Optimal Debt Maturity and Self-Fulfilling Crises

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What drove the 2010-2012 spike in Italian sovereign spreads?

- **Fundamentals View:** Worsening economy, low GDP growth
- **Self-Fulfilling View:** Investors' beliefs triggered the crisis

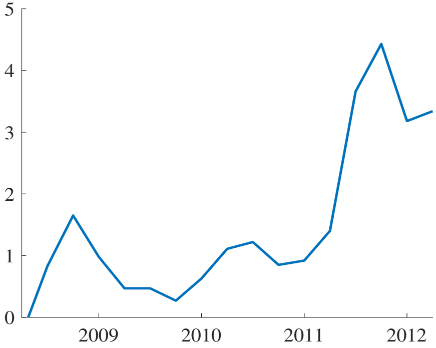

Figure 1: Italian Bond Spread (2008-2012)

Bocola and Dovis (2019)

- Maturity choice reveals the government's perceived risk
- Find limited role for self-fulfilling risk
- Italy shortened maturity, which suggest confidence risk was low

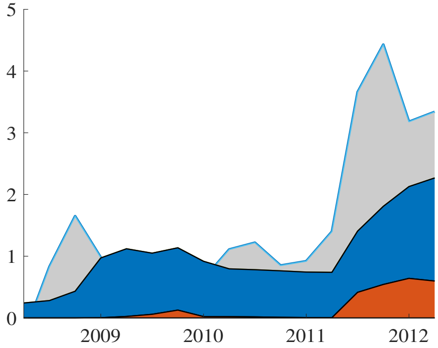

Figure 2: Spread Decomposition from BD's

This Project

Revisit the question and propose a Calvo-style model (Lorenzoni and Werning (2019))

- endogenous maturity structure and fiscal policy

Cole-Kehoe vs. Calvo

- CK (Bocola and Dovis (2019)): confidence crisis implies immediate default
- Calvo-style: crisis can occur without default

Result Preview

- Threat of self-fulfilling crisis leads government to
  1. cut next-period spending (Conesa and Kehoe (2024))
  2. lengthen maturity
- This framework reveals a larger role for self-fulfilling risk

Model

Timing

- Exogenous shocks are realized: (y_t, χ_t, λ_t)
- Government chooses next-period spending g_{t+1} and short-term debt $b_{S,t+1}$
- Sunspot, $\omega_t \in \{0, 1\}$, is realized, then risk-averse investors bid on prices of long-term debt
- Given prices, government adjust long-term debt $b_{L,t+1}$ to satisfy budget constraint or defaults $d_t = 1$

Stochastic Processes

- **Fundamental Shocks:** Log-output, $y_t = \log(Y_t)$, and term premium shock, χ_t, follow a correlated VAR(1) process:

$$\begin{bmatrix} y_{t+1} \\ \chi_{t+1} \end{bmatrix} = \begin{bmatrix} (1-\rho_y)\mu_y \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_y & 0 \\ 0 & \rho_\chi \end{bmatrix} \begin{bmatrix} y_t \\ \chi_t \end{bmatrix} + \begin{bmatrix} \epsilon^y_{t+1} \\ \epsilon^\chi_{t+1} \end{bmatrix}$$

  - $[\epsilon^y, \epsilon^\chi]'$ normally distributed with covariance matrix Σ.

- **Non-Fundamental Shocks:** Probability of bad sunspot today, λ_t, is drawn i.i.d. from a fixed grid Λ
  - Bad sunspot, $\omega_t = 1$, is realized with probability λ_t

Government

Preferences:

$$\mathbb{E}\sum_t \beta^t \frac{g_t^{1-\sigma}}{1-\sigma},$$

- $\beta \in (0,1)$ discount factor
- g_t spending chosen last period

Short- and Long-term Bonds: (Arellano and Ramanarayanan (2012))

- b_S: one-period bond
- b_L: long-term bond with $\kappa, (1-\delta)\kappa, (1-\delta)^2\kappa, ...$

Budget Constraints

Under repayment:

$$g_t + b_{St} + \kappa b_{Lt} = \tau Y_t + q_{St} b_{St+1} + q_{Lt}(b_{Lt+1} - (1-\delta) b_{Lt})$$

- τ constant tax rate

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Under default:

$$g_D \leq \tau Y_t$$

- $b_{St} = b_{Lt} = b_{St+1} = b_{Lt+1} = 0$
- $g_D < \tau Y_{\text{min}}$ and $g_D < g_{\text{min}}$
- reenter market next period

International Investors

Non-arbitrage conditions:

$$q_{St} = \mathbb{E}_t \left[M_{t,t+1}(1 - d_{t+1}) \right]$$

$$q_{Lt} = \mathbb{E}_t \left[M_{t,t+1} \left\{ (1 - d_{t+1})(\kappa + (1 - \delta) q_{Lt+1}) + d_{t+1} \frac{\nu_{t+1}}{b_{t+1}} \right\} \right]$$

- Stochastic discount factor $M_{t,t+1}$
- Recovery value in default: $\nu_t = \phi(\tau Y_t - g_D)$ with $\phi \in [0, 1]$.

Details on SDF

Recursive Formulation

Government's Problem

Let $S = (b_S, b_L, g, y, \chi)$

Value function:

$$V(S,\lambda) = \max_{b'_S, g'} \left\{ (1-d)u(g) + d(u(g_D)) + \beta\mathbb{E}[V(S',\lambda')] \right\}$$

$$s.t. \quad b'_L = B_L(S,\omega,b'_S,g'), \quad d = D(S,b'_S,g')$$

- Solution gives $\mathcal{B}_S(S,\lambda)$ and $\mathcal{G}(S,\lambda)$

Price Functions

$$Q_L(y, \chi, b'_S, b'_L, g') = \mathbb{E}\Bigg[M(y, \chi, y', \chi')$$

$$\Bigg\{(1 - d')(\kappa + (1 - \delta) Q_L(b''_S, b''_L, g'', \lambda'')) + d' \frac{\nu'}{b'_L}\Bigg\}\Bigg],$$

$$Q_S(y, \chi, b'_S, b'_L, g') = \mathbb{E}\Big[M(y, \chi, y', \chi')(1 - d')\Big],$$

- $d' = D(S', b''_S, g'')$
- $b''_S = \mathcal{B}_S(S', \lambda')$, $g'' = \mathcal{G}(S', \lambda')$
- $b''_L = B_L(S', \omega', b''_S, g'')$
- $\nu' = \phi(\tau Y' - g_D)$

The Solvency Constraint

Maximum revenue:

$$m(b_S, b_L, y, \chi, b'_S, g') = \max_{\overline{b_L}} \Big\{ Q_L(y, \chi, b'_S, \overline{b_L}, g')(\overline{b_L} - (1-\delta)b_L) + Q_S(y, \chi, b'_S, \overline{b_L}, g')b'_S - b_S - \kappa b_L \Big\}$$

Default Rule:

$$D(S, b'_S, g') = \begin{cases} 1 & \text{if} \quad g > \tau Y + m(b_S, b_L, y, \chi, b'_S, g'), \\ 0 & \text{otherwise} \end{cases}$$

Long-term Debt Rule:

- If $D(S, b'_S, g') = 0$ then

$$B_L(S, \omega, b'_S, g') = \begin{cases} \max \mathbb{B}_L(S, b'_S, g') & \text{if} \quad \omega = 1, \\ \min \mathbb{B}_L(S, b'_S, g') & \text{if} \quad \omega = 0 \end{cases}$$

- If $D(S, b'_S, g') = 1$, then $B_L(S, \omega, b'_S, g') = 0$

Set of possible debt values:

$$\mathbb{B}_L(S, b'_S, g') = \Big\{ b'_L : g + b_S + \kappa b_L = \tau Y + Q_s(y, \chi, b'_S, b'_L, g') b'_S$$
$$+ Q_L(y, \chi, b'_S, b'_L, g')(b'_L - (1-\delta) b_L) \Big\}$$

Markov Perfect Equilibrium

Definition A Markov perfect equilibrium consists of

- a value function for the government V
- a set of policies $\mathcal{B}_S, \mathcal{G}, B_L, D$
- and bond price functions Q_S, Q_L

such that:

i Given bond price function Q_S, Q_L, the policy functions $\mathcal{B}_S, \mathcal{G}, B_L, D$ and the value function V solve the Bellman equation.

ii Given government policies, the bond price functions Q_S, Q_L satisfies the non-arbitrage conditions.

Quantitative Analysis

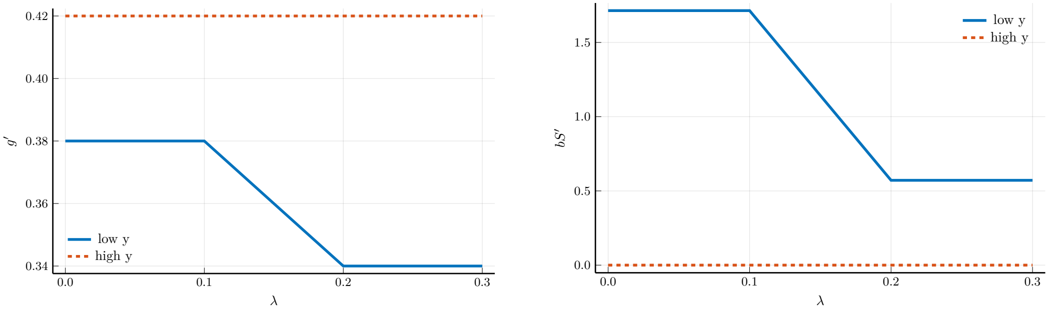
Calibration

Parameter	Value	Targets
Predetermined parameters		
σ	2	Conventional value
δ	0.033	Long-term bond duration
τ	0.41	Tax revenues over GDP
μ_y	-0.0002	GDP process
ρ_y	0.9668	GDP process
σ_y	0.008	GDP process
Calibrated parameters		
β	0.98	Method of simulated moments
ϕ	0.25	Method of simulated moments
g_D	0.026	Method of simulated moments

Empirical Targets

Moment	**Data**	**Model**
Average debt-to-GDP ratio (%)	87.9	58.4
Average spread (basis points)	61	52
Average debt maturity (years)	6.8	7.4

State-Dependent Fiscal and Debt Policies



- Preemptive austerity as in Conesa and Kehoe (2024)

European Debt Crises

Nonlinear State-Space System

Following Bocola and Dovis (2019):

$$Y_t = g(S_t) + \eta_t,$$
$$S_t = f(S_{t-1}, \epsilon_t)$$

- Y_t: vector of observables (detrended GDP, data counterpart to χ_t, maturity, spread)
- $S_t = [b_{St}, b_{Lt}, g_t, y_t, \chi_t, \lambda_t, \omega_t]$
- η_t : measurement errors, set variance of $\eta_{y,t}, \eta_{\chi,t}$ to zero
- ϵ_t : structural shocks

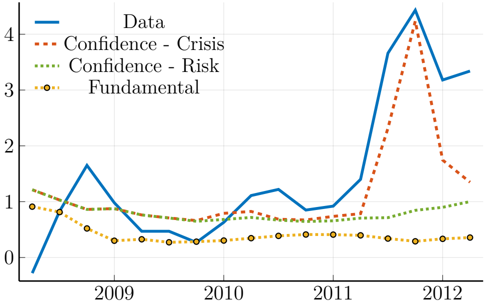
Particle Filter (Sequential Monte Carlo)

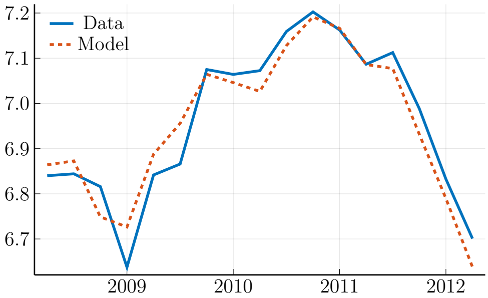
Goal: estimate unobserved states given the observed data

Algorithm: For $t = 1, \ldots, T$, we have N particles $\{S_{t-1}^{(k)}\}_{k=1}^{N}$:

1. **Prediction:** For each k, draw new shocks $(\lambda_t^{(k)}, \omega_t^{(k)})$ and combine with previous state to form $\{S_{t|t-1}^{(k)}\}_{k=1}^{N}$.
2. **Weighting:** For each k, use model to compute implied observables, $Y_t^{(k)} = g(S_{t|t-1}^{(k)})$. Assign weight based on likelihood of observed data, and normalize.
3. **Resampling:** Draw N new particles $\{S_t^{(k)}\}_{k=1}^{N}$ by sampling from $\{S_{t|t-1}^{(k)}\}_{k=1}^{N}$ with normalized weights.

Spread Decomposition





- Set $\omega_t = 0$ to obtain "good-sunspot component"
- The difference between observed and filtered is "unexplained"

Other Variables

Conclusion

- Proposed a Calvo-style model
  - endogenous maturity and fiscal policy
  - crises can occur without default
- Found a larger role for self-fulfilling risk during the Eurozone crisis
- Underlying crisis mechanism is critical for quantitative findings and has important implications for policy

Appendix

Stochastic Discount Factor (SDF)

The SDF, $M_{t,t+1}$, follows a log-normal process:

$$\log(M_{t,t+1}) = -(\phi_0 + \phi_1 \chi_t) - \frac{1}{2}\kappa_t^2 + \kappa_t \epsilon_{t+1}^{\chi},$$

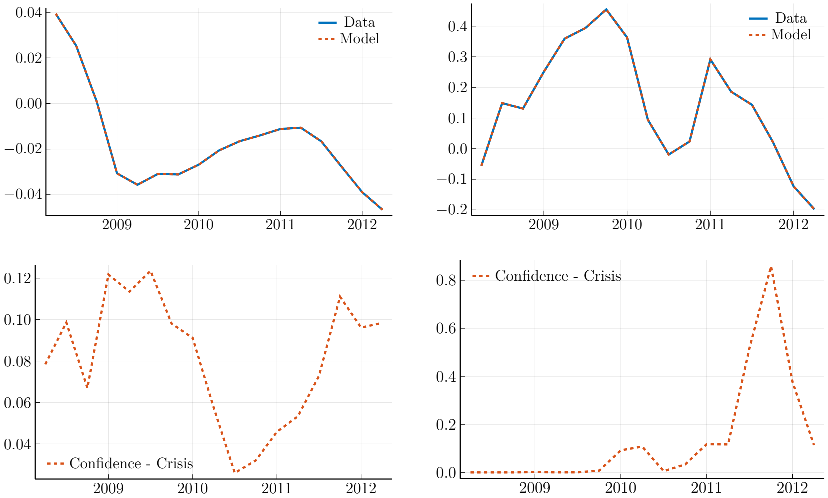
where κ_t depends on term premium shock, χ_t:

$$\kappa_t = \kappa_0 + \kappa_1 \chi_t$$

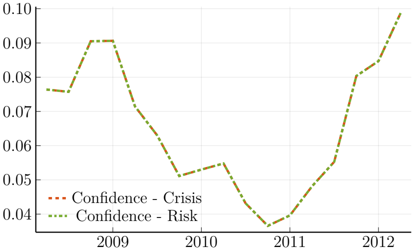
- A high realization of χ_t increases the market price of risk, κ_t
- This makes the SDF more volatile and increases the term premium investors demand for holding long-term debt

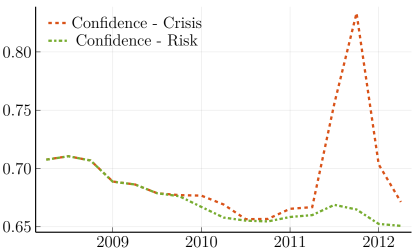
back

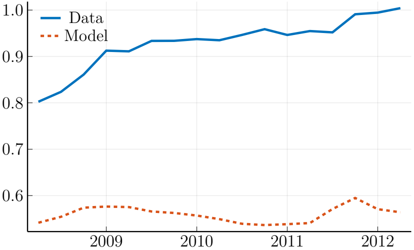
Other Variables



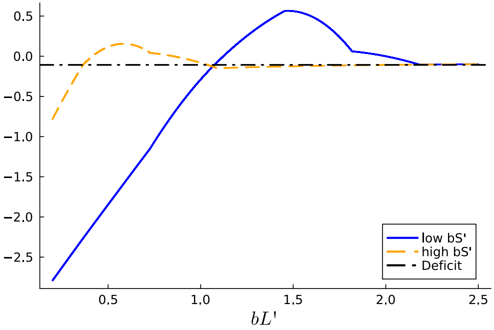
Other Variables Cont. ⏩ back

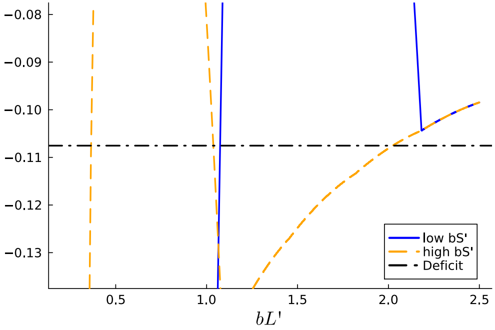






Laffer Curves





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