Mathematics Analysis and Approach HL Exploration

MAKING A BIRTHDAY CAKE WITH MINIMUM EXPENSES BY USING OPTIMIZATION METHODS

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Introduction

Optimization comes from the same root as “optimal”, both of them are originated from the Latin word “optimus” which means “the best”. In math, optimization or (mathematical optimization) refers to a branch of applied mathematics. Optimization methods are developed to ensure that the optimal choice is made in a given situation. In other words, optimization lead one to pick the option with the smallest possible cost or with the maximum reward. In our present day, this principle is more important than ever, considering the alarming situation of the world. Climate change, global warming and other environmental issues are the outcomes of the decisions made in previous years. Thus, we need to consider eco-friendly measures to create a sustainable environment. Mathematical optimization enables us to make the “best” choice, not only for ourselves, but for the environment as well.

Rationale

This year, I want to give the biggest birthday party ever because turning 18 has always been something that I looked forward to. I want to bake the birthday cake by myself for the 50 people I invited. As an aspiring “green chef”, I wish to be environmentally friendly as much as I can while baking. Besides, I am aware that not everyone has the opportunity to throw such parties or baking a birthday cake. However, I believe that minimizing the amount of ingredients would be helpful for many to afford baking a cake. Thanks to my Math HL class, I can now appreciate the various real-life applications of the calculus in everyday life. Thus, I decided to apply my mathematical knowledge to develop a recipe of a cake that yields 50 slices with minimum ingredients, so that it can be affordable for many people and reduces the amount of waste ingredients.

Aim and Methodology

In this investigation, I aim to design a birthday cake for 50 people with maximum volume and minimum surface area possible by employing optimization concepts. By considering various shapes, I will determine the best possible design which requires minimum frosting and thus, minimum ingredients for its surface. The volume of the cake will be kept constant for serving adequately to 50 people. First of all, I will determine the overall volume of the cake, based on the measurements of an average slicing apparatus. I find constrained optimization concepts to be applicable for deriving relations in which frosting would be minimized and verifies the volume of the cake. After deriving generalized conditions for basic shaped designs, I will determine the required values for the certain dimensions by applying my findings. After developing the recipe for an "optimum cake" I will publish it on my personal website. I aim to reach many people to raise awareness upon "green-cooking" and share my knowledge on how to bake an affordable cake by minimizing the expenses as well as potential waste material used for frosting.

Table 1: Definitions of the symbols used

Represented Quantity	**Notation**
Radius of Circular Base	r
Optimized Radius of Circular Base	r_{opt}
Vertical Height	h
Optimized Vertical Height	h_{opt}
Surface Area	S
Minimized Surface Area	S_{min}
Volume	V
Length	x
Width	y
Breadth	z

The Average Volume of a Slice of Cake

To calculate the total volume of the birthday cake, the volume of one slice is required. A slice is assumed to be a perfect triangular prism. The values regarding the dimensions of the prism is calculated with reference to the cake slicer seen in the Figure 2.A. Based on the apparatus, a diagram showing the dimensions of the slice is drawn as seen in the Figure 2.B.

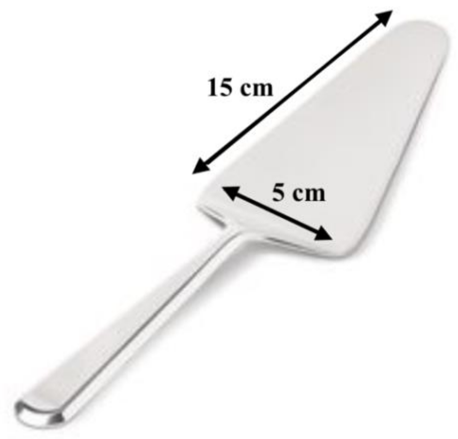

Figure 1.A: Reference for an average slice of cake

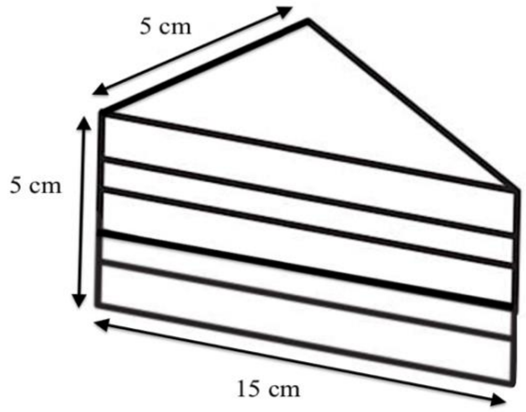

Figure 2.B: Diagram showing the dimensions of a slice of cake (not to scale)

The necessary formula for the volume of a triangular prism is given below:

$$Triangular\ Prism: V = xyh \text{ "}(Eq. 1) \mid \text{" } x, y, h \in R+$$

→ Solving ***Eq. 1*** and multiplying it by 50 to find the overall volume

$$V_{slice} = 15\ x\ 5\ x\ 5 = 375\ cm^3$$

$$V_{cake} = 375\ x\ 50 = 18750\ cm^3$$

Evaluation of a Cylindrical Cake Design

The necessary formulae for the volume of a cylindrical prism cake are given below:

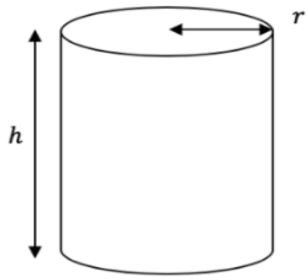

Figure 3: Labelled Cylinder

$$Cylinder = \begin{cases} V = \pi r^2 h \ \textbf{\textit{(Eq. 2.1)}} \\ S = 2\pi r^2 + 2\pi r h \ \textbf{\textit{(Eq. 2.2)}} \end{cases} \Bigg| \ h, r \in \mathbb{R}^+$$

→ I re-arrange the ***Eq. 2.1,*** so I can express h in terms of V:

$$h = \frac{V}{\pi r^2}$$

→ I use the Power Rule: $\frac{d}{dx}\,(x^n) = n\,x^{n-1}$ to differentiate ***Eq. 2.2*** with respect to r,

$$\frac{dS}{dr} = 4\pi r - \frac{2V}{r^2}$$

→ At critical points, surface area is a minimum/maximum, therefore I set the derivative to zero

$$4\pi r - \frac{2V}{r^2} = 0$$

Rearranging for V gives the equation $V = 2\pi r_{opt}^3$, which expresses the volume in terms of optimal radius.

→ By using second derivative, I will verify that the surface area is minimum for the abovementioned condition

$$\frac{d^2S}{dr^2} = 4\pi + \frac{4(V)}{r^3}$$

$$\frac{d^2S}{dr^2} = 4\pi + \frac{4(2\pi r^3)}{r^3} = 4\pi + 8\pi$$

$$\frac{d^2S}{dr^2} = 12\pi, > 0.$$

Since $\frac{d^2S}{dr^2} > 0$, surface area is minimum for $V = 2\pi r_{opt}^3$

→ By equating $V = 2\pi r_{opt}^3$ and $V = \pi r^2 h$, I obtain the relation of h and r

$$\pi r^2 h = 2\pi r_{opt}^3$$

$$h_{opt} = 2r_{opt}$$

Hence, the minimum surface area is found when "**height = diameter"** of the cylinder's circular base.

Frosting a Cylindrical Cake

A general model is drawn using Tinkercad, however, the specific values of dimensions are not determined and not displayed on Figure 4.

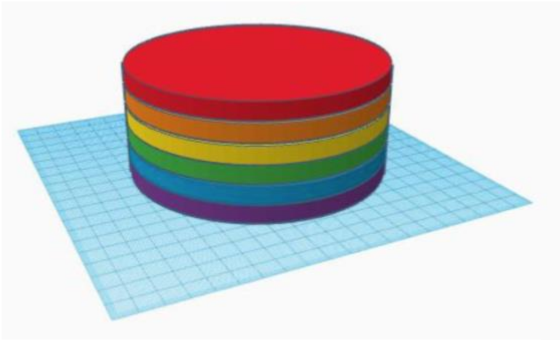

Figure 4. A birthday cake model drawn by myself using Tinkercad

→ Assuming the volume of this model is 18750 cm^3 as calculated above

$$V = 2\pi r_{opt}^3$$

$$r_{opt} = \left(\frac{18750}{2\pi}\right)^{1/3}$$

$$r_{opt} = 14{,}396\ cm$$

Using the final deduction of $h_{opt} = 2r_{opt}$, the height for minimum surface area is $28{,}792\ cm$. Thus, surface area can be found by substituting known values in the ***Eq. 2.2.***

$$S = 2\pi r^2 + 2\pi rh = 6\pi r^2$$

$$S_{min} = 3906.473\ cm^2$$

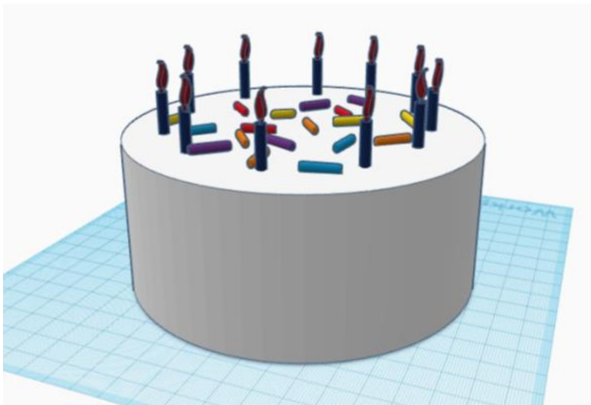

Figure 5. Illustration of frosting (candles and cake decorations are not included)

2.Assessing a Rectangular Prism Cake Design

The necessary formulae for the volume of a rectangular prism cake are given below:

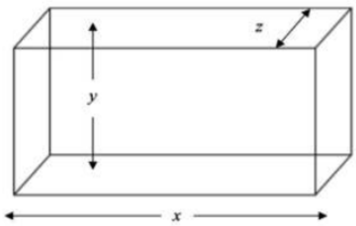

Figure 6. Labelled rectangular prism

$$Rectangular\ Prism = \begin{cases} V = xyz\ \textbf{\textit{(Eq.3.1)}} \\ S = 2\,(xy + yz + xz)\ \textbf{\textit{(Eq. 3.2)}} \end{cases} \Bigg|\ x, y, z \in \mathbb{R}^+$$

Since the V constant is a constraint that minimizes the surface area, Lagrange Multipliers are employed. The following equations are defined as two separate functions.

$$V\,(xyz) = xyz\ \textbf{\textit{(Constrait)}}$$
$$S\,(x, y, z) = 2\,(xy + yz + xz)\textbf{\textit{(Minimize)}}$$

To find the relation between a function's gradient and its constraints, Lagrange Multipliers are employed. This enables to determine the point when a function's gradient and its restrictions' gradient are identical. If those two gradients are equivalent of each other, their functions and constraints must be parallel, as well as pointing in the same direction. To calculate the gradient, we use "the slope of a line perpendicular to the curve itself". Thus, "if the two functions are parallel, the lines perpendicular to the function are parallel as well". (West Texas A&M University MathLab)

If two vectors are parallel, each has an integral multiplier which is equal to the other. Thus, their magnitudes vary by a constant factor. Consequently, the gradient of a function and its restrictions may be connected by a common multiplier λ. When we set the difference between the two gradients 0, this limits the value of the constraint. The specific point demonstrates an extremum of the function under the particular constraint;

$$\nabla S(x, y, z) = \lambda \nabla V(x, y, z)$$

To determine that, partial derivatives can be employed since they are practical when determining maximum and minimum points of the surfaces and writing partial differential equations.

→ Employing partial derivation for S and V with respect to x, y and z

$$\frac{\partial S}{\partial x} = \lambda \frac{\partial V}{\partial x}$$

$$\frac{\partial S}{\partial y} = \lambda \frac{\partial V}{\partial y}$$

$$\frac{\partial S}{\partial z} = \lambda \frac{\partial V}{\partial z}$$

Table 2: *Table summarizing the partial differential equations obtained*

Condition	Equation
When the constant is $\boldsymbol{S'(xy) = 0}$	$2(x + y) = \lambda xy$
When the constant is $\boldsymbol{S'(yz) = 0}$	$2(y + z) = \lambda yz$
When the constant is $\boldsymbol{S'(xz) = 0}$	$2(x + z) = \lambda xz$

→ Equating the three equations to each other and the common multiplier λ

$$\frac{(x + y)}{xy} = \frac{(y + z)}{yz} = \frac{(x + z)}{xz} \; (= \lambda)$$

By common sense, $x = y = z$ is a solution. The constraint function $V(xyz) = \; xyz$ can be used to calculate other solution sets. By rearranging the equation of the function V;

$$x = \frac{V}{yz}$$

→ Substituting x in the pairs of equations that are equal to λ

$$\frac{(y + z)}{yz} = \frac{\left(\frac{V}{yz}\right) + y}{\left(\frac{V}{yz}\right) y} \; (\boldsymbol{Eq. 3.4})$$

$$\frac{(y + z)}{yz} = \frac{\left(\frac{V}{yz}\right) + z}{\left(\frac{V}{yz}\right) z} \; (\boldsymbol{Eq. 3.5})$$

→ Solving the **Eq. 3.4**

$$\frac{(y+z)}{yz} = \frac{V + zy^2}{Vy}$$

$$\frac{(y+z)}{yz} = \frac{1}{y} + \frac{zy}{V}$$

$$\frac{(y+z)}{yz} - \frac{1}{y} = \frac{(y+z-y)}{yz} = \frac{zy}{V}$$

$$\frac{1}{y} = \frac{zy}{V}; \boldsymbol{V = y^2 z}$$

When **Eq. 3.5** solved by following the same steps, the equation $\boldsymbol{V = yz^2}$ is obtained. Deriving two equations relates *y*, *z* and *V*. Thus, solving for *y*, *z* gives $y = \sqrt[3]{V}$ and $z = \sqrt[3]{V}$. Similarly, substituting for *x* gives $x = \sqrt[3]{V}$. Thus, we can conclude that in order for a rectangular prism with certain volume to have minimum surface area it must be a cube, which is a special form of rectangular prism.

By employing *the Hessian Matrix* test for the second partial derivatives, we can prove that this condition yields the minimum surface area. The Hessian Matrix holds "the second-order partial derivatives of a function". Let Matrix H define the Hessian Matrix that captures the information of minimum surface area.

$$H = \begin{bmatrix} \frac{\partial^2 S}{\partial x^2} & \frac{\partial}{\partial x}\left(\frac{\partial S}{\partial y}\right) \\ \frac{\partial}{\partial y}\left(\frac{\partial S}{\partial x}\right) & \frac{\partial^2 S}{\partial y^2} \end{bmatrix}_{(x,y)}$$

The equation $S\ (x, y, z) = 2\ (xy + yz + xz)$, which is indented to be minimized is used to obtain different partial derivatives.

→ I substitute $z = \frac{V}{xy}$,

$$S = \left(xy + \frac{(x+y)\ V}{xy}\right)$$

$$\begin{cases} \dfrac{\partial S}{\partial x} = \dfrac{\partial}{\partial x}\left(yz + \dfrac{(y+z)\,V}{yz}\right) = 2y - \dfrac{V}{x^2} \mid \dfrac{\partial^2 S}{\partial x^2} = \dfrac{4\,V}{x^3} \\ \dfrac{\partial S}{\partial y} = \dfrac{\partial}{\partial y}\left(yz + \dfrac{(y+z)\,V}{yz}\right) = 2y - \dfrac{V}{y^2} \mid \dfrac{\partial^2 S}{\partial y^2} = \dfrac{4\,V}{y^3} \end{cases}$$

$$and$$

$$\begin{cases} \dfrac{\partial}{\partial x}\left(\dfrac{\partial S}{\partial y}\right) = 2 \\ \dfrac{\partial}{\partial y}\left(\dfrac{\partial S}{\partial x}\right) = 2 \end{cases}$$

$$thus,$$

$$H(\sqrt[3]{V}, \sqrt[3]{V}) = \begin{bmatrix} \dfrac{\partial^2 S}{\partial x^2} & \dfrac{\partial}{\partial x}\left(\dfrac{\partial S}{\partial y}\right) \\ \dfrac{\partial}{\partial y}\left(\dfrac{\partial S}{\partial x}\right) & \dfrac{\partial^2 S}{\partial y^2} \end{bmatrix}_{(\sqrt[3]{V}, \sqrt[3]{V})} = \begin{bmatrix} \dfrac{4\,V}{x^3} & 2 \\ 2 & \dfrac{4\,V}{y^3} \end{bmatrix}_{(\sqrt[3]{V}, \sqrt[3]{V})} = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

Since $H = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ is a symmetric matrix, Sylvester's Criterion is employed to understand that the derived condition is minimum. Sylvester's Criterion has two conditions:

1) All diagonal elements must be positive $\left(\frac{\partial^2 S}{\partial x^2} > 0\right)$
2) All determinants must be positive $\left[\left(\frac{\partial^2 S}{\partial x^2} \cdot \frac{\partial^2 S}{\partial x^2}\right) - \left(\frac{\partial^2 S}{\partial x \partial y}\right)^2 > 0\right]$

→ Checking for the first condition,

$$\frac{\partial^2 S}{\partial x^2} = 4, > 0 \quad \checkmark$$

→ Checking for the second condition,

$$\left(\frac{\partial^2 S}{\partial x^2} \cdot \frac{\partial^2 S}{\partial x^2}\right) - \left(\frac{\partial^2 S}{\partial x \partial y}\right)^2 = 2, > 0 \quad \checkmark$$

Since both of the conditions are satisfied with the values of Matrix *H*, we can prove that relation $x = y = z = \sqrt[3]{V}$ is correct. Thus, the deduction of "a rectangular prism with certain volume *must be a cube* in order to have minimum surface area" is also correct.

Frosting a Rectangular Prism Cake

The volume of the cake is 18750 cm^3. Using derived condition, $x = \sqrt[3]{V}$, the optimum length of the dimensions was calculated:

$$x = y = z = \sqrt[3]{18750}$$

$$= 26{,}567$$

→ Surface area of the rectangular prism cake can be calculated using *Eq. 3.2* (or simply we can use the formula $6a^2, a\ being\ a\ side\ of\ the\ cube$, which gives the surface area specifically of a cube)

$$S = 2\ (xy + yz + xz) = 6a^2$$

$$S_{min} = \ 6\ x\ (26{,}567)^2 = \ 4234.833\ cm^2$$

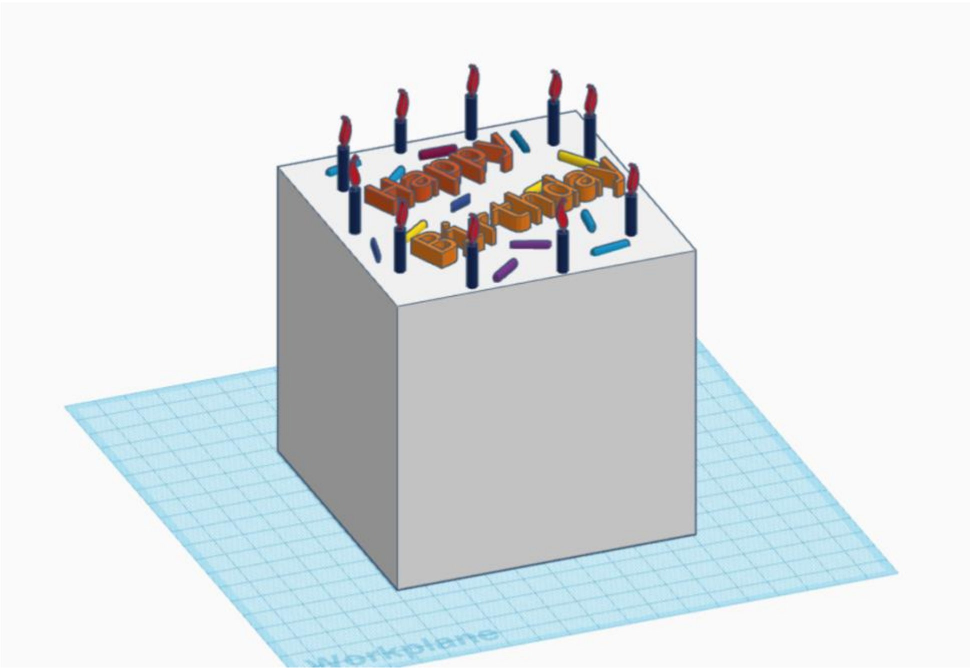

Figure7. Illustration of frosting (candles and cake decorations are not included)

Evaluation of a Conical Cake Design

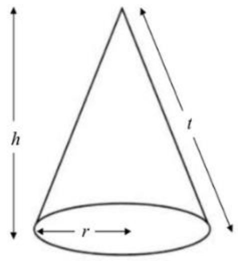

Figure 3 Labelled coni

Since I was born in January, I decided to design a winter themed cake, in which I modelled a pine tree. The necessary formulae of conical prisms are given below.

$$Cone = \begin{cases} V = \frac{1}{3}\pi r^2 h \; \textbf{\textit{(Eq. 4.1)}} \\ S = \pi r t + \pi r^2 \; \textbf{\textit{(Eq. 4.2)}} \end{cases} \Bigg| \; h, r, t \in \mathbb{R}^+$$

→ I express *h* in terms of *V* for ***Eq.4.1***

$$h = \frac{3V}{\pi r^2}$$

→ For expressing *S* in terms of *r* and *h*, I substitute *t* in the ***Eq. 4.2***

$$S = \pi r\sqrt{r^2 + h^2} + \pi r^2$$

$$S = \pi r\sqrt{r^2 + \left(\frac{3V}{\pi r^2}\right)^2} + \pi r^2 = \pi r\sqrt{r^2 + \frac{9V^2}{\pi^2 r^4}} + \pi r^2$$

$$S = \sqrt{r^2 + \frac{9V^2}{\pi^2 r^4}} + \pi r^2$$

→ Applying quotient rule $\left[\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{u'(x)\,v(x) - u(x)\,v'(x)}{[v(x)]^2}\right]$ for differentiating S with respect to r

$$\frac{dS}{dr} = \frac{d}{dr}\left[\frac{\sqrt{\pi^2 r^6 + 9V^2} + \pi r^3}{r}\right]$$

$$= \frac{\frac{d}{dr}\left[\sqrt{\pi^2 r^6 + 9V^2} + \pi r^3\right] \cdot r - \left(\sqrt{\pi^2 r^6 + 9V^2} + \pi r^3\right) \cdot \frac{d}{dr}[r]}{r^2}$$

$$= \frac{-\sqrt{\pi^2 r^6 + 9V^2} - \pi r^3 + \left(0.5(\pi^2 r^6 + 9V^2)^{-\frac{1}{2}} \cdot \frac{d}{dr}[\pi^2 r^6 + 9V^2] + 3\pi r^2\right) \cdot r}{r^2}$$

$$= \frac{-\sqrt{\pi^2 r^6 + 9V^2} - \pi r^3 + \left(\frac{6\pi^2 r^5 + 0}{2\sqrt{\pi^2 r^6 + 9V^2}} + 3\pi r^2\right) \cdot r}{r^2}$$

→ I carry out simplification,

$$\frac{dS}{dr} = \frac{-\sqrt{\pi^2 r^6 + 9V^2} + r \cdot \left(\frac{3\pi^2 r^5 + 0}{\sqrt{\pi^2 r^6 + 9V^2}} + 3\pi r^2\right) - \pi r^3}{r^2}$$

$$\frac{dS}{dr} = \left(\frac{\left(\frac{3\pi^2 r^5 + 0}{\sqrt{\pi^2 r^6 + 9V^2} + 3\pi r^2}\right)}{r}\right) - \left(\frac{\sqrt{\pi^2 r^6 + 9V^2} + \pi r^3}{r^2}\right)$$

→ Further simplified by the Lowest Common Factor,

$$\frac{\partial S}{\partial r} = \frac{2\pi r^3\left(\sqrt{\pi^2 r^6 + 9V^2} + \pi r^3\right) - 9V^2}{r^2 \cdot \sqrt{\pi^2 r^6 + 9V^2}}$$

→ When this derivative is equal to 0, it reveals the minimum point

$$\frac{2\pi r^3\left(\sqrt{\pi^2 r^6 + 9V^2} + \pi r^3\right) - 9V^2}{r^2 \cdot \sqrt{\pi^2 r^6 + 9V^2}} = 0$$

$$2\pi r^3\left(\sqrt{\pi^2 r^6 + 9V^2} + \pi r^3\right) - 9V^2 = 0$$

→ Rearranging

$$9V^2 = 2\pi r^3\left(\sqrt{\pi^2 r^6 + 9V^2} + \pi r^3\right)$$

$$\frac{9V^2}{2\pi r^3} - \pi r^3 = \sqrt{\pi^2 r^6 + 9V^2}$$

$$\left(\frac{9V^2}{2\pi r^3} - \pi r^3\right)^2 = \pi^2 r^6 + 9V^2$$

$$\frac{81V^4}{4\pi^2 r^6} + \pi^2 r^6 - \frac{9V^2 \pi r^3}{\pi r^3} = \pi^2 r^6 + 9V^2$$

$$18V^2 = \frac{81V^4}{4\pi^2 r^6}$$

→ Solving for *V*

$$V = \sqrt{\left(\frac{8\pi^2 r^6}{9}\right)}$$

→ I apply the quotient rule again; calculated the second derivative to prove that the condition yields the minimum value

$$\frac{2\pi r^2\left(6\pi^2 r^8 + 6\pi r^5\sqrt{\pi^2 r^6 + 9V^2} + 27V^2 r^2\right) - \frac{(5\pi^2 r^7 + 18rV^2)\left[2\pi r^3\left(\sqrt{\pi^2 r^6 + 9V^2} + \pi r^3\right) - 9V^2\right]}{\sqrt{\pi^2 r^6 + 9V^2}}}{r^4(\pi^2 r^6 + 9V^2)}$$

→ Carrying out simplification

$$\frac{2\pi^4 r^{13} + \left(2\pi^3 r^{10}\sqrt{\pi^2 r^6 + 9V^2}\right) + 117\pi^2 r^7 V^2 + \left(18\pi r^4 V^2\sqrt{\pi^2 r^6 + 9V^2}\right) + 162rV^4}{r^4(\pi^2 r^6 + 9V^2)\cdot\sqrt{\pi^2 r^6 + 9V^2}}$$

→ After factoring out the common factor r in numerator, I cancelled both with r

$$\frac{2\pi^4 r^{12} + \left(2\pi^3 r^9\sqrt{\pi^2 r^6 + 9V^2}\right) + 117\pi^2 r^6 V^2 + \left(18\pi r^3 V^2\sqrt{\pi^2 r^6 + 9V^2}\right) + 162V^4}{r^3(\pi^2 r^6 + 9V^2)\cdot\sqrt{\pi^2 r^6 + 9V^2}}$$

→ According to the principle of second derivative,

$$\frac{d^2S}{dr^2} > 0 \; for \; V = \sqrt{\left(\frac{8\pi^2 r^6}{9}\right)}$$

→ Deriving a relation between height and radius,

$$\frac{\pi^2 rh}{3} = \sqrt{\left(\frac{8\pi^2 r^6}{9}\right)}$$

$$\frac{\pi^2 r^4 h^2}{9} = \frac{8\pi^2 r^6}{9}$$

$$r^4 h^2 = 8r^6$$

$$h_{opt} = 2\sqrt{2}r_{opt}$$

In conclusion, optimal height can be obtained with multiplying the value of radius with $2\sqrt{2}$.

Frosting a Cylindrical Cake Design

Using the obtained formula for the certain volume, optimum radius for minimizing the surface area can be calculated.

$$V = \sqrt{\left(\frac{8\pi^2 r^6}{9}\right)} = 18750\ cm^3$$

$$r_{opt} = 18.499\ cm$$

$$h_{opt} = 2\sqrt{2}\ x\ (18.499) = 53.322\ cm$$

→ Writing *t* in terms of *r* and *h* by using the Pythagoras Theorem,

$$t = \sqrt{r^2 + h^2}$$

→ Substituting the relevant values in the equation $S = \pi rt + \pi r^2 = \pi$

$$S = \pi\ (18.499)(\sqrt{(18.499)^2 + (53.322)^2}) + \pi(18.499)^2$$

$$S_{min} = 4355.166\ cm^2$$

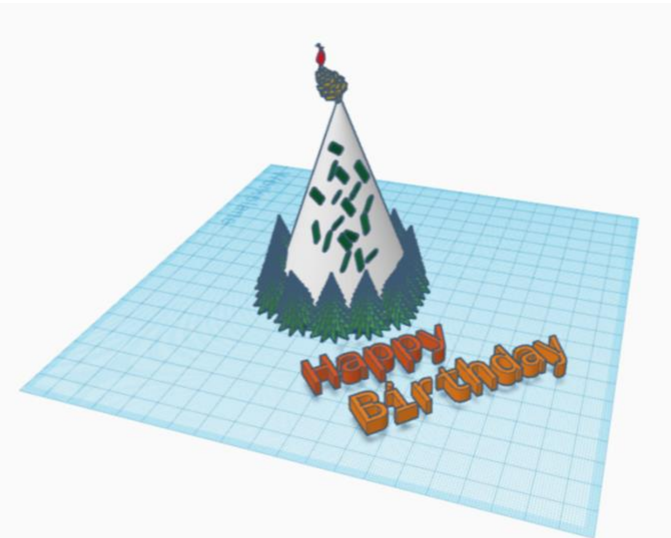

Figure 2 Illustration of frosting (text and cake decorations are not included)

Evaluation and Conclusion

Table 2: A table showing the minimum possible surface areas of designed cake models

Cake Model	**Conditions for Minimal Surface Area**	**Minimum Surface Area Possible**
Cylindrical Cake	$V = 2\pi r_{opt}^3$ $h_{opt} = 2r_{opt}$	$3906.473\ cm^2$
Rectangular Cake	$x = y = z = \sqrt[3]{V}$	$4234.833\ cm^2$
Conical Cake	$V = \sqrt{\left(\frac{8\pi^2 r^6}{9}\right)}$ $h_{opt} = 2\sqrt{2}r_{opt}$	$4355.166\ cm^2$

As seen in *Table 2*, minimum surface area is obtained in the cylindrical cake design. This might be the reason why cylindrical cakes are more prevalent in patisseries. The conditions I derived may be used not only as green-cooking methods, but also other industrial areas, such as industrial painting or wrapping.

In implementing my methodology, I observed a few limitations as seen in *Table 3*.

Table 3: Limitations of the study and possible improvements

Limitations	Improvements
Cakes may be subject to structural damage during baking.	Manually measuring the dimensions of the cake and re-calculating the surface area.
Not all the cakes have geometric-shaped designs.	Mathematically modelling the composite shapes of the cakes and deriving formulae accordingly.
Frosting the bottom area also costs extra frosting material.	Extracting the bottom area of the cakes from the surface area obtained.

Through calculus, I showed how different three-dimensional shapes of a birthday cake could be optimized to reduce the amount of frosting. The frosting material wasted each cake is not significant, however, if we consider the total amount wasted per birthday, it makes a significant difference. With this exploration, I have demonstrated how concepts in mathematics can help gastronomy to reduce the waste material and decrease the cost. This exploration enabled me to understand the possible real-life applications of calculus in green-cooking practices and encouraged me to be a more responsible member of the society.

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