Centre Number	Candidate Number	Candidate Name

NAMIBIA SENIOR SECONDARY CERTIFICATE

MATHEMATICS ADVANCED SUBSIDIARY LEVEL 8227/1

PAPER 1 2 hours

Marks 75 **2022**

Additional Materials: Geometrical instruments
Non programmable calculator
Formulae and notations list

INSTRUCTIONS AND INFORMATION TO CANDIDATES

- Candidates answer on the Question Paper in the spaces provided.
- Write your Centre Number, Candidate Number and Name in the spaces at the top of this page.
- Write in dark blue or black pen.
- You may use a soft pencil for any diagrams or graphs.
- Do not use correction fluid.
- Do not write in the margin *For Examiner's Use*.

- Answer **all** questions.

- **Formulae and notations list is provided on page 21 for your use.**
- If working is needed for any question it must be shown below, or where working is indicated.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- The number of marks is given in brackets [ ] at the end of each question or part question.
- Non-programmable calculators may be used.
- If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three** significant figures. Give answers for angle sizes to **one** decimal place but angles in radians to **three** significant figures, unless a different level of accuracy is specified in the question.
- For π, use your calculator value.

For Examiner's Use

Marker	
Checker	

This document consists of **24** printed pages

Republic of Namibia

MINISTRY OF EDUCATION, ARTS AND CULTURE

For Examiner's Use

1 The equation of a curve is $y = -x^2 + 3x + 5$.

(a) Express $-x^2 + 3x + 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants.

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(b) Hence, or otherwise, write down the coordinates of the turning point of the curve.

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For Examiner's Use

2 **(a)** Solve the equation $3x^2 + 5x + 4 = 6$.

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(b) Hence solve the equation $3y + 5y^{\frac{1}{2}} + 4 = 6$

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For Examiner's Use

3 Given that the coefficient of x^3 in the expansion of $(1 - 3x)^6 + (1 + ax)^4$ is -1040.

Find the value of the constant a.

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For Examiner's Use

4 The quadratic equation $2x^2 - 5x + 8k^2 = 0$ has unequal real roots.
Find the set of possible values of k.

[4]

For Examiner's Use

5 (a) In an arithmetic progression, the third term is 14 and the sum of the first eight terms is 148.

Find the first term and the common difference.

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For Examiner's Use

(b) Evaluate

$$\sum_{r=3}^{\infty} \frac{5}{4}(3^{1-r})$$

[3]

For Examiner's Use

6 The function f is defined by $\mathrm{f}: x \mapsto \ln(x - 2)$ for $x > k$.

(a) Find the smallest possible value of k.

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(b) Find the inverse function, f^{-1}.

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(c) Sketch the graphs of f and f^{-1} on the same diagram, clearly showing any intersections with the axes.

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For Examiner's Use

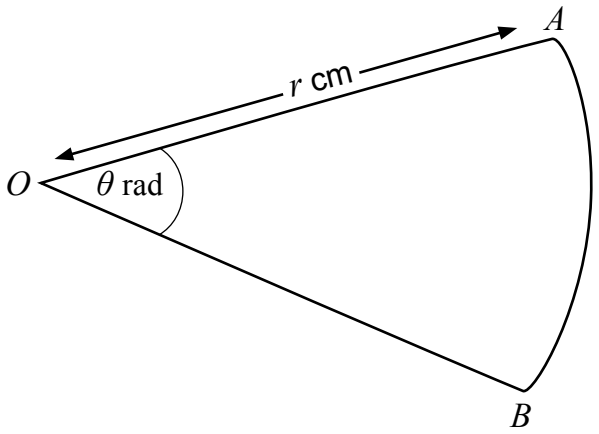
7 A curve has equation $y = x^4 - 5x - 3$ and a line has equation $y = -9x + a$ where a is a constant.

If the line is a tangent to the curve at point P, find the coordinates of P and the value of a.

[4]

For Examiner's Use

8



A flexible piece of wire, 60 cm long, is bent to form the perimeter of a sector of a circle AOB, as shown in the diagram.
The radius of the circle is r cm and the angle AOB is θ radians.

(a) Express θ in terms of r and show that the area, A cm^2, of the sector is given by

$$A = 30r - r^2.$$

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For Examiner's Use

(b) Given that r can vary, find the stationary value of A and determine its nature.

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For Examiner's Use

9 **(a)** Show that the equation $3 \tan x = -2 \cos x$ can be written in the form, $2 \sin^2 x - 3 \sin x - 2 = 0$.

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(b) Hence solve the equation $3 \tan x = -2 \cos x$ for $0 \leq x \leq 2\pi$.

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For Examiner's Use

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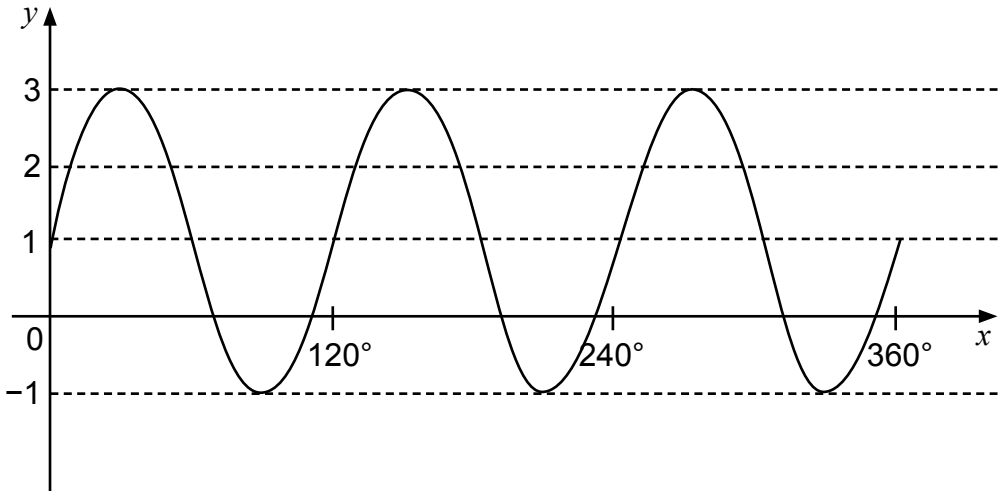

The diagram shows the curve $y = a\sin(bx) + c$ for $0° \leq x \leq 360°$.

(a) Find the values of the integers a, b and c.

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(b) Write down the period of the curve.

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(c) If the curve is now reflected in the x-axis.

Write down the new equation of the curve.

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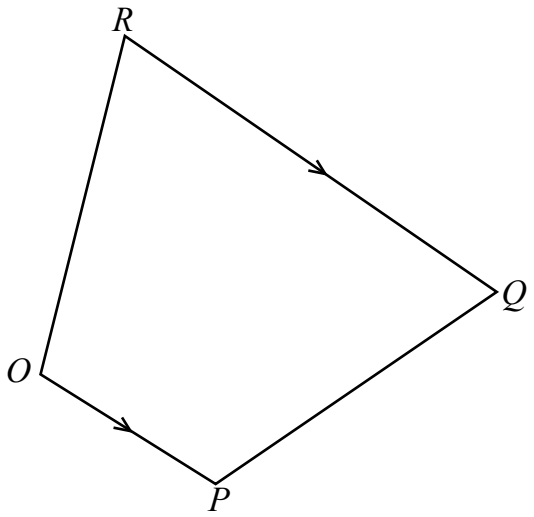

The diagram shows a trapezium $OPQR$ in which OP is parallel to RQ. The position vectors of P and Q relative to the origin O are given by $\overrightarrow{OP} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$. The magnitude of $\overrightarrow{RQ}$ is twice the magnitude of $\overrightarrow{OP}$.

(a) Show that the position vector of R is given by $\overrightarrow{OR} = \begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix}$.

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For Examiner's Use

(b) Use a scalar product to calculate the size of angle POR.

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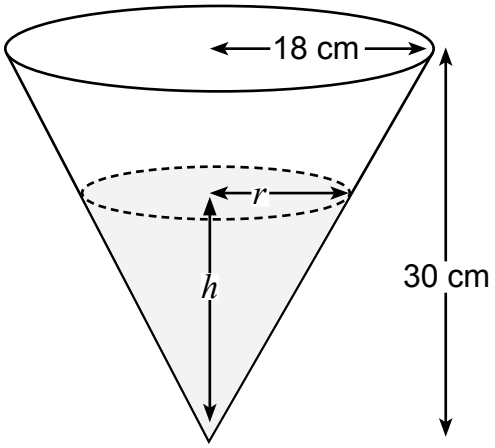
For Examiner's Use

12



The diagram shows a container in the shape of an inverted cone with its circular base upright and horizontal. The height of the container is 30 cm and the base radius is 18 cm. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

[The volume of a cone is $\frac{1}{3}\pi r^2 h$.]

(a) Express r in terms of h and hence show that $V = \frac{3\pi h^3}{25}$.

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Water flows into the container at a rate of 36 cm^3 per second.

(b) Find, in terms of π, the rate of change of h when h = 15 cm.

[4]

For Examiner's Use

13 A function $f(x)$ is defined for $x > 0$ and is such that $f'(x) = 4x + \frac{6}{x^2}$.

The curve $y = f(x)$ passes through the point $P(-2, -1)$.

Find the equation of the curve.

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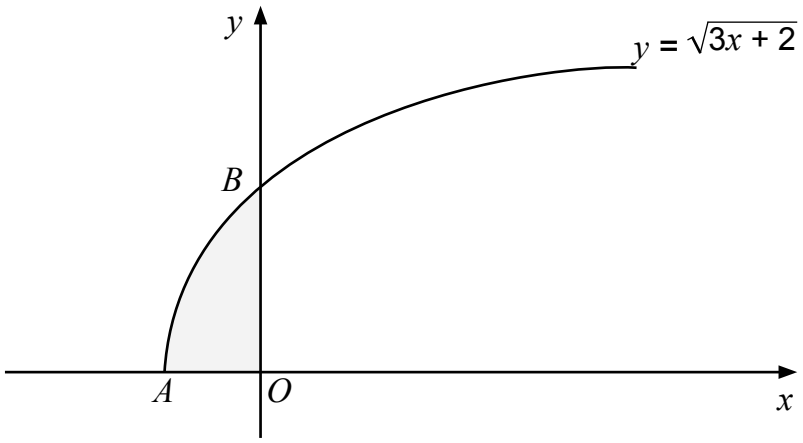
For Examiner's Use

14



The diagram shows the curve $y = \sqrt{3x + 2}$ meeting the x-axis at A and the y-axis at B.

(a) Write down the coordinates of A and B.

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(b) The region AOB is rotated through 360° about the x-axis.

Find the volume of the shaded region, giving your answer in terms of π.

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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FORMULAE AND NOTATIONS LIST

PURE MATHEMATICS

Mensuration

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

Volume of cone or pyramid = $\frac{1}{3} \times$ base area $\times$ height

Area of curved surface of cone = $\pi r \times$ slant height

Arc length of circle = $r\theta$ (θ in radians)

Area of a sector of a circle = $\frac{1}{2}r^2\theta$ (θ in radians)

Algebra

For the quadratic equation: $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

For an arithmetic series:

$$u_n = a + (n-1)d, \qquad S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

For a geometric series:

$$u_n = ar^{n-1}, \qquad S_n = \frac{a(1-r^n)}{1-r} \ (r \neq 1), \qquad S_\infty = \frac{a}{1-r} \qquad (|r| < 1)$$

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \ldots,$$ where n is rational and $|x| < 1$

Trigonometry

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta \equiv 1, \quad 1 + \tan^2\theta \equiv \sec^2\theta, \qquad \cot^2\theta + 1 \equiv \operatorname{cosec}^2\theta$$

$$\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Principal values:

$$-\frac{1}{2}\pi \le \sin^{-1} x \le \frac{1}{2}\pi, \qquad 0 \le \cos^{-1} x \le \pi; \qquad -\frac{1}{2}\pi < \tan^{-1} x < \frac{1}{2}\pi$$

Differentiation

f(x)	**f'(x)**
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
uv	$u\frac{dv}{dx} + v\frac{du}{dx}$
$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $x = f(t)$ and $y = g(t)$ then $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

Integration

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\mathbf{d}x$	
x^n	$\frac{x^{n+1}}{n+1} + c$	$(n \neq -1)$
$\frac{1}{x}$	$\ln \lvert x \rvert + c$	
e^x	$e^x + c$	
$\sin x$	$-\cos x + c$	
$\cos x$	$\sin x + c$	
$\sec^2 x$	$\tan x + c$	
$\frac{1}{x^2 + a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$	$(x > a)$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert$	$(\lvert x \rvert < a)$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\,\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\mathrm{d}x = \ln\left|\mathrm{f}(x)\right| + c$$

Vectors

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a}.\mathbf{b} = a_1\,b_1 + a_2\,b_2 + a_3b_3 = \left|\mathbf{a}\right|\left|\mathbf{b}\right|\cos\theta$

Numerical integration

Trapezium rule:

$$\int_a^b \mathrm{f}(x)dx \approx \frac{1}{2}h\left\{y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n\right\}, \text{ where } h = \frac{b-a}{n}$$

Operations

$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{N}$ $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $r \in \mathbb{N}$, $0 \leq r \leq n$
	$\frac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{N}$