Optimal Maturity and Self-Fulfilling Crises*

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Abstract

This paper revisits the quantitative importance of self-fulfilling risk in the 2010−2012 European debt crisis. While a benchmark study by Bocola and Dovis (2019) finds a limited role using a Cole-Kehoe framework, we use a Calvo-style sovereign debt model. In our setting, a rollover crisis can occur without a government default—a mechanism more consistent with the empirical observation that countries like Italy never fully lost market access. Calibrating our model to pre-crisis Italy, we filter observed sovereign spreads. In sharp contrast to previous findings, our decomposition attributes a substantially larger role to shifts in investor beliefs, demonstrating that the specific modeling of the crisis mechanism is quantitatively critical.

Keywords: Sovereign default, financial repression

JEL classification: E42, F31, F32, F34, F41, P48.

*NOT FOR CIRCULATION.

1 Introduction

The European sovereign debt crisis of 2010-2012 saw dramatic spikes in government bond yields, with countries like Italy experiencing borrowing costs that threatened fiscal sustainability. Although European policymakers invoked a "confidence crisis" narrative to justify interventions, arguing that pessimistic investor beliefs had become self-reinforcing, recent quantitative work by Bocola and Dovis (2019) finds only a modest role for confidence risk in explaining Italian spreads. Their key insight is that a government's choice of debt maturity reveals its perception of risk. The authors argue Italy's decision to shorten its debt maturity during the crisis signals that the government perceived the risk of a confidence-driven default to be low.

This disagreement may arise from the choice of theoretical framework. The literature features two distinct approaches to modeling confidence in sovereign debt crises. The Cole and Kehoe (2000) framework, motivated by Mexico's 1994-95 experience, features a complete exclusion of the government from the bond market. In contrast, Calvo-style models allow for intermediate outcomes where markets remain open but price deteriorates sharply. This distinction matters empirically: while Mexico almost lost market access entirely, Italy continued issuing debt throughout the crisis, albeit at elevated yields. The persistent but expensive market access observed in Europe aligns more naturally with the Calvo mechanism.

Motivated by this fact, we revisit the role of confidence risk in the European crisis using a quantitative Calvo-style sovereign debt model based on Lorenzoni and Werning (2019). We enrich the benchmark model by allowing for an endogenous choice of debt maturity and fiscal policy. We show that allowing for the possibility that a confidence crisis materialized during the European debt crises implies a higher role for the confidence risk.

In our framework, shifts in investor sentiment can trigger self-reinforcing debt spirals. If lenders anticipate high future borrowing, they demand higher yields on long-term bonds, forcing the government to issue more debt to meet its budget and thus validating the pessimistic expectation. Such a dynamic allows a confidence crisis to materialize through soaring yields without causing an immediate default. This mechanism, we argue, better captures the European experience where the central question was not whether governments could issue debt, but at what price.

Governments in our model anticipate this possibility and can deploy two preemptive instruments. First, they can commit to future fiscal consolidation, a form of "preventive austerity" as in (Conesa and Kehoe (2024)), to reassure markets of their commitment to sustainability. Second, they can alter their debt maturity structure, reducing their exposure to confidence risk. This choice involves a crucial trade-off. Issuing more short-term bonds lessens the immediate reliance on the volatile long-term market, thereby lowering the probability of a confidence crisis in the current period. The cost, however, is increased future rollover pressure and a greater risk of subsequent crises. As noted by Lorenzoni and Werning (2019) among others, a longer maturity portfolio provides insurance against the future risk of self-fulfilling crises. Debt maturity thus becomes an active policy tool, and its observable changes provide a powerful signal of the government's perceived risk.

We calibrate the model to match key features of Italy's economy prior to the crisis and use a particle-filter approach to infer the sequence of unobserved shocks from the data. Our decomposition of sovereign spreads reveals a different picture from that of Bocola and Dovis (2019). Our model successfully matches both the evolution of the maturity choice of the government and the spike in the spreads over the Euro crises.

Two things are key to our result. First, an important restriction of a Cole and Kehoe (2000) framework for the case of Italy is that under Cole-Kehoe type of runs the confidence crises did not materialize. The realization of a confidence crisis implies a run that leads to an immediate default, which we did not observe in Italy. Therefore, the role of confidence risk is limited to the impact that changes in the probability of facing future crises have on today's bond prices. Instead, in our model, the confidence risk could materialize, without implying a default, manifesting instead as an increase in yields of government bonds. According to our estimation, this channel was an important factor in the increase in Italian spread during the crises.

Second, we model an increase in confidence risk as an increase in the probability of facing a confidence crisis in the current period. In our model, a higher confidence risk calls for a shorter maturity. This mechanism of our model helps us to reconcile the observation that the government shortened the maturity while the probability of a confidence crisis increases. The mechanism can also help rationalize the empirical evidence in Broner et al. (2013), according to which governments tend to reduce their maturity during debt crises.

Finally, we analyze the use of the two preemptive policies that the government has at hand. In our baseline calibration, the government primarily uses the maturity structure to reduce the risk of confidence crises. However, in some cases, it also uses the preemptive austerity as a complement, where the government tries to reduce the risk of a confidence crisis by both shortening the maturity and committing to lower future spending.

Beyond this re-evaluation of the Eurozone experience, our findings carry broader implications for sovereign debt management and fiscal policy design. They highlight that governments facing fragile market confidence may have incentives to reduce future spending or alter their maturity structures not solely because of fundamentals but to shape market expectations themselves. More generally, our results demonstrate that the quantitative conclusions drawn from self-fulfilling debt models are highly sensitive to how the crisis mechanism is modeled.

Related Literature. Our analysis is most closely related to the literature on multiplicity of equilibria in sovereign debt models and self-fulfilling crises, where investor beliefs can become a source of risk independent of fundamentals. Two main theoretical frameworks have emerged. The first, pioneered by Cole and Kehoe (2000), models crises as a sunspot-driven event where the government is completely excluded from credit markets if investors coordinate on a pessimistic equilibrium. The second, in the tradition of Calvo (1988), models crises as a "rollover crisis" where markets remain open but borrowing costs spike, potentially triggering a debt spiral. Our work adopts this second approach, building on the specific "slow moving debt crisis" framework of Lorenzoni and Werning (2019), as we argue it better captures the European experience where countries like Italy faced punishingly high yields but never fully lost market access.

A crucial element of our model is the government's ability to strategically choose its debt maturity structure. A rich literature has explored this choice. Arellano and Ramanarayanan (2012) show how governments trade off the insurance benefits of long-term debt against its higher cost, while Hatchondo et al. (2016) highlight the risk of debt dilution associated with long-term issuance. In these frameworks, maturity is not just a passive feature of the debt stock but an active policy tool used to manage rollover risk.

The most direct antecedent to our work is the influential quantitative study by Bocola and Dovis (2019). They integrate an endogenous maturity choice into a quantitative Cole-Kehoe-style model to analyze the European debt crisis. Their key finding is that self-fulfilling risk played only a modest role in explaining Italian spreads. This conclusion hinges on a powerful insight: the government's maturity choice reveals its perception of risk. They argue that because Italy shortened its debt maturity during the crisis, the government must have perceived the risk of a confidence-driven crisis to be low. In their framework, facing a high probability of a *future* crisis would incentivize the government to issue long-term debt. This presents a significant challenge to the "confidence crisis" narrative.

This paper resolves the tension identified by Bocola and Dovis (2019). We show that within a Calvo-style framework where crisis risk is *imminent* rather than distant, a government's optimal response to high perceived risk is to shorten its debt maturity, thus reconciling Italy's actions with the confidence crisis narrative.

Outline. The paper is organized as follows. Sections 2 and 3 present our Calvo-style sovereign debt model, detailing the government's problem, investor bond pricing, and the equilibrium rollover crisis mechanism. Section 4 details the quantitative analysis, including our calibration strategy and the filtering methodology used to decompose sovereign spreads. Section 5 presents our main findings, comparing our decomposition of Italian sovereign spreads with the reference in the literature. Section 6 concludes.

2 Model

We consider a small open economy with discrete time and an infinite horizon. The periods are indexed by $t \in \{0, 1, 2, ...\}$. The economy is populated by the benevolent government and a continuum of risk-averse foreign investors. We assume that the government chooses its spending one period in advance and faces stochastic tax revenue.

The government faces the risk of liquidity defaults. Given the level of spending inherited from the last period, the government defaults whenever it cannot raise enough revenue from financial operations to satisfy its budget constraint.

We follow Lorenzoni and Werning (2019) timing, assuming that the government cannot adjust spending in response to changes in the price of government bonds, leading to multiple equilibria.

The timing is as follows. At the beginning of each period, the government generates tax revenues, which are a constant share τ of the output produced in the economy, Y_t. Given its revenue, the government chooses next-period spending g_{t+1} and the position of short-term debt b_{St+1}. Next, the sunspot variable $\omega_t \in \{0, 1\}$ is realized, and the lenders bid on the price of long-term debt. Given the price, the government issues new long-term debt b_{Lt+1} or defaults $d_t = 1$, at the end of the period.

Long-term debt issuance and default are not choices of the government. The government's only choices are spending for the next period g_{t+1} and short-term debt b_{St+1}. Given these choices, if there is no long-term debt level that can clear the budget, the government defaults and enters the next period with zero debt. Otherwise, it has to issue the amount of debt necessary to satisfy the budget.

2.1 Stochastic Processes

Uncertainty in the model is driven by two components of exogenous states: *fundamental* and *non-fundamental* (or "confidence") shocks.

First, fundamental risk is determined by a vector of economic shocks $\{y_t, \chi_t\}$, where $y_t = \log(Y_t)$ is log-output and χ_t is a term premium shock. These two shocks are correlated and follow a joint Vector Autoregression of order 1 (VAR(1)) process. Tax revenues in the model are a constant fraction of output, Y_t, and are therefore driven by the fundamental shock y_t.

Second, the government faces confidence risk, which arises from the possibility of multiple equilibria in financial markets. This risk is governed by two related shocks. At the beginning of each period, a shock λ_t is drawn from a fixed discrete set Λ. This shock, λ_t, represents the probability that investors will coordinate on a ”bad” equilibrium. Following this, a sunspot variable, $\omega_t \in \{0, 1\}$, is realized. With probability λ_t, a bad sunspot occurs ($\omega_t = 1$), leading to low bond prices. Conversely, with probability $1-\lambda_t$, a good sunspot occurs ($\omega_t = 0$), corresponding to high bond prices.

2.2 Government

The government values a stochastic stream of spending according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{g_t^{1-\sigma}}{1-\sigma},$$

where $\beta \in (0, 1)$ is the discount factor, σ is the risk-aversion parameter and $\mathbb{E}_t$ denotes the expectation operator conditional on the information set available at time t.

As in Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016), the government can issue short- and long-term bonds. Short-term bond is a one-period discount bond, specifying a price q_{St} and a loan face value b_{St+1} such that the borrower receives $q_{St}b_{St+1}$ units of goods in period t and promises to pay, conditional on not defaulting, b_{St+1} units of goods in period $t + 1$. Long-term bonds have geometrically decreasing coupons. A unit of a long-term bond issued this period schedules an infinite stream of debt service payments, starting next period, given by

$$\kappa, (1-\delta)\kappa, (1-\delta)^2\kappa, ...$$

where $\kappa > 0$ and $\delta \in (0, 1)$. Long-term debt dynamics can be represented by the following law of motion:

$$b_{Lt+1} = (1-\delta)b_{Lt} + i_t$$

where b_{Lt} is the stock of long-term bonds at time t, and i_t is the amount of long-term bonds issued in period t. The government issues these bonds at a price q_{Lt}, which in equilibrium depends on the government's portfolio and spending decisions.

The government begins every period with a portfolio $\{b_{St}, b_{Lt}\}$, a predetermined level of spending g_t, and some tax revenue τY_t. Under repayment, the government's budget constraint is

$$g_t + b_{St} + \kappa b_{Lt} = \tau Y_t + q_{St} b_{St+1} + q_{Lt}(b_{Lt+1} - (1-\delta) b_{Lt}).$$

The government finances its spending g_t, payments on outstanding debt $b_{St} + \kappa b_{Lt}$ with tax revenue τY_t and new issuances of short- and long-term debt $q_{St} b_{St+1} + q_{Lt} i_t$.

We assume that the government honors its debt whenever possible, so that default occurs only if the tax revenue and new borrowing are insufficient to cover spending and refinance outstanding debt. Let $d_t = 1$ denote default and $d_t = 0$ denote repayment. If the government defaults, its spending drops to $g_D < \tau Y_{\min}$. We also assume that after a default event both short- and long-term bondholders receive a fraction $\phi \in [0, 0.5]$ of the government's net surplus in default state as a recovery value:

$$v_t = \phi(\tau Y_t - g_D),$$

which is divided equally among all bondholders. Our specification implies that default is costly because of lower public spending.

2.3 International Investors

The lenders value flows using the stochastic discount factor $M_{t,t+1}$. Hence, the value of a stochastic stream of payments $\{p_t\}_{t=0}^{\infty}$ at time zero is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t} d_t,$$

where $M_{0,t} = \Pi_{j=0}^{t} M_{j-1,j}$. Given a default policy of the government, the non-arbitrage condition of international investors implies:

$$q_{St} = \mathbb{E}_t \left[M_{t,t+1} \left\{ (1 - d_{t+1}) + d_{t+1} \frac{v_{t+1}}{b_{St+1}} \right\} \right],$$

$$q_{Lt} = \mathbb{E}_t \left[M_{t,t+1} \left\{ (1 - d_{t+1})(\kappa + (1-\delta) q_{Lt+1}) + d_{t+1} \frac{v_{t+1}}{b_{Lt+1}} \right\} \right].$$

3 Recursive Government Problem

We consider equilibria with a Markovian structure. We denote $S = (b_S, b_L, g, y, \chi)$ for notational convenience.

3.1 Value Function

The value of the government is given by

$$V(S,\lambda) = \max_{b'_S, g'} \left\{ (1-d)u(g) + d(u(g_D)) + \beta \mathbb{E}_{\omega, y', \chi', \lambda'}[V(S', \lambda')] \right\} \tag{1}$$

$$s.t. \quad b'_L = B_L(S, \omega, b'_S, g'), \quad d = D(S, b'_S, g').$$

The solution to the government problem yields decision rules for short-term debt $\mathcal{B}_S(S,\lambda)$ and spending $\mathcal{G}(S,\lambda)$. Given these decision rules, the default function $D(S, b'_S, g')$ and the long-term debt function $B_L(S, \omega, b'_S, g')$ are derived from the liquidity constraint and the fiscal budget. In a rational expectations equilibrium (defined below), lenders use these decision rules to price debt contracts. If the government defaults on some choices of b'_S and g', we assume that the choice of g' stay the same, and $b'_S = 0$.

3.2 Price Functions

To be consistent with lenders' portfolio conditions, the bond price schedules must satisfy

$$Q_L(y, \chi, b'_S, b'_L, g') = \mathbb{E}\left[M(y, \chi, y', \chi') \left\{ (1-d')(\kappa + (1-\delta) Q_L(b''_S, b''_L, g'', \lambda'')) + d' \frac{v'}{b'_L} \right\} \right], \tag{2}$$

$$Q_S(y, \chi, b'_S, b'_L, g') = \mathbb{E}\left[M(y, \chi, y', \chi') \left\{ (1-d') + d' \frac{v'}{b'_S} \right\} \right], \tag{3}$$

where $d' = D(S', b''_S, g'')$, $b''_S = \mathcal{B}_S(S', \lambda')$, $g'' = \mathcal{G}(S', \lambda')$, $b''_L = B_L(S', \omega', b''_S, g'')$, and $v' = \phi(\tau Y' - g_D)$.

3.3 Default Rule

Given the bond price functions, we define the Laffer curve from debt issuance to be

$$\mathcal{L}(b_S, b_L, y, \chi, b'_S, b'_L, g') = Q_L(y, \chi, b'_S, b'_L, g')(b'_L - (1-\delta)b_L) + Q_S(y, \chi', b'_S, b'_L, g')b'_S - b_S - \kappa b_L. \tag{4}$$

The government's net financial revenue comes from new issuances of short- and long-term debt less outstanding obligations. We also define the maximum possible revenue as follows:

$$m(b_S, b_L, y, \chi, b'_S, g') = \max_{\overline{b_L}} \left\{ Q_L(y, \chi, b'_S, \overline{b_L}, g')(\overline{b_L} - (1-\delta)b_L) + Q_S(y, \chi, b'_S, \overline{b_L}, g')b'_S - b_S - \kappa b_L \right\}. \tag{5}$$

The default function of the government can be characterized using $m(b_S, b_L, \chi, b'_S, g')$ as follows:

$$D(S, b'_S, g') = \begin{cases} 1 & \text{if} \quad g > \tau Y + m(b_S, b_L, y, \chi, b'_S, g'), \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

3.4 Long-Term Debt Rule

We define $B_L(S, \omega, b'_S, g')$ as the equilibrium long-term debt as a function of the relevant states. Whenever $D(S, b'_S, g') = 1$, the government is excluded from the financial markets, so we have $B_L(S, \omega, b'_S, g') = 0$. Conversely, whenever $D(S, b'_S, g') = 0$, the function $B_L(S, \omega, b'_S, g')$ selects the equilibrium debt from the candidate b'_L that satisfy the government's budget constraint. Formally we define:

$$\mathbb{B}_L(S, b'_S, g') = \left\{ b'_L : g + b_S + \kappa b_L = \tau Y + Q_s(y, \chi, b'_S, b'_L, g')b'_S + Q_L(y, \chi, b'_S, b'_L, g')(b'_L - (1-\delta)b_L) \right\}. \tag{7}$$

This set contains all possible debt values that can be part of the equilibrium for any initial state. For some initial portfolios, $\mathbb{B}_L$ contains only one value, in which case the equilibrium is unique. However, the set could contain multiple values for some combinations of debt and reserves.

Formally, the long-term debt rule is:

$$B_L(S,\omega,b'_S,g') = \begin{cases} \max \mathbb{B}_L(S,b'_S,g') & \text{if} \quad D(S,b'_S,g')=0 \quad \text{and} \quad \omega=1, \\ \min \mathbb{B}_L(S,,b'_S,g') & \text{if} \quad D(S,b'_S,g')=0 \quad \text{and} \quad \omega=0, \\ 0 \quad \text{otherwise.} \end{cases} \tag{8}$$

3.5 Markov Equilibrium

Now we are ready to define the equilibrium concept

Definition 1 (Markov Perfect Equilibrium)**.** Given an initial state $S_0 = (b_S, b_L, g, y, \chi)$ and the law of motion for the fundamentals and the risk of confidence, a perfect Markov equilibrium consists of a value function for the government V, policy functions B_S, G, B_L, D, and bond price functions Q_S, Q_L, such that:

i) Given the price functions of bonds (Q_S, Q_L), the policy functions (B_S, G, B_L, D) and the value function V solve the Bellman problem of the government:

$$V(S,\lambda) = \max_{b'_S, g'} \left\{(1-d)u(g) + d\,u(g_D) + \beta\,\mathbb{E}_{\omega,y',\chi',\lambda'}\left[V(S',\lambda')\right]\right\}$$

subject to budget constraints, liquidity/default restrictions $D(S,b'_S,g') \in \{0,1\}$, and long-term debt selection rule $B_L(S,\omega,b'_S,g')$ implied by budget feasibility.

ii) Given government policies and default rule, bond price functions satisfy the lenders' non-arbitrage (pricing) conditions:

$$Q_S(y,\chi,b'_S,b'_L,g') = \mathbb{E}\left[M(y,\chi,y',\chi')\left\{(1-d') + d'\,\frac{v'}{b'_S}\right\}\right],$$

$$Q_L(y,\chi,b'_S,b'_L,g') = \mathbb{E}\left[M(y,\chi,y',\chi')\left\{(1-d')\big(\kappa + (1-\delta)Q_L(S'')\big) + d'\,\frac{v'}{b'_L}\right\}\right],$$

where $d' = D(S', B_S(S',\lambda'), G(S',\lambda'))$, $S'' = (b''_S, b''_L, g'', y', \chi')$, with $b''_S = B_S(S',\lambda')$, $g'' = G(S',\lambda')$, and $b''_L = B_L(S',\omega',b''_S,g'')$.

iii) If $D(S,b'_S,g')=0$, the long-term debt chosen from the budget-feasible set $B_L(S,b'_S,g')$

is selected by the sunspot ω:

$$B_L(S, \omega, b'_S, g') = \begin{cases} \max \mathbb{B}_L(S, b'_S, g'), & \omega = 1, \\ \min \mathbb{B}_L(S, b'_S, g'), & \omega = 0, \end{cases}$$

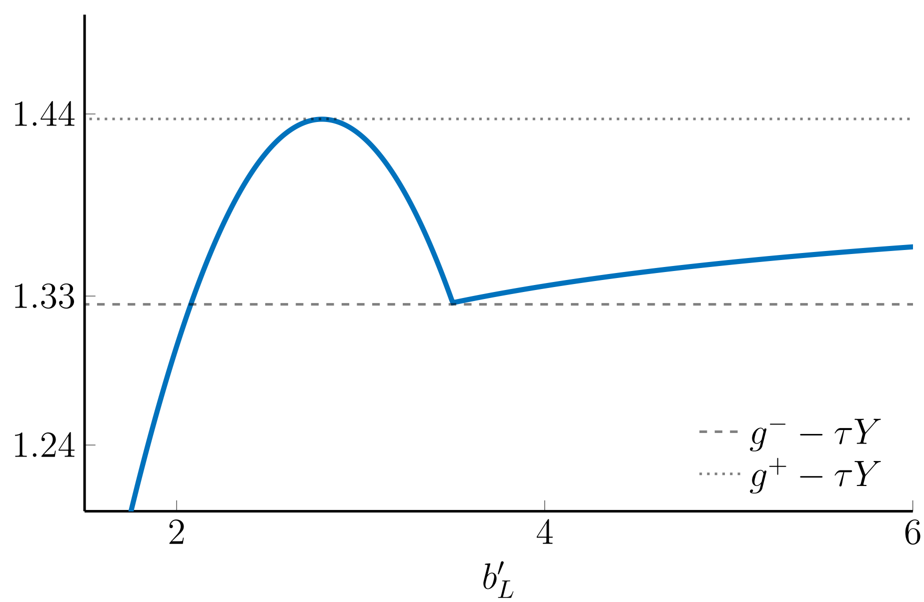
and $B_L(S, \omega, b'_S, g') = 0$ if $D(S, b'_S, g') = 1$.

3.6 The Laffer Curve and Equilibrium Multiplicity

This section explains the core mechanism that generates multiplicity in our model. We show that the government's ability to raise revenue from long-term debt issuance is a non-monotonic function of its total debt burden. This non-monotonicity, which we refer to as the Laffer curve, creates a region where investor expectations can be self-fulfilling.

The Laffer curve, $\mathcal{L}(\cdot)$ defined in equation (4), plots the total revenue the government raises from issuing new short- and long-term bonds net of its liabilities against the level of long-term debt it attempts to issue. Figure 1 illustrates a typical shape for this curve.

Figure 1: Laffer Curve and Equilibrium Regions



On the initial upward-sloping part, when debt is low, increasing the amount of debt raises total revenue. Markets are not worried, so the price is high and stable. However, after a certain point, the curve becomes downward-sloping. As the government tries to issue more debt, the perceived risk of a rollover crisis increases, causing the bond price to collapse. This price collapse is so severe that it overwhelms the increase in the quantity of debt, causing total government revenue to fall, even as the face value of issued debt rises.

As in Lorenzoni and Werning (2019), this non-monotonic shape is driven by the interaction of three forces: (i) the increase in the debt stock; (ii) the fall in debt prices, which reduces the value of new bonds (b'_L and b'_S); and (iii) the price decrease's impact on the value of old bonds $(1-\delta)b'_L$. The first and third effects increase net revenue, while the second decreases it. This interaction can generate the "multi-peaked" Laffer curve shown in Figure 1.

This Laffer curve structure is what allows for multiple equilibria. An equilibrium is an intersection between the revenue the government can raise (the Laffer curve) and the revenue it needs to raise (its net deficit, $D(\cdot) = g - \tau Y$). As the figure shows, for a range of deficits, multiple intersections are possible. Formally, we characterize the state space as follows:

Definition 2 (Regions)**.** Let $S = (b_S, b_L, g, y, \chi)$ be the initial states at the beginning of the period. Given a choice of (b'_S, g'), the state space can be partitioned into the following regions:

$$\mathcal{S}_{\mathcal{L}} = \left\{S : B_L(S, \omega, b'_S, g') = \mathbb{B}_L(S, b'_S, g'), F(B_L) < 1\right\}$$
$$\mathcal{S}_{\mathcal{M}} = \left\{S : B_L(S, \omega = 0, b'_S, g') = \max \mathbb{B}_L(S, b'_S, g'), F(B_L) = 1; \right.$$
$$\left. B_2(S, \omega = 1, b'_S, g') = \min \mathbb{B}_L(S, b'_S, g'), F(B_L) < 1\right\}$$
$$\mathcal{S}_{\mathcal{H}} = \left\{S : B_L(b_1, a_1, \omega_1) = \mathbb{B}_L(S, b'_S, g'), F(B_L) = 1\right\}$$

This construction partitions the initial state space into three regions. To describe these regions, we use current spending g as our illustrative state variable, holding all others constant. This implies the net deficit $D(g)$ is an increasing function of g. We define two critical thresholds for this deficit, g^+ and g^-, corresponding to the local peak and local trough of the Laffer curve, respectively.

In the Low-Debt Region $\mathcal{S}_{\mathcal{L}}$, where $g < g^-$, the net deficit intersects the Laffer curve only once, at a low b^*_L on the first increasing portion. Equilibrium is unique and characterized by low long-term debt issuance and high bond prices. In the High-Debt Region $\mathcal{S}_{\mathcal{H}}$, where $g > g^+$, the equilibrium is also unique buy features high long-term debt issuance, low bond prices, and default probability of one in the next period. Finally, the Multiplicity Region $\mathcal{S}_{\mathcal{M}}$, where $g^- < g < g^+$, allows for multiple equilibria[1]. At these intermediate deficit levels, two stable equilibria

[1]Following Lorenzoni and Werning (2019), we discard the equilibrium on the locally decreasing portion of the Laffer curve as unstable.

are possible: one with low borrowing and high prices and another with high borrowing and low prices.

A key result, formally proven in Appendix A, is that the Laffer curve is always increasing once b'_L is sufficiently large to make default next period certain. This ensures the existence of the stable, high-debt "crisis" equilibrium that forms the basis for $\mathcal{S}_{\mathcal{M}}$ and $\mathcal{S}_{\mathcal{H}}$.

Lemma 1. *In Multiplicity Region $\mathcal{S}_{\mathcal{M}}$, there always exist two stable equilibria.*

Lemma 1, also formally proven in Appendix A, is crucial. It guarantees that as the government faces a higher deficit, the possibility of a "bad" equilibrium—characterized by high debt issuance and low prices—is always present. This prevents a perverse incentive: without this co-existing bad equilibrium, a government might be tempted to commit to irresponsibly high spending, knowing that doing so might to a unique, low-debt equilibrium. The guaranteed existence of a stable bad equilibrium in the Multiplicity Region ensures that such a gamble is never risk-free.

4 Quantitative Analysis

To assess the quantitative importance of self-fulfilling risk within our Calvo-style framework, we calibrate the model to match key features of the Italian economy prior to the 2010-2012 crisis. Our quantitative analysis closely follows the parameterization strategy in Bocola and Dovis (2019), an approach that allows us to compare our results directly to theirs. We then use this calibrated model to filter the observed data on Italian sovereign spreads and decompose them into components attributable to economic fundamentals and to shifts in investor beliefs.

4.1 Calibration

We calibrate the model's parameters in two steps. First, a set of parameters is predetermined based on conventional values in the literature or direct empirical counterparts. Second, the remaining parameters are jointly calibrated using the method of simulated moments to match key long-term averages of the Italian economy.

Fundamental Risk The fundamental shocks to the economy are logarithmic output, $y_t = \log(Y_t)$, and a term premium shock, χ_t. These follow a correlated VAR(1) process:

$$\begin{bmatrix} y_{t+1} \\ \chi_{t+1} \end{bmatrix} = \begin{bmatrix} (1-\rho_y)\mu_y \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_y & 0 \\ 0 & \rho_\chi \end{bmatrix} \begin{bmatrix} y_t \\ \chi_t \end{bmatrix} + \begin{bmatrix} \epsilon^y_{t+1} \\ \epsilon^\chi_{t+1} \end{bmatrix},$$

where $[\epsilon^y, \epsilon^\chi]'$ are normally distributed with a covariance matrix Σ. The parameters of the GDP process—its mean (μ_y), persistence (ρ_y), and standard deviation (σ_y)—are estimated directly from detrended Italian GDP data.

The Stochastic Discount Factor (SDF) of risk-averse international investors, $M_{t,t+1}$, is modeled as a log-normal process that depends on the term premium shock.

$$\log(M_{t,t+1}) = -(\phi_0 + \phi_1 \chi_t) - \frac{1}{2}\kappa_t^2 + \kappa_t \epsilon_{\chi t+1},$$

where the market price of risk, κ_t, is itself a function of the shock:

$$\kappa_t = \kappa_0 + \kappa_1 \chi_t.$$

A high realization of χ_t increases the market price of risk, making the SDF more volatile and increasing the demand for long-term long-term debt premiums by investors.

The remaining parameters—the government's discount factor β, the recovery rate parameter ϕ, and the level of spending under default g_D—are calibrated jointly to match three empirical targets from the Italian data: the average debt-to-GDP ratio, the average sovereign spread. and the average maturity of the debt. In Appendix C we provide details on the definitions of debt and maturity that we use in the calibration.

4.2 Baseline

The baseline parameters are detailed in Table 1. The utility curvature parameter, σ, is set to 2, a standard value in macroeconomic models. The parameter governing the decay of the long-term bond, δ, is set to 0.033, which implies an average duration for long-term government debt consistent with the data. The tax rate, τ, is set at 0.41, which corresponds to the average ratio of tax revenues to GDP in Italy.

Table 1: Model Parameters

Parameter	**Value**	**Targets**
Predetermined parameters		
σ	2	Conventional value
δ	0.033	Long-term bond duration
τ	0.41	Tax revenues over GDP
μ_y	-0.0002	GDP process
ρ_y	0.9668	GDP process
σ_y	0.008	GDP process
Calibrated parameters		
β	0.98	Method of simulated moments
ϕ	0.25	Method of simulated moments
g_D	0.026	Method of simulated moments

The other parameters are calibrated to match the moments detailed in Table 2. We close match the spread and the maturity, which are the main variables of interest of the paper, but the debt/GDP is lower in the model compared to the data.

Table 2: Empirical Targets: Data v.s. Model

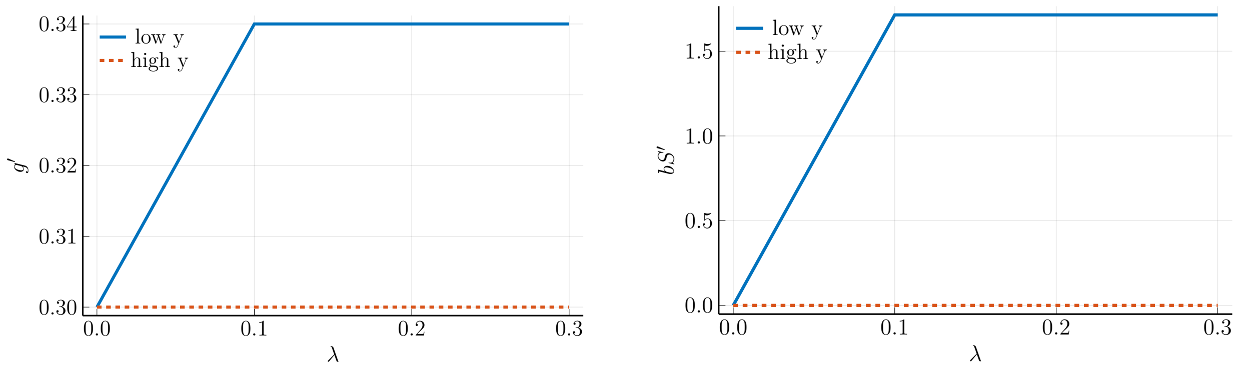
Moment	**Data**	**Model**
Average debt-to-GDP ratio (%)	87.9	57.8
Average spread (basis points)	61	62
Average debt maturity (years)	6.8	7.3

4.3 Policy Functions

Before analyzing the behavior of sovereign spreads and debt maturity during the crisis, we first study the government's strategic policy choices. We examine how optimal next-period spending g' and short-term debt issuance bS' vary with exogenous states: output y and sunspot probability λ.

Figure 2 plots the policy functions for next-period spending g' and short-term debt issuance bS' in dimensions y and λ, keeping all other states constant. For this particular state, when output is high (high state y), both planned government spending and short-term debt issuance are insensitive to sunspot probability λ. However, in the low y state, the policy changes significantly: as the probability of a future crisis increases, the government commits to *shorten* its portfolio maturity and *increase* its future spending simultaneously. This behavior suggests that the government opts to use debt maturity as a primary lever when facing the risk of a self-fulfilling crisis.

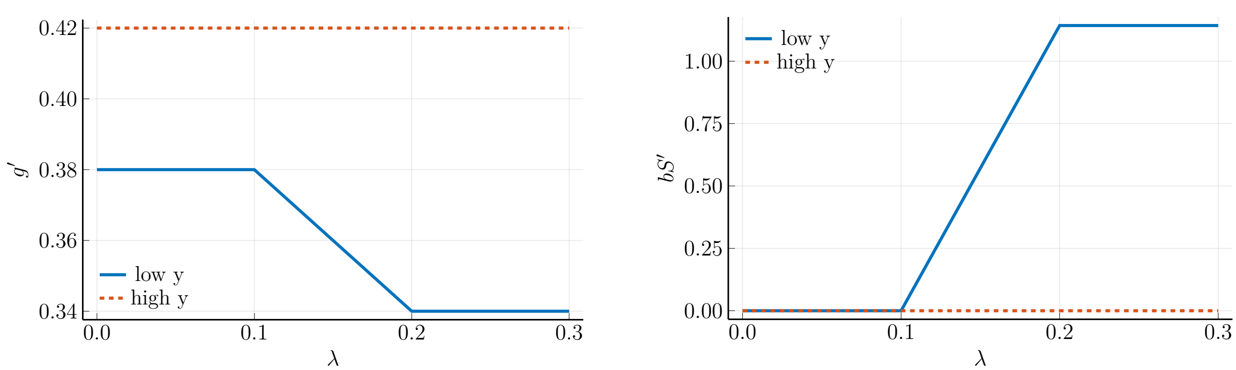
Figure 2: Policy Functions of g' (increasing) and bS' (increasing)



Although committing to higher future spending amid rising crisis risk may seem counterintuitive, our model suggests a clear mechanism. By aggressively shortening its debt maturity, the government so powerfully mitigates the immediate rollover risk that it creates the policy space to credibly promise future stimulus. In this state, managing maturity is such an effective tool that it not only contains the crisis risk, but also allows for expansionary future commitments.

However, in some other states, debt maturity alone is not enough. As Figure 3 illustrates, for this particular state, the government keeps both g' and bS' constant in the high y state for all λ. But in the low y state, as the probability of a future crisis increases, the government commits to *lower* future spending in addition to *shortening* its portfolio maturity. This state-dependent result illustrates a "preemptive austerity" mechanism (Conesa and Kehoe (2024)) and shows that the government uses two distinct policy levers to manage self-fulfilling risk: it can commit to future austerity (lower g') and reduce its immediate rollover exposure (lower bS'). Our model shows that these tools can be complements and that their use is highly state-dependent.

Figure 3: Policy Functions of g' (decreasing) and bS' (increasing)



5 Decomposing Italian Spreads

To use our model to interpret the data from the European debt crisis, we must infer the evolution of the unobserved state variables, particularly the sunspot shock ω_t, which captures changes in investor sentiment. Following Bocola and Dovis (2019), we cast our model as a non-linear state-space system. We used a Sequential Monte Carlo (SMC) particle filter to estimate the sequence of unobserved states given the observed data.

5.1 The State-Space Representation

The system consists of a vector of observable variables, X_t, and a vector of state variables S_t:

$$X_t = g(S_t) + \eta_t,$$
$$S_t = f(S_{t-1}, \epsilon_t).$$

Based on our model and the filtering implementation, these are:

- **Observables** X_t: The vector of observable data at time t contains detrended GDP (y_t), the term premium shock (χ_t), the debt maturity, and the sovereign spread.
- **State Vector** S_t: The state at time t includes all the variables necessary to describe the economy. This includes fundamentals observed at t (y_t, χ_t), the unobserved exogenous shocks (λ_t, ω_t), and the endogenous state variables determined in the previous period (b_{St}, b_{Lt}, g_t).

The functions $g(\cdot)$ and $f(\cdot)$ in the standard state-space form are defined by our model's complex, non-linear equilibrium conditions and policy functions. In Appendix D, we provide details on the algorithm.

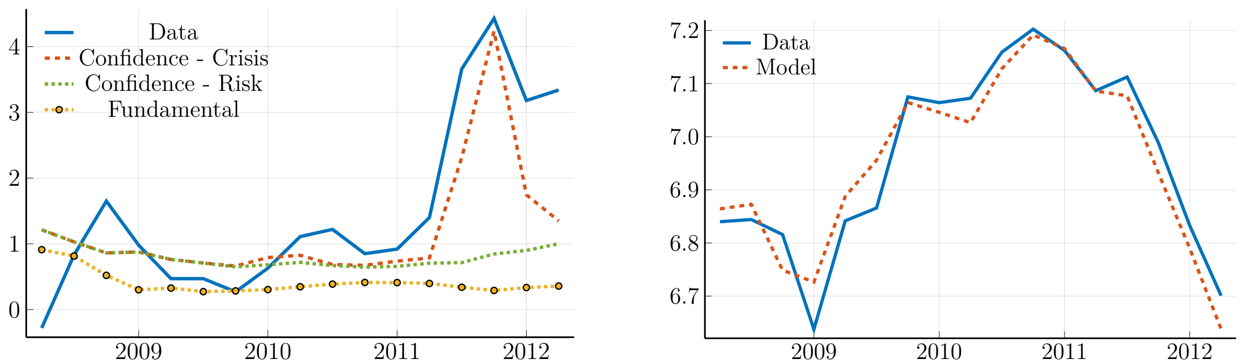
5.2 Spread Decomposition

This filtering procedure provides us with a time series of estimated distributions for the unobserved states, including the probability of a "bad" sunspot realization ($\omega_t = 1$) for each period.

The main goal is to decompose the observed Italian sovereign spread into two components: one driven by economic fundamentals and another driven by self-fulfilling beliefs. In particular, we decompose the spread into three components. "Fundamental", which captures the contribution of fundamentals alone. The "Confidence Risk" is then the contribution of the increased probability of a confidence crisis (λ). And "Confidence Crises" which captures the contribution of the "bad" realization of the sunspot (ω). This decomposition allows us to quantify the extent to which the increase in Italian borrowing costs can be attributed to a shift in investor sentiment, as captured by our Calvo-style crisis mechanism, rather than to a deterioration in fundamentals.

In Figure 4, we present the main result of the filter. Our estimation captures closely follows the evolution of the maturity and the spike in the government spread. Consistent with Bocola and Dovis (2019), we find a modest effect of fundamental risk during crises.

Figure 4: Italian Crisis



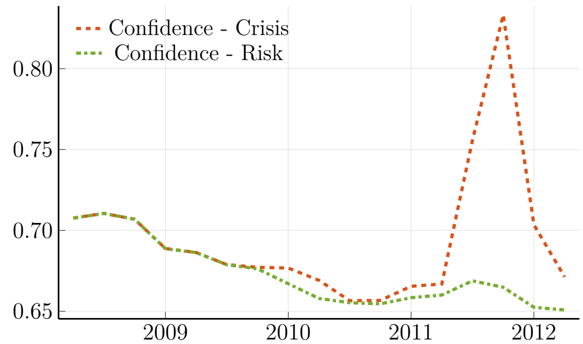
(a) Spread

(b) Maturity

Consistent with Bocola and Dovis (2019), the effect of the increased probability of a crisis is also small. Therefore, the most important factor to explain the increase in the spreads is the "bad" realization of the sunspot. Consistent with arguments used by policymakers during the crises, according to the model, nervous investors were pressuring the spreads of Italy up during the European debt crisis, which in turn made the government to increase their debt given support for those high interest rates that we observed in the data.

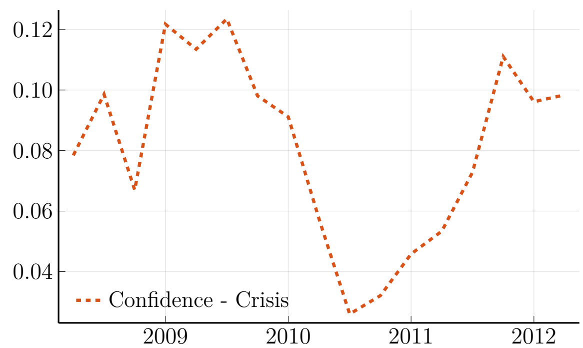
To further investigate the dynamic of the model during the crises in Figure 5, we plot the debt dynamics in the model. Panels (a) and (b) show the evolution of short term and long term bonds, respectively. The shorten in maturity during the crises was the combination of two factors, one the government was issuing more short term bonds during the crisis, consistent with the intuition that in response to higher confidence risk (panel (c)). In addition, the government issue more long term bonds. In panel (c) we compare the model implied issuance of long term bonds when the realization of the sunspot is zero to the baseline case; we can see that the realization of the confidence crises (panel (d)), implied that the government had to increase their issuance of long term bonds to clear the budget which contributed to longer the maturity. The implication is that the increase of the probability of a bad equilibrium contributed to explain the changes in maturity along the crises while the realization of the sunspot contributed to explain the changes in the spreads.

Figure 5: Debt Structure and Sunspot Dynamics

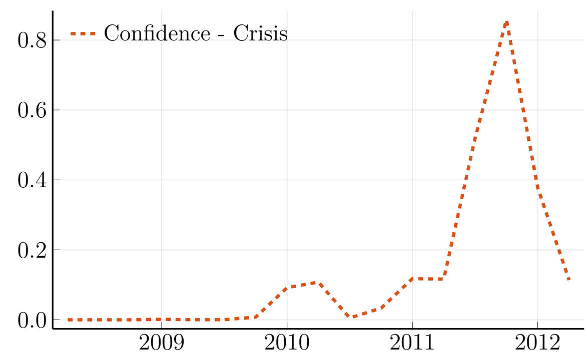


(a) Long-Term Debt

(b) Short-Term Debt



(c) Sunspot Probability



(d) Sunspot Realization

6 Conclusions

This paper revisits the quantitative importance of self-fulfilling risk in the European sovereign debt crisis. We find that shifts in investor beliefs played a substantially larger role in driving Italian sovereign spreads than previously concluded by benchmark studies. This contrasting result stems from our use of a Calvo-style sovereign debt model, which, consistent with the European experience, allows for crises to manifest as costly debt spirals without immediate market exclusion. Crucially, our framework resolves the puzzle of Italy's debt maturity choice: we show that when crisis risk is imminent, the government's optimal policy is to shorten its maturity to avoid locking in punitively high long-term yields. This reinterprets Italy's observed actions not as evidence of low perceived risk, but as a rational response to a high and immediate threat.

Our findings carry broader implications for both policy and future research. They highlight that governments facing fragile market confidence have active policy levers—such as committing to future austerity or altering maturity structures—to manage expectations. In this environment, debt maturity is not merely a signal of risk perceptions but an active tool for mitigating self-fulfilling shocks. More generally, our results demonstrate that the quantitative conclusions drawn from self-fulfilling debt models are highly sensitive to how the crisis mechanism is modeled. The choice between a Cole-Kehoe market exclusion and a Calvo-style debt spiral is not a theoretical formality but a decision with first-order implications for attributing risk, underscoring the need to carefully align our models with the empirical realities of a crisis.

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A Proofs

Lemma 2. *Let $\bar{b}_L$ be the level of long-term debt such that for all $b'_L \geq \bar{b}_L$, the probability of default in the next period is one (i.e., $d'(\cdot) = 1$). For all $b'_L \geq \bar{b}_L$, the Laffer curve $\mathcal{L}(\cdot)$ is strictly increasing in b'_L.*

Proof. When bL' is sufficiently large such that the default probability $d' = 1$, the price of new bonds is determined by the expected discounted recovery value, $\mathbb{E}\Big[M(\cdot)v'\Big]$. The Laffer curve becomes

$$\mathcal{L}(\cdot) = Q_L(y, \chi, b'_S, b'_L, g')(b'_L - (1-\delta)b_L) + Q_S(y, \chi', b'_S, b'_L, g')b'_S - b_S - \kappa b_L \tag{9}$$

$$= \left(\frac{\mathbb{E}\Big[M(\cdot)v'\Big]}{b'_L}\right)(b'_L - (1-\delta)b_L) + \left(\frac{\mathbb{E}\Big[M(\cdot)v'\Big]}{b'_S}\right)b'_S - b_S - \kappa b_L \tag{10}$$

$$= \mathbb{E}\Big[M(\cdot)v'\Big] - \mathbb{E}\Big[M(\cdot)v'\Big](1-\delta)\frac{b_L}{b'_L} + \mathbb{E}\Big[M(\cdot)v'\Big] - b_S - \kappa b_L \tag{11}$$

$$= 2\mathbb{E}\Big[M(\cdot)v'\Big] - \mathbb{E}\Big[M(\cdot)v'\Big](1-\delta)\frac{b_L}{b'_L} - b_S - \kappa b_L \tag{12}$$

Taking the derivative with respect to b'_L, and noting that $\mathbb{E}\Big[M(\cdot)v'\Big]$, b_S, b_L, and κ are constant with respect to b'_L in this region:

$$\frac{\partial \mathcal{L}(\cdot)}{\partial b'_L} = \frac{\partial}{\partial b'_L}\left(-\mathbb{E}\Big[M(\cdot)v'\Big](1-\delta)\frac{b_L}{b'_L}\right) = \mathbb{E}\Big[M(\cdot)v'\Big](1-\delta)\frac{b_L}{(b'_L)^2} > 0. \tag{13}$$

□

B Numerical Solution

To address the divergent behavior during computation, we follow Dvorkin et al. (2021) and use standard discrete choice methods. We require that the short-term debt and spending policies take values in a discrete set and subject each potential choice to an iid draw from a Gumbel distribution, as in the multinomial logit model. First, we introduce the net-of-taste-shocks value of choosing (b'_S, g') in state $(S, \lambda) = (b_S, b_L, g, y, \chi, \lambda)$

$$W(S, \lambda, b'_S, g') = (1-d)u(g) + d(v(g_D)) + \beta\mathbb{E}_{\omega, y', \chi', \lambda'}[V(S', \lambda')],$$

where $d = D(S, b'_S, g')$ and $b'_L = B_L(S, \omega, b'_S, g')$. Then, the government's value function becomes

$$V(S, \lambda) = \mathbb{E}_{\epsilon_{(b'_S, g')}} \max_{b'_S, g'} \Big\{W(S, \lambda, b'_S, g') + \epsilon_{(b'_S, g')}\Big\},$$

and the resulting choice probabilities and ex-ante value are

$$\Pr(b'_S = b_{Si}, g' = g_i | S, \lambda) = \frac{\exp\left[\frac{W(S,\lambda,b_{Si},g_i)}{\rho}\right]}{\sum_j \exp\left[\frac{W(S,\lambda,b_{Sj},g_j)}{\rho}\right]},$$

and

$$V(S,\lambda) = \rho \log\left\{\sum_j \exp\left[\frac{W(S,\lambda,b_{Sj},g_j)}{\rho}\right]\right\},$$

respectively. The parameter ρ controls the magnitude of taste shocks.

Finally, the bond price functions consistent with these choice probabilities is given by

$$Q_L(b'_S, b'_L, g', \lambda') = \frac{1}{R^\star}\mathbb{E}_{\omega',T',\lambda''}\Bigg[\sum_j \Pr(b''_S = b_{Sj}, g'' = g_j | b'_S, b'_L, g', T', \lambda') \\ \left\{(1-d')(\kappa + (1-\delta)Q(b_{Sj}, b''_L, g_j, \lambda'')) + d'\frac{v'}{b'_L}\right\}\Bigg],$$

and

$$Q_S(b'_S, b'_L, g', \lambda') = \frac{1}{R^\star}\mathbb{E}_{\omega',T',\lambda''}\left[\sum_j \Pr(b''_S = b_{Sj}, g'' = g_j | b'_S, b'_L, g', T', \lambda')(1-d')\right],$$

where $d' = D(b'_S, b'_L, g', T', b''_S, g'', \lambda'')$.

Next we describe the model's solution algorithm and numerical implementation.

1. Construct a grid for $T_t, T_{t+1}, g, b_t, b_{t+1}, a_{t+1}$. Make the grids of x_{t+1} bigger than grids for x_t for all variables.

2. Construct a matrix for policy functions $V, Q, \mathbb{T}, \mathbb{B}, \mathbb{A}, \mathbb{D}$ of dimensions $n_g x(n_a n_T)$.

2. Guess the price function $Q = \frac{1}{R}$, the continuation value $V = 0$, the policy function of reserves $\mathbb{A} = a_t$, $\mathbb{D} = 0$ and $\mathbb{B}_{t+1} = b_t$

3. Update the policy function for taxes. Given the optimal policy of reserves $\mathbb{A}$ and each state; grid search $\mathbb{T}$; For each value solve $\mathbb{B}_{t+1}$ using equilibrium conditions and get welfare. Find the maximum.

4. Update the policy function of reserves. Given the optimal policy of tax $\mathbb{T}$ and each state; grid search $\mathbb{A}$; For each value solve $\mathbb{B}_{t+1}$ using equilibrium conditions and get welfare. Find the maximum. Update the value function and the default function.

5. Update the price function. Given $\mathbb{D}$ use the break-even condition of lenders to update the price function

6. Check convergence of Q, V, if not convergence go back to 3

C Empirical Definitions

The average maturity of the debt is calculated as a weighted average of the maturities of the short- and long-term debt instruments, where the weights are the face value of each security. Short-term debt has a maturity of 1, while Long-term debt has a risk-free Macaulay duration of $1/\delta$. We use risk-free duration in order to focus on the changes in quantities, rather than movements in bond prices.

The average spread in the model is computed as a weighted average of the spreads on short- and long-term debt, where the weights are the relative market value of each instrument. Individual spreads are calculated as annualized basis points. The short-term spread is the difference between the gross yield of the one-period bond and the gross risk-free rate of the one-period:

$$s_S = 400 \times \left(\frac{1}{q_S} - R_{f,S} \right).$$

The long-term spread is the difference between the yield-to-maturity and the long-term risk-free rate:

$$s_L = 400 \times (YTM_L - R_{f,L}),$$

where

$$YTM_L = \frac{\kappa}{q_L} - \delta.$$

and the average maturity of the debt. The average maturity of the debt is calculated as a weighted average of the maturities of the short- and long-term debt instruments, where the weights are the face value of each security. Short-term debt has a maturity of 1, while Long-term debt has a risk-free Macaulay duration of $1/\delta$. We use risk-free duration in order to focus on the changes in quantities, rather than movements in bond prices.

D Implementation via Particle Filter

We solve the model's policy functions for g', bS', and bL' over the full state grid beforehand. To accelerate the filter, we create multi-dimensional interpolation objects. This allows us to rapidly evaluate the implied policy choices for any particle (i.e., any point in the state-space) without re-solving the model.

The filter then proceeds iteratively for each time period t from $2008Q1$ onward, using a set of $N = 20,000$ particles.

1. **Initialization ($t = 1$):** We initialize the N particles by setting the first period's observed fundamentals (y_1, χ_1) and debt levels (b_{S1}, b_{L1}) from the data, adding small noise to the debt. The unobserved

states (λ_1, ω_1) are drawn from their respective prior distributions.

2. **Prediction (Move to $t+1$):** The resampled particles from period t, $S_{t|t}$ are propagated forward to create a set of N predicted particles for $t+1$, $S_{t+1|t}$. In our guided filter, we take the observed fundamentals y_{t+1} and χ_{t+1} directly from the data. We draw new unobserved shocks $(\lambda_{t+1}, \omega_{t+1})$ for each particle. The endogenous state $(b_{St+1}, b_{Lt+1}, g_{t+1})$ are set using the policy function interpolants, evaluated at the particle's state S_t.

3. **Weighting:** For each particle k, we use its state $S^{(k)}_{t+1|t}$ and the policy function interpolants to calculate the implied observables: $\text{Maturity}^{(k)}_{t+1}$ and $\text{Spread}^{(k)}_{t+1}$. We then compute the squared distance between these implied observables and the actual observables from the data, X^{data}_{t+1}, using the measurement error covariance matrix H.

4. **Resampling:** We check for particle degeneracy by computing the Effective Sample Size (ESS). If the ESS is too low, we apply a rejuvenation step. Finally, we draw a new set of N particles $S_{t+1|t+1}$ by sampling with replacement from the predicted particles $S_{t+1|t}$, using their normalized weights. This new set of particles represents our best estimate of the state distribution at $t+1$ and forms the initial states for the next iteration.