Series EF1GH/2

SET~1

प्रश्न-पत्र कोड
Q.P. Code **65/2/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 80*

Time allowed : 3 hours — *Maximum Marks : 80*

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।*
Please check that this question paper contains 23 printed pages.

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।*
Please check that this question paper contains 38 questions.

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and follow them :

(i) *This Question Paper contains* ***38*** *questions.* ***All*** *questions are compulsory.*

(ii) *Question paper is divided into* ***FIVE*** *Sections – Section* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A*** *– Question Nos.* ***1*** *to* ***18*** *are Multiple Choice Questions (MCQs) and Question Nos.* ***19*** *&* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B*** *– Question Nos.* ***21*** *to* ***25*** *are Very Short Answer (VSA) type questions of* ***2*** *marks each.*

(v) *In* ***Section C*** *– Question Nos.* ***26*** *to* ***31*** *are Short Answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D*** *– Question Nos.* ***32*** *to* ***35*** *are Long Answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* ***Section E*** *– Question Nos.* ***36*** *to* ***38*** *are source based/case based/passage based/integrated units of assessment questions carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in* ***2*** *questions in Section* ***B****,* ***3*** *questions in Section* ***C****,* ***2*** *questions in Section* ***D*** *and* ***2*** *questions in Section* ***E****.*

(ix) *Use of calculators is* ***NOT*** *allowed.*

SECTION – A
(Multiple Choice Questions)
Each question carries 1 mark.

1. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to **1**

(A) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

2. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$ = P + Q, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to **1**

(A) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

3. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and a ∈ A, then the set A is **1**

(A) $\mathbb{R}$

(B) {0}

(C) {4}

(D) $\mathbb{R} - \{4\}$

4. If |A| = |kA|, where A is a square matrix of order 2, then sum of all possible values of k is **1**

(A) 1

(B) –1

(C) 2

(D) 0

5. If $\frac{d}{dx}[f(x)] = ax + b$ and f(0) = 0, then f(x) is equal to **1**

(A) a + b

(B) $\frac{ax^2}{2} + bx$

(C) $\frac{ax^2}{2} + bx + c$

(D) b

6. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is **1**

(A) 2

(B) 1

(C) not defined

(D) 0

7. The integrating factor of the differential equation

$(1-y^2)\frac{dx}{dy} + yx = ay, (-1 < y < 1)$ is **1**

(A) $\frac{1}{y^2-1}$ (B) $\frac{1}{\sqrt{y^2-1}}$

(C) $\frac{1}{1-y^2}$ (D) $\frac{1}{\sqrt{1-y^2}}$

8. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are (2, 1, –1) and (4, 4, –7), is **1**

(A) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$

(C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

9. Position vector of the mid-point of line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B is **1**

(A) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$ (B) $4\hat{i} + \hat{j} - 2\hat{k}$

(C) $5\hat{i} + 5\hat{j} - 7\hat{k}$ (D) $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$

10. Projection of vector $2\hat{i} + 3\hat{j}$ on the vector $3\hat{i} - 2\hat{j}$ is **1**

(A) 0 (B) 12

(C) $\frac{12}{\sqrt{13}}$ (D) $\frac{-12}{\sqrt{13}}$

11. Equation of a line passing through point (1, 1, 1) and parallel to z-axis is **1**

(A) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (B) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$

(C) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$ (D) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

12. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is : **1**

(A) $\frac{1}{9}$ (B) $\frac{4}{9}$

(C) $\frac{1}{18}$ (D) $\frac{1}{2}$

13. Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ with respect to x is **1**

(A) $\sec^2\left(\frac{\pi}{4} - x\right) + c$ (B) $-\sec^2\left(\frac{\pi}{4} - x\right) + c$

(C) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$ (D) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$

14. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to **1**

(A) $2\Delta^2$ (B) $4\Delta^2$

(C) 2Δ (D) 4Δ

15. The function $f(x) = x|x|$ is **1**

(A) continuous and differentiable at $x = 0$.

(B) continuous but not differentiable at $x = 0$.

(C) differentiable but not continuous at $x = 0$.

(D) neither differentiable nor continuous at $x = 0$.

16. If $\tan\left(\frac{x + y}{x - y}\right) = k$, then $\frac{dy}{dx}$ is equal to **1**

(A) $\frac{-y}{x}$ (B) $\frac{y}{x}$

(C) $\sec^2\left(\frac{y}{x}\right)$ (D) $-\sec^2\left(\frac{y}{x}\right)$

17. The objective function Z = ax + by of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true ? **1**

(A) a = 9, b = 1 (B) a = 5, b = 2

(C) a = 3, b = 5 (D) a = 5, b = 3

18. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If Z = 30$x$ + 24y is the objective function, then (maximum value of Z – minimum value of Z) is equal to **1**

(A) 40 (B) 96

(C) 120 (D) 136

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 & 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

19. **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2. **1**

Reason (R) : Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

20. **Assertion (A) :** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. **1**

Reason (R) : The sum of squares of the direction cosines of a line is 1.

SECTION – B

This section comprises Very Short Answer Type questions (VSA) of **2** marks each.

21. (a) Evaluate $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$. **2**

OR

(b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range. **2**

22. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x–coordinate 1, y–coordinate is changing twice as fast at x–coordinate. Find the value of a. **2**

23. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then find the angle between $\vec{a}$ and $\vec{b} - \vec{c}$. **2**

24. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin. **2**

25. (a) If $y = \sqrt{ax + b}$, prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$. **2**

OR

(b) If $f(x) = \begin{cases} ax + b & ; \quad 0 < x \le 1 \\ 2x^2 - x & ; \quad 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2), then find the values of a and b. **2**

SECTION – C

This section comprises Short Answer type questions (SA) of **3** marks each.

26. (a) Evaluate $\int_0^{\pi/4} \log(1 + \tan x)\, dx$. **3**

OR

(b) Find $\int \frac{dx}{\sqrt{\sin^3 x \cos(x - \alpha)}}$. **3**

27. Find $\int e^{\cot^{-1} x}\left(\frac{1 - x + x^2}{1 + x^2}\right) dx$. **3**

28. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} dx$ **3**

29. (a) Find the general solution of the differential equation :

$(xy - x^2)\, dy = y^2\, dx$. **3**

OR

(b) Find the general solution of the differential equation :

$(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$ **3**

30. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable. **3**

OR

(b) A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of wining, if A starts the game first. **3**

31. Solve the following linear programming problem graphically : **3**

Minimize : $Z = 5x + 10y$

subject to constraints : $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$,

$x \geq 0$, $y \geq 0$

SECTION – D

This section comprises Long Answer type questions (LA) of **5** marks each.

32. (a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations :

$x - 2y = 3$

$2x - y - z = 2$

$-2y + z = 3$ **5**

OR

(b) If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$ **5**

33. (a) Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, –6), Q(5, –3, 1), R(12, 4, 5) and S(11, 9, –2). Use these equations to find the point of intersection of diagonals. **5**

OR

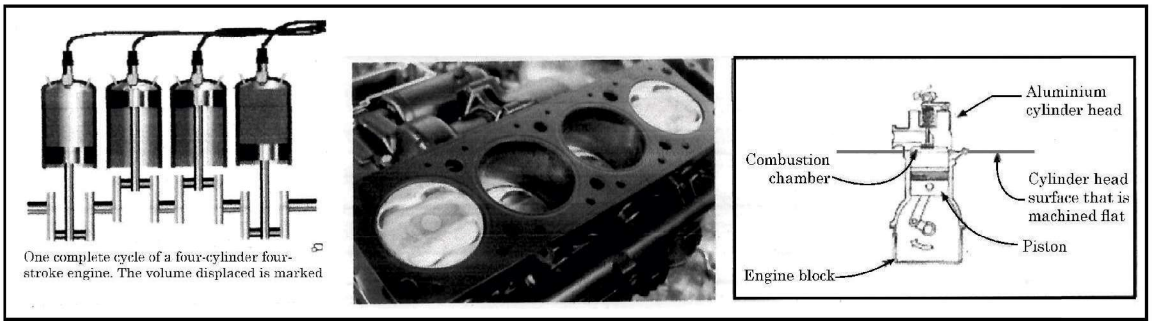
(b) A line l passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin. **5**

34. Using integration, find the area of region bounded by line $y = \sqrt{3}x$, the curve $y = \sqrt{4 - x^2}$ and y–axis in first quadrant. **5**

35. A function f: [– 4, 4] → [0, 4] is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$. **5**

This section comprises 3 source based/case-based/passage based/integrated units of assessment questions of **4** marks each.

36. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the above information, answer the following questions :

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r. **1**

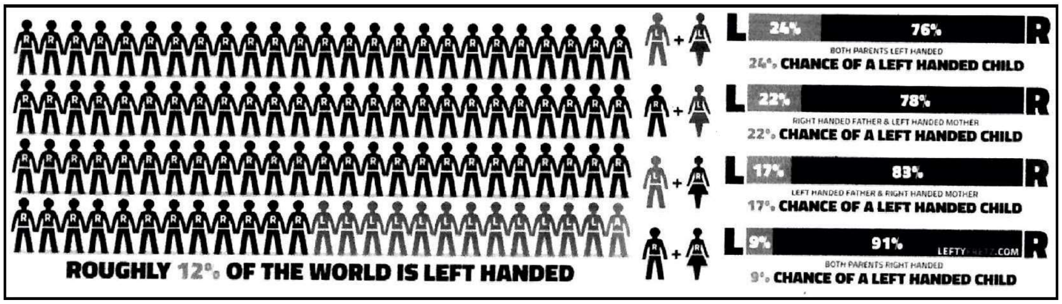
(ii) Find $\frac{dV}{dr}$. **1**

(iii) (a) Find the radius of cylinder when its volume is maximum. **2**

OR

(b) For maximum volume, h > r. State true or false and justify. **2**

37. आधुनिक अध्ययन यह सुझाते हैं कि दुनिया की आबादी में लगभग 12% लोग वामहस्तिक हैं ।



Depending upon the parents, the chances of having a left handed child are as follows :

A : When both father and mother are left handed :
Chances of left handed child is 24%.

B : When father is right handed and mother is left handed :
Chances of left handed child is 22%.

C : When father is left handed and mother is right handed :
Chances of left handed child is 17%.

D : When both father and mother are right handed :
Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions :

(i) Find P(L/C) **1**

(ii) Find P($\overline{\text{L}}$/A) **1**

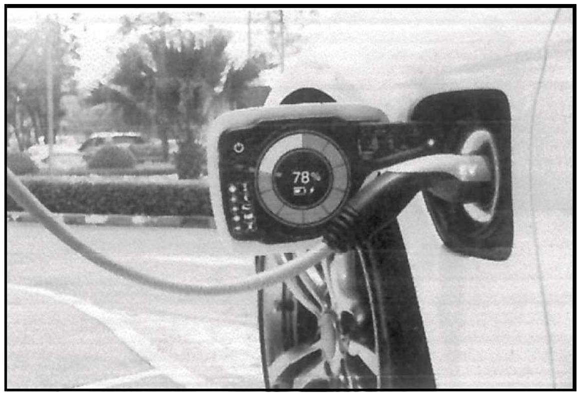
(iii) (a) Find P(A/L) **2**

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. **2**

38. विद्युत वाहनों का प्रयोग अंत में वायु प्रदूषण पर नियंत्रण कर लेगा ।

38. The use of electric vehicles will curb air pollution in the long run.



The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify. **2**

(ii) Prove that the function V(t) is an increasing function. **2**