Centre Number	Candidate Number	Candidate Name

NAMIBIA SENIOR SECONDARY CERTIFICATE

MATHEMATICS ADVANCED SUBSIDIARY LEVEL 8227/2

PAPER 2 2 hours

Marks 75 **2022**

Additional Materials: Geometrical instruments
Non programmable calculator
Formulae and notation list

INSTRUCTIONS AND INFORMATION TO CANDIDATES

- Candidates answer on the Question Paper in the spaces provided.
- Write your Centre Number, Candidate Number and Name in the spaces at the top of this page.
- Write in dark blue or black pen.
- You may use a soft pencil for any diagrams or graphs.
- Do not use correction fluid.
- Do not write in the margin *For Examiner's Use*.

- Answer **all** questions.

- **Formulae and notations list is provided on page 15 for your use.**
- If working is needed for any question it must be shown below, or where working is indicated.
- If additional space is needed, you should use the lined page at the end of this booklet: the question number or numbers must be clearly shown.
- The number of marks is given in brackets [ ] at the end of each question or part question.
- Non-programmable calculators may be used.
- If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three** significant figures. Give answers for angle sizes to **one** decimal place but angles in radians to **three** significant figures, unless a different level of accuracy is specified in the question.
- For π, use your calculator value.

For Examiner's Use	
Marker	
Checker	

This document consists of **18** printed pages and **2** blank pages.

Republic of Namibia

MINISTRY OF EDUCATION, ARTS AND CULTURE

For Examiner's Use

1 **(a)** Solve the equation $|3x - 1| = 4$.

[3]

(b) Hence, using logarithms, solve the equation $|3\times5^{y} - 1| = 4$

[2]

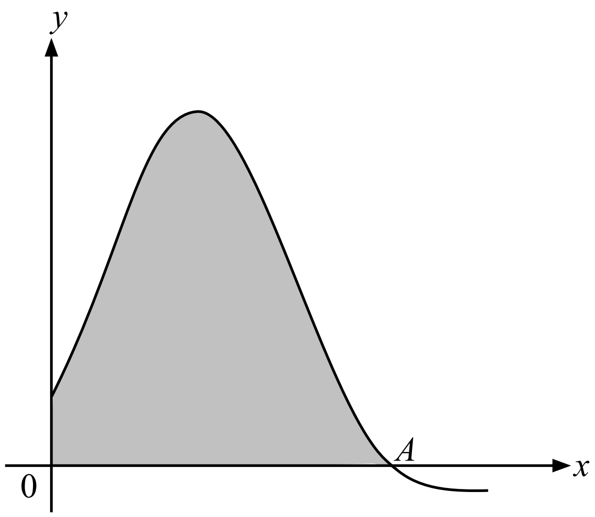
For Examiner's Use

2 Solve the equation $2e^{2x} - 17e^{x} + 8 = 0$, giving your answers in the form $x = a \ln 2$, where a is a constant.

[4]

For Examiner's Use

3



The diagram shows part of a curve whose equation is

$$y = 2\sin^2 x + 7\sin x + 3,$$

where x is measured in radians. The curve crosses the x-axis at the point A and the shaded region is bounded by the curve and the lines $x = 0$ and $y = 0$.

(a) Find the exact x-coordinate of A.

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(b) Find the exact gradient of the curve at A.

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For Examiner's Use

(c) By expressing $2\sin^2 x$ in terms of $\cos 2x$, find the area of the shaded region.

[5]

For Examiner's Use

4 The polynomials $f(x)$ and $g(x)$ are defined by

$f(x) = x^3 + ax^2 + b$ and $g(x) = x^3 + bx^2 - a$,

where a and b are constants. It is given that $(x + 3)$ is a factor of $f(x)$. It is also given that, when $g(x)$ is divided by $(x + 1)$, the remainder is 6.

(a) Find the values of a and b.

.... [5]

(b) When a and b are these values, find the smallest possible value of $g(x) - f(x)$ as x varies.

.... [2]

For Examiner's Use

5 The equation of a curve is $y = 6xe^{\frac{1}{3}x}$.
At the point on the curve with x-coordinate p, the gradient of the curve is 20.

(a) Show that $p = 3\ln\left(\frac{10}{p+3}\right)$.

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(b) Show by calculation that $2.0 < p < 2.2$.

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(c) Use the iterative formula $P_{n+1} = 3\ln\left(\frac{10}{P_n+3}\right)$ with $P_1 = 2.1$, to find the value of p correct to 3 decimal places.
Give the result to each iteration to 5 decimal places.

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For Examiner's Use

6 **(a)** Express $2\sin\theta + \sqrt{5}\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

Give the value of α correct to 2 decimal places.

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(b) Hence

(i) solve the equation $2\sin\theta + \sqrt{5}\cos\theta = 2.4$ for $0° < \theta < 360°$,

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(ii) state the greatest and least values of

$$2\sin\theta + \sqrt{5}\cos\theta,$$

as θ varies.

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For Examiner's Use

7 **(a)** Show that $\operatorname{cosec} 2x + \cot 2x \equiv \cot x$.

[4]

(b) Find $\int \sin 2x(\operatorname{cosec} 4x + \cot 4x)dx$.

[3]

For Examiner's Use

8 It is given that $\displaystyle\int_{a}^{2a} \frac{3}{3x-5}\,dx = \ln\frac{13}{4}$.

Find the value of the positive constant a.

[6]

For Examiner's Use

9 The function f is such that $\mathrm{f}(x) = \log_2(x - 3)$, for the domain $x > a$, where a is a constant.

(a) Find the minimum value of a.

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.... [1]

(b) For the value of a in **(a)**,

(i) find an expression for f^{-1},

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.... [2]

(ii) state the range of f^{-1}.

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.... [1]

(c) The function g is defined by $\mathrm{g}(x) = 3x - 2$.

(i) Evaluate $\mathrm{gf}^{-1}(3)$.

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(ii) Show that $\mathrm{fg}(1)$ cannot be evaluated.

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10 The parametric equations of a curve are

$$x = 4\cos 2\theta + 3\sin\theta, \quad y = 5\cos\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

(a) Find the gradient of the curve at the point for which $\theta = 1$ radian.

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(b) Find the value of $\sin\theta$ at the point on the curve were the tangent is parallel to the y-axis.

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

FORMULAE AND NOTATIONS LIST

PURE MATHEMATICS

Mensuration

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

Volume of cone or pyramid = $\frac{1}{3} \times$ base area × height

Area of curved surface of cone = πr × slant height

Arc length of circle = $r\theta$ (θ in radians)

Area of a sector of a circle = $\frac{1}{2}r^2\theta$ (θ in radians)

Algebra

For the quadratic equation: $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

For an arithmetic series:

$$u_n = a + (n-1)d, \qquad S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

For a geometric series:

$$u_n = ar^{n-1}, \qquad S_n = \frac{a(1-r^n)}{1-r}\ (r \neq 1), \qquad S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

Binomial expansion:

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \dots$, where n is rational and $|x| < 1$

Trigonometry

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta \equiv 1, \quad 1 + \tan^2\theta \equiv \sec^2\theta, \qquad \cot^2\theta + 1 \equiv \text{cosec}^2\theta$$

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2\sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Principal values:

$$-\frac{1}{2}\pi \le \sin^{-1} x \le \frac{1}{2}\pi, \qquad 0 \le \cos^{-1} x \le \pi; \quad -\frac{1}{2}\pi < \tan^{-1} x < \frac{1}{2}\pi$$

Differentiation

f(*x*)	**f'(*x*)**
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\cot x$	$-\text{cosec}^2 x$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
uv	$u\frac{dv}{dx} + v\frac{du}{dx}$
$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $x = f(t)$ and $y = g(t)$ then $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

Integration

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathbf{d}x$	
x^n	$\dfrac{x^{n+1}}{n+1}+c$	$(n \neq -1)$
$\dfrac{1}{x}$	$\ln\|x\| + c$	
e^x	$e^x + c$	
$\sin x$	$-\cos x + c$	
$\cos x$	$\sin x + c$	
$\sec^2 x$	$\tan x + c$	
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\|\dfrac{x-a}{x+a}\right\|$	$(x > a)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\|\dfrac{a+x}{a-x}\right\|$	$(\|x\| < a)$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\,\frac{du}{dx}\,dx$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

Vectors

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a.b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Numerical integration

Trapezium rule:

$$\int_a^b f(x)dx \approx \frac{1}{2}h\,\{y_0 + 2(y_1 + y_2 + ... + y_{n-1}) + y_n\}, \text{ where } h = \frac{b-a}{n}$$

Operations

$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots. + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{N}$ $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $r \in \mathbb{N}$, $0 \leq r \leq n$ $\frac{n(n-1)\ldots(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{N}$

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