 10 out of 50 as a percentage.

9. Express 6 out of 15 as a percentage.

10. Express 22 out of 44 as a percentage.

11. Express 5 out of 10 as a percentage.

12. Express 17 out of 20 as a percentage.

13. Express 150 cm as a percentage of 2 meters.

14. Express 4 kilograms as a percentage of 5,000 grams.

15. Express 3.5 liters as a percentage of 1,000 milliliters.

16. Express 500 milliliters as a percentage of 5 liters.

17. Express 2.5 hours as a percentage of 3 hours.

18. Express 40 kilometers as a percentage of 200 meters.

19. Express 600 grams as a percentage of 2 kilograms.

20. Express 1.2 meters as a percentage of 150 centimeters.

21. Express 1.5 liters as a percentage of 2,000 milliliters.

22. Express 8 ounces as a percentage of 1 pound.

23. Express 10 kilometers as a percentage of 5,000 meters.

24. Express 0.5 kilometers as a percentage of 2,000 meters.

Chapter 3.5 Increase, Decrease, Profit and Loss with Percentage

Profit is calculated as the selling price minus the cost price.

Loss is calculated as the cost price minus the selling price.

Percentage change involves expressing one quantity as a percentage of another:

- **Percentage change** = $\frac{\text{change}}{\text{original value}} \times 100\%$
- **Percentage profit** = $\frac{\text{profit}}{\text{cost price}} \times 100\%$
- **Percentage loss** = $\frac{\text{loss}}{\text{cost price}} \times 100\%$

Percentage error is calculated as the difference between the actual value and the measured or estimated value, expressed as a percentage of the actual value.

- **Percentage error** = $\frac{\text{error}}{\text{actual value}} \times 100\%$

Practice Questions

1. Find:

a. 50% of 36

b. 20% of 45

c. 5% of 60

d. 2% of 150

e. 15% of 880

f. 45% of 88

2. Find:

a. 130% of 10

b. 200% of 40

c. 125% of 54

d. 320% of 16

3. Find:

a. $33\frac{1}{3}$% of 16 liters of orange juice

b. $66\frac{2}{3}$% of 3000 marbles

c. $12\frac{1}{2}$% of \$64 pair of jeans

d. 37.5% of 120 doughnuts.

4. Find:

    a. 20% of 90 minutes

    b. 30% of 150 kg

    c. 40% of 2 weeks

5. Find the new value when:

a. $400 is increased by 10%

b. $240 is increased by 15%

c. $80 is decreased by 8%

d. $42000 is decreased by 2%

e. $5000 is increased by 8%

f. $60.60 is increased by 60%

g. $15 is decreased by 10%

h. $84 is decreased by 40%

Word Problems

1. A product is bought for $120 and sold for $150. Calculate the profit.

2. A product is bought for $80 and sold for $60. Calculate the loss.

3. The cost price of an item is $50, and it was sold for $45. Calculate the percentage loss.

4. The cost price of an item is $75, and it was sold for $100. Calculate the percentage profit.

5. An item is purchased for $200 and sold for $250. Calculate the percentage profit.

6. A book is bought for $12 and sold for $10. What is the percentage loss?

7. The selling price of a gadget is $300, and the cost price is $250. Calculate the profit.

8. The cost price of a shirt is $40, and it is sold for $45. Find the percentage profit.

9. A bicycle is bought for $250 and sold for $275. Calculate the percentage change in price.

10. A phone was bought for $400 and sold for $350. What is the percentage loss?

11. A company estimates its sales at $10,000, but the actual sales amount to $9,500. Calculate the percentage error.

12. A student estimated the length of a table to be 120 cm, but the actual length was 125 cm. Calculate the percentage error.

13. An item is purchased for $150 and sold for $180. Calculate the percentage profit.

14. A factory's cost price for producing an item is $75, and it sells for $90. Calculate the percentage profit.

15. A shopkeeper bought a product for $30 and sold it for $35. What is the percentage change in price?

CHAPTER 4 MEASUREMENT AND PYTHAGORAS THEOREM

Table of Contents

Chapter 4.1 Measurement of Quadrilaterals

The **perimeter of a quadrilateral** is the total length of all its sides. To find the perimeter, simply add up the lengths of all four sides.

Formula for Perimeter of a Quadrilateral

$$\text{Perimeter} = \text{Side } 1 + \text{Side } 2 + \text{Side } 3 + \text{Side } 4$$

Types of Quadrilaterals and Their Perimeters

1. **Square** – All sides are equal, so
$$P = 4 \times \text{side length}$$
2. **Rectangle** – Opposite sides are equal, so
$$P = 2(\text{length} + \text{width})$$
3. **Parallelogram** – Opposite sides are equal, so
$$P = 2(\text{base} + \text{side length})$$
4. **Trapezium** – No fixed formula, just add up all four sides.
$$P = a + b + c + d$$

Area of Quadrilaterals

The **area** of a quadrilateral is the amount of space it covers. Different quadrilaterals have different formulas for area.

Common Quadrilateral Area Formulas

1. **Square**
$$A = side^2$$
(Multiply the side length by itself)

2. **Rectangle**

$$A = \text{length} \times \text{width}$$

(Multiply the two adjacent sides)

3. **Parallelogram**

$$A = \text{base} \times \text{height}$$

(Multiply the base by the perpendicular height)

4. **Trapezium (Trapezoid)**

$$A = \frac{1}{2}(\text{base}_1 + \text{base}_2) \times \text{height}$$

(Find the average of the two parallel sides, then multiply by the height)

5. **Rhombus & Kite**

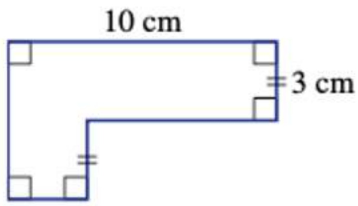
$$A = \frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$$

(Multiply the diagonals and divide by 2)

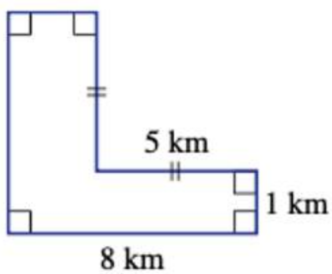
Practice Questions

1. Find perimeter of the shape.

a.



b.



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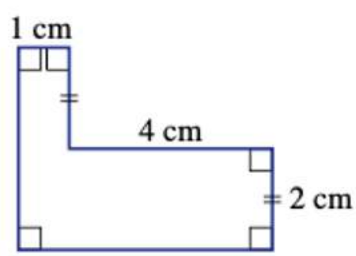
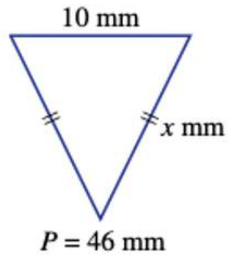


2. Find x when given the perimeter of the shape.

a.



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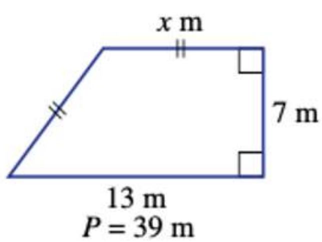
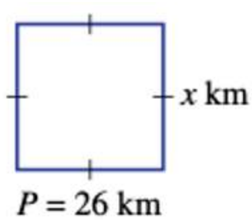
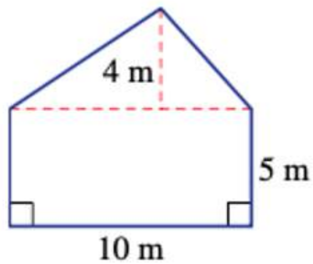


c.



3. Find the area of these composite shapes by using addition or subtraction.

a.



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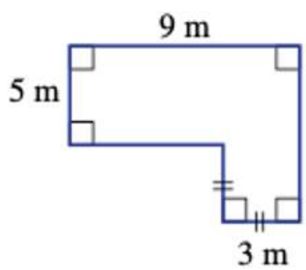
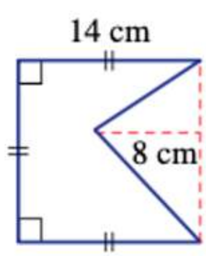
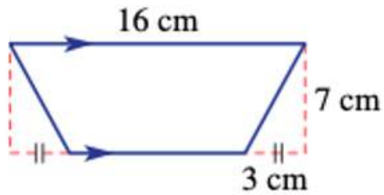


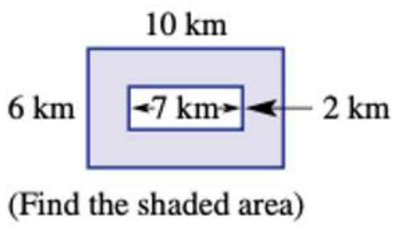
c.



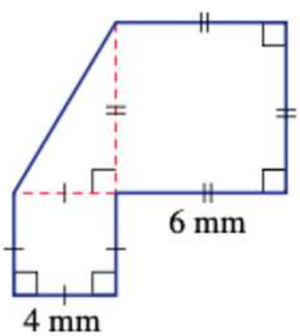
d.



e.

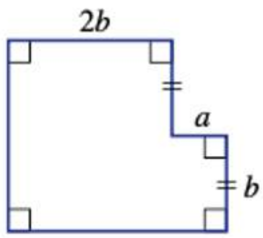


f.



4. Write down expressions for the area of these shapes in simplest form using the letters a and b (e.g. $A = 2ab + a^2$).

a.



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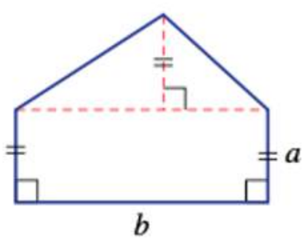
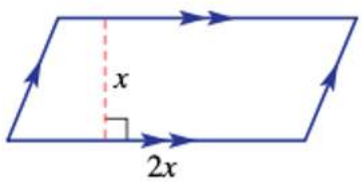
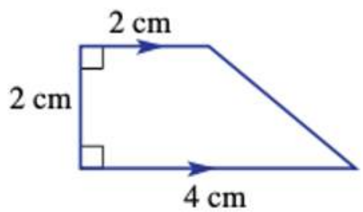


c.



5. These trapeziums have one side at right angles to the two parallel sides. Find the area of each.

a.



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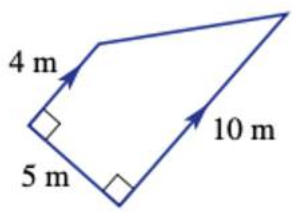

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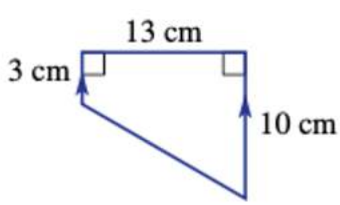
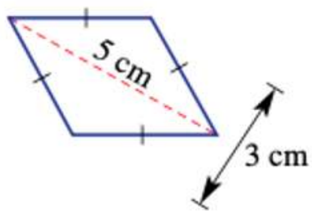


6. Find the area of these special quadrilaterals. First state the name of the shape.

a.



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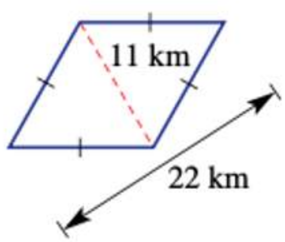

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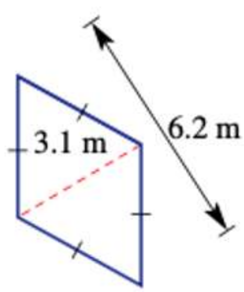

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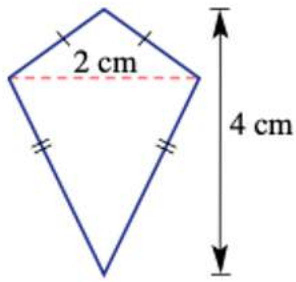

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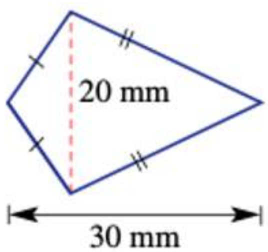

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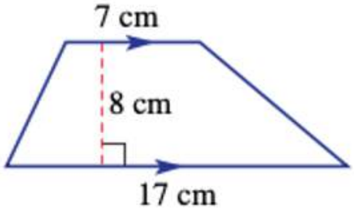


Word Problems

1. Samantha wants to build a rectangular garden in her backyard. The garden is 12 m long and 8 m wide. She plans to put a fence around the entire garden. How many metres of fencing does she need?

2. A park has a rectangular walking track that is 250 m long and 150 m wide. Emma jogs around the entire track once. How far does she run?

3. A square-shaped swimming pool has a side length of 5.5 m. A rope is placed around the edge of the pool for safety. What is the total length of the rope?

4. A kite has two pairs of equal sides: one pair is 9 cm each, and the other pair is 6 cm each. Find the perimeter of the kite.

5. A parallelogram-shaped garden has a base of 18 m and covers an area of 216 m^2. Find the height of the garden.

6. A trapezium-shaped swimming pool has bases measuring 12 m and 20 m, with a height of 5 m. Calculate the area of the pool.

7. A rectangular billboard has an area of 72 m^2. If its width is 8 m, find its length.

8. A kite-shaped field has diagonals of 16 m and 30 m. Find the area of the field.

9. A rhombus has an area of 150 cm^2, and one of its diagonals is 10 cm. Find the length of the other diagonal.

10. A farm has a trapezium-shaped paddock with side lengths of 50 m, 70 m, 40 m, and 60 m. How much fencing is needed to enclose the paddock?

11. A rectangular school playground is twice as long as it is wide. If the perimeter is 144 m, find the length and width of the playground.

12. A trapezium-shaped plot of land has two parallel sides measuring 18 m and 30 m, and the other two non-parallel sides measure 14 m and 16 m. Find the total length of fencing needed to enclose the land.

13. A parallelogram has a base of 24 cm and a side length of 17 cm. If the perimeter of the parallelogram is increased by 50%, what is the new perimeter?

14. A rectangular swimming pool has a width that is 5 metres shorter than its length. If the total perimeter is 58 metres, find the dimensions of the pool.

15. A rhombus has a perimeter of 64 cm, and one of its diagonals is 24 cm. Find the length of one side of the rhombus.

Chapter 4.2 Circles

Features of a Circle

- **Diameter (d):** The distance across the centre of a circle.
- **Radius (r):** The distance from the centre to the edge of the circle (**d = 2r**).
- **Circumference (C):** The distance around a circle, given by:

$$C = 2\pi r \text{ or } C = \pi d$$

- **Pi (π):** Approximately **3.14159**, with common approximations of **3.14** or **22/7**.
- A more precise value of **π** can be found using calculators or online sources.
- **For a circle:**

$$\pi = \frac{C}{d}$$

The **area (A) of a circle** is the amount of space inside the circle. It is given by the formula:

$$A = \pi r^2$$

where:

- **A** = area of the circle
- **r** = radius of the circle
- **π** ≈ **3.14159** (or approximated as **3.14** or **22/7**)

Key Points

- The **radius (r)** is the distance from the centre to the edge.
- The **diameter (d) = 2r**, so an alternative formula is:

$$A = \pi \left(\frac{d}{2}\right)^2$$

- Always express the final answer in **square units** (e.g., cm^2, m^2).

Formula for Sector Area:

$$\text{Sector Area} = \frac{\theta}{360} \times \pi r^2$$

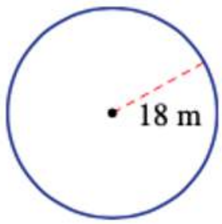
where:

- **θ** = angle of the sector
- **r** = radius of the circle
- **π** ≈ 3.14159

Practice Questions

1. Find the circumference of these circles, correct to two decimal places. Use a calculator for the value of π.

a.



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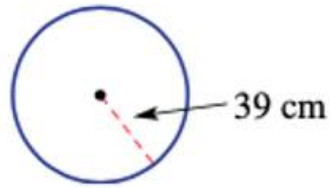

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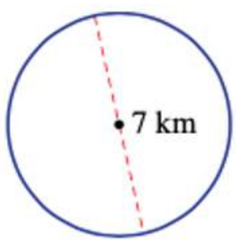

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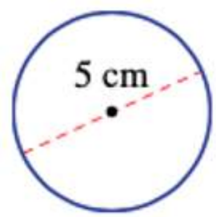
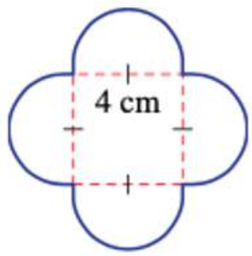


2. Calculate the perimeter of these shapes, correct to two decimal places.

a.



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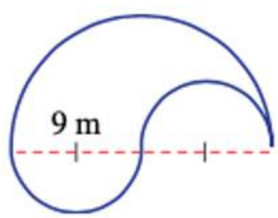

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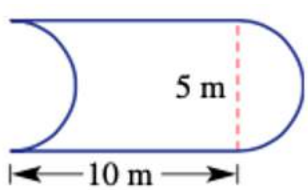
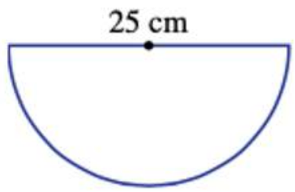


3. These shapes are semicircles. Find the perimeter of these shapes including the straight edge and round the answer to two decimal places.

a.



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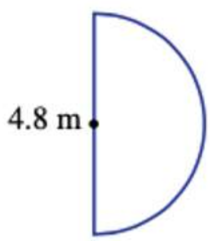

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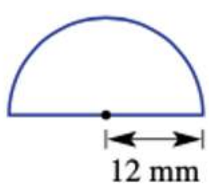
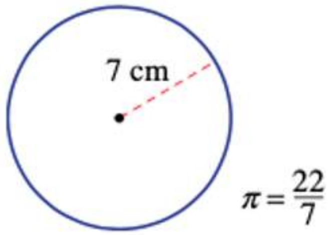


4. Find the area of these circles, using the given approximate value of π.

a.



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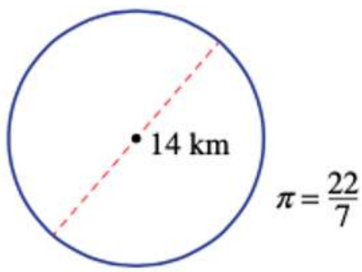

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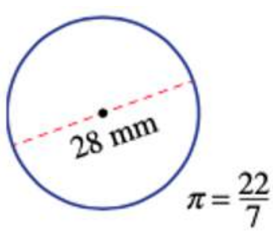

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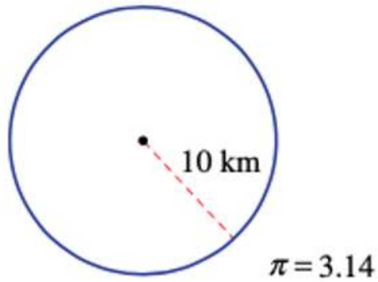

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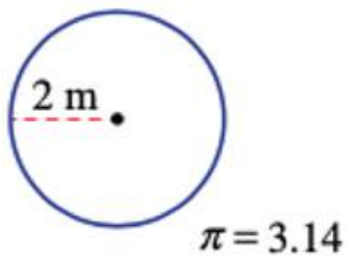

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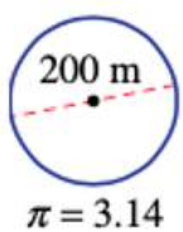
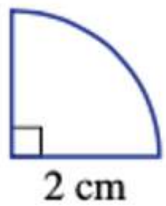


5. Find the area of these quadrants and semicircles, correct to two decimal places.

a.



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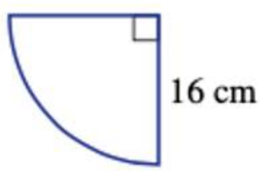

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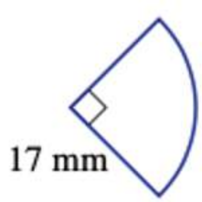

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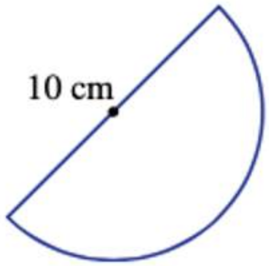

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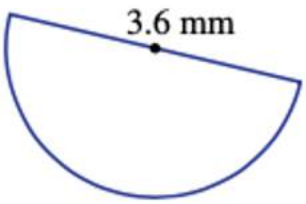

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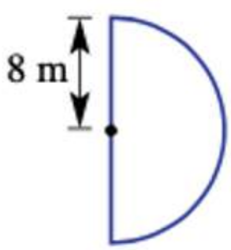
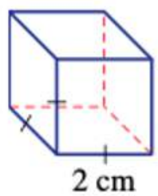


6. Find the surface area of these right prisms.

a.



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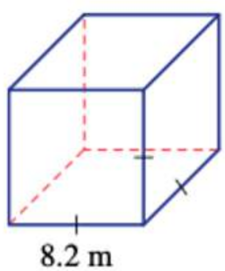

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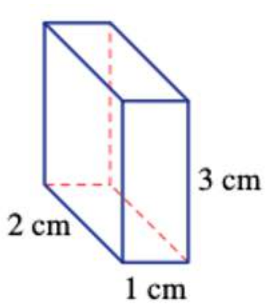

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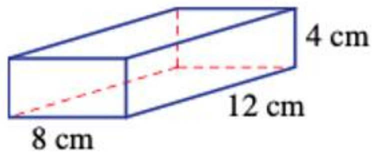

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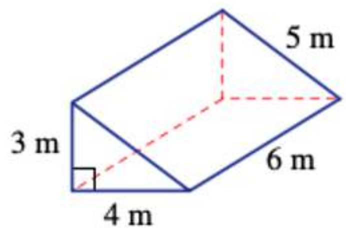

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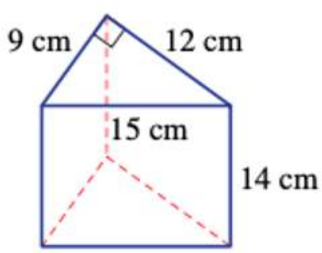
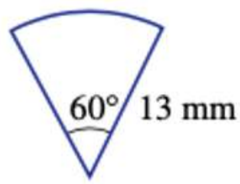


7. Find the area of these sectors, correct to two decimal places.

a.



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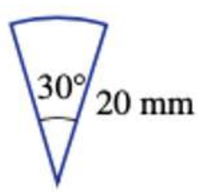

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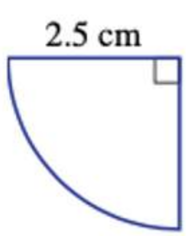

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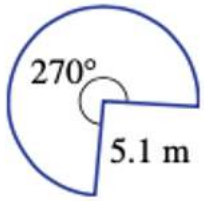

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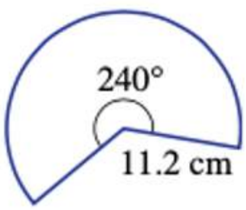

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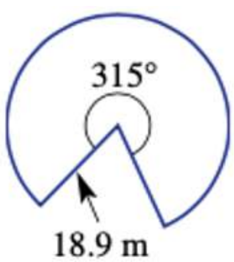


Word Problems

1. A wheel has a radius of 35 cm. Find the circumference of the wheel using $\pi \approx 3.14$.

2. A circular park has a diameter of 120 metres. Calculate the total distance around the park.

3. A pizza has a radius of 14 cm. What is the total length of the crust (circumference)?

4. A bike tire completes one full rotation. If the tire's diameter is 66 cm, how far does the bike travel in one rotation?

5. A clock face has a radius of 10 cm. Find the distance around the outer edge of the clock.

6. A circular garden has a radius of 7 m. What is the area of the garden? (Use $\pi \approx 3.14$.)

7. A circular table has a diameter of 1.2 m. Calculate the area of the table’s surface.

8. A circular swimming pool has a radius of 15 m. How much space is inside the pool?

9. A flower bed is shaped like a circle with a radius of 4.5 m. What is the area of the flower bed? (Use $\pi \approx 3.14$.)

10. A round pizza has a diameter of 40 cm. If you want to cover the pizza with a topping, what is the area of the pizza you need to cover?

11. A pizza is cut into 8 equal slices. If the pizza has a radius of 12 cm and the entire pizza has an area of $\pi \times 12^2$ cm^2, what is the area of one slice?

12. A sector of a circular garden has an angle of 90° and a radius of 10 m. What is the area of the sector?

13. A circular track has a radius of 20 m. A runner runs 120° around the track. Calculate the area of the sector the runner covers.

Chapter 4.3 Volume

Common Conversions

- **1 mL = 1 cm^3**
- **1 L = 1000 mL**
- **1 kL = 1000 L = 1 m^3**

Volume Formulas

- **Volume of a Rectangular Prism:**

$$V = \text{length} \times \text{width} \times \text{height}\ (V = lwh)$$

- **Volume of a Cube:**

$$V = \text{side}^3\ (V = t^3)$$

Volume of a Prism:

$$V = \text{Area of Cross-Section} \times \text{Perpendicular Height}\ (V = A \times h)$$

Volume of a Cylinder:

$$V = \pi r^2 h$$

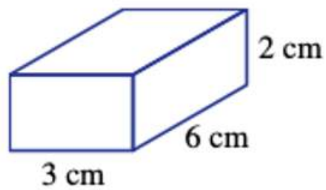
where:

- **r** = radius of the base of the cylinder
- **h** = height of the cylinder
- **π** ≈ 3.14159

Practice Questions

1. Find the volume of these rectangular prisms.

a.



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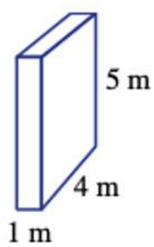

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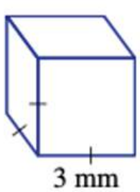
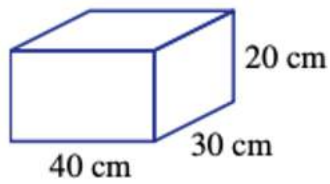


2. Find the capacity of these containers, converting your answer to litres.

a.



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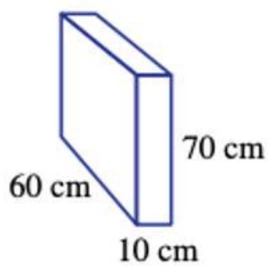

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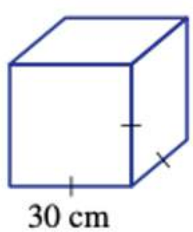

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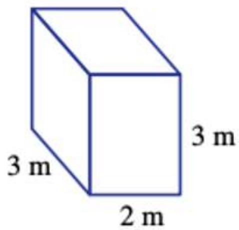

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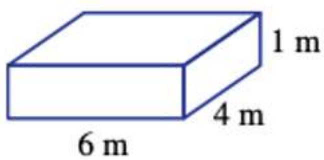

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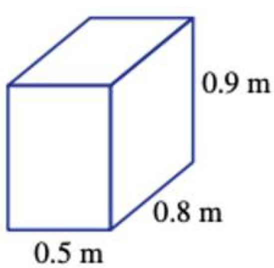
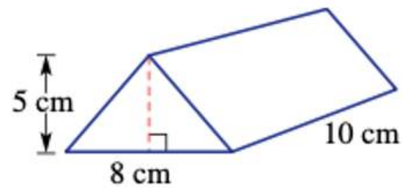


3. Find the volume of these prisms.

a.



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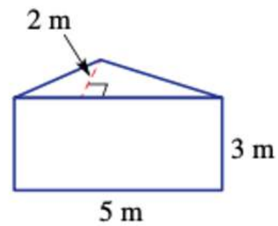

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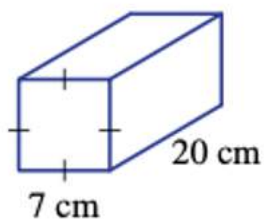

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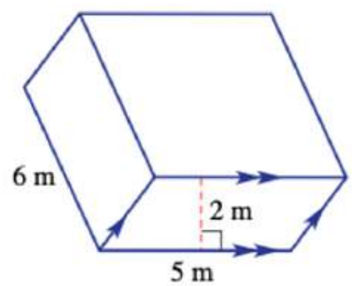
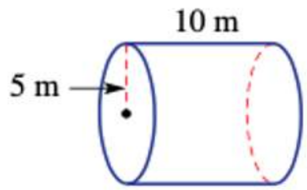
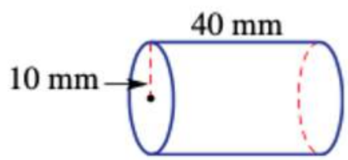


4. Find the volume of these cylinders. Round the answer to two decimal places.

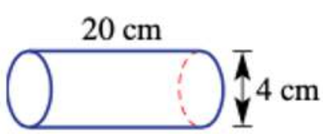
a.



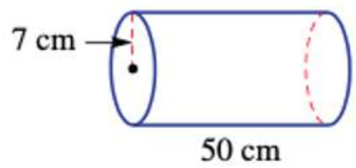
b.



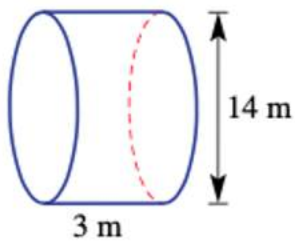
c.



d.



e.



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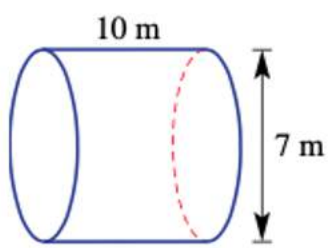


Word Problems

1. A rectangular swimming pool has a length of 8 m, a width of 4 m, and a depth of 2 m. What is the volume of the pool?

2. A cube-shaped box has a side length of 5 cm. What is the volume of the box?

3. A tank in the shape of a rectangular prism has dimensions of 12 m by 6 m by 5 m. How much water can the tank hold?

4. A kitchen jug can hold 2 L of water. How many milliliters can the jug hold?

5. A fish tank is in the shape of a cube with a side length of 20 cm. What is its volume?

6. A cylinder-shaped container holds 10 kL of liquid. How many litres does it hold?

7. A rectangular box has a length of 4 cm, width of 3 cm, and height of 7 cm. What is the volume of the box?

8. A rectangular room has dimensions of 5 m by 3 m by 2.5 m. What is the volume of the room?

9. A kitchen storage container has a length of 30 cm, width of 20 cm, and height of 10 cm. What is the volume of the container?

10. A water tank has a volume of 6000 L. How many cubic metres does the tank hold? (Hint: 1 kL = 1 m^3)

11. A rectangular prism has a base area of 50 cm^2 and a height of 10 cm. What is the volume of the prism?

12. A cylinder-shaped water tank has a radius of 3 m and a height of 8 m. Calculate the volume of the tank.

13. A cylinder has a radius of 5 cm and a height of 12 cm. What is its volume? (Use $\pi \approx 3.14$.)

14. The volume of a prism is given as 1800 cm^3. If the area of its cross-section is 60 cm^2, find its height.

15. A prism has a cross-sectional area of 100 cm^2 and a height of 15 cm. What is the volume of the prism?

Chapter 4.4 Time Units

Common Time Units:

- 1 minute (min) = 60 seconds (s)
- 1 hour (h) = 60 minutes (min)
- 1 day = 24 hours (h)
- 1 week = 7 days
- 1 year = 12 months

Smaller Time Units:

- Millisecond = 0.001 second
- Microsecond = 0.000001 second

Practice Questions

1. Find the value of a in these polygons.

   a. 10:30 a.m. and 1:20 p.m.

   b. 9:10 a.m. and 3:30 p.m.

   c. 2:37 p.m. and 5:21 p.m.

   d. 10:42 p.m. and 7:32 a.m.

   e. 1451 and 2310 hours

   f. 1940 and 0629 hours

2. Convert 6:30 p.m. to the 24-hour clock format.

3. What is 00:45 in 12-hour time?

4. Convert 2:15 a.m. to the 24-hour clock format.

5. What is 13:30 in 12-hour time?

6. Convert 8:00 p.m. to the 24-hour clock format.

7. What is 18:20 in 12-hour time?

8. Convert 4:50 a.m. to the 24-hour clock format.

9. What is 22:10 in 12-hour time?

10. Convert 11:00 p.m. to the 24-hour clock format.

11. What is 05:25 in 12-hour time?

Word Problems

1. Convert 4.75 hours into hours, minutes, and seconds.

2. How many seconds are there in 3 minutes and 45 seconds?

3. A race lasts for 2 hours, 30 minutes, and 45 seconds. How many total seconds did the race last?

4. Convert 90,000 milliseconds into seconds.

5. How many microseconds are there in 0.02 seconds?

6. A meeting starts at 10:30 a.m. and lasts for 2 hours, 45 minutes. What time does the meeting end?

7. A flight departs at 11:00 a.m. and travels for 5 hours and 30 minutes. What time will the flight arrive?

8. Convert 5 days, 12 hours, 30 minutes into hours.

9. A movie runs for 2 hours and 15 minutes. How many total minutes is the movie?

10. If it's 2:15 p.m., what time will it be 3 hours and 45 minutes later?

Chapter 4.5 Pythagoras

The **hypotenuse** is the longest side of a right-angled triangle and is opposite the right angle.

Pythagoras' Theorem states that the square of the hypotenuse (c) is equal to the sum of the squares of the other two sides (a and b):

$$a^2 + b^2 = c^2$$

A **Pythagorean triple** is a set of three whole numbers that satisfy Pythagoras' theorem, e.g., **3, 4, 5** because:

$$3^2 + 4^2 = 5^2$$

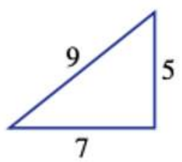
A triangle can be classified based on its side lengths a, b, and c (in increasing order) as:

- **Right-angled** if $c^2 = a^2 + b^2$
- **Acute** if $c^2 < a^2 + b^2$
- **Obtuse** if $c^2 > a^2 + b^2$

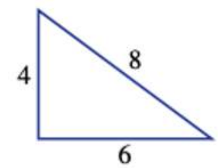
Practice Questions

1. Classify the following triangles as right-angles, acute or obtuse based on their side lengths. Note that the triangles are not drawn to scale.

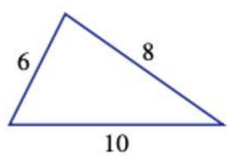
a.



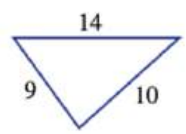
b.



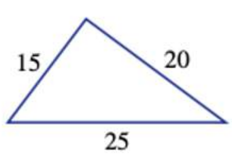
c.



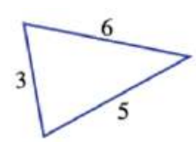
d.



e.

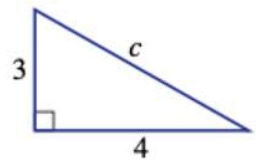


f.



2. Find the length of the hypotenuse of these right-angled triangles.

a.



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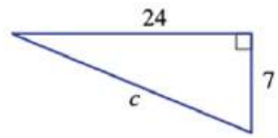
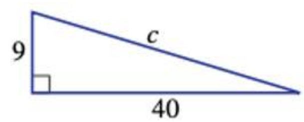
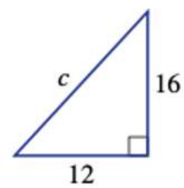


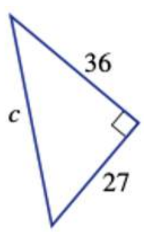
c.



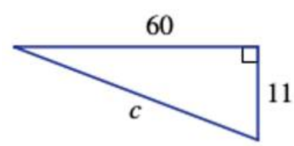
d.



e.

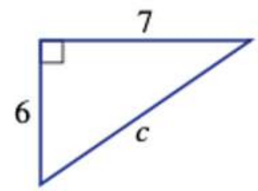


f.

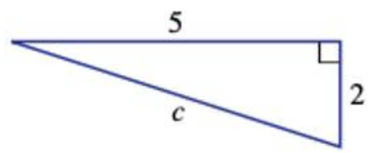


3. Find the length of the hypotenuse of these right-angled triangles correct to two decimal places.

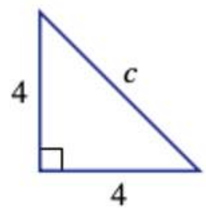
a.



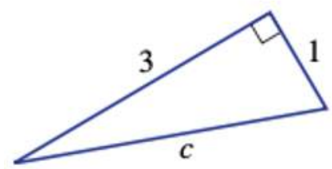
b.



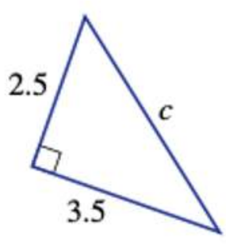
c.



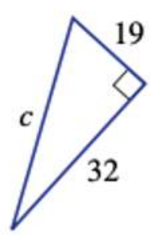
d.



e.

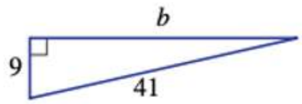


f.

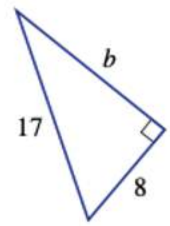


4. Find the length of the unknown side in these right-angled triangles.

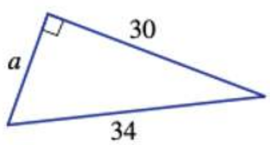
a.



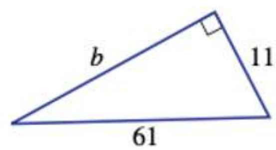
b.



c.



d.



Word Problems

1. A right-angled triangle has one side of length 6 cm and the other side of length 8 cm. What is the length of the hypotenuse?

2. The base of a right-angled triangle is 9 m and the height is 12 m. Find the length of the hypotenuse.

3. A ladder leans against a wall. The ladder is 15 feet long, and the distance from the base of the ladder to the wall is 9 feet. How high up the wall does the ladder reach?

4. A rectangular garden has a length of 7 m and a width of 24 m. What is the length of the diagonal of the garden?

5. A right-angled triangle has legs of length 5 cm and 12 cm. Find the length of the hypotenuse.

6. A right-angled triangle has a hypotenuse of length 13 cm and one leg of length 5 cm. Find the length of the other leg.

7. The diagonal of a rectangular room measures 10 m, and the room's length is 6 m. What is the width of the room?

8. A right-angled triangle has a base of 24 cm and a hypotenuse of 25 cm. What is the length of the height?

9. A right-angled triangle has sides of length 8 cm and 15 cm. What is the length of the hypotenuse?

10. The diagonal of a square is 12 cm. What is the length of each side of the square?

CHAPTER 5 ALGEBRA

Table of Contents

Chapter 5 Introduction

Algebra is a branch of mathematics that uses symbols (variables) to represent numbers and express relationships through equations and expressions.

It helps solve problems, find unknown values, and analyze patterns using operations like addition, subtraction, multiplication, and division.

- **Pronumerals (variables)** represent numbers in algebra.
- **Multiplication** is written as ab instead of $a \times b$. **Product:** $a \times b$
- **Division** is written as a/b instead of $a \div b$. **Quotient:** $a \div b$
- **Sum:** $a + b$
- **Difference:** $a - b$
- **Square:** a^2
- An **expression** combines numbers, pronumerals, and operations (e.g., $3x + 2yz$).
- A **term** is a single part of an expression (e.g., $9a$, $10cd$).
- A **coefficient** is the number before a pronumeral:
  - (e.g., in $3x + y - 7z$, the coefficients are 3, 1, and -7).
- A **constant** is a term without variables.
- **Substitution** is to replace a pronumeral with a number.

Chapter 5.1 Adding and Subtracting Algebraic Terms and Fractions

Algebraic Terms

- **Like terms** have the same pronumerals with the same powers, regardless of order (e.g., $4ab$ and $7ba$).
- **Like terms can be combined** when added or subtracted to simplify expressions (e.g., $3xy + 5xy = 8xy$).
- **The sign stays with the term** when rearranging (e.g., $3x + 7y - 2x + 3y + x - 4y = 2x + 6y$).

Algebraic Fractions

- An **algebraic fraction** has an algebraic expression in the numerator or denominator.
- The **lowest common denominator (LCD)** is the smallest multiple of the denominators.
- **Adding and subtracting algebraic fractions** requires a common denominator.

Example 1: Adding algebraic fractions

$$\frac{3x}{4} + \frac{5x}{4} = \frac{3x + 5x}{4} = \frac{8x}{4} = 2x$$

Example 2: Subtracting algebraic fractions

$$\frac{6y}{7} - \frac{2y}{7} = \frac{6y - 2y}{7} = \frac{4y}{7}$$

Practice Questions

Adding & Subtracting Algebraic Terms

1. Simplify the following by combining like terms.

   a. $7f + 2f + 8 + 4$

   b. $2a + 5a + 13b - 2b$

   c. $10 + 5x + 2 + 7x$

   d. $10x + 31y - y + 4x$

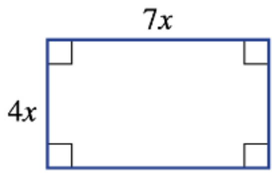
e. $7x^2y + 5x + 10yx^2$

f. $-4x^2 + 3x^2$

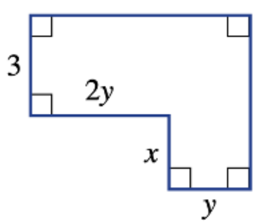
g. $10 + 7q - 3r + 2q - r$

2. Write expressions for the perimeters of the following shapes in simplest form.

a.



b.



Adding & Subtracting Algebraic Fractions

1. Simplify the following sums.

a. $\frac{x}{4} + \frac{2x}{4}$

b. $\frac{5a}{3} + \frac{2a}{3}$

c. $\frac{4k}{3} + \frac{k}{3}$

d. $\frac{a}{2} + \frac{a}{3}$

e. $\frac{p}{2}+\frac{p}{5}$

f. $\frac{q}{4}+\frac{q}{2}$

g. $\frac{2m}{5}+\frac{2m}{3}$

h. $\frac{7p}{6}+\frac{2p}{5}$

2. Simplify the following differences.

a. $\frac{3y}{5} - \frac{y}{5}$

b. $\frac{7p}{13} - \frac{2p}{13}$

c. $\frac{8q}{5} - \frac{2q}{5}$

d. $\frac{p}{2} - \frac{p}{3}$

e. $\frac{9u}{11} - \frac{u}{2}$

f. $\frac{8y}{3} - \frac{5y}{6}$

g. $\frac{6u}{7} - \frac{7u}{6}$

h. $\frac{9u}{1} - \frac{3u}{4}$

3. Simplify the following expressions, giving your final answer as an algebraic fraction (*Hint*: $4x$ is the same as $\frac{4x}{1}$).

   a. $4x + \frac{x}{3}$

   b. $3x + \frac{x}{2}$

   c. $\frac{a}{5} + 2a$

d. $\frac{8p}{3} - 2p$

e. $\frac{10u}{3} + \frac{3v}{10}$

f. $\frac{7y}{10} - \frac{2x}{5}$

Word Problems

Adding & Subtracting Algebraic Terms

1. Emma has $5x$ dollars, and her brother gives her $7x$ more. How much money does she have in total?

2. A rectangle has a length of $4a + 3a$ cm. What is its total length?

3. A car travels 12m kilometers east and then 5m kilometers west. What is the net distance traveled?

4. A company produces $8y$ laptops in one month and $6y$ the next month. How many laptops are produced in total?

5. Jason collected $9c$ coins but lost $3c$ of them. How many coins does he have left?

6. The perimeter of a triangle is given by the expression $3x + 5y - 2z$ cm. If one side is $x + 2y - z$ cm and another side is $2x + y$ cm, find the length of the third side.

7. A manufacturer produces $12a + 5b$ mobile phones in one factory and $7a - 2b$ mobile phones in another factory. How many mobile phones are produced in total?

8. The height of a building is given as $15m + 4n$ meters, while the height of a second building is $9m - 2n$ meters. What is the difference in height between the two buildings?

9. A football team scored $8x + 3y$ goals in the first half and $5x - 2y$ goals in the second half. How many goals did the team score in total?

10. A school cafeteria uses $20p + 6q$ liters of milk in the morning and $15p - 3q$ liters in the afternoon. How much more milk is used in the morning than in the afternoon?

Adding & Subtracting Algebraic Fractions

1. A farmer divides his land into two sections. One section covers $\frac{3x}{4}$ hectares, and the other covers $\frac{5x}{6}$ hectares. What is the total area of the farmer's land in hectares?

2. A tank is $\frac{2a}{3b}$ full of water, but $\frac{a}{4b}$ leaks out. How much water remains in the tank?

3. A cyclist travels $\frac{x+2}{x^2-4}$ kilometers in the morning and $\frac{3}{x-2}$ kilometers in the afternoon. What is the total distance traveled?

4. A construction worker has $\frac{5y}{2x}$ meters of steel but uses $\frac{3y}{4x}$ meters for a project. How much steel does he have left?

5. A school cafeteria uses $\frac{4m}{m+1}$ liters of milk in the morning, $\frac{3}{m+1}$ liters at lunch, and $\frac{2m}{m+1}$ liters in the afternoon. How much milk is used in total?

Chapter 5.2 Multiplying and Dividing Algebraic Terms and Fractions

Algebraic Terms

- **Multiplication**: The order of multiplication does not matter (e.g., $2 \times a \times 4 \times b = 2 \times 4 \times a \times b$).
- **Exponents**: x^2 means $x \times x$, and x^3 means $x \times x \times x$.
- **Division**: Cancel out common factors when simplifying fractions.

Example 1:

$$\frac{12xy}{18x} = \frac{12 \cdot x \cdot y}{18 \cdot x} = \frac{2y}{3}$$

Example 2:

$$\frac{15a^2b}{25ab} = \frac{15 \cdot a \cdot a \cdot b}{25 \cdot a \cdot b} = \frac{3a}{5}$$

Algebraic Fractions

- **Multiplication:** multiply numerators and denominators of two algebraic fractions separately. Then cancel out common factors.
- **Cancel** any **common factors** in the numerator and denominator.
- The **reciprocal** of a fraction is formed by swapping the numerator and denominator.
- **Division:** take the reciprocal of the second fraction and then multiply.

Example of Multiplication:

$$\frac{2x}{3} \times \frac{4}{5x}$$

Multiply the numerators and denominators:

$$\frac{2x \times 4}{3 \times 5x} = \frac{8x}{15x}$$

Cancel out the common factor of x:

$$\frac{8}{15}$$

Example of Division using Reciprocal:

$$\frac{2x}{3} \div \frac{4}{5x}$$

Take the reciprocal of the second fraction and multiply:

$$\frac{2x}{3} \times \frac{5x}{4} = \frac{10x^2}{12}$$

Simplify the fraction:

$$\frac{5x^2}{6}$$

Practice Questions

Multiplying and Dividing Algebraic Terms

1. Simplify the following.

   a. $8ab \times 3c$

   b. $3d \times d$

   c. $7x \times 2y \times x$

   d. $4xy \times 2xz$

e. $12x^2y \times 4x$

f. $3x^2y \times 2x \times 4y$

g. $-5xy \times 2yz$

h. $4xy^2 \times 4y$

2. Simplify the following divisions by cancelling any common factors.

a. $\frac{5a}{10a}$

b. $\frac{10xy}{12y}$

c. $\frac{7xyz}{21yz}$

d. $\frac{-5x}{10yz^2}$

e. $\frac{-4a^2}{8ab}$

f. $\frac{-21p}{-3p}$

Multiplying and Dividing Algebraic Fractions

1. Simplify the following products, remembering to cancel any common factors.

a. $\frac{6x}{5} \times \frac{7y}{6}$

b. $\frac{2b}{5} \times \frac{7d}{6}$

c. $\frac{8a}{5} \times \frac{3b}{4c}$

d. $\frac{9d}{2} \times \frac{4e}{7}$

e. $\frac{3x}{2} \times \frac{1}{6x}$

f. $\frac{4}{9k} \times \frac{3k}{2}$

2. Simplify the following divisions, cancelling any common factors.

a. $\frac{3a}{4} \div \frac{1}{5}$

b. $\frac{9a}{10} \div \frac{1}{4}$

c. $\frac{4}{5} \div \frac{2y}{3}$

d. $\frac{4a}{7} \div \frac{2}{5}$

e. $\frac{2x}{5} \div \frac{4y}{3}$

f. $\frac{5}{12x} \div \frac{7x}{2}$

Word Problems

1. A rectangle has a length of $3x$ meters and a width of $4y$ meters. What is the area of the rectangle in terms of x and y?

2. Simplify the expression $\frac{6x^2}{3y} \div \frac{2xy}{5}$.

3. A car travels at a speed of $2a$ kilometers per hour for $4b$ hours. What is the total distance traveled by the car?

4. Simplify $\frac{5x^3}{6y^2} \div \frac{2x^2}{3y}$.

5. A gardener is planting flowers in rows. Each row has $5x$ plants, and there are $3y$ rows. How many plants are there in total?

6. Simplify the expression $\frac{7a^2b}{10c} \div \frac{2ab}{5c}$.

7. The cost of one notebook is $4x$ dollars. If a student buy $5y$ notebooks, what is the total cost?

8. Solve $\frac{3x^2y}{7} \div \frac{6xy^2}{5}$.

9. A box has a volume of $2a \times 3b \times 4c$. What is the volume of the box in terms of a, b, and c?

10. Simplify $\frac{9x^4y}{6z^3} \div \frac{3x^2y^2}{2z}$.

Chapter 5.3 Expanding Brackets using Distributive Law

Distributive law

The distributive law (also known as the distributive property) states that when you multiply a number by a sum or difference, you multiply each term inside the parentheses by the number outside.

In algebraic form, the distributive law is written as:

$$a(b+c) = ab + ac \quad or \quad a(b-c) = ab - ac$$

Example:

Simplify $3(x+4)$.

Using the distributive law:

$$3(x+4) = 3 \times x + 3 \times 4 = 3x + 12$$

So, $3(x+4) = 3x + 12$.

Practice Questions

1. Use the distributive law to expand the following.

a. $-5(9+g)$

b. $-7(5b+4)$

c. $-9(u-9)$

d. $-8(5-h)$

e. $8z(k - h)$

f. $-6j(k + a)$

g. $4u(2r - q)$

h. $4m(5w - 3a)$

2. Simplify the following by expanding and then collecting like terms.

a. $7(9f + 10) + 2f$

b. $8(2 + 5x) + 4x$

c. $4(2a + 8) + 7a$

d. $6(3v + 10) + 6v$

e. $7(10a + 10) + 6a$

f. $6(3q - 5) + 2q$

g. $6(4m - 5) + 8m$

h. $4(8 + 7m) - 6m$

3. Simplify the following by expanding and then collecting like terms.

   a. $3(3 + 5d) + 4(10d + 7)$

   b. $10(4 + 8f) + 7(5f + 2)$

   c. $2(9 + 10j) + 4(3j + 3)$

d. $2(9 + 6d) + 7(2 + 9d)$

e. $6(10 - 6j) + 4(10j - 5)$

f. $8(5 + 10g) + 3(4 - 4g)$

Word Problems

1. Simplify the expression: A box contains $5(x + 3)$ pencils. How many pencils are there in total?

2. A farmer has $7(a + 2)$ baskets of apples. How many apples does the farmer have in total?

3. Expand and simplify: The cost of one notebook is $4x$ dollars, and the student buys $6(x + 2)$ notebooks. What is the total cost?

4. A car travels $3(t+5)$ kilometers every day. How far does the car travel in one day?

5. A recipe requires $2(y+4)$ cups of sugar. How much sugar is needed in total for the recipe?

6. Simplify: The total score in a game is calculated by $2x+3(x+4)$. What is the total score?

7. A company manufactures 5 different types of sports equipment. Each type of equipment requires x hours to produce, and the company produces $(x + 3)$ items of each type. How many hours will the company spend producing all the equipment in total?

8. A group of a students are working on a project. Each student spends 3 hours per day studying, but they also spend an additional 2 hours per day meeting with their mentor. How many total hours does the group of students spend on the project each day?

9. A bookstore sells books that cost $8y$ dollars each. They are running a promotion where a customer can buy 4 books of the same type, plus 2 extra books at a discounted price of y dollars each. What is the total cost of the purchase?

10. A movie theater charges 12 dollars for each ticket and a service fee of 2 dollars per ticket. A family buys $(x + 4)$ tickets for a movie. How much will the family pay in total for all the tickets?

Chapter 5.4 Factorising Expressions

HCF (Highest Common Factor): The largest factor that divides into each term.

- Example:
  - HCF of $15x$ and $21y = 3$
  - HCF of $10a$ and $20c = 10$
  - HCF of $12x$ and $18xy = 6x$
- Factorisation Process:
  1. Find the HCF of the terms outside the brackets.
  2. Divide each term by the HCF.
  3. Leave the result inside the brackets.
- Check the Answer:
  - Expand the factorized form to verify.
  - Example: $5(2x + 3y) = 10x + 15y$

Practice Questions

1. Find the highest common factor (HCF) of the following pairs of terms.

   a. 15 and $10x$

   b. $27a$ and $9b$

   c. $-2yz$ and $4xy$

   d. $8qr$ and $-4r$

   e. $14p$ and $25pq$

2. Factorise the following by first finding the highest common factor. Check your answers by expanding them.

a. $3x + 6$

b. $8v + 40$

c. $15x + 35$

d. $10z + 25$

e. $40 + 4w$

f. $5j - 20$

g. $9b - 15$

h. $12 - 16f$

i. $5d - 30$

3. Factorise the following expressions.

a. $10cn + 12n$

b. $14jn + 10n$

c. $10h + 4z$

d. $40y + 56ay$

e. $21hm - 9mx$

f. $28u - 42bu$

Word Problems

1. A rectangle has a length of $12x^2 + 18x$ and a width of 6x. Factorise the expression for the perimeter of the rectangle.

2. A group of students are selling tickets for a fundraiser. The total revenue from ticket sales can be expressed as $15x^2 + 25x$. Factorise the expression to find the common factor.

3. The area of a rectangular garden is given by the expression $20x^3 + 30x^2$. Factorise the expression to find the common factor that can be taken out.

4. The difference between two numbers is given by the expression $24a - 36b$. Factorise this expression to show the common factor.

5. A company sells two types of products. The total cost for both products is represented by $18x + 24y$. Factorise this expression to find the common factor of the costs.

6. A quadratic equation is written as $4x^2 + 8x$. Factorise the expression to find the common factor and rewrite it in factorised form.

7. The expression $35x^3 + 14x^2$ represents the total number of books sold in two different categories. Factorise this expression to find the common factor.

8. The total revenue from selling two products is given by the expression $12x^2 + 16x + 20$. Factorise the expression to reveal the greatest common factor.

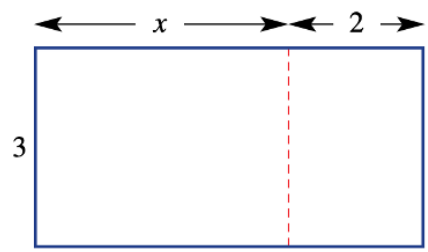
9. A factory produces two types of machines. The cost of producing both types is given by $16a^2 + 24a$. Factorise this expression to find the common factor.

10. The area of a trapezoid is given by $30x^2 - 45x$. Factorise this expression and show the common factor.

Chapter 5.5 Applying Algebra

Practice Questions

1.



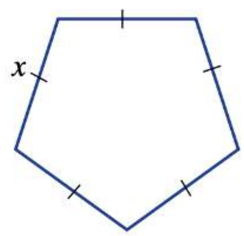
a. Write an expression for the total area of the shape shown.

b. If $x = 9$, what is the area?

2. An electrician charges a call-out fee of $30 and $90 per hour. Which of the following represents the total cost, in dollars, for x hours?

   a. $x(30 + 90)$
   b. $30x + 90$
   c. $30 + 90x$
   d. $120x$

3.



   a. Give an expression for the perimeter of this regular pentagon.

   b. If each side length were doubled, what would the perimeter be?

c. If each side length were increased by 3, write a new expressions for the perimeter.

4. An indoor soccer pitch costs $40 per hour to hire plus a $30 booking fee.

   a. Write an expression for the cost, in dollars, or hiring the pitch for x hours.

   b. Hence, find the cost of hiring the pitch for an 8-hour round-robin tournament.

5. A plumber says that the cost, in dollars, to hire her for x hours is $50 + 60x$.

   a. What is her call-out fee?

   b. How much does she charge per hour?

   c. If you had $200, what is the longest period you could hire the plumber?

6. A repairman says the cost, in dollars, to hire his services for x hours is $20(3+4x)$.

   a. How much would it cost to hire him for 1 hour?

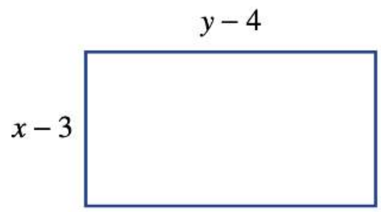
   b. Expand the expression he has given you.

   c. Hence, state:

      i. his call-out fee

      ii. the amount he charges per hour.

7. Roberto draws a rectangle with unknown dimensions. He notes that the area is $(x-3)(y-4)$.



a. If $x=5$ and $y=7$, what is the area?

b. What is the value of $(x-3)(y-4)$ if $x=1$ and $y=1$?

c. Roberto claims that this proves that if $x=1$ and $y=1$, then his rectangle has an area of 6. What is wrong with his claim? (*Hint*: Try to work out the rectangle's perimeter.)

8. Tamir notes that whenever he hires an electrician, they charge a call-out fee, $\$F$, and an hourly rate of $\$H$ per hour.

   a. Write an expression for the cost, in dollars, of hiring an electrician for one hour.

   b. Write an expression for the cost, in dollars, of hiring an electrician for two hours.

   c. Write an expression for the cost, in dollars, of hiring an electrician for 30 minutes.

   d. How much does it cost to hire an electrician for t hours?

Chapter 5.6 Index Laws for Multiplication and Division

Index Notation: Repeated multiplication can be written as a base and index.

- Example: $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Expanded Form: An expression like $4x^3$ can be written as $4 \times x \times x \times x$.

Multiplying Powers (Same Base):

- Law: $a^m \times a^n = a^{m+n}$
- Example: $a^4 \times a^2 = a^6$

Dividing Powers (Same Base):

- Law: $\frac{a^m}{a^n} = a^{m-n}$
- Example: $\frac{a^8}{a^5} = a^3$

Practice Questions

1. Simplify the following using the index law for multiplication.

   a. $4m^2 \times 5m^3$

   b. $7x^2 \times 4x^{12}$

   c. $m^2 \times n^3 \times m^4 \times n^7$

   d. $3r^3s^2 \times s^5$

e. $11x \times 10x^3$

f. $2x^2y^2 \times 4x^3y^5$

g. $-7x^2y^3 \times 2x^5y$

h. $-4ab^2 \times a^4b$

i. $7x \times 12x^3y^5$

2. Simplify the following, giving your answers in index form.

a. $\frac{3^7}{3^2}$

b. $\frac{10^{15}}{10^7}$

c. $\frac{2^{10}}{2^5}$

d. $\frac{5^{100}}{5^{98}}$

3. Simplify the following using the index law for division.

a. $\frac{m^5}{m^2}$

b. $\frac{z^5}{z^2}$

c. $\frac{q^{10}}{q^3}$

d. $\frac{r^{10}}{r}$

e. $\frac{m^5n^7}{m^3n^2}$

f. $\frac{a^{10}b^5}{a^5b^2}$

g. $\frac{x^3y^{10}z^5}{x^2y^4z^3}$

h. $\frac{x^4y^7z^3}{x^2y^4}$

i. $\frac{4k^{10}}{k^7}$

j. $\frac{10m^{20}}{5m^7}$

k. $\frac{30x^{20}y^{12}}{18x^2y^5}$

l. $\frac{a^3b}{2ab}$

Word Problems

1. A gardener has a plot of land where the area of one section is represented by x^4 square meters, and the area of another section is represented by x^3 square meters. If both areas are multiplied together, what is the total area?

2. A factory produces boxes, and the total number of boxes in one batch is represented by $5a^2$. In another batch, the total is represented by $3a^5$. How many boxes are produced when the two batches are combined?

3. A student is dividing the total number of books in a library, represented by y^7, into smaller groups of y^3 books. How many smaller groups are formed?

4. A factory produces $2m^6$ items in one batch and m^2 items in another batch. How many items are produced when both batches are combined?

5. A teacher is dividing a total of $4x^8$ markers among groups of x^5 markers each. How many groups of markers are there?

6. A construction company has a set of materials represented by $3b^5$ and another set represented by b^2. If the two sets are combined, what is the total amount of material?

7. A librarian is organizing books into groups, with each group containing y^4 books. If the total number of books is $6y^9$, how many groups of books does the librarian have?

8. A small business sells $7x^3$ units of product A and $2x^4$ units of product B. How many units of both products are sold in total?

9. A scientist is dividing a substance represented by p^6 into portions of p^3 each. How many portions does the scientist have?

10. A bakery produces $10a^4$ loaves of bread in one batch and a^3 loaves in another batch. How many loaves of bread are produced in total when both batches are combined?

Chapter 5.7 The Zero Index and Power of a Power

$a^0 = 1$ for every value of a, except when $a = 0$.

- Example: $4^0 = 1$ and $(7^3xy)^0 = 1$

Power of a Power: Multiply the indices to simplify.

- Example: $(x^2)^5 = x^{10}$

Expanding Powers: When expanding expressions with powers, apply the power to both the number and the variable.

- Example: $(3x)^4 = 3^4 \times x^4$

Practice Questions

1. Simplify the following.

   a. 7^0

   b. $5^0 \times 3^0$

   c. $5b^0$

   d. $12x^0y^2z^0$

e. $(3x^2)^0$

f. $13(m+3n)^0$

g. $2(x^0y)^2$

h. $4x^0(4x)^0$

i. $3(a^5y^2)^0a^2$

2. Simplify the following.

a. $(2^3)^4$

b. $(5^2)^8$

c. $(6^4)^9$

d. $(d^3)^3$

e. $(k^8)^3$

f. $(m^5)^{10}$

3. Simplify the following. Large numerical powers like 5^4 should be left in index form.

a. $(3x^5)^2$

b. $(2u^4)^3$

c. $(5x^5)^4$

d. $(12x^5)^3$

e. $(4x^4)^2$

f. $(7x^2)^2$

g. $(9x^7)^{10}$

h. $(10x^2)^5$

4. Simplify the following using the index laws.

a. $(x^3)^2 \times (x^5)^3$

b. $(2k^4)^2 \times (5k^5)^3$

c. $4(x^3)^2 \times 2(x^4)^3$

d. $\frac{(y^3)^4}{y^2}$

e. $\frac{(2p^5)^3}{2^2p^2}$

f. $\frac{8h^{20}}{(h^3)^5}$

Word Problems

1. A number is raised to the power of 0. If the number is 5, what is the result?

2. If a variable x is raised to the power of 0, what is the value of x^0?

3. A school has a rule that each class can have up to 4^0 students in a group. How many students can be in a group according to this rule?

4. A scientist is measuring a substance and finds that $(2a)^0 = 1$. What is the value of $(2a)^0$?

5. A factory produces $(m^2)^3$ items in one day. How many items are produced in total?

6. In an experiment, the formula $(x^3)^2$ represents the result. What is the simplified form of this expression?

7. A builder is working on a project and finds that 6^0 represents a specific value. What is the value of 6^0?

8. A student is calculating the power of a number and encounters $(y^4)^2$. What is the simplified result of this calculation?

9. A computer program calculates $(3x)^0$. What is the result of this calculation?

10. A researcher is working with the expression $(7a^2)^3$. What is the simplified form of this expression?

CHAPTER 6 RATIOS AND RATES

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Chapter 6.1 Simplifying and Solving Ratios

Simplifying Ratios

- **Simplifying Ratios**: Divide both numbers by their **HCF** (Highest Common Factor).
  Example: $15 : 25 \rightarrow 3 : 5$
- **Equivalent Ratios**: Multiply or divide both numbers by the same value.
  Example: $4 : 12, 8 : 24$, and $1 : 3$ are equivalent
- **Simplest Form**: Ratios should be in whole numbers with a **HCF of 1**.
- **Ratios with Fractions**: Convert to whole numbers by multiplying by the **LCD (Lowest Common Denominator)**.
- **Same Units**: Before simplifying, make sure all values are in the **same unit**.

Solving Ratio Problems

- **Using Equivalent Ratios:** Multiply or divide both numbers in the ratio to find missing values.
  Example: If the ratio is **2:3** and one person gets $**40,** multiply by **20 → 40:60, so the** second person gets $**60.**
- **Understanding Parts in a Ratio:**
  Example: **2:3** means **2 parts of one quantity** for **3 parts of another,** total **5 parts.**
- **Unitary Method for Dividing a Quantity:**
  1. Add up the total parts in the ratio.
  2. Find the value of **one part** by dividing the total by the number of parts.
  3. Multiply to find each share.
- **Using Fractions to Divide a Quantity:**
  1. Find the **fraction of the total** for each part **→ (number in ratio) ÷ (total parts).**
  2. Multiply by the total to find each share.

Practice Questions

1. Simplify the following ratios.

a. $10:50$

b. $8:10$

c. $21:28$

d. $18:14$

e. $45:35$

f. $51:17$

g. $1200:100$

h. $200:125$

2. Simplify the following ratios. (Note: You can divide all three numbers by the highest common factor.)

   a. $2:4:6$
   b. $42:60:12$
   c. $12:24:36$
   d. $270:420:60$

3. Simplify the following ratios involving fractions and mixed numerals.

a. $\frac{1}{3} : \frac{1}{2}$

b. $\frac{2}{5} : \frac{3}{4}$

c. $\frac{3}{8} : \frac{1}{4}$

d. $\frac{11}{10} : \frac{2}{15}$

e. $1\frac{1}{2} : \frac{3}{4}$

f. $3\frac{1}{3} : 1\frac{2}{5}$

4. First change the quantities to the same unit, and then express each pair of quantities as a ratio in simplest form.

a. 120 m to 1 km

b. 200 g to 2.5 kg

c. 400 mL to 1 L

d. 3 days to 8 hours

e. 4 days to 4 weeks

f. $7.50 to 25 cents

5. In a child-care centre the adult-to-child ratio is 1:4.

   a. Find the number of children that can be cared for by 2 adults.

   b. Find the number of children that can be cared for by 10 adults.

   c. Find the number of adults that are required to care for 12 children.

   d. Find the number of adults that are required to care for 100 children.

6. In a nut mixture the ratio of peanuts to walnuts to cashews is 2:3:1.

   a. If the number of peanuts is 20, find the number of walnuts and cashews.

   b. If the number of walnuts is 6, find the number of peanuts and cashews.

   c. If the number of cashews is 8, find the number of peanuts and cashews.

7. Divide:

   a. 40 m in the ratio of 2:3

   b. 14 kg in the ratio of 4:3

   c. $60 in the ratio of 2:3

d. 48 kg in the ratio of 1:5

e. 72 m in the ratio of 1:2

f. $110 in the ratio of 7:4

8. Divide:

   a. $200 in the ratio of 1:2:2

   b. $400 in the ratio of 1:3:4

   c. 12 kg in the ratio of 1:2:3

d. 88 kg in the ratio of 2:1:5

e. 329 kg in the ratio of 12:13:15

f. $50 000 in the ratio of 1:2:3:4

Word Problems

1. A company distributes a $2,400 bonus among three employees in the ratio 3:5:7. How much does each employee receive?

2. A recipe for fruit juice uses orange, pineapple, and mango in the ratio 4:3:2. If 1.8 liters of juice is made, how much of each ingredient is used?

3. Two business partners split their profits in the ratio 7:9. If one partner receives $8,400, how much does the other partner get, and what was the total profit?

4. A school has students in three houses: Red, Blue, and Green, in the ratio 5:6:9. If there are 300 students in Green House, how many students are in the school?

5. A metal alloy is made by mixing copper, zinc, and tin in the ratio 5:3:2. If 50 kg of zinc is used, how much copper and tin are needed, and what is the total weight of the alloy?

Chapter 6.2 Scale Drawings

Scale drawings show the same shape as the real object but at a different size.

Scale ratio is written as:

- Drawing length : Actual length

Examples of scale ratios:

- **1:100** → The real object is **100 times bigger** than the drawing.

  Example*:* A 2 cm drawing represents a 200 cm (2 m) object.

- **20:1** → The drawing is **20 times bigger** than the real object.

  Example*:* A 40 cm model represents a 2 cm object.

Conversions:

- **To find the real size:** Multiply the drawing length by the scale factor.
- **To find the drawing size:** Divide the real length by the scale factor.

Unit conversions to remember:

- **1 km = 1000 m**
- **1 m = 100 cm**
- **1 cm = 10 mm**

Practice Questions

1. Find the actual distance for each of the following scaled distances. Give your answer in an appropriate unit.

   a. Scale 1:10 000

      i. 2 cm

      ii. 4 mm

      iii. 7.3 cm

   b. Scale 1:4000

      i. 16 mm

      ii. 72 cm

      iii. 0.03 m

c. Scale 1:2

i. 44 m

ii. 310 cm

iii. 2.5 mm

d. Scale 1:0.5

i. 12 cm

ii. 3.2 mm

iii. 400 m

2. Find the scaled distance for each of these actual distances.

a. Scale 1:200

i. 200 m

ii. 4 km

iii. 60 cm

b. Scale 1:10 000

i. 1350 m

ii. 45 km

iii. 736.5 m

e. Scale 1:250 000

i. 5000 km

ii. 750 000 m

iii. 1250 m

f. Scale 1:0.1

iv. 30 cm

v. 5 mm

vi. 0.2 mm

Word Problems

1. A model airplane is built to a scale of 1:50. If the wingspan of the actual airplane is 30 meters, what is the wingspan of the model?

2. A map uses a scale of 1:25,000. If two towns are 6 cm apart on the map, what is the actual distance between them in kilometers?

3. A blueprint of a building is drawn with a scale of 1:200. If a wall on the blueprint is 12 cm long, how long is the actual wall in meters?

4. A scale drawing of a park has a scale factor of 1:500. If the width of a pond is 8 meters in real life, how wide is it on the drawing in centimeters?

5. A toy car is designed at a scale of 5:1, meaning the model is five times larger than the real car. If the model car is 45 cm long, what is the actual length of the real car?

6. A city planner creates a map with a scale of 1:10,000. A road on the map measures 15 cm in length. What is the actual length of the road in kilometers?

7. An architect designs a model of a skyscraper using a scale of 1:300. If the real skyscraper is 210 meters tall, how tall is the model in centimeters?

8. A photo of a beetle is enlarged using a scale of 20:1. If the beetle in the photo appears 30 cm long, what is the actual length of the beetle in millimeters?

9. A model ship is built at a scale of 1:75. If the length of the model is 1.2 meters, what is the actual length of the real ship in meters?

10. A theme park map is drawn to a scale of 1:5,000. If the distance between two attractions is 7.5 cm on the map, how far apart are they in real life in meters?

Chapter 6.3 Rates and Solving Rate Problems

Rates compare quantities in different units.

- Example: 20 km / h (20 kilometers per hour)

Rates must include units for each quantity.

- Example: 50 miles / hour

The slash (/) means 'per'

- Example: 60 km / hour = 60 kilometers per hour

Write rates in their simplest form (only one unit for the second quantity).

- Example: 80 km / 2 hours = 40 km per hour (simplified)

Average rate = total change in one quantity ÷ total change in the second quantity.

- Example: If you travel 100 km in 2 hours, the average rate = 100 km ÷ 2 hours = 50 km per hour.

A rate shows a change in one quantity.

- Example: 10 km / hour means for each hour, you travel 10 kilometers.

An equivalent change must happen in the other quantity.

- Example: If you travel at 10 km/h for 2 hours, you cover 20 km (10 km * 2 hours).

Always consider the units carefully when answering.

- Example: 50 miles / hour means 50 miles for each hour, not per minute or day.

Practice Questions

1. Express each of the following as a simplified rate.

   a. 12 km in 4 years

   b. $180 in 6 hours

   c. $126 000 to purchase 9 acres

   d. 12 000 revolutions in 10 minutes

   e. 60 minutes to run 15 kilometers

2. Find the average rate of change for each situation.

a. Relma drove 6000 kilometers in 20 days.

b. Holly saved $420 over three years.

c. A cricket team scored 78 runs in 12 overs.

d. Saskia grew 120 centimeters in 16 years.

e. Russell gained 6 kilograms in 4 years.

f. The temperature dropped 5℃ in 2 hours.

3. If 30 salad rolls were bought to feed 20 people at a picnic, and the total cost was $120, find the following rates.

    a. salad rolls / person

    b. cost / person

    c. cost / roll

4. The number of hours of sunshine was recorded each day for one week in April. The results are: Monday 6 hours, Tuesday 8 hours, Wednesday 3 hours, Thursday 5 hours. Friday 7 hours, Saturday 6 hours, Sunday 7 hours. Find the average number of hours of sunshine:

    a. per weekday

    b. per weekend day

    c. per week

    d. per day

5. A factory produces 40 plastic bottles per minute.

    a. How many bottles can the factory produce in 60 minutes?

    b. How many bottles can the factory produce in an 8 hour day of operation?

6. Mario is a professional home painter. When painting a new home he uses an average of 2 litres of paint per hour. How many litres of paint would Mario use in a week if he paints for 40 hours?

7. A flywheel rotates at a rate of 1500 revolutions per minute.

    a. How many revolutions does the flywheel make in 15 minutes?

    b. How many revolutions does the flywheel make in 15 seconds?

    c. How long deos it take for the flywheel to complete 15 000 revolutions?

    d. How long does it take for the flywheel to complete 150 revolutions?

8. Putra is an elite rower. When training, he has a steady working heart rate of 125 beats per minute (bpm). Putra's resting heart rate is 46 bpm.

    a. How many times does Putra's heart beat during a 30 minute workout?

    b. How many times does Putra's heart beat during 30 minutes of 'rest'?

    c. If his coach says that he can stop his workout once his heart has beaten 10 000 times, for how long would Putra need to train?

Word Problems

1. A car travels 120 kilometers in 2 hours. How far will it travel in 5 hours at the same rate?

2. A swimmer swims 400 meters in 8 minutes. How long will it take them to swim 1.2 kilometers at the same rate?

3. A printer can print 150 pages in 30 minutes. How many pages can it print in 1 hour?

4. A truck travels 240 miles in 6 hours. What is the truck's average speed in miles per hour?

5. A bakery bakes 48 loaves of bread in 4 hours. How many loaves will the bakery bake in 10 hours at the same rate?

6. A runner completes 3 laps around a track in 9 minutes. How many laps will the runner complete in 30 minutes at the same rate?

7. A machine produces 500 widgets in 5 hours. How many widgets does the machine produce in 8 hours?

8. A cyclist travels 90 kilometers in 3 hours. How long will it take the cyclist to travel 150 kilometers at the same speed?

9. A factory uses 120 liters of paint to cover 500 square meters. How many liters of paint are needed to cover 1,000 square meters?

10. A bus travels 150 kilometers in 2.5 hours. How far will the bus travel in 7 hours at the same rate?

Chapter 6.4 Speed

Speed formula as a fraction:

- Average speed = Distance travelled ÷ Time taken becomes:

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Rearranged formulas as fractions:

- Distance = Average speed × Time remains the same:

$$\text{Distance} = \text{Average speed} \times \text{Time}$$

- Time = Distance ÷ Average speed becomes:

$$\text{Time} = \frac{\text{Distance}}{\text{Average speed}}$$

Practice Questions

1. Calculate the average speed of:

    a. A sprinter running 200 m in 20 seconds.

    b. A skateboarder travelling 840 m in 120 seconds.

c. A truck travelling 400 km in 8 hours.

d. A tram travelling 15 km in 20 minutes.

2. Calculate the distance travelled by:

   a. A cyclist travelling at 12 m/s for 90 seconds.

   b. A jogger running at 2.5 m/s for 3 minutes.

c. A bushwalker who has walked for 8 hours at an average speed of 4.5 km/h.

d. A tractor ploughing fields for 2.5 hours at an average speed of 20 km/h.

3. Calculate the time taken by:

   a. A sports car to travel 1200 km at an average speed of 150 km/h.

   b. A bus to travel 14 km at an average speed of 28 km/h.

c. A plane to fly 6900 km at a constant speed of 600 km/h.

d. A ball moving through the air at a speed of 12 m/s to travel 84 m.

Word Problems

1. A plane travels 600 kilometers in 1.5 hours. What is the average speed of the plane in kilometers per hour?

2. A cyclist travels at an average speed of 25 km/h. How long will it take the cyclist to cover a distance of 150 kilometers?

3. A car travels 240 kilometers in 4 hours. How long will it take the car to travel 600 kilometers at the same speed?

4. A runner completes a marathon (42.195 kilometers) in 3 hours. What is the average speed of the runner in kilometers per hour?

5. A boat travels at a speed of 18 km/h. How far will the boat travel in 5 hours?

6. A car drives at a speed of 80 km/h. How long will it take the car to travel 320 kilometers?

7. A train travels 500 kilometers in 10 hours. What is the speed of the train in kilometers per hour?

8. A motorbike travels 75 kilometers in 1.5 hours. How fast is the motorbike traveling?

9. A truck travels 120 kilometers in 3 hours. How long will it take to travel 400 kilometers at the same speed?

10. A swimmer swims 800 meters in 16 minutes. What is the swimmer's speed in meters per minute?

Chapter 6.5 Unitary Method: Rates and Ratio

Unitary method: Find the value of one unit, then use it to solve the problem.

- Example: If 5 phones cost $25, find the cost of 1 phone.

  Cost of 1 phone = $25 ÷ 5 = $5.

With ratios, find the value of 1 part.

- Example: Ratio of phones to televisions is 5:2.

  For 15 phones, find the number of televisions:

  1 phone = 2 ÷ 5 = 0.4 televisions.

  For 15 phones: 0.4 × 15 = 6 televisions.

With rates, find the rate per 1 unit.

- Example: Pedro earns $64 for 4 hours of work.

  Wage rate = $64 ÷ 4 hours = $16 per hour ($16/h).

Practice Questions

1. Solve the following problems.

    a. If 8 kg of chicken fillets cost $72, how much would 3 kg of chicken fillets cost?

    b. If three pairs of socks cost $12.99, how much would 10 pairs of socks cost?

2. Solve the following ratio problems.

   a. The required staff to student ratio for an excursion is 2:15. If 10 teachers attend the excursion, what is the maximum number of students who can attend?

   b. A rectangle has length and width dimensions in a ratio of 3:1. If a particular rectangle has a length of 21 m, what is its width?

3. Solve the following rate problems.

   a. A tap is dripping at a rate of 200 mL every 5 minutes. How much water drips in 13 minutes?

   b. A snail travelling at a constant speed travels 400 mm in 8 minutes. How far does it travel in 7 minutes?

4. Convert the following rates into the units given in the brackets.

a. $15/h (c/min)

b. 3.5 L/min (L/h)

c. 0.5 kg/month (kg/year)

d. 60 g/c (kg/$)

e. 108 km/h (m/s)

5. Convert the following speeds to m/s.

   a. 36 km/h

   b. 180 km/h

   c. 660 m/min

   d. 4 km/s

6. Convert the following speeds to km/h.

   a. 15 m/s

   b. 2 m/s

   c. 12 m/min

   d. 1 km/s

Word Problems

1. The ratio of boys to girls in a school is 3:5. If there are 240 boys in the school, how many girls are there?

2. A worker earns $120 for 8 hours of work. How much will they earn for 15 hours at the same rate?

3. A recipe uses 3 cups of flour to make 12 cookies. How many cups of flour are needed to make 48 cookies?

4. The ratio of apples to oranges in a basket is 7:3. If there are 84 apples, how many oranges are there?

5. A car travels 180 kilometers in 3 hours. How far will the car travel in 5 hours at the same speed?

6. The ratio of the length to the width of a rectangle is 4:3. If the length is 12 meters, what is the width of the rectangle?

7. A factory produces 200 toys in 5 hours. How many toys will it produce in 9 hours?

8. A cyclist rides 48 kilometers in 2 hours. How long will it take the cyclist to ride 72 kilometers at the same rate?

9. The ratio of the number of red marbles to blue marbles in a bag is 5:7. If there are 35 red marbles, how many blue marbles are there?

10. A painter can paint 45 square meters of wall in 3 hours. How many square meters can the painter paint in 7 hours at the same rate?

CHAPTER 7 EQUATIONS AND INEQUALITIES

Table of Contents

Chapter 7.1 Equations and Equivalent Equations

Two equations are equivalent if substituting values for the pronumerals makes both true or both false

Example:

$3x + 5 = 17$ and $3x = 12$ are equivalent if $x = 4$ (both true) or false if $x \neq 4$.

To check a solution, substitute the value of the unknown into both sides and see if they are equal.

For $5(6) + 2 = 32$, LHS $= 32$, RHS $= 32$ (both sides are equal).

Practice Questions

1. Someone has attempted to solve the following equations. State whether the solution is correct (C) or incorrect (I).

   a. $5 + 2x = 4x - 1$,
   proposed solution: $x = 3$

   b. $4 + q = 3 + 2q$,
   proposed solution: $q = 10$

   c. $13 - 2a = a + 1$,
   proposed solution: $a = 4$

   d. $b \times (b + 3) = 4$,
   proposed solution: $b = -4$

2. State a solution to each of the following equations.

a. $5 + x = 12$

b. $4v + 2 = 14$

c. $10x = 20$

d. $4u + 1 = 29$

e. $3 + a = 2a$

3. Write equations for each of the following problems. You do not need to solve the equations.

   a. A number x is doubled and then 7 is added. The result is 10.

   b. Aston's age is a. His father, who is 25 years older, is twice as old as Aston.

   c. Coffee costs $\$c$ per cup and tea costs $\$t$. Four cups of coffee and three cups of tea cost a total of 21.

4. Copy and complete the following to solve the given equation algebraically.

a.

$$10x = 30$$

$\div 10$ $\qquad$ $\div 10$

$$x = \_\_$$

b.

$$q + 5 = 2$$

-5 $\qquad$ -5

$$\_\_ = \_\_$$

c.

$$4x + 2 = 22$$

-2 $\qquad$ -2

$$4x = \_\_$$

$\div 4$ $\qquad$ $\div 4$

$$\_\_ = \_\_$$

d.

$30 = 4p + 2$

-2 -2

$\_\_ = \_\_$

☐ ☐

$\_\_ = \_\_$

e.

$20 - 4x = 8$

-20 -20

$-4x = -12$

$\div(-4)$ $\div(-4)$

$x = \_\_$

f.

$p \div 3 + 6 = 8$

-6 -6

$p \div 3 = 2$

☐ ☐

$\_\_ = \_\_$

5. Solve the following equations algebraically and check your solutions.

a. $20 - 4d = 8$

b. $21 - 7a = 7$

c. $13 - 8k = 45$

d. $13 = -3b + 4$

e. $6a - 4 = -16$

6. The following equations do not all have whole number solutions. Solve the following equations algebraically, giving each solution as a fraction.

a. $2x + 3 = 10$

b. $12 = 10b + 7$

c. $15 = 10 - 2x$

d. $22 = 9 + 5y$

e. $1 - 3y = -1$

Chapter 7.2 Solving Equations with Fractions

To solve equations with fractions:

1. **Eliminate fractions**: Multiply both sides of the equation by the denominator to get rid of the fraction.
   - Example: $\frac{x}{3} = 5$ becomes $x = 5 \times 3 \rightarrow x = 15.$
2. **Simplify**: After multiplying, simplify the equation to solve for the unknown.
   - Example: $\frac{3x}{4} = 12 \rightarrow$ Multiply both sides by $4 \rightarrow 3x = 48 \rightarrow x = 16.$
3. **If there are multiple fractions**: Find the least common denominator (LCD) and multiply the entire equation by it.
   - Example: $\frac{x}{2} + \frac{x}{3} = 5 \rightarrow \text{LCD} = 6$, multiply through by $6 \rightarrow 3x + 2x = 30 \rightarrow 5x = 30 \rightarrow x = 6.$

Practice Questions

1. Solve the following equations algebraically.

   a. $\frac{7w}{10} = -7$

   b. $\frac{3s}{2} = -9$

   c. $\frac{-7j}{5} = 7$

   d. $\frac{-6f}{5} = -24$

2. Solve the following equations algebraically.

a. $\frac{a+2}{5} = 2$

b. $\frac{b}{5} + 2 = 6$

c. $\frac{c}{4} - 2 = 1$

d. $\frac{d-4}{3} = 6$

3. Solve the following equations algebraically.

a. $\frac{4u+1}{7} = 3$

b. $\frac{3k}{2} + 1 = 7$

c. $8 + \frac{2x}{3} = 14$

d. $4 + \frac{c}{2} = 10$

4. Solve the following equations algebraically. Check your solutions by substituting.

a. $\frac{t-8}{2} = -10$

b. $\frac{a+12}{5} = 2$

c. $1 = \frac{2-s}{8}$

d. $3 = \frac{7v}{12} + 10$

e. $\frac{7q+12}{5} = -6$

f. $15 = \frac{3-12l}{5}$

g. $-6 = \frac{8-5x}{7}$

h. $\frac{5k+4}{-8} = -3$

Word Problems

1. A factory produces $\frac{2x}{3}$ items per hour. After 5 hours, the total production is 100 items. How many items are produced per hour?

2. A car's fuel consumption is $\frac{3x}{4}$ liters per 100 kilometers. If the car uses 30 liters of fuel over a 400-kilometer trip, how much fuel does it consume per 100 kilometers?

3. A recipe calls for $\frac{5x}{7}$ teaspoons of salt for 2 servings. If 10 servings are needed, how many teaspoons of salt are required?

4. A tank can fill at a rate of $\frac{x}{12}$ liters per minute. After 36 minutes, the tank is filled with 72 liters of water. How many liters are added per minute?

5. A runner completes $\frac{4x}{5}$ kilometers per hour. After running for 3 hours, they cover 36 kilometers. What is their running speed in kilometers per hour?

6. A school bus travels $\frac{7x}{9}$ kilometers per hour. If the bus travels for 4.5 hours and covers a distance of 63 kilometers, what is the speed of the bus in kilometers per hour?

7. The total cost for producing $\frac{x}{6}$ units of a product is $\$24$. If 72 units are produced, how much does it cost to produce 1 unit?

8. A book contains $\frac{5x}{8}$ pages per chapter. If the book has 12 chapters and the total number of pages in the book is 360, how many pages are in each chapter?

9. A factory produces $\frac{3x}{4}$ kilograms of steel per day. After 20 days, it produces a total of 180 kilograms. What is the daily production rate of the factory?

10. A rectangular garden has a length of $\frac{7x}{10}$ meters and a width of $\frac{3x}{5}$ meters. If the area of the garden is 42 square meters, what is the value of x?

Chapter 7.3 Equations with Pronumerals on Both Sides

Adding or subtracting the same term on both sides of an equation keeps the equation equivalent.

- Example: $10 + 5a = 13 + 2a$ → subtract $2a$ from both sides: $10 + 3a = 13$.

If pronumerals are on both sides of an equation, move them to one side by adding or subtracting terms.

- Example: $4b + 12 = 89 - 3b$ → add $3b$ to both sides: $7b + 12 = 89$.

Practice Questions

1. Solve the following equations.

a. $12 - 8n = 8 - 10n$

b. $21 - 3h = 6 - 6h$

c. $13 - 7c = 8c - 2$

d. $10a + 32 = 2a$

e. $18 + 8c = 2c$

f. $6n - 47 = 9 - 8n$

2. Solve the following equation, giving your solutions as improper fractions where necessary.

a. $3x + 5 = x + 6$

b. $5k - 2 = 2k$

c. $3 + m = 6 + 3m$

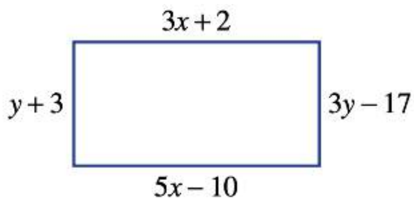
d. $9j + 4 = 5j + 14$

e. $3 - j = 4 + j$

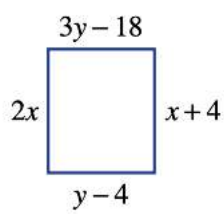
f. $2z + 3 = 4z - 8$

3. Find the value of x and y in the following rectangles.

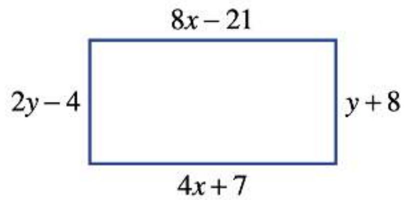
a.



b.



4. Find the area and the perimeter of this rectangle.



Chapter 7.4 Equations with Brackets

Distributive Law for expanding brackets:

- $a(b + c) = ab + ac$
  Example: $3(x + 4) = 3x + 12$
- $a(b - c) = ab - ac$
  Example: $4(b - 2) = 4b - 8$

Like terms are terms that contain the same pronumerals and can be combined to simplify expressions.

- Example: $5x + 10 + 7x$ simplifies to $12x + 10$.

Practice Questions

1. Solve the following equations by expanding and combining like terms.

a. $4(3y + 2) + 2y = 50$

b. $4(5 + 3w) + 5 = 49$

c. $28 = 4(3d + 3) - 4d$

d. $23 = 4(2p - 3) + 1$

e. $49 = 3(2c - 5) + 4$

2. Solve the following equations by expanding brackets on both sides.

a. $5(4x - 4) = 5(3x + 3)$

b. $5(5f - 2) = 5(3f + 4)$

c. $2(5h + 4) = 3(4 + 3h)$

d. $4(3r - 2) = 4(2r + 3)$

e. $3(2a + 1) = 11(a - 2)$

3. Solve the following equations algebraically.

   a. $2(3 + 5r) + 6 = 4(2r + 5) + 6$

   b. $2(3x - 5) + 16 = 3 + 5(2x - 5)$

   c. $4(4y + 5) - 4 = 6(3y - 3) + 20$

Word Problems

1. A carpenter is building a bookshelf. The total length of wood used is 3 times the length of a shelf plus 4 meters. If the length of wood used to build the bookshelf is equal to twice the length of another shelf minus 5 meters, plus 8 meters, what is the length of one shelf?

2. A farmer is planting rows of vegetables in two fields. The total number of plants in one field is five times the number of plants per row minus 3, plus 7. The total number of plants in the second field is equal to three times the number of plants per row, plus four, minus twice the number of plants per row. How many plants are in each row?

3. A painter is painting two walls. The total area painted on the first wall is 4 times the area of the second wall, plus 2 square meters. The total area painted on the second wall is equal to twice the area of the third wall, minus 7 square meters, plus 10 square meters. What is the area of the second wall?

4. A student is organizing her notes. She has 2 times the number of notes she had before, plus 6, but has also lost 3 notes. She now has 5 times the number of notes she lost, plus 7, minus twice the number of notes she found. How many notes does she have?

5. A company is making packages of cookies. The number of cookies in one package is 3 times the number of cookies in another, plus 2. If the number of cookies in the second package is equal to 4 times the number of cookies in the first, minus 1, plus 2, how many cookies are in each package?

Chapter 7.5 Solving Simple Quadratic Equations

If $\boldsymbol{x}^2$ = a positive number, there are **two answers**:

- Example: $\boldsymbol{x}^2 = \mathbf{9} \rightarrow \boldsymbol{x} = \pm\mathbf{3}$ (because $3^2 = 9$ and $(-3)^2 = 9$).
- **Shortcut:** $x = \pm 9$.

If $\boldsymbol{x}^2 = \mathbf{0}$, there is **only one answer**:

- Example: $\boldsymbol{x}^2 = \mathbf{0} \rightarrow \boldsymbol{x} = \mathbf{0}$.

If $\boldsymbol{x}^2$ = a negative number, there is **no solution** (because squaring any real number always gives a positive result or 0).

Steps to solve $\boldsymbol{ax}^2 = \boldsymbol{c}$:

1. Divide by "a" to make x^2 alone.
2. Take the square root of both sides and **write $\pm$ for two answers.**

Practice Questions

1. Solve the following equations.

   a. $2x^2 = 8$

   b. $5x^2 = 45$

   c. $2x^2 = 288$

   d. $3x^2 = 363$

2. State the number of solutions for these equations.

a. $x^2 = 10$

b. $x^2 = 3917$

c. $x^2 = -94$

d. $a^2 = 0$

3. By first dividing both sides by the coefficient of x^2, solve these simple quadratic equations.

a. $-2x^2 = -8$

b. $-3x^2 = -3$

c. $-5x^2 = -45$

d. $-3x^2 = -12$

e. $-2x^2 = -50$

f. $-7x^2 = 0$

g. $-6x^2 = -216$

h. $-10x^2 = -1000$

Chapter 7.6 Formulas and Relationships

The subject of an equation is the variable **by itself** on the **left side**.

- Example: In $\boldsymbol{V = 3x + 2y}$, $\boldsymbol{V}$ is the subject.

A formula is an equation with **two or more variables**, where one is the subject.

- Example: $\boldsymbol{A = l \times w}$ (Area formula, A is the subject).

Using a formula:

1. **Substitute** the known values.
2. **Solve** for the unknown variable.

Practice Questions

1. Consider the relationship $y = 2x + 4$.

   a. Find y if $x = 3$.

   b. By solving an appropriate equation, find the value of x that makes $y = 16$.

   c. Find the value of x if $y = 0$.

2. Use the formula $P = mv$ to find the value of m when $P = 22$ and $v = 4$.

3. Assume that x and y are related by the equation $4x + 3y = 24$.

   a. If $x = 3$, find y by solving an equation.

b. If $x = 0$, find the value of y.

c. If $y = 2$, find x by solving an equation.

d. If $y = 0$, find the value of x.

4. Consider the formula $G = k(2a + p) + a$.

   a. If $k = 3, a = 7$ and $p = -2$, find the value of G.

   b. If $G = 78$, $k = 3$ and $p = 5$, find the value of a.

Word Problems

1. The area A of a triangle is given by the formula $A = \frac{1}{2}bh$, where b is the base and h is the height. If a triangle has an area of $72\ \mathrm{cm}^2$ and a base of $12\ \mathrm{cm}$, find the height.

2. The formula for converting temperature from Fahrenheit F to Celsius C is $C = \frac{5}{9}(F - 32)$. If the temperature is $86°F$, what is the temperature in Celsius?

3. The perimeter P of a rectangle is given by $P = 2l + 2w$, where l is the length and w is the width. If the perimeter is $50\ \mathrm{cm}$ and the width is $8\ \mathrm{cm}$, find the length.

4. The volume V of a cylinder is given by $V = \pi r^2 h$. If a cylinder has a volume of 314 cm^3 and a radius of 5 cm, find its height (use $\pi \approx 3.14$).

5. The total cost C of renting a car is given by $C = 50 + 0.25d$, where d is the number of kilometers driven. If the total cost is $\$90$, how many kilometers were driven?

6. The speed formula is $s = \frac{d}{t}$, where s is speed, d is distance, and t is time. If a car travels 240 km in 3 hours, what is its speed?

7. The force F acting on an object is given by $F = ma$, where m is mass and a is acceleration. If a force of 50 N is applied to an object with an acceleration of 5 m/s^2, find the mass.

8. The pressure P in a liquid is given by $P = \rho gh$, where ρ is density, g is gravity, and h is height. If $\rho = 1000 \text{ kg/m}^3$, $g = 9.8 \text{ m/s}^2$, and $P = 49000\text{Pa}$, find h.

9. The surface area S of a sphere is given by $S = 4\pi r^2$. If the surface area is 452.16 cm^2, find the radius (use $\pi \approx 3.14$).

10. The energy stored in a stretched elastic band is given $by\ E = \frac{1}{2}kx^2$, where k is the stiffness of the band and x is the stretch. If $k = 200 \text{ N/m}$ and the energy stored is 50 J, find x.

Chapter 7.7 Inequalities and Solving Inequalities

An inequality compares two values using symbols:

- **"Greater than" ($>$)** → Example: $\mathbf{5 > 2}$
- **"Greater than or equal to" ($\geq$)** → Example: $\mathbf{7 \geq 7}$
- **"Less than" ($<$)** → Example: $\mathbf{2 < 10}$
- **"Less than or equal to" ($\leq$)** → Example: $\mathbf{5 \leq 5}$

Inequalities can be written in different ways but mean the same thing:

- Example: $\mathbf{3 < x}$ is the same as $\mathbf{x > 3}$

Number line representation:

- **Use an open circle (○) if the value is NOT included** ($>$ or $<$).
- **Use a closed circle (●) if the value is included** ($\geq$ or $\leq$).

Solving inequalities (like equations, but with extra care):

- **Add or subtract** on both sides.
- **Multiply or divide** by a **positive number** (symbol stays the same).
- **Multiply or divide** by a **negative number** (symbol **flips**!).

Practice Questions

1. Represent the following inequalities on separate number lines.

a. $x > 3$

b. $x < 10$

c. $x < -5$

d. $x < -9$

e. $x \geq -3$

f. $x \leq 5$

g. $2 < x$

h. $5 \geq x$

2. Represent the following inequalities on separate number lines.

a. $1 \leq x \leq 6$

b. $4 \leq x \leq 11$

c. $-2 < x \leq 6$

d. $-8 \leq x \leq 3$

e. $2 < x \leq 5$

f. $-8 < x < -1$

g. $7 < x \leq 8$

h. $0 < x < 1$

3. Solve the following inequalities.

a. $x + 9 > 12$

b. $8g - 3 > 37$

c. $9k + 3 > 21$

d. $8a - 9 > 23$

e. $9 + 2d \geq 23$

f. $10 + 7r \leq 24$

4. Solve the following inequalities involving fractions.

a. $\frac{d-9}{2} > 10$

b. $\frac{x-3}{4} > 2$

c. $\frac{q+4}{2} \leq 11$

d. $\frac{2x+4}{3} > 6$

e. $\frac{4+6p}{4} \geq 4$

5. Solve the following inequalities involving negative numbers. Remember to reverse the inequality when multiplying or dividing by a negative number.

a. $6 - 2x < 4$

b. $43 - 4n > 23$

c. $2 - 9v \leq 20$

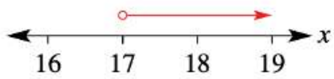
d. $48 - 8c \geq 32$

e. $7 - 8s > 31$

f. $10 - 4v \geq 18$

6. Match the following inequalities with their solutions depicted on a number line.

i. $5x + 2 \geq 17$ ii. $\frac{x+1}{6} > 3$ iii. $9(x+4) < 45$ iv. $5 - 2x < 3$

a.


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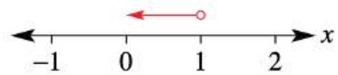

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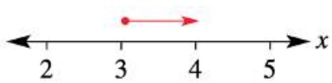
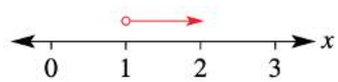


d.


7. In AFL football the score is given by $6g + b$ where g is the number of goals and b is the number of behinds. A team scored 4 behinds and their score were less than or equal to 36.

   a. Write an inequality to describe this situation.

   b. Solve the inequality.

   c. Given that the number of goals must be a whole number, what is the maximum number of goals that they could have scored?

Chapter 7.8 Application

Practice Questions

1. A combination of 6 chairs and a table costs \$3000. The table alone costs \$1740.

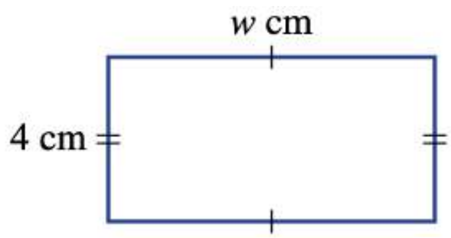
   a. Define a pronumeral for the cost of one chair.

   b. Write an equation to describe the problem.

c. Solve the equation algebraically.

d. Hence state the cost of one chair.

2. The perimeter of this rectangle is 72 cm.



a. Write an equation to describe the problem, using w for the width.

b. Solve the equation algebraically.

c. Hence state the width of the rectangle.

3. The plumber charges a $70 call-out fee and $52 per hour. The total cost of a particular visit was $252.

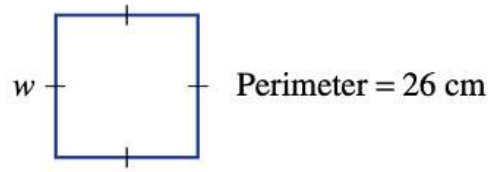
   a. Define a variable to stand for the length of the visit in hours.

   b. Write an equation to describe the problem.

   c. Solve the equation algebraically.

   d. State the length of the plumber’s visit, giving your answer in minutes.

4. A square has a perimeter of 26 cm.

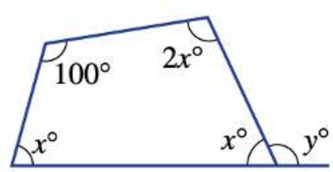


a. Solve an equation to find its width.

b. State the area of the square.

5. Alison and Flynn's combined age is 40. Given that Flynn is 4 years older than Alison, write an equation and use it to find Alison's age.

6. Recall that in a quadrilateral the sum of all angles is 360°. Find the values of x and y in the diagram shown.



7. The average of two numbers can be found by adding them and dividing by 2.

   a. If the average of x and 10 is 30, what is the value of x?

   b. If the average of x and 10 is 2, what is the value of x?

   c. If the average of x and 10 is some number R, create a formula for the value of x.

CHAPTER 8 PROBABILITIES AND STATISTICS

Table of Contents

Chapter 8.1 Equations and Equivalent Equations

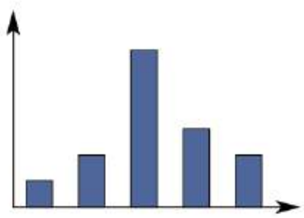
Numerical data → Numbers

- **Discrete** → Specific, countable values
- **Continuous** → Any value in a range

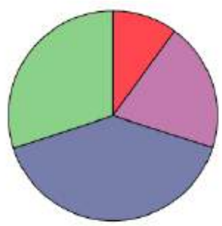
Categorical data → Groups or labels

Data can be shown in tables or graphs

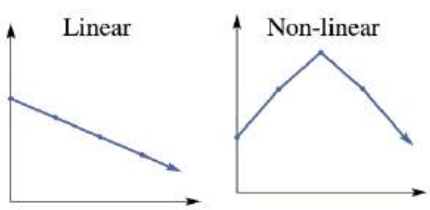
Most common types of graphs:



Column graphs



Pie charts (also called sector graphs)

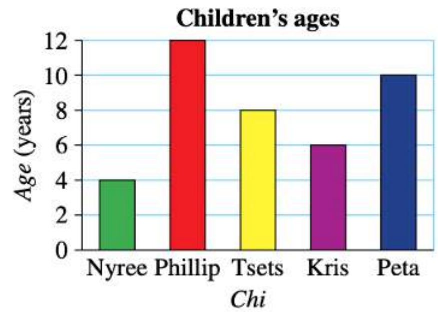


Line graphs

Divided bar graphs

Practice Questions

1. The column graph shows the age of five children.



   a. How old is Peta?

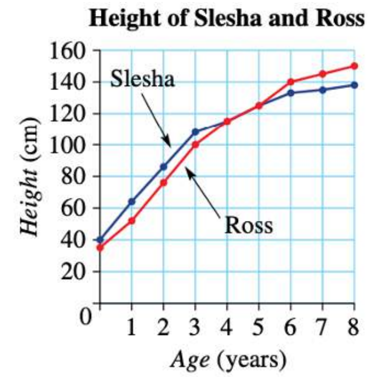
   b. How old is Kris?

   c. Who is the oldest of the five children?

   d. Who is the youngest of the five children?

   e. What is the difference in age between Tsets and Nyree?

2. The line graph shows the height of Slesha and her twin brother Ross from the time they were born.



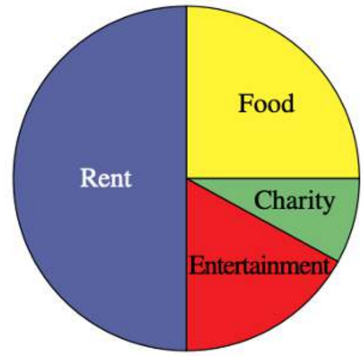
a. Which of the children was taller on their first birthday?

b. Which of the children was taller on their eighth birthday?

c. How old were the children when they were the same height?

d. Would you describe the general shape of the graphs as linear (straight line) or non-linear?

3. This pie chart shows one person's spending in a month.



   a. What is the largest expense in that month?

   b. What is the smallest expense in that month?

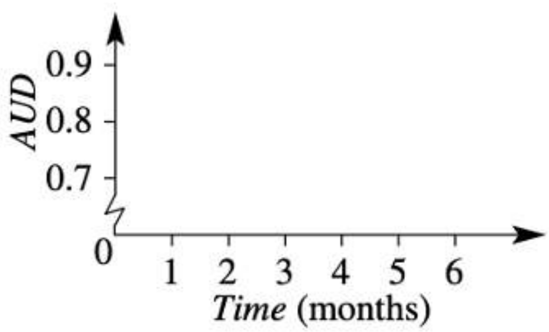
   c. What percentage of the month's spending was on rent?

   d. If the person spent a total of $600 on food in the month, what was their total spending?

4. This table shows the value of the Australian dollar against the US dollar over a period of 6 months.

Time	0	1	2	3	4	5	6
AUD	0.90	0.87	0.85	0.81	0.78	0.76	0.72

a. Plot a graph of the value of the Australian dollar against the US dollar over the 6-month period. Use these axes to help you get started.



b. Would you describe the general shape of the graph as linear (straight line) or non-linear?

c. By how much did the Australian dollar decrease in:

i. The first 3 months?

ii. The second month?

d. Assuming this trend continued, what would the Australian dollar be worth after:

i. 7 months?

ii. 9 months?

Word Problems

1. A school surveyed 120 students about their favorite type of music: Pop, Rock, Jazz, or Classical. The results need to be displayed in a table and a bar graph. How would you organize and represent this data?

2. A weather station recorded the daily temperatures for a week. The temperatures were 24°C, 27°C, 22°C, 30°C, 28°C, 26°C, and 25°C. Create a table to display this data and choose an appropriate graph to represent it.

3. A shop tracks its weekly sales of three different products over a month. The number of items sold per week is recorded. How can this data be organized in a table and what type of graph would best represent the trends over time?

4. A teacher collects data on how long (in minutes) students take to complete a maths test. The results range from 35 to 65 minutes. How can this data be grouped into a frequency table, and what type of graph would be best for displaying the distribution?

5. A survey asks 50 people about their mode of transport to school/work (bus, car, train, bicycle, or walking). How can this categorical data be presented in a clear table and graph?

6. A factory records monthly production of bicycles over a year. The production numbers vary each month. What type of graph would best show the changes over time, and how would you set up the table?

7. A football team records the number of goals scored per game over 15 matches. The values range from 0 to 5 goals per game. How would you organize the data in a table, and what type of graph would best represent the performance of the team?

8. A class records the number of hours spent studying per week by each student. The values vary from 2 to 10 hours. How can this data be grouped, and what type of graph would best show the distribution?

9. A hospital monitors the number of patients visiting the emergency room each day for a month. How would you organize this data in a table, and what graph would best display trends in daily visits?

10. A bakery tracks sales of different flavors of cakes over a week. Some flavors are more popular than others. How can this data be displayed in a table and what type of graph would best compare the popularity of each flavor?

Chapter 8.2 Frequency Tables and Tallies

Tally Marks

- Used for counting as data is gathered.
- Every 5th mark is crossed (e.g., **4 = ||||**, **7 = |||| ||**).

Frequency Tables

- Show how often a value appears.
- Can list **individual values** or **intervals** (e.g., **0–9, 10–19, 20–29**).

Frequency Graphs

- **Y-axis** represents frequency.
- **Bars may have spaces** if the first bar doesn't start at zero.

Dot Plots

- Used for small sets of discrete data.

Outliers

- A value that is far from the rest of the data.

Practice Questions

1. Braxton surveys a group of people to find out how much time they spend watching television each week. They give their answers rounded to the nearest hour.

Number of hours	0–1	2–4	5–9	10–14	15–19	20–24	25–168
Tally	卌	III	卌 卌 II	卌 卌 卌	卌 IIII	IIII	II

   a. Draw a frequency table of his results, converting the tallies to numbers.

   b. How many people did he survey?

c. How many people spend 15-19 hours per week watching television?

d. How many people watch television for less than 5 hours per week?

e. How many people watch television for less than 2 hours per day on average?

2. The heights of a group of 21 people are shown below, given to the nearest cm.

174	179	161	132	191	196	138	165	151	178	189
147	145	145	139	157	193	146	169	191	145	

a. Complete the frequency table below.

Height (cm)	130-139	140-149	150-159	160-169	170-179	180-189	190+
Tally							
Frequency							

b. How many people are in the range 150-159 cm?

c. How many people are 180 cm or taller?

d. How many people are between 140 cm and 169 cm tall?

3. Five different classes are in the same building in different rooms. The ages of students in each room are recorded in the frequency table below.

Age	Room A	Room B	Room C	Room D	Room E
12	3	2	0	0	0
13	20	18	1	0	0
14	2	4	3	0	10
15	0	0	12	10	11
16	0	0	12	10	11
17	0	0	0	1	0

a. How many students are in room C?

b. How many students are in the building?

c. How many 14-year-olds are in the building?

4.

Aces	Frequency
0	1
1	3
2	4
3	2
4	6
5	1
10	1

a. Construct a dot plot for the frequency table showing the number of aces served by a tennis player.

b. Use your dot plot to find the most common number of aces served.

c. Use your dot plot to identify any outliers.

5. Represent the frequency tables below as frequency graphs.

a.

Number	Frequency
0	5
1	3
2	5
3	2
4	4

b.

Score	Frequency
0–19	1
20–39	4
40–59	10
60–79	12
80–100	5

6. A set of results is shown below for a quiz out of 10.

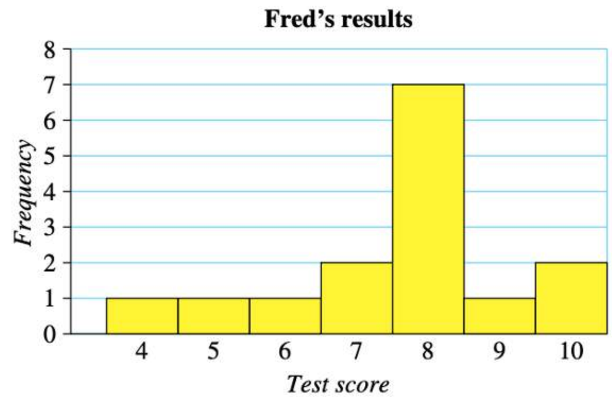
4, 3, 8, 9, 7, 1, 6, 3, 1, 1, 4, 6, 2, 9, 7, 2, 10, 5, 5, 4.

a. Create a frequency table.

b. Draw a frequency graph to represent the results.

c. Construct a dot plot to represent the results.

7. Edwin records the results for his spelling tests out of 10. They are 3, 9, 3, 2, 7, 2, 9, 1, 5, 7, 10, 6, 2, 6, 4.



a. Draw a frequency graph for his results.

b. Is he better or a worse speller generally than Fred, whose results are given by the graph shown?

Chapter 8.3 Measures of Centre and Spread

Measures of Centre (Location of Data)

- **Mean** → Average (**sum of values ÷ number of values**).
- **Median** → Middle value when data is ordered.
- **Mode** → Most common value (can be more than one).

Outliers

- A value much smaller or larger than the rest.
- **Mean is affected by outliers**, but **median and mode are not**.

Range

- **Formula: Range = Highest value − Lowest value**
- **Shows total spread of data**
- **Affected by outliers**

Interquartile Range (IQR)

- **Steps to find IQR:**
  1. Put data in order.
  2. If odd number of values, remove the middle one.
  3. Split into two equal groups.
  4. **Lower quartile** = Median of lower half.
  5. **Upper quartile** = Median of upper half.
  6. **IQR = Upper quartile − Lower quartile**
- **Not affected by outliers**
- **Better measure of spread than range**

Practice Questions

Centre

1. For each of the following sets of numbers, find the mean and the mode.

    a. 2, 2, 1, 2, 1, 4, 2

    b. 4, 3, 3, 10, 10, 2, 3

    c. 13, 15, 7, 7, 20, 9, 15, 15, 11, 17

    d. 20, 12, 15, 11, 20, 3, 18, 2, 14, 16

    e. 18, 12, 12, 14, 12, 3, 3, 16, 5, 16

    f. 18, 5, 14, 5, 19, 12, 13, 5, 10, 3

2. Find the mean of the following sets of numbers.

   a. $-10, -4, 0, 0, -2, 0, -5$

   b. $3, -6, 7, -4, -3, 3$

3. Find the value of x and y in the following rectangles.

a. 6, −10, 8, 1, 15, 8, 3, 1, 2

b. 5, −7, 12, 7, −3, 7, −3, 11, 12

c. 12, 17, 7, 10, 2, 17, −2, 15, 11, −8

d. −2, −1, −3, 15, 13, 11, 14, 17, 1, 14

4. Some people's ages are placed into a stem-and-leaf plot.

Stem	Leaf
1	8 9
2	0 3 5 7
3	1 2 2 7

2|3 means 23 years old

a. Write this set of data out as a list in ascending order. (Recall 1|8 means 18, 2|0 means 20 etc.)

b. Find the median.

c. Calculate the mean.

d. State the mode.

Spread

1. Find the range of the following sets.

   a. 9, 3, 9, 3, 10, 5, 0, 2

   b. 4, 13, 16, 9, 1, 6, 5, 8, 11, 10

   c. 16, 7, 17, 13, 3, 12, 6, 6, 3, 6

   d. 16, −3, −5, −6, 18, −4, 3, −9

   e. 3.5, 6.9, −9.8, −10.0, 6.2, 0.8

   f. −4.6, 2.6, −6.1, 2.6, 0.8, −5.4

2. Find the IQR of the following sets. Remember to sort first.

a. 0, 12, 14, 3, 4, 14

b. 14, 0, 15, 18, 0, 3, 14, 7, 18, 12, 9, 5

c. 6, 11, 3, 15, 18, 14, 13, 2, 16, 7, 7

d. 6, 4, 4, 5, 14, 8, 10, 18, 16, 6

e. 18, −15, 17, −15, −1, 2

f. −12, −17, −12, 11, 15, −1

g. −19, 8, 20, −10, 6, −16, 0, 14, 2, −2, 1

h. −4, −9, 17, 7, −8, −4, −16, 4, 2, 5

Chapter 8.4 Surveying and Sampling

Population = Entire group being studied.

Sample = A smaller selected group from the population.

Simple Random Sample → Randomly chosen from the whole population.

Stratified Sample → Randomly chosen from different groups within the population.

Convenience Sample → Easily available but may not represent the whole population.

Survey → Uses a sample to study a larger group.

Census → Collects data from the entire population.

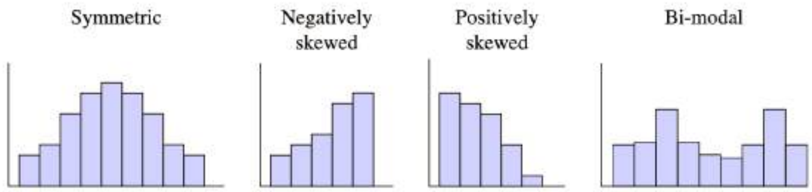
Sample size → Larger samples give more reliable results.

Bias → If the sample isn't representative, results may be inaccurate.

Measurement errors → Mistakes in data collection can create outliers.

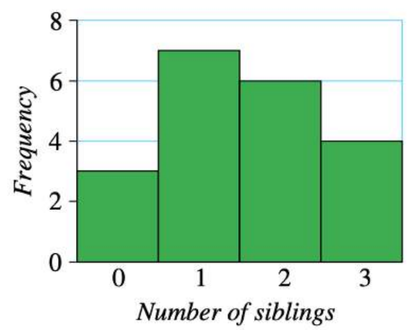
Data Distributions:

- **Symmetric** → Mean ≈ Median.
- **Skewed** → Mean & Median are different.
- **Bi-modal** → Two common values in the data.



Practice Questions

1. A survey is conducted to determine how many siblings children have. A simple random sample of 20 children are surveyed. The results are shown in this graph.



   a. Assume that this sample is representative of the population.

      i. What proportion of the child population has one sibling?

      ii. In a group of 60 children, how many would you expect to have two siblings?

b. Describe the shape of the distribution.

c. Which of the following methods of conducting the survey is more likely to lead to bias?
Method 1: Choosing 20 children at random from a school.
Method 2: Asking 20 children waiting outside a primary school at the end of a day.

2. Imagine you wanted to know the average number of subjects taken by a student in your high school. You cannot ask everyone in the school but want a sample of 20-30 students.

   a. For the following sampling methods, classify them as ‘simple random’, ‘convenience’ or ‘stratified’.

      i. Method 1: Ask the first 24 students you see

      ii. Method 2: Randomly select 4 students from each year level (Year 7, Year 8, up to Year 12).

      iii. Method 3: Randomly select 24 students from a list of all students in the school.

3. In a factory producing chocolate bars, a sample of bars is taken and automatically weighed to check whether they are between 50 and 55 grams. The results are shown in a frequency table.

Weight (g)	49	50	51	52	53	54	55	108
Frequency	2	5	10	30	42	27	11	1

a. Which weight value is an outlier?

b. How could the automatic weighing mechanism have caused this measurement error?

c. Disregarding the 108 gram result, is this distribution skewed or symmetric?

d. To find the spread of weights, the machine can calculate the range, or the IQR. Which would be a better value to use? Justify your answer.

e. If measurement errors are not removed, would the mean or the median be a better guide to the 'central weight' of the bars?

Chapter 8.5 Probabilities

Probability Basics

- **Experiment** → A chance-based situation with results (e.g., flipping a coin).
- **Trial** → A repeatable process (e.g., rolling a die).
- **Outcome** → A possible result (e.g., getting heads on a coin flip).
- **Event** → One or more outcomes (e.g., rolling a 3 or rolling a 3, 4, or 5).

Probability Formula

- **Pr(event) =** (favourable outcomes) ÷ (total outcomes)
- Written as a **fraction, decimal, or percentage**
- Probability ranges between **0 (impossible)** and **1 (certain)**.

Key Concepts

- **Sample Space** → All possible outcomes (e.g., rolling a die → {1,2,3,4,5,6}).
- **Complement of an event (E')** → The event NOT happening.
  - Pr(E) + Pr(E')=1 (e.g., Pr(rolling a 3) + Pr(not rolling a 3) = 1).

Common Probability Terms

- **'At least 3'** → {3, 4, 5, …}.
- **'At most 7'** → {…, 5, 6, 7}.
- **'Or' (e.g., even number or 5)** → {2, 4, 5, 6}.
- **'And' (e.g., even number and prime number)** → {2}.

Practice Questions

1. Five cards have one of the numbers 1, 2, 3, 4 and 5 written on them and all have a different number. One of the cards is chosen at random.

   a. List the sample space.

   b. Find the probability of selecting:

      i. An odd number

      ii. The number 2

      iii. A number which is at most 4

      iv. A number which is at least 2.

c. Using phrases like 'at least' and 'at most', give descriptions for the following events.

i. {1, 2}

ii. {4, 5}

iii. {2, 3, 4, 5}

iv. {1, 2, 3, 4}

2.

a. Find $\Pr(A')$ if $\Pr(A) = \frac{1}{4}$

b. Find $\Pr(A')$ if $\Pr(A) = 0.3$

c. Find $\Pr(D')$ if $\Pr(D) = \frac{4}{7}$

d. Find $\Pr(X')$ if $\Pr(X) = 0.95$

3. There are five red marbles, two green marbles and three black marbles. The 10 marbles are placed into a hat and one is picked out.

    a. What is $\Pr(\text{red})$? That is, what is the probability that the picked marble is red?

    b. Find $\Pr(\text{green})$.

    c. Find $\Pr(\text{black})$.

    d. Find $\Pr(\text{a black or a red marble is drawn})$.

e. Find $\mathrm{Pr}(\mathrm{red}')$. That is, find the probability of not choosing a red marble.

f. Find $\mathrm{Pr}(\mathrm{black}')$.

g. Give an example of an event that has a probability of 0.

4. Jake has a collection of equally shaped and sized marbles in his pocket. Some are blue, some are green and some are white. It is known there is a 0.3 chance of a green marble being chosen, and a 0.75 chance of not choosing a blue marble.

   a. What is the probability of not choosing a green marble?

   b. What is the probability of choosing a blue marble?

   c. Find the probability of choosing a white marble. (*Hint*: The sun of the three colours' probabilities is 1.)

   d. What is the minimum number of marbles Jake could have in his pocket?

Chapter 8.6 Two-Step Experiments

If an experiment has **two steps**, outcomes can be listed in a **table**.

Probability formula:

- **Pr(event) = (favourable outcomes) ÷ (total possible outcomes)**.

Practice Questions

1. A letter is chosen from the word LINE and another is chosen from the word RIDE.

   a. Draw a table to list the sample space.

   b. How many possible outcomes are there?

   c. Find $\Pr(\mathrm{NR})$, i.e. the probability that N is chosen from LINE and R is chosen from RIDE.

d. Find $\Pr(\mathrm{LD})$.

e. Find the probability that two vowels are chosen.

f. Find the probability that two consonants are chosen.

g. Find the probability that the two letters chosen are the same.

2. A letter from the word EGG is chosen at random and then a letter from ROLL is chosen at random. The sample space is shown below.

	R	O	L	L
E	ER	EO	EL	EL
G	GR	GO	GL	GL
G	GR	GO	GL	GL

a. Find Pr(ER).

b. Find Pr(GO).

c. Find Pr(both letters are vowels).

d. Find Pr(both letters are consonants).

3. Between 2 and 12.

   a. Draw a table to describe the sample space.

   b. Find the probability that the two dice add to 5.

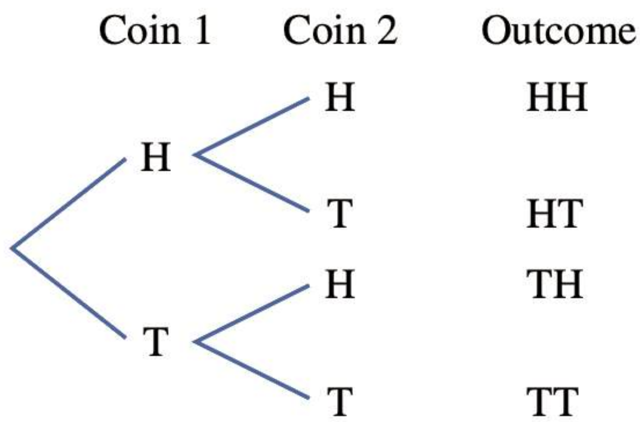
   c. Find the probability that the two dice do not add to 5. Recall that complementary events add to 1.

d. What is the most likely sum to occur?

e. What are the two least likely sums to occur between 2 and 12?

Chapter 8.7 Tree Diagrams

A tree diagram can be used to list the outcomes of experiments that involve two or more steps.



Practice Questions

1. A spinner with numbers 1, 2 and 3 is spun twice.

   a. Show the sample space in a tree diagram.

   b. Find $\Pr(1 \text{ then } 1)$.

   c. Find $\Pr(1 \text{ then } 2)$.

d. Find Pr(1 and 2 spun in either order).

e. Find Pr(both show the same number).

f. Find Pr(numbers add to 4).

2. A coin is tossed three times.

   a. Draw a tree diagram to represent the sample space.

   b. Find $\mathrm{Pr}(3\text{ tails})$.

   c. Find $\mathrm{Pr}(\text{at least one head})$. (*Hint:* This is the complement of 3 tails.)

d. Find $\text{Pr}(2 \text{ tails then } 1 \text{ head})$.

e. Find $\text{Pr}(2 \text{ tails then } 1 \text{ head, in any order})$.

f. Which is more likely: getting exactly 3 tails or getting exactly 2 tails?

g. Find the probability of getting at least 2 tails.

3. Two letters are chosen from the word CAR. Once a letter is chosen it cannot be chosen again.

   a. Draw a tree diagram of the six possible outcomes.

   b. What is the probability that A and C will be chosen?

c. Find $\Pr(2 \text{ consonants})$.

d. Find $\Pr(2 \text{ vowels})$.

e. What is the probability that the letters chosen will be different?

4. The letters of the word PIPE are placed on four cards. Two of the cards are chosen.

   a. Draw a tree diagram showing all 12 outcomes.

   b. Find $\Pr(2 \text{ vowels})$.

c. Find Pr(two same letter is one the 2 cards).

d. What is Pr(at least one letter is a P)?

Chapter 8.8 Venn Diagrams and Two-Way Tables

A **two-way table** shows the number of outcomes in different **categories**, with totals in the last row and column.

It helps in **finding probabilities**, e.g.:

- **Pr(like Maths) = 33/100**
- **Pr(like Maths and not English) = 5/100 = 1/20**

A **Venn diagram** is a visual version of a two-way table **without totals**.

"Or" can mean:

- **Inclusive or** (A, B, or both).
- **Exclusive or** (A or B, but not both).

Mutually exclusive events cannot happen at the same time (e.g., rolling an odd number and rolling a 6).

Practice Questions

1. In a group of 40 dogs, 25 had a name tag and a collar, 7 had only a name tag and 4 had only a collar.

    a. How many dogs had neither a name tag nor a collar?

    b. Represent the survey findings in a Venn diagram.

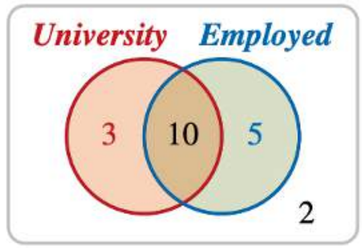
    c. How many dogs had a name tag?

d. How many dogs had either a name tag or a collar or both?

e. How many dogs had either a name tag and a collar but not both?

f. Represent the survey findings in a two-way table.

2. Consider this Venn diagram showing the number of people who have a university degree and the number who are now employed.



a. What is the total number of people in the survey who are employed?

b. Complete the two-way table shown below.

	Employed	Unemployed	Total
University degree			
No university degree			
Total			

c. If the 10 in the centre of the Venn diagram changed to an 11, which cells in the two-way table would change?

3. The two-way table below shows the results of a survey on car and home ownership at a local supermarket.

	Own car	Do not own car	Total
Own home	8	2	10
Do not own home	17	13	30
Total	25	15	40

a. Represent the two-way table above as a Venn diagram.

b. Find Pr(randomly selected person owns a car and a home).

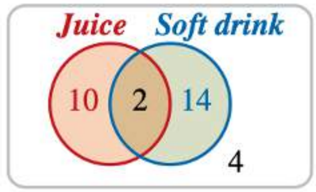
c. Find Pr(randomly selected person owns a car but not a home).

d. What is the probability that a randomly selected person owns their own home?

e. If a person from the group is selected at random and they own a car, what is the probability that they also own a home?

f. If a person from the group is selected at random and they own a home, what is the probability that they also own a car?

4. The Venn diagram shows the number of people who like juice and/or soft drinks.



a. What is the total number of people who like juice?

b. What is the probability that a randomly selected person likes neither juice nor soft drink?

c. What is the probability that a randomly selected person likes either juice or soft drink or both?

d. What is the probability that a randomly selected person likes just one of the two drink types listed?

e. What is the probability that a randomly selected person likes juice, given that they like soft drink?

f. Are ‘liking juice’ and ‘liking soft drink’ mutually exclusive? Why or why not?

Chapter 8.9 Experimental Probability

Experimental probability = (Number of times the event occurs) ÷ (Total trials).

Expected occurrences = Probability × Number of trials.

Simulations use random tools like **coins, dice, spinners, or random number generators** to model complex events.

Experimental probability is also called **relative frequency**.

Practice Questions

1. A group of households are surveyed on how many cars they own. The results are shown.

0 cars	1 car	2 cars	3 cars	4 cars
𝍸 𝍸 II	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 II	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 I	𝍸 III	II

   a. Write the tallied results as a frequency table.

   b. How many households were surveyed?

c. What is the experimental probability that a randomly chosen household owns no cars?

d. What is the experimental probability that a randomly chosen household owns at least 2 cars?

2. A die is painted so that 3 faces are blue, 2 faces are red and 1 face is green.

   a. What is the probability that it will display red on one roll?

   b. How many times would you expect it to display red on 600 rolls?

   c. How many times would you expect it to display blue on 600 rolls?

3. A spinner displays the numbers 1, 2, 3 and 4 on four sectors of different sizes. It is spun 20 times and the results are 1, 3, 1, 2, 2, 4, 1, 1, 3, 1, 2, 4, 4, 2, 4, 3, 1, 1, 3, 2.

   a. Give the experimental probability that the spinner landed on:

      i. 1

      ii. 2

      iii. 3

iv. 4

b. On the basis of this experiment, what is the expected number of times in 1000 trials that the spinner will land on 3?

4. A number of marbles are placed in a bag – some are red and some are green. A marble is selected from the bag and then replaced after its colour is noted. The results are shown in the table.

Red	Green
28	72

Based on the experiment, give the most likely answer to the following questions.

a. If there are 10 marbles in the bag, how many are red?

b. If there are 6 marbles in the bag, how many are red?

c. If there are 50 marbles, how many are red?

d. If there are 4 marbles, how many are green?

e. If there are 14 green marbles in the bag, how many marbles are there in total?

f. If there are 3 red marbles, how many green marbles are there?

CHAPTER 9 LINEAR RELATIONSHIPS

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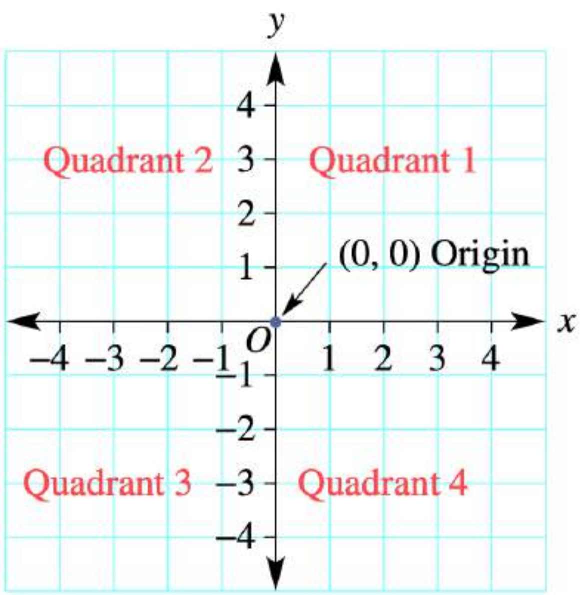
Chapter 9.1 The Cartesian Plane

Coordinates: (x,y)

Origin: (0,0)



Practice Questions

1. Draw a number plane extending from −4 to 4 on both axes, and then plot and label these points.

   a. $A(2,3)$

   b. $B(0.1)$

   c. $C(-2,2)$

   d. $D(-3,0)$

   e. $E(-3,-4)$

   f. $F(4,-3)$

2. Using a scale extending from −5 to 5 on both axes, plot and then join the points for each part. Describe the basic picture formed.

   a. $(-2,2), (2,-2), (2,2), (1,3), (1,4), \left(\frac{1}{2}, 4\right), \left(\frac{1}{2}, 3\frac{1}{2}\right), (0,4), (-2,2), (-2,-2)$

   b. $(2,1), (0,3), (-1,3), (-3,1), (-4,1), (-5,2), (-5,-2), (-4,-1), (-3,-1),$
   $(0,-3), (2,-1), (1,0), (2,1)$

3. The midpoint of a line segment (or interval) is the point that cuts the segment in half. Find the midpoint of the line segment joining these pairs of points.

   a. $(1,3)$ and $(3,5)$

   b. $(-4,1)$ and $(-6,3)$

   c. $(-2,-3)$ and $(0,-2)$

   d. $(3,-5)$ and $(6,-4)$

Chapter 9.2 Rules, Tables and Graphs with Linear Relationships

A **rule** is an equation showing the relationship between variables.

Linear relationships create **straight-line graphs**.

Linear equations are often written as **$y = mx + b$**, e.g., **$y = 2x - 3$**.

Special lines:

- **Horizontal** (constant y)
- **Vertical** (constant x)

Graphing steps:

1. Create a **table of values** by substituting x-values into the equation.
2. **Plot the points** on a graph.
3. **Draw a straight line** through the points.

Practice Questions

1. For each rule, construct a table like the one shown here and draw a graph.

x	−3	−2	−1	0	1	2	3
y							

a. $y = x + 1$

b. $y = x - 2$

c. $y = 2x - 3$

d. $y = 2x + 1$

e. $y = -2x + 3$

f. $y = -3x - 1$

g. $y = -x$

h. $y = -x + 4$

2. Plot the points given to draw these special lines.

a. Horizontal $(y = 2)$

x	−2	−1	0	1	2
y	2	2	2	2	2

b. Vertical $(x = -3)$

x	−3	−3	−3	−3	−3
y	−2	−1	0	1	2

3. For x-coordinates from -3 to 3, construct a table and draw a graph for these rules. For parts c and d remember that subtracting a negative number is the same as adding its opposite; for example, that $3-(-2)=3+2$.

   a. $y=\frac{1}{2}x+1$

   b. $y=-\frac{1}{2}x-2$

   c. $y=3-x$

   d. $y=1-3x$

4. For the graphs of these rules, state the coordinates of the two points at which the line cuts the x- and y-axes. If possible, you can use graphing software to draw these graphs.

a. $y = x + 1$

b. $y = 2 - x$

c. $y = 2x + 4$

d. $y = 10 - 5x$

e. $y = 2x - 3$

f. $y = 7 - 3x$

5. A straight line graph is to be drawn from a rule. The table is shown below with some values missing.

x	-2	-1	0	1	2	3
y			3		7	

a. Copy and complete the table..

b. Decide whether the point $(5,13)$ would be on the graph.

c. What would be the value of y when $x = -10$?

Chapter 9.3 Rules with Table of Values

A **rule** is an equation showing how x and y are related in a table or graph.

Example:

- **y = 3x + 2**

Coefficient of x = how much y increases when x increases by 1.

- If y **decreases**, the coefficient is **negative**.

Constant = the value of y when x = 0.

If x = 0 is **missing**, substitute another x, y pair to find the constant.

x	−2	−1	0	1	2
y	−1	1	3	5	7

+2 +2 +2 +2

$y = 2x + 3$

x	−2	−1	0	1	2
y	1	0	−1	−2	−3

−1 −1 −1 −1

$y = \ x - 1$

Practice Questions

1. Find the rule for these tables of values.

a.

x	−2	−1	0	1	2
y	0	2	4	6	8

b.

x	−3	−2	−1	0	1
y	4	3	2	1	0

2. Find the rule for these tables of values.

a.

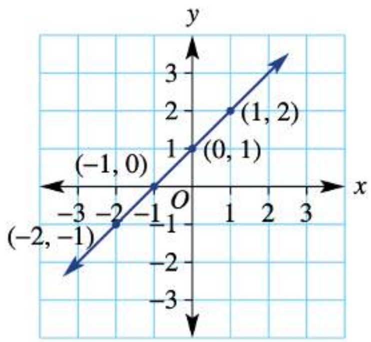
x	1	2	3	4	5
y	5	9	13	17	21

b.

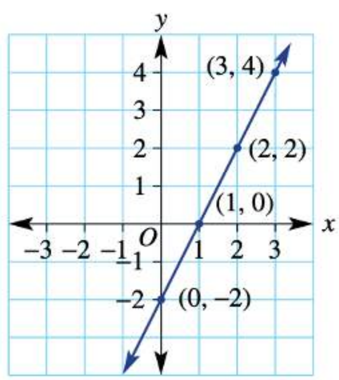
x	5	6	7	8	9
y	−12	−14	−16	−18	−20

3. Find the rule for these graphs by first constructing a table of (x, y) values.

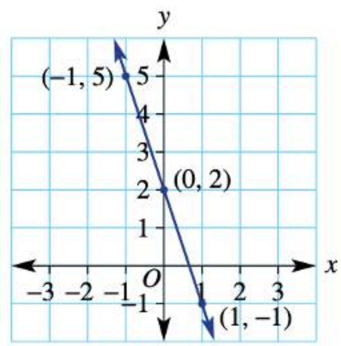
a.



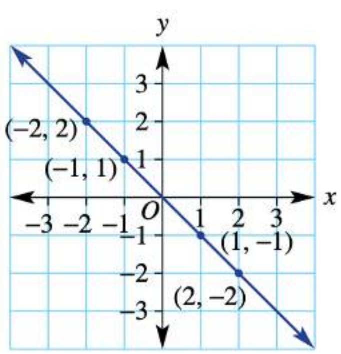
b.



c.



d.



4. The table below shows two points on a straight line graph, $(0,2)$ and $(2,8)$.

x	−2	−1	0	1	2
y			2		8

a. Find the missing three numbers.

b. State the rule for this graph.

5.

a. A straight line graph passes through the two points $(0, -2)$ and $(1,6)$. What is the rule of the graph?

b. A straight line graph passes through the two points $(-2,3)$ and $(4, -3)$. What is the rule of the graph?

Chapter 9.4 Graphs to Solve Linear Equations

The **x-coordinate** of each point on a straight line is a solution to its linear equation.

To create a linear equation, substitute a chosen **y-coordinate** into the relationship.

- **Example:** If $y = 2x - 1$ and $y = 4$, the equation becomes $2x - 1 = 4$.
- **Solution:** The point (2.5, 4) shows that $x = 2.5$ is the solution to $2x - 1 = 4$.

A point **(x, y)** is a solution to the equation if substituting the coordinates makes the equation true.

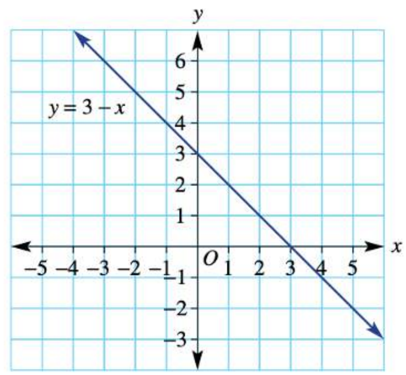
- **Equation is true** if **LHS = RHS** after substituting the coordinates
- Every point on the line satisfies the equation.
- Points not on the line **do not** satisfy the equation.

The **intersection** of two lines is the **only point** that satisfies both equations.

- Example: **(1, 3)** satisfies both $y = 6 - 3x$ and $y = 2x + 1$.

Practice Questions

1. Use the graph of $y = 3 - x$, shown here, to solve each of the following equations.



a. $3 - x = 1$ (*Hint:* Find x for $y = 1$.)

b. $3 - x = 5.5$

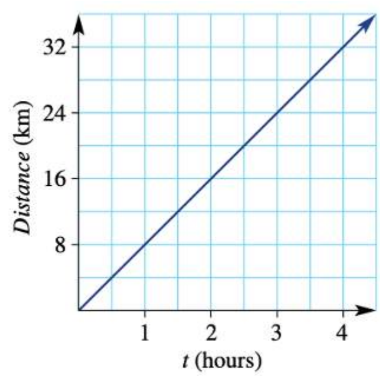
c. $3 - x = 0$

d. $3 - x = 3.5$

e. $3 - x = -1$

f. $3 - x = -2$

2. This graph shows the distance travelled by a cyclist over 4 hours. Use the graph to answer the following.



a. How far has the cyclist travelled after:

i. 2 hours?

ii. 3.5 hours?

b. How long does it take for the cyclist to travel:

i. 24 km?

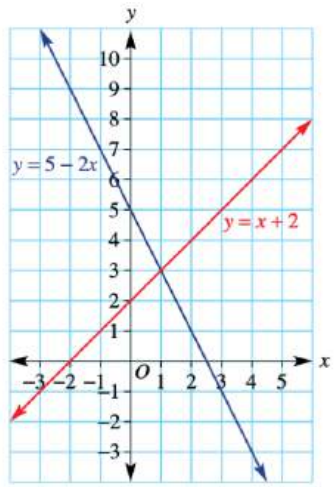
ii. 12 km?

3. Use digital technology to sketch a graph of each pair of lines and find the coordinates of the points of intersection. Round answers to two decimal places.

   a. $y = 0.25x + 0.58$ and $y = 1.5x - 5.4$

   b. $y = 2 - 1.06x$ and $y = 1.2x + 5$

4. Use the graphs of $y = 5 - 2x$ and $y = x + 2$, shown here, to answer the following questions.



a. Write the coordinates of four points (x, y) on the line with equation $y = 5 - 2x$.

b. Write the coordinates of four points (x, y) on the line with equation $y = x + 2$.

c. Write the coordinates of the intersection point and show that it satisfies both equations.

d. Solve the equation $5 - 2x = x + 2$ from the graph.

Chapter 9.5 Graphs to Solve Linear Inequalities

Horizontal line: Rule is **y = c**, where **c** is any number.

Vertical line: Rule is **x = k**, where **k** is any number.

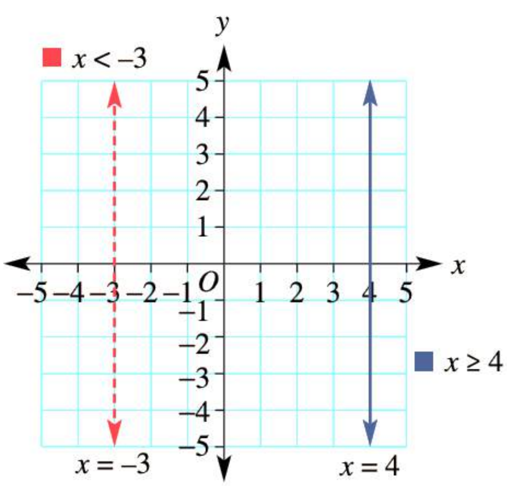
Region in the plane: Described using an inequality.

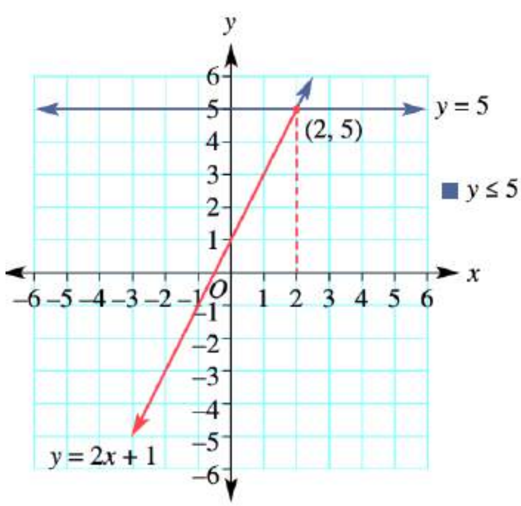
- Points satisfying the inequality form the **shaded region**.
- **Dashed line**: Indicates points on the line are **not included** (used for **<** or **>** inequalities).
- **Full line**: Indicates points on the line **are included** (used for **≤** or **≥** inequalities).

Graphs and inequalities:

- To solve **2x + 1 ≤ 5**:
  - Graph **y = 2x + 1** and the region **y ≤ 5**.
  - The solution is **x ≤ 2** (where the line is part of the shaded region).
- To solve **2x + 1 > 5**:

The solution is **x > 2** with a dashed line for **y = 5**.

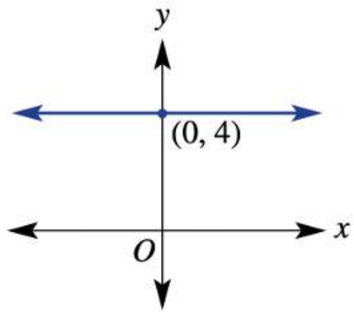




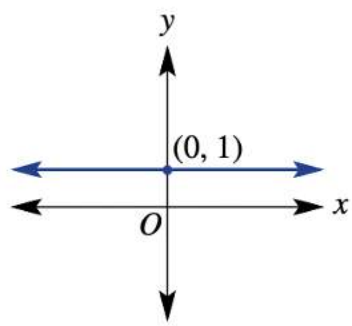
Practice Questions

1. Write the rule for these horizontal and vertical lines.

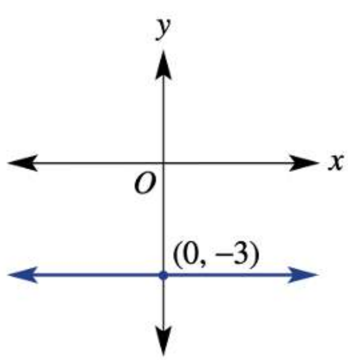
a.



b.



c.



2. Sketch the following regions.

a. $y \geq 1$

b. $y > -2$

c. $y < 3$

d. $y \leq 4$

e. $x < 2$

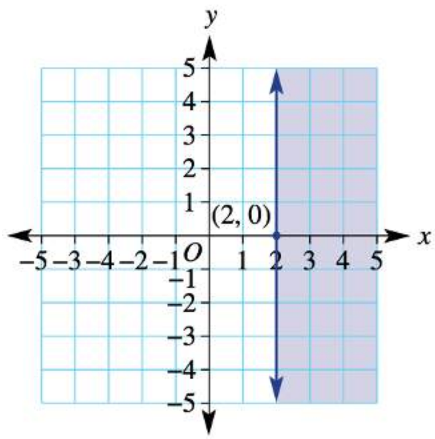
f. $x \leq -4$

g. $x \geq 2$

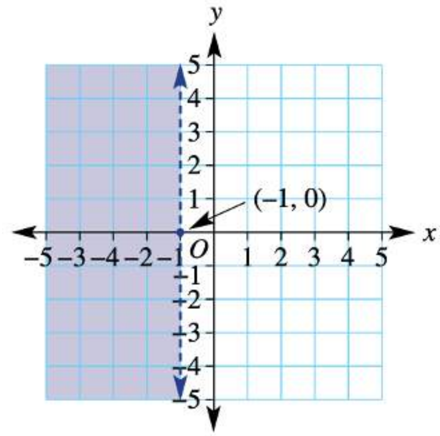
h. $x > -1$

3. Write the inequalities matching these regions.

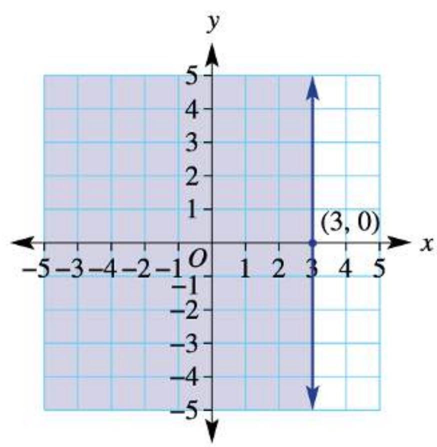
a.



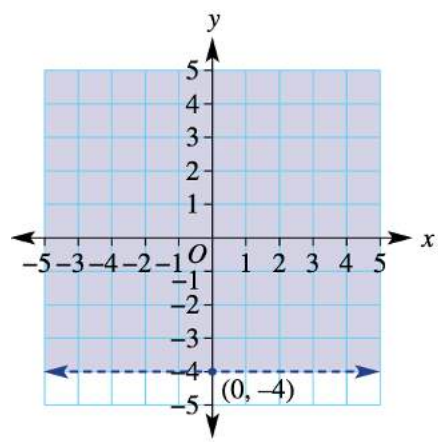
b.



c.

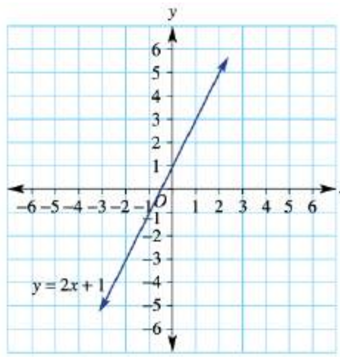


d.

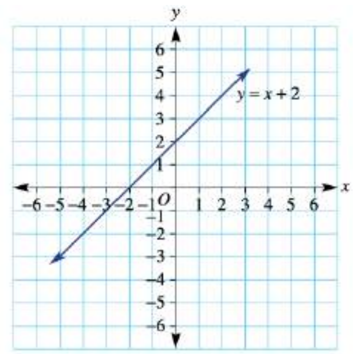


4. Solve the following inequalities using the given graphs.

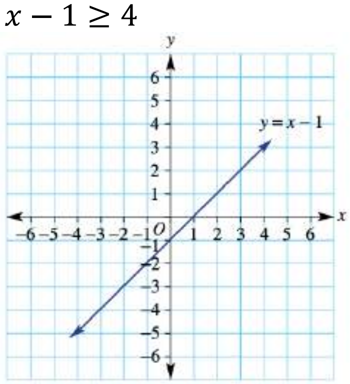
a. $2x + 1 \leq 5$



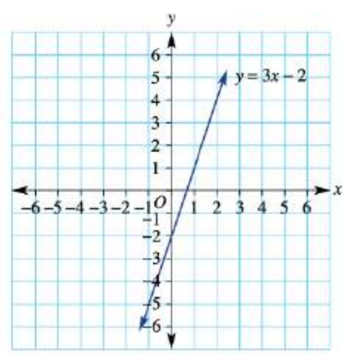
b. $x + 2 \leq 3$



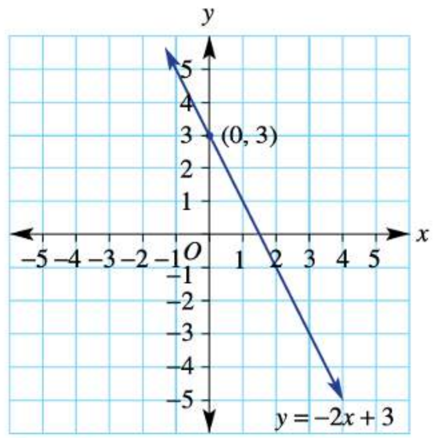
c. $x - 1 \geq 4$



d. $3x - 2 > 4$



5. This diagram shows the graph of $y = -2x + 3$. Use the graph to solve the following equations and inequalities.



a. $-2x + 3 = 5$

b. $-2x + 3 = -1$

c. $-2x+3=0$

d. $-2x+3<0$

e. $-2x+3\geq 0$

f. $-2x+3\geq 1$

Chapter 9.6 X- and Y-intercepts

y-intercept: Point where **x = 0**.

x-intercept: Point where **y = 0**.

Practice Questions

1. Find the x- and y-intercepts for the following rules. Note that some coordinates will involve fractions.

   a. $y = 2x - 5$

   b. $y = 3x - 7$

   c. $y = -2x + 4$

   d. $y = -4x + 8$

   e. $y = -4x + 6$

   f. $y = -3x - 8$

2. The rule $y = 2x + 6$ has an x-intercept at $(-3,0)$ and y-intercept at $(0,6)$. Sketch the graph of $y = 2x + 6$.

3. Find the x- and y-intercepts and then sketch the graphs of these rules.

a. $y = x + 1$

b. $y = x - 4$

c. $y = 2x - 10$

d. $y = 3x + 9$

e. $y = -2x - 4$

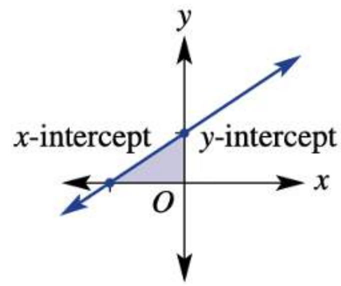
f. $y = -4x + 8$

g. $y = -x + 3$

h. $y = -x - 5$

i. $y = -3x - 15$

4. Find the area of the triangle enclosed by the x-axis, the y-axis and the given line. You will first need to find the x- and y- intercepts.



a. $y = 2x + 4$

b. $y = 3x - 3$

c. $y = -x + 5$

d. $y = -4x - 8$

5. Consider the rule $y = 2x + 5$.

   a. Find the y-intercept.

   b. Give an example of two other rules with the same y-intercept.

6. Consider the rule $y = 3x - 6$.

   a. Find the x-intercept.

   b. Give an example of two other rules with the same x-intercept.

7. A straight line graph passes through the points $(2,8)$ and $(3,10)$. Find its x-intercept and its y-intercept.

8. The height of water (H cm) in a tub is given by $H = -2t + 20$, where t is the time in seconds.

   a. Find the height of water initially (i.e. at $t = 0$).

   b. How long will it take for the tub to empty?

Chapter 9.7 Gradient and Gradient Intercept Form

Gradient measures slope.

- It shows the increase in **y** as **x** increases by 1.
- Formula: **Gradient = rise / run**.
- **Rise**: change in **y** (can be positive, negative, or zero).
- **Run**: change in **x** (always positive).

Positive gradient: **y** increases as **x** increases.

Negative gradient: **y** decreases as **x** increases.

Horizontal line gradient: 0.

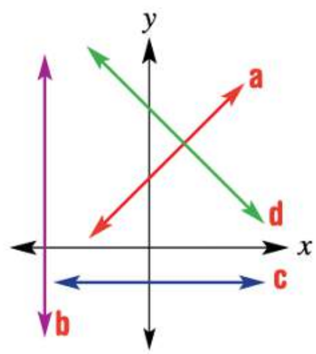
Vertical line gradient: Undefined.

Rule for straight line: **y = mx + c**.

- **m** is the gradient.
- **(0, c)** is the y-intercept.

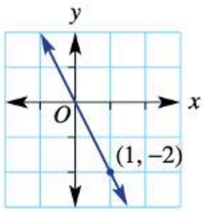
Practice Questions

1. Decide if the lines labelled a, b, c and d on this graph have a positive, negative, zero or undefined gradient.

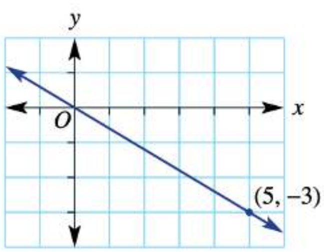


2. Find the gradient of these lines.

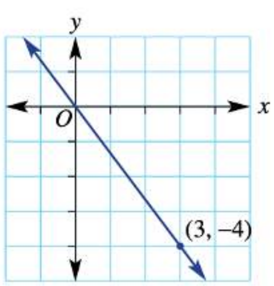
a.



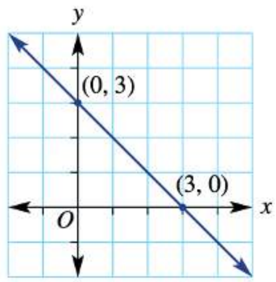
b.



c.



d.



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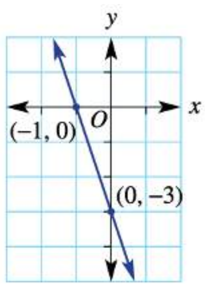
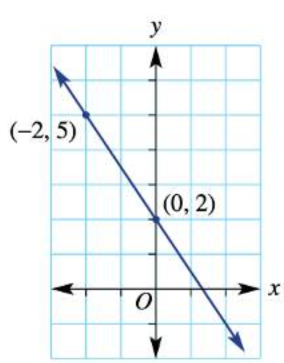


f.



3. Find the gradient of the line joining these pairs of points.

a. $(0,2)$ and $(2,7)$

b. $(0,-1)$ and $(3,4)$

c. $(-3,7)$ and $(0,-1)$

d. $(-5,6)$ and $(1,2)$

e. $(-2,-5)$ and $(1,3)$

f. $(-5,2)$ and $(5,-1)$

4. State the gradient and y-intercept for the graphs of these rules.

a. $y = 4x + 2$

b. $y = 3x + 7$

c. $y = \frac{1}{2}x + 1$

d. $y = \frac{2}{3}x + \frac{1}{2}$

e. $y = -2x + 3$

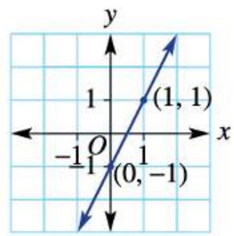
f. $y = -4x + 4$

g. $y = -x - 6$

h. $y = -\frac{2}{3}x - \frac{1}{2}$

5. Find the rule for these graphs by first finding the gradient (m) and the y-intercept (c).

a.



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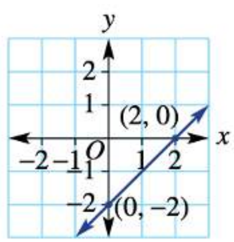
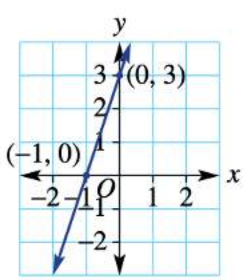


c.



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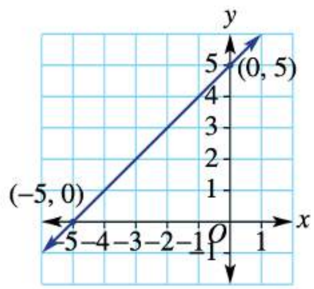
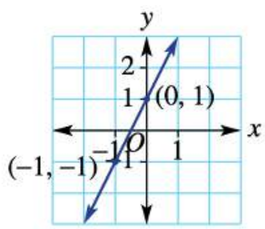


e.



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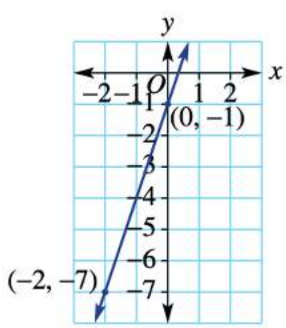
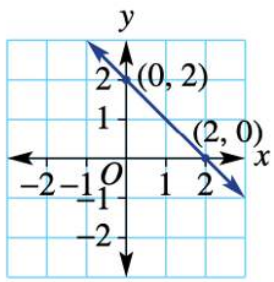
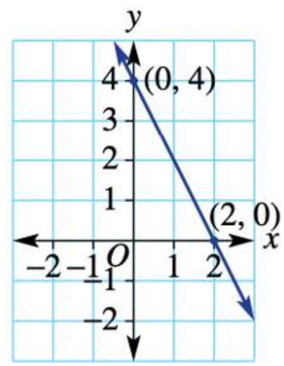


6. Find the rule for these graphs by first finding the values of m and c.

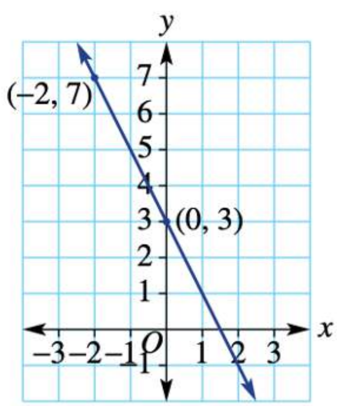
a.



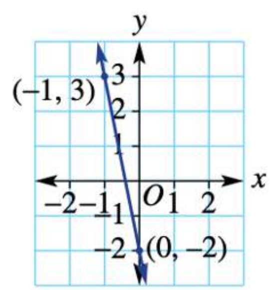
b.



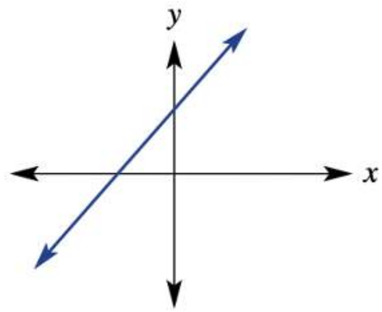
c.

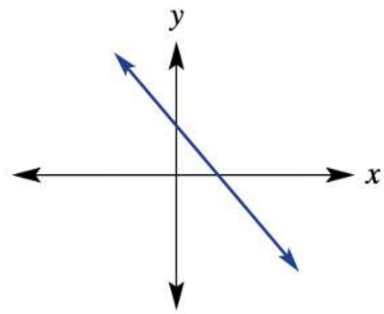


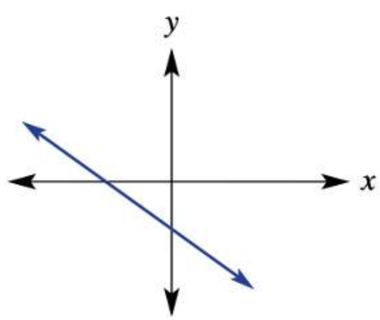
d.

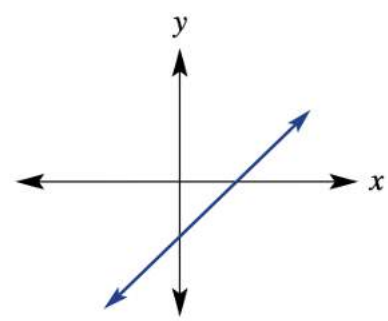


7. The graphs below are shown without any numbers. The x-axis and y-axis do not necessarily have the same scale. Choose the correct rule for each from the choices:
$y = x - 4, y = -2x + 3, y = 3x + 4, y = -4x - 1$

a.


b.


c.


d.


8. Find the rule for the graph of the lines connecting these pairs of points.

a. $(0,0)$ and $(2,6)$

b. $(-1,5)$ and $(0,0)$

c. $(-2,5)$ and $(0,3)$

d. $(0,-4)$ and $(3,1)$

Chapter 9.8 Applications of Straight Line Graphs

A **linear relationship** means one variable changes at a constant rate with respect to another.

In graphs, different letters can be used instead of **x** and **y** to match the variables.

Practice Questions

1. A paddle steamer moves up the Murray River at a constant rate of 5 kilometers per hour for 8 hours.

   a. Draw a table of values using t for time in hours and d for distance in kilometers. Use t values between 0 and 8.

   b. Draw a graph by plotting the points given in the table in part a.

c. Write a rule linking d with t.

d. Use your rule to find the distance travelled after 4.5 hours.

e. Use your rule to find how long it takes to travel 20km.

2. A weather balloon at a height of 500m starts to descend at a rate of 125m per minute for 4 minutes.

   a. Draw a table of values using t for time in minutes and h for height in metres.

   b. Draw a graph by plotting the points given in the table in part a.

c. Write rule linking h with t.

d. Use your rule to find the height of the balloon after 1.8 minutes.

e. Use your rule to find how long it takes for the balloon to fall to a height of 125 m.

3. A BBQ gas bottle starts with 3.5 kg of gas. Gas is used at a rate of 0.5 kg per hour for a long lunch.

   a. Write a rule for the mass of gas M in terms of time t.

   b. How long will it take for the gas bottle to empty?

   c. How long will it take for the mass of the gas in the bottle to reduce to 1.25 kg?

4. A cyclist races 50 km at an average speed of 15 km per hour.

   a. Write a rule for the distance travelled d in terms of time t.

   b. How long will it take the cyclist to travel 45 km?

   c. How long will the cyclist take to complete the 50-km race? Give your answer in hours and minutes.

5. An oil well starts to leak and the area of an oil slick increases by 8 km^2 per day. How long will it take the slick to increase to 21 km^2? Give your answer in days and hours.

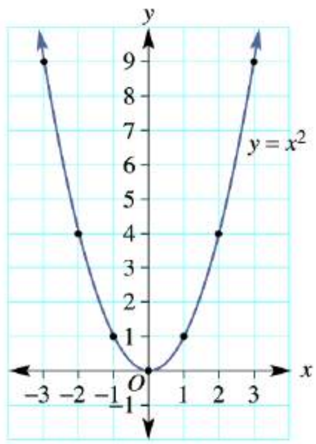
Chapter 9.9 Non-Linear Graphs

Non-linear graphs do not form straight lines.

To plot a **non-linear curve**:

- Make a **table of values** using the rule.
- **Plot the points** on a graph.
- **Join the points** with a smooth curve.

Parabolas are a common type of non-linear graph, like **y = x²**, where **y** is the square of **x**.

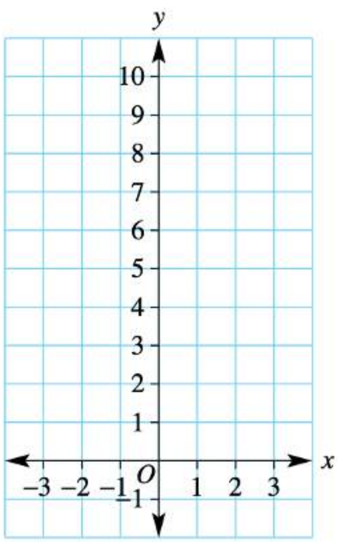


Practice Questions

1. Plot points to draw the graph of each of the given rules. Use the table and set of axes as a guide.

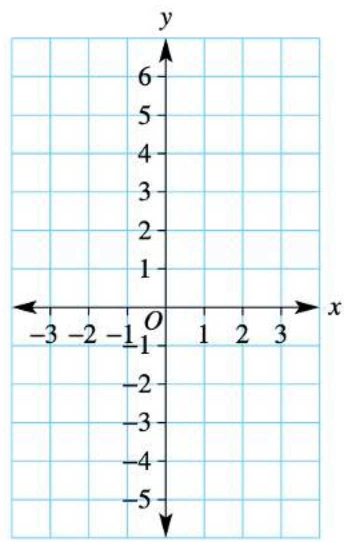
a. $y = x^2$

x	−3	−2	−1	0	1	2	3
y	9						



b. $y = x^2 - 4$

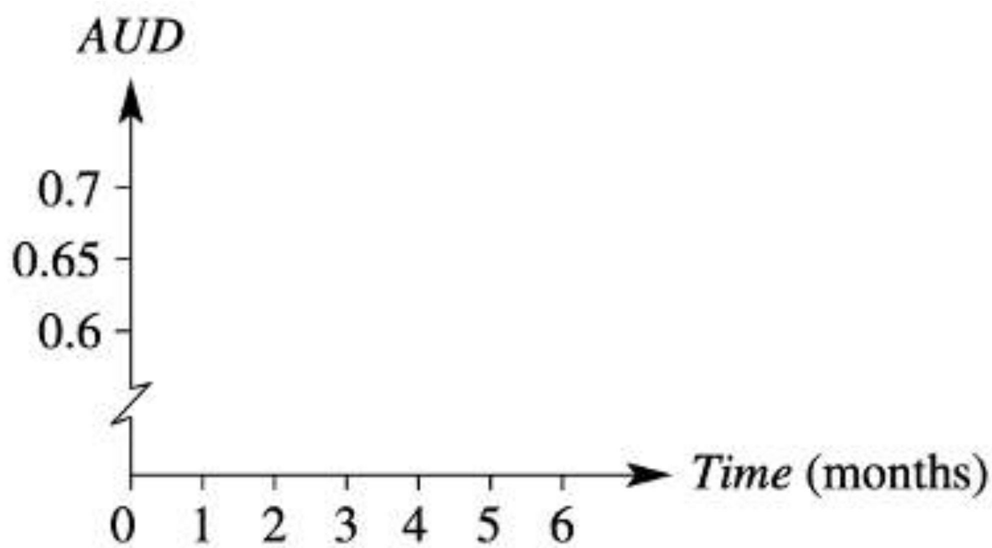
x	−2	−1	0	1	2
y	5				



2. The behaviour of the Australian dollar against the British pound over a 6-month period is summarized by the data in this table.

Time	0	1	2	3	4	5	6
AUD	0.69	0.64	0.61	0.6	0.61	0.64	0.69

a. Plot the data on the given graph and join to form a smooth curve.



b. Describe the shape of your graph.

c. By how much has the Australian dollar:

   i. Decreased in the first month?

   ii. Increased in the fifth month?

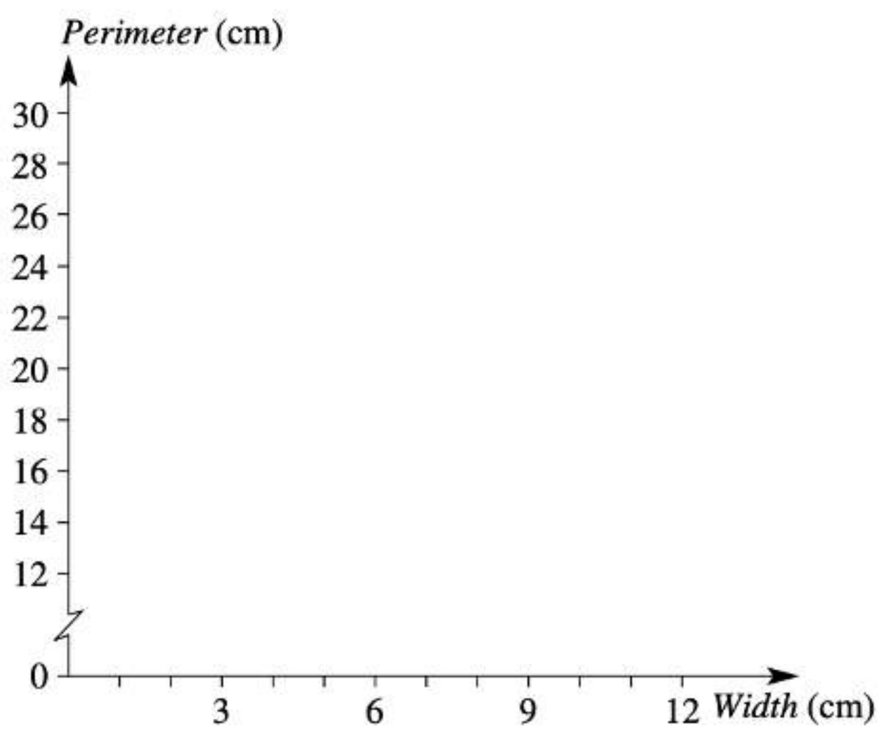
d. Estimate the value of the Australian dollar after 7 months.

3. Lara has enough paint to cover 12 cm^2 of space. She intends to paint a rectangular area.

a. For the given values, complete this table.

Width (cm)	1	2	3		6	
Length (cm)				3		
Perimeter (cm)						26

b. Plot the $Perimeter$ against $Width$ to form a graph.



c. Would you describe the curve to be linear or non-linear?

d. Look at the point where there is a minimum perimeter.

   i. Estimate the width of the rectangle at this point.

   ii. Estimate the perimeter at this point.

CHAPTER 10 TRANSFORMATIONS AND CONGRUENCE

Table of Contents

Chapter 10.1 Reflection

Reflection (Mirror Flipping)

Reflection means flipping a shape over a mirror line to create a mirror image.

The shape stays the same size and shape — it just looks reversed.

The mirror line could be:

- A straight line (like the x-axis or y-axis)
- A diagonal (like the line y = x)

Each point on the original shape and its reflected image are the same distance from the mirror line.

Example:

- Reflecting point (3, 2) over the x-axis gives (3, –2).

Key idea: The shape doesn't stretch, shrink, or rotate — it just flips.

Translation (Sliding Shapes)

Translation means sliding a shape across the grid — no turning or flipping.

Every point on the shape moves the same distance in the same direction.

This is shown using a vector, like (4, –2), which means:

- Move 4 right
- Move 2 down

Example:

- Point (2, 5) → translate by (4, –2) → becomes (6, 3)

Key idea: The shape keeps the same size, direction, and angles — just shifts position.

Rotation (Turning Around a Point)

Rotation means turning a shape around a fixed point, called the centre of rotation.

You must know:

- The centre of rotation
- The angle (like 90°, 180°, or 270°)
- The direction (clockwise or anticlockwise)

Example:

- A shape rotated 90° clockwise around the origin changes position, but the shape and size stay the same.

Key idea: Rotated shapes stay congruent to the original.

Practice Questions

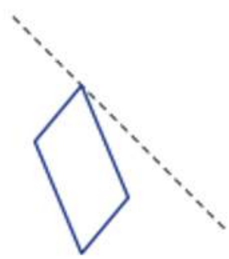
1. Copy the diagram and draw the reflected image over the given mirror line.

a.

b.

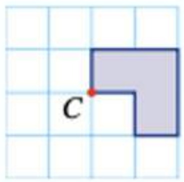
c.

d.

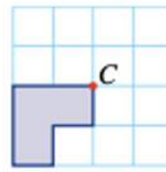


2. Rotate these shapes about the point C by the given angle and direction.

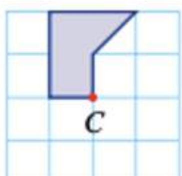
a. Clockwise by 90°



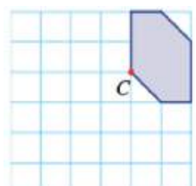
b. Anticlockwise by 90°



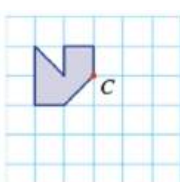
c. Anticlockwise by 180°



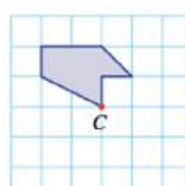
d. Clockwise by 90°



e. Anticlockwise by 180°

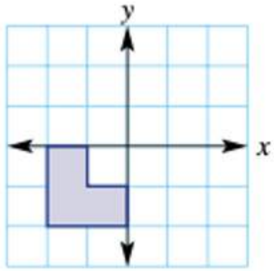


f. Clockwise by 180°

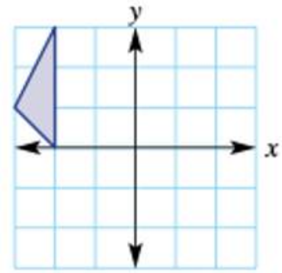


3. Copy the diagrams and draw the image of the shapes translated by the given vectors.

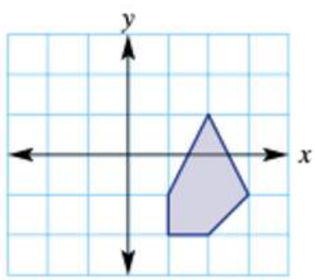
a. Vector (2,3)



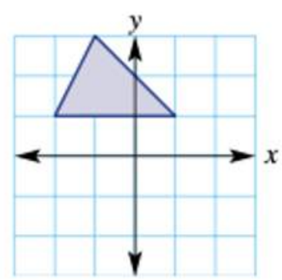
b. Vector (4,–2)



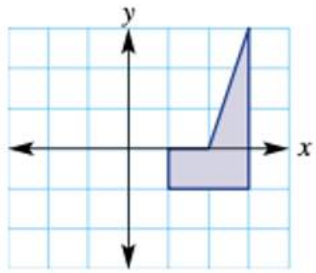
c. Vector



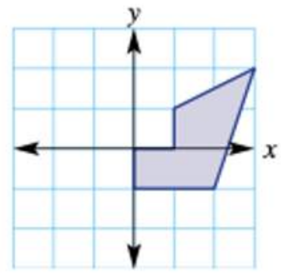
d. Vector (0,–3)



e. Vector $(-4,-1)$



f. Vector $(-3,0)$



Intermediate-Level Questions

1. Reflect the point A(3, 4) over the x-axis. What are the new coordinates?

2. Reflect the point B(–2, 5) over the y-axis.

3. Triangle XYZ has vertices X(1, 2), Y(4, 2), Z(2, 5). Reflect it over the x-axis and give the new coordinates.

4. Translate point P(2, 3) using the vector (–1, 4). What are the new coordinates?

5. Translate triangle ABC with points A(1, 1), B(3, 1), and C(2, 4) by the vector (2, –3).

6. Describe what happens to a point when it is reflected over the line y = 0.

7. Rotate the point (3, 2) 90° clockwise about the origin. What are the new coordinates?

8. Rotate the point (–4, 1) 180° about the origin.

9. Translate the point (–2, –3) using the vector (4, 5), then reflect it across the x-axis. What are the final coordinates?

10. Rotate triangle MNP with M(0, 1), N(1, 3), P(2, 1) 90° counter-clockwise about the origin.

Hard-Level Questions

1. Reflect triangle ABC with A(–2, 2), B(0, 5), C(2, 2) over the line y = –1. Give the image coordinates.

2. A shape is reflected over y = x. Reflect the point (4, –2) and state the image.

3. Translate a point (–6, 2) by the vector (3, –5), then rotate the new point 180° about the origin.

4. A triangle is rotated 270° clockwise about the origin. One vertex is (2, 0). Where does it land?

5. Reflect triangle PQR with P(3, –1), Q(4, 2), R(2, 1) over x = 2. Give the new coordinates.

6. A shape is reflected over y = x and then translated using vector (–2, 3). If a point starts at (1, 5), where does it end up?

7. Rotate the point (–3, –2) 90° clockwise about the origin, then reflect it over the x-axis.

8. Translate triangle DEF with D(–3, 1), E(–1, 4), F(–2, 3) by (2, –3), then rotate it 180° about the origin.

9. A hexagon is reflected over the y-axis, then rotated 90° clockwise about the origin. If one point started at (3, 2), what is its final position?

10. A composite transformation consists of reflecting a point across y = –x and then translating it by (–3, 2). Apply this to point (4, –1) and give the final result.

Chapter 10.2 Congruent Figures (Same Shape and Size)

Two shapes are congruent if they:

- Have the same size
- Have the same shape

They might be turned or flipped, but everything still matches — same sides, same angles.

You can move one onto the other using:

- Reflection
- Translation
- Rotation

Example:

- If triangle ABC ≡ triangle DEF, then their sides and angles match exactly.

Key idea: Congruent figures are exact copies, even if they're facing different directions.

Instead of checking all sides and angles, we use shortcuts to tell if two triangles are congruent.

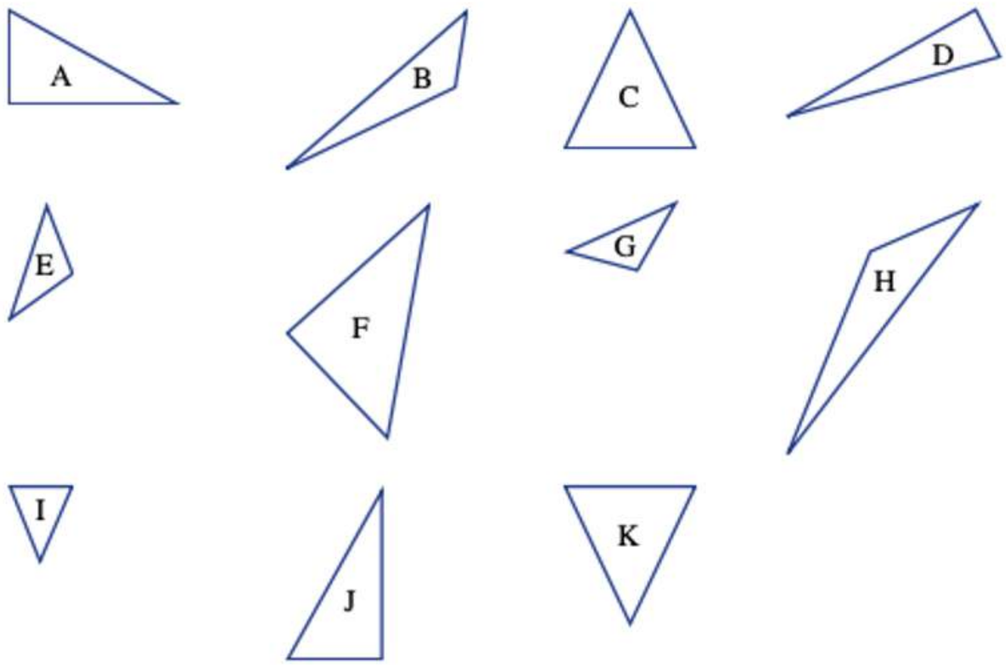
The main rules:

- SSS: All three sides match
- SAS: Two sides and the angle between them match
- AAS: Two angles and one side match
- RHS: Right-angle triangle with same hypotenuse and one side

Key idea: These shortcuts save time and prove that two triangles are exactly the same, even if they look different.

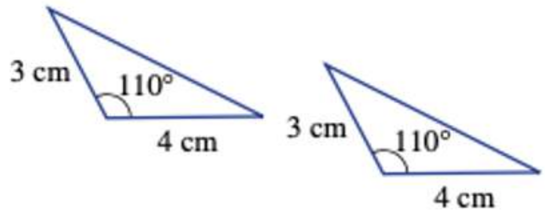
Practice Questions

1. List the pairs of the triangles below that look congruent.

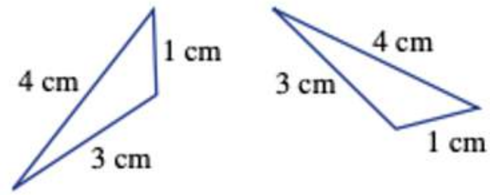


2. Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these triangles?

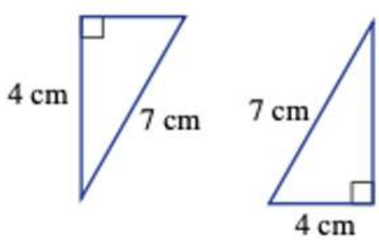
a.



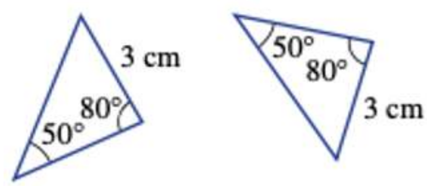
b.



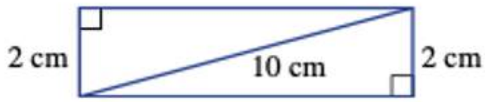
c.



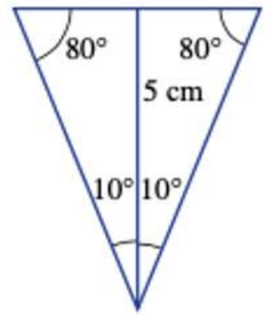
d.



e.



f.



Intermediate-Level Questions

1. Define what it means for two shapes to be congruent.

2. Determine whether these two rectangles are congruent:

   a. Rectangle A: 5 cm by 3 cm

   b. Rectangle B: 3 cm by 5 cm

3. If triangles have the same size and shape but are in different positions, are they still congruent? Explain why.

4. Triangle ABC has side lengths AB = 5 cm, BC = 6 cm, and AC = 7 cm. Triangle DEF has sides DE = 5 cm, EF = 6 cm, and DF = 7 cm. Are the triangles congruent?

5. Name three transformations that can map one congruent shape onto another.

6. State whether the following pairs of triangles are congruent:

   a. Triangle 1: angles 40°, 60°, 80°

   b. Triangle 2: angles 40°, 60°, 80°, sides not given.

7. Two figures are congruent. What can you say about their corresponding angles and side lengths?

8. Triangle PQR has sides 4 cm, 5 cm, and 6 cm. Triangle XYZ has the same side lengths. Which congruence rule proves they are congruent?

9. What does the “SAS” rule stand for in triangle congruence?

10. Identify the congruence rule used if two triangles have two equal angles and one equal side.

Hard-Level Questions

1. Prove two triangles are congruent using the SSS rule, given the side lengths:
   Triangle A: 6 cm, 7 cm, 9 cm
   Triangle B: 6 cm, 7 cm, 9 cm

2. Triangle ABC has ∠A = 60°, ∠B = 40°, and side AB = 5 cm. Triangle DEF has ∠D = 60°, ∠E = 40°, and side DE = 5 cm. Are the triangles congruent? Which rule applies?

3. Two right-angled triangles have a hypotenuse of 10 cm and one leg of 6 cm. Prove the triangles are congruent using the RHS rule.

4. Explain why SSA (Side-Side-Angle) is not a valid rule for proving congruence.

5. A triangle has sides 5 cm and 7 cm with an included angle of 65°. Another triangle has the same. Use the SAS rule to prove or disprove congruence.

6. Triangle A has vertices at A(1, 1), B(4, 1), C(4, 4). Triangle B is rotated and reflected but has sides and angles matching Triangle A. Are they congruent? Explain why.

7. Given triangle ABC and triangle DEF with ∠A = ∠D, AB = DE, and ∠B = ∠E, prove they are congruent using ASA.

8. A triangle has angles 45°, 45°, and 90°, and a leg of 6 cm. Another triangle has the same angles and leg. Prove congruence using AAS.

9. Triangle XYZ is reflected over the y-axis to create triangle X′Y′Z′. Are the two triangles congruent? Why?

10. Using coordinate geometry, prove triangle ABC with A(0,0), B(3,0), and C(0,4) is congruent to triangle DEF with D(0,0), E(–3,0), F(0,4).

Chapter 10.3 Similar Figures

Similar figures are shapes that have the same shape but different sizes. This means:

- All corresponding angles are equal
- All corresponding sides are in the same ratio (called the scale factor)

For example, when you look at something through binoculars or zoom in on a photo, the object looks bigger or smaller but keeps its shape — that's a similar figure. If the scale factor is 1, the shapes are exactly the same (this is congruence). You can check if shapes are similar by:

1. Matching angle sizes
2. Comparing side lengths This is very useful in real-life applications like enlarging images or designing scaled models in architecture.

Similar Triangles

Similar triangles are a powerful mathematical tool, especially when you need to find distances or heights that are hard to measure — like the height of a tree or the width of a river. You just need to form two triangles that are similar in shape, then use ratios of corresponding sides to calculate missing lengths.

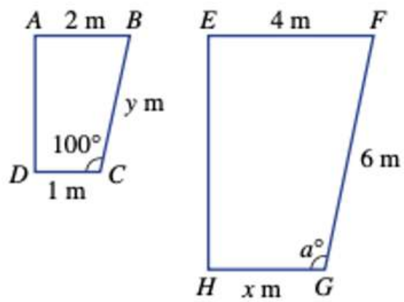
There are four ways to prove that two triangles are similar:

- AAA: All angles are equal
- SSS: All sides are in the same ratio
- SAS: Two sides are in the same ratio and the angle between them is equal
- RHS: For right-angled triangles, the hypotenuse and one other side are in the same ratio

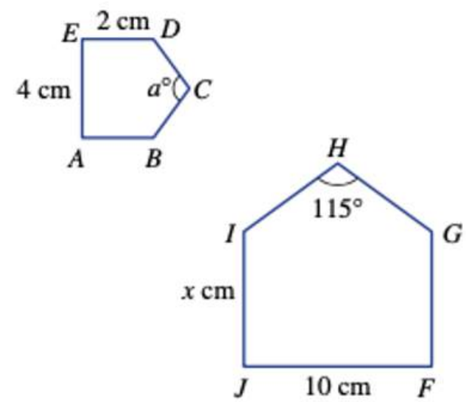
Practice Questions

1. For the following pairs of similar figures, complete these tasks.
   i. List the pairs of corresponding sides.
   ii. List the pairs of corresponding angles.
   iii. Find the scale factor.
   iv. Find the values of the pronumerals.

a.



b.



2. Define what it means for two figures to be similar.

3. Two rectangles have dimensions:
    - Rectangle A: 4 cm by 6 cm
    - Rectangle B: 6 cm by 9 cm

    Are these rectangles similar?

4. A square has sides of length 3 cm. A similar square has sides of 12 cm. What is the scale factor from the smaller to the larger square?

5. A triangle has angles 60°, 60°, and 60°. Another triangle has the same angles but different side lengths. Are they similar or congruent?

6. A rectangle is enlarged using a scale factor of 2.5. If the original width is 8 cm, what is the new width?

7. A photo measuring 10 cm by 15 cm is enlarged to 20 cm wide. What is the new height if the figure stays similar?

8. Two shapes are similar. One has a side length of 12 cm, and the other has a corresponding side length of 9 cm. What is the scale factor?

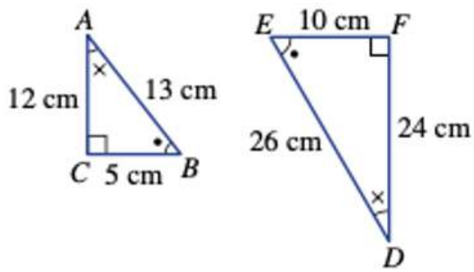
9. A triangle is enlarged by a scale factor of 3. If a side in the original triangle is 5 cm, how long is the corresponding side in the image?

10. Explain why two equilateral triangles are always similar.

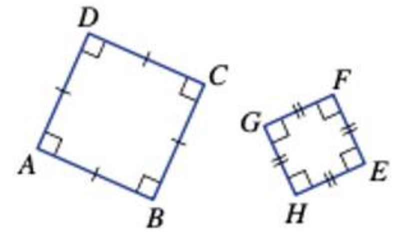
11. If two figures have corresponding angles equal but side lengths not in the same ratio, are they still similar?

12. Decide if these shapes are similar by considering corresponding angles and the ratios of sides.

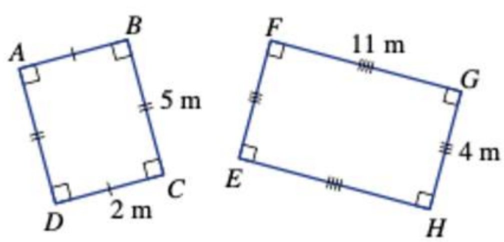
a.



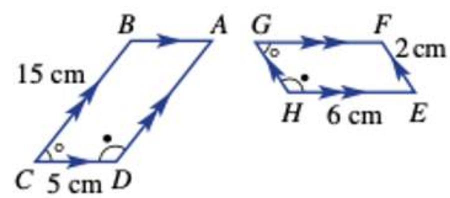
b.



c.



d.



Hard-Level Questions

1. Triangle ABC is similar to triangle DEF. If AB = 6 cm, BC = 9 cm, and AC = 12 cm, and the scale factor from triangle ABC to triangle DEF is 1.5, find the lengths of sides DE, EF, and DF.

2. Prove two triangles are similar using the AA (angle-angle) rule.

3. A tower casts a 15 m shadow. At the same time, a 2 m stick casts a 1.5 m shadow. Use similar triangles to find the height of the tower.

4. Two similar triangles have side lengths in the ratio 5:3. If the perimeter of the larger triangle is 60 cm, what is the perimeter of the smaller triangle?

5. In triangle ABC and triangle XYZ, angle A = angle X, angle B = angle Y, and side AB = 8 cm, side XY = 12 cm. Prove the triangles are similar and calculate the length of AC if XZ = 15 cm.

6. A triangle is reduced using a scale factor of 0.25. If the original triangle has sides 12 cm, 16 cm, and 20 cm, find the new side lengths.

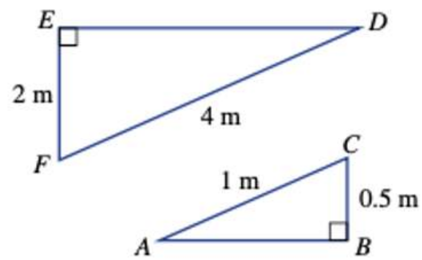
7. Triangle ABC and triangle DEF are similar. If the area of triangle ABC is 25 cm^2 and the scale factor from ABC to DEF is 3, find the area of triangle DEF.

8. Two right-angled triangles are similar. One has a hypotenuse of 10 cm and one leg of 6 cm. The corresponding hypotenuse of the second triangle is 15 cm. Find the missing side.

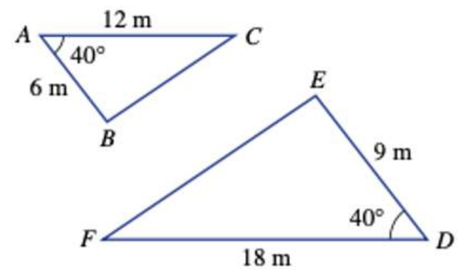
9. Explain why all isosceles right-angled triangles are similar to each other.

10. Given triangle PQR with coordinates P(0, 0), Q(4, 0), and R(0, 3), and triangle XYZ with X(0, 0), Y(8, 0), and Z(0, 6), prove that the two triangles are similar and determine the scale factor.

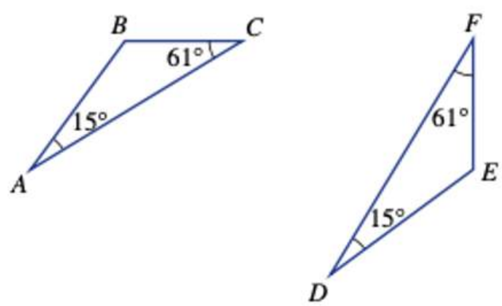
a.



b.



c.



d.