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Magnel Diagrams for Prestressed Concrete Beams

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Abstract: Magnel's graphical method for the analysis of a prestressed concrete beam is very useful for the determination of safe prestressing force and eccentricity. The paper presents the characteristics of the original Magnel diagram, reviews the writers' modifications to and extensions of the method, discusses relevant work by others, and offers a further simplification for the plotting of the safe zone, illustrated by a worked example.

Introduction

The use of graphical methods for the selection of prestressing force and tendon location, and for checking the adequacy of the concrete section, were very useful in prestressed concrete beam design when first introduced in the forties. Designers have continued to use them and investigators have tried to improve them over the years. In this paper, Magnel's original diagram and subsequent modifications are reviewed and a further improvement is proposed.

Magnel Diagram

Magnel (5) said it all when he pointed out the linear relationship between the eccentricity, e, and the reciprocal of the prestressing force, P, in the expression for the stress, f, in the extreme fiber at a distance, c, from the centroidal axis (CA) of a prestressed concrete section of cross-sectional area, A, and moment of inertia, I, about the CA, subjected to a bending moment, M, as in Fig. 1:

$$f = \frac{P}{A} + \frac{Pec}{I} + \frac{Mc}{I} \quad \text{(1)}$$

Compressive stress in concrete and, thus, tendon pretension are positive; e, c, and other distances are positive measured upwards from the CA; and moment is positive when causing compression in the top fiber.

The linear relationship becomes obvious when Eq. 1 is rearranged as

$$e = (fS - M)\left(\frac{1}{P}\right) - \frac{S}{A} \quad \text{(2)}$$

in which S = the section modulus, I/c.

There are four constraints that Eqs. 1–2 must satisfy, corresponding to the stress limits which may not be exceeded at the top and bottom

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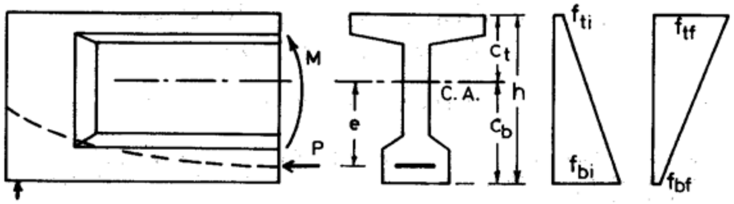

FIG. 1.—Prestressed Concrete Beam: Half Longitudinal Elevation, Cross Section, Initial Distribution, and Final Stress Distribution

fibers under the initial condition (i.e., prestress plus self-weight and other dead loads), and the final condition (i.e., reduced prestress after losses, plus full dead and live loads). The following four limiting stress equations may be drawn as straight lines in $(1/P)$ and e axes as in Fig. 2(*a*)

$$\text{Line 1:}\quad e = (f_{ti}S_t - M_i)\left(\frac{1}{P}\right) - \frac{S_t}{A} \qquad (3a)$$

$$\text{Line 2:}\quad e = (f_{bi}S_b - M_i)\left(\frac{1}{P}\right) - \frac{S_b}{A} \qquad (3b)$$

$$\text{Line 3:}\quad e = \left(\frac{f_{tf}S_t - M_f}{R}\right)\left(\frac{1}{P}\right) - \frac{S_t}{A} \qquad (3c)$$

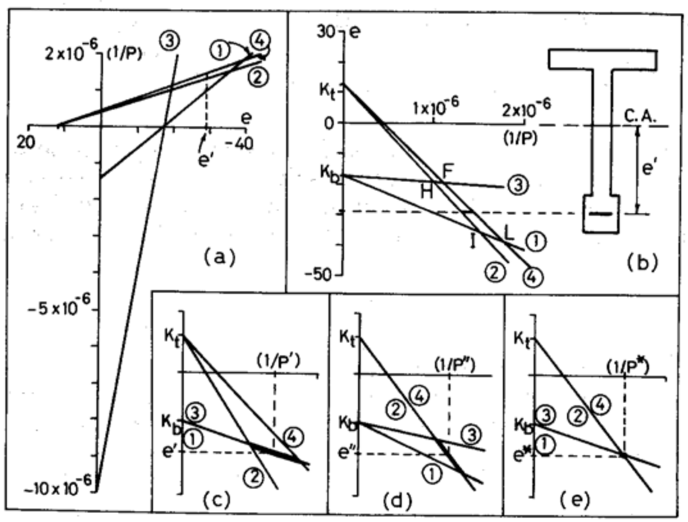

FIG. 2.—(*a*) Magnel Diagram; (*b*) Writer's Modification; (*c*) Case of Actual S_t = Required S_t; (*d*) Case of Actual S_b = Required S_b; (*e*) Case of Actual S_t, S_b = Required S_t, S_b. Values Marked in (*a*) and (*b*) are in Inch and Pound Units. (1 in. = 25.4 mm, 1 lb = 4.45 N)

$$\text{Line 4:}\quad e = \left(\frac{f_{bf}S_b - M_f}{R}\right)\left(\frac{1}{P}\right) - \frac{S_b}{A} \qquad (3d)$$

in which the subscripts t and b refer to top and bottom fibers; subscripts i and f refer to the initial and final stress and loading conditions; and R = the effective prestress ratio, after losses. It may be noted that S_b is negative as c_b is negative.

When the concrete section is adequate, the region enclosed by the four lines will be the "safe zone" of valid combinations of e and P.

Magnel used a different sign convention, and expressed the P/A and M/S terms as stresses. He plotted the diagram with the e-axis horizontal, and the A/P-axis vertical. The lines were located by their calculated intercepts on the coordinate axes and extended to intersect and define the safe zone, as indicated in Fig. 2(*a*).

Writer's Modifications

To achieve greater plotting efficiency and practical significance, the writer proposed plotting Eqs. 3*a*–*d* in a modified graphical format and procedure (3). The e-axis was plotted vertically with the beam cross section drawn alongside with its CA located in line with the horizontal $(1/P)$ axis, as in Fig. 2(*b*). This enabled the safe zone to be immediately and visually located with respect to the beam cross section for checking or selection of feasible e and P combinations. A series of such diagrams and cross sections would also assist in the development of a suitable tendon profile along the length of the beam.

The safe zone was developed as follows: In Eqs. 3, S_t/A and S_b/A may be recognized as k_t and k_b, the "kern distances" below and above the CA. A prestress beyond these distances would cause tension in the extreme fibers at the top and bottom, respectively.

Lines 3a and 3c intersect at K_b on the e-axis at a distance, k_t, below the CA. Lines 3b and 3d intersect at K_t on the e-axis at a distance, k_b, above the CA, k_b being negative as S_b is negative.

Two corners [I and F in Fig. 2(*b*)] of the safe zone are determined as the points corresponding to "full utilization of the beam cross section" in the initial and final stages, in which stresses f_{ti} and f_{bi} are simultaneously achieved under the initial loading, and stresses f_{tf} and f_{bf} are simultaneously achieved under the final loading.

The relevant values of the required pretension are determined by simultaneous solution of Eqs. 3(*a*)–(*b*), and of Eqs. 3(*c*)–(*d*), respectively, as

$$P_i = \frac{A}{h}(c_t f_{bi} - c_b f_{ti}) \qquad (4a)$$

$$\text{and}\quad P_f = \frac{A}{Rh}(c_t f_{bf} - c_b f_{tf}) \qquad (4b)$$

in which h = the beam depth. The corresponding values of e_i and e_f may be obtained by substitution of P_i for P into Eq. 3(*a*) or 3(*b*), and of P_f for P into Eq. 3(*c*) or 3(*d*), respectively.

With these modifications, not only the wasteful plotting on the negative side of the $(1/P)$ axis is avoided, but also the safe zone is determined as short extensions of the lines from K_t and K_b through I and F more precisely than in the original Magnel diagram, as evident from a comparison of Figs. 2(*a*)–(*b*).

In a second paper (4), the writer recast his modified equations in nondimensional form, pointed out a number of practically significant features of the modified Magnel diagram, and presented typical design charts and examples.

Influence of Section Dimensions

Prestressed concrete is different from other structural materials in that: (1) The concrete section may be adequate to take the stresses and yet the steel tendon cannot be positioned within the section; and (2) there is an upper limit as well as a lower limit to the prestressing force, and here more does not mean safer.

These and other limitations involving section dimensions can be detected and avoided or overcome by the Magnel diagram or its modifications.

Thus, in the writer's modified (or equivalent) format [Fig. 2(*b*)], whether the safe zone would permit a valid tendon location within the concrete section would be immediately obvious. Further, for any particular eccentricity, e', in Fig. 2(*a*) or 2(*b*), the valid range of prestressing force is limited to that portion of the straight line, drawn at $e = e'$ and parallel to the $(1/P)$ axis, which is intercepted within the safe zone. Not only a smaller P but even a larger P than this range would violate at least one of the limiting stresses.

To check the adequacy of the concrete section, expressions for the required upper and lower section moduli may be developed by simultaneous solution of Eqs. 3*a*–*c* and Eqs. 3*b*–*d*, respectively, as follows:

$$S_t = \frac{M_f - RM_i}{f_{tf} - Rf_{ti}} \quad \text{(5a)}$$

$$\text{and} \quad S_b = \frac{M_f - RM_i}{f_{bf} - Rf_{bi}} \quad \text{(5b)}$$

As long as the actual section moduli provided are not smaller than the required values by Eqs. 5*a*–*b*, the specified stresses will not be exceeded. The graphical counterpart of this section check for simply supported beams would be for the four lines of Eqs. 3*a*–*d* to bound the safe zone in the 1-2-3-4 clockwise order.

If a beam provides exactly the required S_t by Eq. 5*a*, Eqs. 3*a*–*c* would be simultaneously satisfied indicating the achievement of the top stress limits, f_{ti} and f_{tf}, under initial and final loadings; thus lines 1 and 3 of Fig. 2(*a*) or 2(*b*) would fuse into a single line. The safe zone would degenerate into the segment—shown thickened in Fig. 2(*c*)—of the combined line (1,3) intercepted between the lines 2 and 4. For a particular eccentricity, e', there would be only one value of P (= P') that would be safe.

Similarly, if a beam provides exactly the required S_b by Eq. 5*b*, Eqs. 3*b*–*d* would be simultaneously satisfied indicating the achievement of the bottom stress limits, f_{bi} and f_{bf}, under initial and final loadings; thus lines 2 and 4 of Fig. 2(*a*) or 2(*b*) would fuse into a single line. The safe zone would degenerate into the segment—shown thickened in Fig. 2(*d*)—of the combined line (2,4) intercepted between lines 1 and 3. Again, for any desired eccentricity, e'', there would be only one safe value of P (= P'').

Theoretically then, to design an ideal or "fully-stressed" beam that would develop all four specified stress limits at the appropriate fibers and loadings, all that would be necessary is for a beam to have I, c_t, and c_b values such that the actual section moduli, I/c_t and I/c_b, are equal to the required values, S_t and S_b, respectively.

However, satisfying both Eqs. 5*a* and 5*b* is easier said than done. It would be extremely difficult, and (if R is also fixed) generally impossible for doubly symmetric sections, to develop a beam with the precise dimensions that would provide the exact moduli required.

For such an ideal beam, the four limiting stress lines would degenerate into two combined lines [(1,3) and (2,4)], with the safe zone being only the point of intersection of the two lines, as in Fig. 2(*e*). This would require that only the particular value, P^*, of the prestressing force corresponding to the intersection point be provided by tendons located precisely at the specific eccentricity, e^*, for that point.

All these rigid requirements are not only very impractical to meet, but also risky in actual use. Thus most prestressed concrete beams end up with a quadrilateral area for a safe zone, usually longer in one direction than in the other.

Other Relevant Work

Many textbooks (2,6) have continued to include the Magnel diagram as a useful design tool, even in this computer era. Naaman (6), author of the latest, mentions many of the points presented herein, and also happens to use the axes and orientation proposed by the writer (3). To facilitate the preliminary selection of section dimensions, tables and charts relating various geometric parameters for typical prestressed concrete sections are available in books (2,6) and papers.

In a recent paper, Somayaji (7) has proposed another variation of the safe zone diagram, and has developed expressions to estimate section moduli for preliminary design. The basis of his formulation can be explained as follows: Eq. 1 may be rewritten as

$$f = f_p + \left(\frac{f_p e}{h}\right)\left(\frac{ch}{r^2}\right) + \frac{M}{S} \quad \text{or} \quad f_p = -\left(\frac{ch}{r^2}\right)\left(\frac{f_p e}{h}\right) + \left(f - \frac{M}{S}\right) \quad \text{(6)}$$

in which f_p = the prestress intensity P/A; and $r^2 = I/A$.

Eq. 6 is of the form

$$Y = mX + C \quad \text{(7)}$$

which is a straight line with slope, m, and intercept, C, in X (= $f_p e/h$) and Y (= f_p) coordinates. This is the form that Somayaji uses, with a

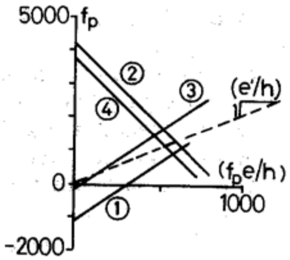

FIG. 3.—Somayaji's Interaction Diagram. Values Marked Are in Inch and Pound Units. (1 in. = 25.4 mm, 1 lb = 4.45 N)

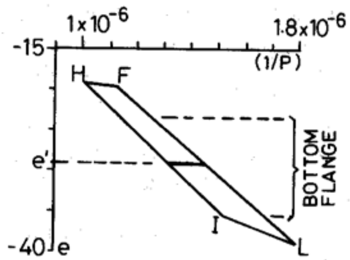

FIG. 4.—Writers Proposed Simplification. Values Marked Are in Inch and Pound Units. (1 in. = 25.4 mm, 1 lb = 4.45 N)

different sign convention from what the writer has used. Again, the four stress constraints give rise to four straight lines, enclosing the region of safe combinations of f_p and e, as in Fig. 3.

The writer's and Somayaji's formulations differ mainly in coordinate systems and, of course, both differ from the Magnel diagram in the plotting technique to locate the safe zone.

Further Simplification

The advantage of the graphical method, especially in conjunction with the display of the beam geometry, lies in its visual presentation of the entire safe domain of combinations of prestress and eccentricity which would satisfy all four stress constraints without violating certain practical requirements. Thus, further improvement of this process may be considered worthwhile from a practical viewpoint.

The ultimate simplification—for the greatest precision—of the safe zone plot would be the determination of the coordinates of its four corners, namely I, F, L, and H marked in Fig. 2(*b*).

The writer has already developed expressions for the coordinates of the two corners, I and F, by Eqs. 4*a*–*b*, followed by the use of Eqs. 3.

The other two corners, L and H, correspond to the "low" and "high" prestress conditions, namely, simultaneous achievement of the minimum specified stresses (f_{ti}, f_{bf}), and of the maximum specified stresses (f_{bi}, f_{tf}), respectively, in the appropriate fibers and under the appropriate loadings. The significance of these two points is also mentioned in Ref. 6.

Simultaneous solution of Eqs. 3*a* and 3*d* for minimum stress constraints gives

$$P_l = \frac{(Rf_{ti}S_t - f_{bf}S_b) + (M_f - RM_i)}{R(k_t - k_b)} \quad \text{(8a)}$$

The corresponding eccentricity, e_l, may be obtained by substitution of P_l for P into Eq. 3*a* or *d*.

Similarly, simultaneous solution of Eqs. 3*b*–*c* for maximum stress constraints gives

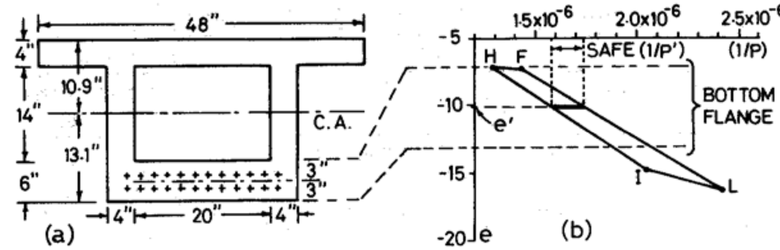

FIG. 5.—Illustrative Example. (*a*) Cross Section for Standard Girder; (*b*) Writer's Simplified Diagram. Values Marked Are in Inch and Pound Units. (1 in. = 25.4 mm, 1 lb = 4.45 N)

$$P_h = \frac{(f_{tf}S_t - Rf_{bi}S_b) - (M_f - RM_i)}{R(k_t - k_b)} \quad \text{(8b)}$$

The corresponding eccentricity, e_h, may be obtained by substitution of P_h for P into Eq. 3*b* or 3*c*.

The safe zone, now defined completely by its four corners, may be plotted in (1/P) and e axes to a large scale, as in Fig. 4. Comparison of all four methods considered thus far, depicted in Figs. 2(*a*)–(*b*), Fig. 3, and Fig. 4, for the same case, namely, the worked example in the writer's first paper (3), should demonstrate the superiority of the last one proposed by the writer: tremendously improving the precision of the solution while retaining the advantage of tendon location capability of the writer's earlier modification. The computations involved are all explicit and present no difficulty at all with today's electronic calculators.

The graphical check for section adequacy now rests on whether the four corners plot in the *L-I-H-F* order clockwise.

Illustrative Example

To demonstrate the application of the proposed method, a standard 24 in. (61 cm) deep section mentioned in Ref. 2 and depicted in Fig. 5(*a*) will be analyzed for the following data:

Concrete f'_c = 6,000 psi (41.37 MPa); beam span 60 ft (18.3 m), simply supported; self-weight and initial dead loads, 600 lb/ft (8.76 kN/m); superimposed loads, 1,050 lb/ft (15.32 kN/m); prestress losses, 15%.

Solution

Stress Constraints.—According to the ACI Code (1) with compression being positive: $f_{ti} = -195$ psi (-1.34 MPa); $f_{bi} = 2{,}520$ psi (17.38 MPa); $f_{tf} = 2{,}700$ psi (18.60 MPa); $f_{bf} = -465$ psi (-3.21 MPa).

Bending Moments.—Initial moment, $M_i = (600)(60^2)(12)/8 = 3.24 \times 10^6$ lb in. (366.1 kN·m). Final moment, $M_f = (600 + 1{,}050)(60^2)(12)/8 = 8.91 \times 10^6$ lb in. (1,006.8 kN·m).

Required Section Moduli.—$R = 1 - 0.15 = 0.85$. By Eqs. 5*a*–*b*, S_t = 2,148 in.^{3} (35,200 cm^3), and $S_b = -2{,}361$ in.^{3} ($-38{,}690$ cm^3).

Section Properties.—A = 472 sq in. (3,045 cm^2); c_t = 10.9 in. (27.7 cm), $c_b = -13.10$ in. (-33.3 cm); I = 34,940 in.^{4} (1.454 × 10^6 cm^4); S_t = 3,205

TABLE 1.—Coordinates of Safe Zone Corners

Point (1)	Equation used (2)	(1/P) 1/lb (3)	1/kN (4)	Eccentricity In inches (5)	In centimeters (6)
I	4*a*, 3*a*	2.041×10^{-6}	4.588×10^{-4}	−14.68	−37.3
F	4*b*, 3*c*	1.426×10^{-6}	3.207×10^{-4}	−7.22	−18.3
L	8*a*, 3*a*	2.411×10^{-6}	5.420×10^{-4}	−16.11	−40.9
H	8*b*, 3*b*	1.287×10^{-6}	2.895×10^{-4}	−7.17	−18.2

in.^{3} (52,520 cm^3), $S_b = -2{,}667$ in.^{3} ($-43{,}700$ cm^3); and $k_t = 6.79$ in. (17.2 cm), $k_b = -5.65$ in. (-14.4 cm).

As actual section moduli are greater than required section moduli, the section is adequate for the stresses.

Safe Zone.—The coordinates of the four corners of the safe zone may be computed from Eqs. 3, 4, and 8, with the results given in Table 1.

The safe zone is shown plotted in Fig. 5(*b*), on which the bottom flange boundaries are also indicated. The clockwise ordering of the corners, *L*, *I*, *H*, *F*, confirm the adequacy of the concrete section. The safe zone overlaps the beam bottom flange assuring a feasible tendon location.

Tendon Selection.—To locate the prestress tendons with their centroid at the center of the bottom flange, the eccentricity, *e*, must be −10.10 in. (−25.7 cm), which is within the valid range of eccentricity.

When a horizontal line is drawn within the safe zone at this chosen, *e*, value in Fig. 5(*b*), the valid range of $(1/P)$ values extends from 1.58×10^6 to 1.75×10^{-6}, corresponding to a *P* range from 633,000 lb (2,815 kN) to 571,000 lb (2,542 kN). Any standard tendon and prestress combination which can supply a pretension within the valid range may be used.

For instance, 24 one-half in. (13 mm) diam, Grade 270 seven-wire strands, each of 24,800 lb (110 kN) capacity, will give 595,000 lb (2,647 kN). These may be laid in two rows of 12 strands each, 1 in. (25 mm) above and 1 in. (25 mm) below the mid-depth of the bottom flange, as indicated in Fig. 5(*a*).

Conclusion

The original Magnel diagram, the writer's earlier modifications, and related work by others are reviewed. A further simplification is presented and illustrated by a worked example. It may be concluded that the simplified technique is also the most precise one for the determination of safe combinations of prestressing force and eccentricity.

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Appendix II.—Notation

The following symbols are used in this paper:

A = cross-sectional area of beam;
C = intercept of straight line;
c = distance of extreme fiber from centroidal axis;
e = eccentricity of prestressing force from centroidal axis;
f = extreme fiber stress;
f'_c = concrete ultimate strength;
f_p = intensity of prestress;
h = depth of beam;
I = moment of inertia of beam about centroidal axis;
k = kern distance, measured from centroidal axis;
M = bending moment in beam, at cross section;
m = slope of straight line;
p = prestressing force;
R = effective prestress ratio, after losses;
r = radius of gyration of beam cross section;
S = section modulus of beam cross section;
X = horizontal coordinate; and
Y = vertical coordinate.

Subscripts

b = bottom fiber;
f = final loading;
h = "high" prestress condition;
i = initial loading;
l = "low" prestress condition; and
t = top fiber.

Superscripts

$', '', *$ = particular values.

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MAGNEL DIAGRAMS FOR PRESTRESSED CONCRETE BEAMS[a]

Discussion by Bohdan K. Boczkaj,[2] M. ASCE

The author is to be commended for his effort in revealing how the evolution process changes the Magnel method from the graphical through the semi-graphical into the analytical. There are, in fact, five constraints which Eqs. 1–2 must satisfy, although the author wrote about only four constraints. In his discussion with Hatcher, Naaman (13) points out:

> The solution of the four stress equations at equality, guarantees a cross section of concrete for which there is at least one feasible point on the Magnel diagram with coordinates P^* and e^*. It does not guarantee, however, that the value of e^* is less that c_b so as to place the prestressing steel inside the cross section. Thus an additional practicality condition of the form $e < c_b - a$, in which a represents the cover from the lower fiber to the centroid of the steel, must generally be added to the four stress equations. Quite often this additional condition is automatically satisfied for the high magnitude of live load as compared to dead load, but may be binding in other cases.

Naaman errs here because the condition he is talking about does not depend on the ratio of live load to dead load; it is a repetition of Guyon's error, but the rest of the statement is correct.

Two statements from the paper, "As long as the actual section moduli provided are not smaller that the required values by Eqs. 5*a*–*b*, the specified stresses will not be exceeded," and, "The concrete section may be adequate to take the stresses and yet the steel tendons cannot be positioned within the section," may serve as proof that Eqs. 5*a*–*b* may form the necessary, but insufficient condition for checking the adequacy of the concrete section. The section which satisfies Eqs. 5*a*–*b* becomes inadequate, when the above given fifth constraint is not satisfied. The form of Eqs. 5*a*–*b* is an iterative one, because to calculate a section moduli it is necessary to know moment M_i which contains the influence of dead load, and it is possible only by iteration or by a different form of Eqs. 5*a*–*b* not given herein.

In the time when the Magnel diagram was created, Guyon (10) had shown that two regions of design exist, below and above the critical span. Later Prasada Rao (15) worked out a method for the design of a fully stressed section leaving an unsolved vast area of beams above the critical span, but Nilson (14) covered a whole area, when he gave a method of design for beams with limited depth in addition to sections fully stressed and with excessive capacity. Hatcher (11) then proposed a method for the design of a fully stressed section and Naaman (13) proved (confirmed by Hatcher in Ref. 12) that a section fully stressed is not always possible. Lately Somayaji (7) proposed formulas for section moduli similar to Eqs. 5*a*–*b*, and gave a partial solution of the problem. The defi-

[a]December, 1983, by Natarajan Krishnamurthy (Paper 18463).
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ciency of Somayaji's solution was proved by the writer (9) through examples. Now, the author takes a position similar to Hatcher, Prasada Rao and Somayaji. In order to avoid the repetition of the arguments, it is necessary to stress that a fully stressed section is possible only when the span of a beam is below critical, or when sufficient depth is at our disposal. All that would be necessary for a fully stressed beam to be adequate is to have exactly satisfied the required section moduli and to have a minimum required depth. The paper, therefore, should be amended with this statement.

After the work of Guyon, the statement, "It would be extremely difficult and (if R is also fixed) generally impossible for double symmetric section to develop a beam with the precise dimensions that would provide the exact moduli required," is, of course, false.

The problem of a section design lies in finding an adequate section which satisfies all five constraints, not in finding the dimensions of a section which exactly satisfy section moduli required by Eqs. 5*a*–*b*.

After the author's improvements, coordinates of all four corners of Magnel quadrangle are found analytically, and only the range of a safe prestressing force P' for chosen e' is found from the diagram. Further improvements are possible when the range of P' is established analytically. It is enough to substitute the chosen value of e' into the proper Eq. 3, and calculate P'. In the particular case of the author's example these are Eqs. 3*a*–*d*. When values of P' are found analytically, calculations of reciprocals $(1/P')$ are not necessary and the diagram of a safe zone may be drawn in coordinates P-e, as was done by the writer (8). The diagram has only illustrative meaning.

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15. Prasada Rao, A. S., discussion of "Direct Flexural Design of Prestressed Concrete," by D. S. Hatcher, *Journal of the Structural Division*, ASCE, Vol. 106, No. ST5, May, 1980, pp. 1222–1223.

Discussion by Andres R. Castella

Krishnamurthy is to be congratulated for his modification to Magnel's diagram, especially with regard to the non-dimensional diagrams published in Ref. 4, A.C.I. SP-26.

In prestressed concrete flexural design, economy through minimum requirement of prestressed force is obtained by locating the prestress centroid as close as possible to the tensile face of the member (under gravity loads only). This location is possible only if the allowable concrete stresses are not exceeded both at the initial and final stages. The SP-26 charts, if made for A.A.S.H.T.O. and A.C.I., specified allowable stresses will yield the location of the prestress centroid with a minimum of calculations and acceptable precision, compatible with that of the design parameters.

Calculation of the eight coordinates defining the safe zone, is not in my opinion the simplest procedure to obtain greater than graphical precision; especially considering that the eccentricity and prestress force will have to be recalculated for the centroids' final location. Only the smallest and largest eccentricities, within which it is possible to place the centroid without exceeding allowable stresses, are of interest.

The maximum eccentricity possible will be that which does not exceed the allowable tensile stress at transfer (initial stage) nor the allowable tensile stress under service (final) loads. By clearing $(1/P)$ and equating this value in Eqs. 3*a* and 3*d* an expression for the maximum eccentricity can be obtained which bypasses calculations for P.

The minimum eccentricity possible will be that which does not exceed the allowable compressive stresses at transfer or under service loads. An expression which is, again, independent of P can be obtained for the minimum eccentricity from Eqs. 3*b* and 3*c*, by the previously described procedure. As a matter of interest, note that this expression becomes invalid if the eccentricity falls within the kern.

The concrete section is inappropriate if the minimum eccentricity is larger than the maximum as the safe eccentricity range disappears. The section is also inadequate if the safe range of eccentricity falls out of its boundaries.

Only after the final eccentricity has been selected within the safe range, is the prestress force calculated using a transposed version of Eq. 3*d*.

The procedure is easily adapted for use with a programmable calculator with either A.C.I. or A.A.S.H.T.O. specified allowable stresses as options. Input would consist of section properties, η, concrete quality, code option, initial load and service load moments. Output would consist of maximum and minimum eccentricities. A second phase of the program could yield the service prestress force once the final selected eccentricity has been input.

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Discussion by David M. F. Orr[4] and Eamonn O'Mahony[5]

The author has developed useful simplifications to the original Magnel diagram, and while it is true that the computations involved can be carried out quite readily on an electronic calculator, it is, nonetheless, worthwhile to produce the diagram using computer graphics. A complete Magnel diagram, together with relevant calculated quantities can be displayed on a graphics terminal screen in a matter of seconds and the design of a cross section can be rapidly modified until it meets not only with design criteria expressed in numerical terms, but also with visual approval as to its suitability for concrete placing.

Also, Eq. 2 may be arranged in the form

$$e = \left(f\frac{I}{y} - M\right)\left(\frac{1}{P}\right) - \frac{r^2}{y} \quad \text{(9)}$$

and applied to stresses at any distances, y, from the centroidal axis. In general there are upper and lower limits to the stresses, which may be allowed in any fiber. In the case of the homogeneous cross section dealt with in the paper, only the extreme fibers need be considered. In composite sections, because of different material properties, stresses in fibers at the interface between in situ slab and precast beam concrete should be considered in addition to stresses in the top and bottom fibers of the total section, Fig. 6. Thus the relationships between prestressing force and eccentricity are required for eight stresses rather than four, as in the case of the homogeneous beam. The general form of the Magnel diagram for these eight stresses is shown in Fig. 7 and the lines are identified in Table 2.

In order to draw the Magnel diagram, the coördinates of two points on each line are required, and to define the feasible region precisely, the coordinates of the intersection points that form the corners of the region must also be calculated. The writers have derived expressions for the coordinates of relevant points and have programmed these expressions to produce a Magnel diagram for the type of cross section shown in Fig. 6. A computed diagram is shown in Fig. 8. In this diagram, Magnel lines 7 and 8 are omitted because they are not relevant to the feasible region. The cross section, drawn to scale, is also displayed on the graphics screen

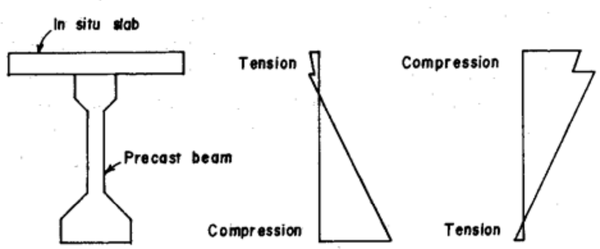

FIG. 6.—Composite Section and Stresses

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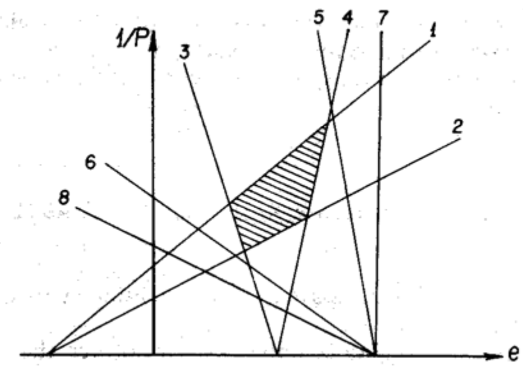

FIG. 7.—Magnel Diagram for Composite Section, General Case

TABLE 2.—Description of Magnel Lines

Line (1)	Description (2)
1	Bottom fiber of precast section, lower stress limit
2	Bottom fiber of precast section, upper stress limit
3	Top fiber of in situ slab, upper stress limit
4	Top fiber of in situ slab, lower stress limit
5	Top fiber of precast section, lower stress limit
6	Top fiber of precast section, upper stress limit
7	Bottom fiber of in situ slab, lower stress limit
8	Bottom fiber of in situ slab, upper stress limit

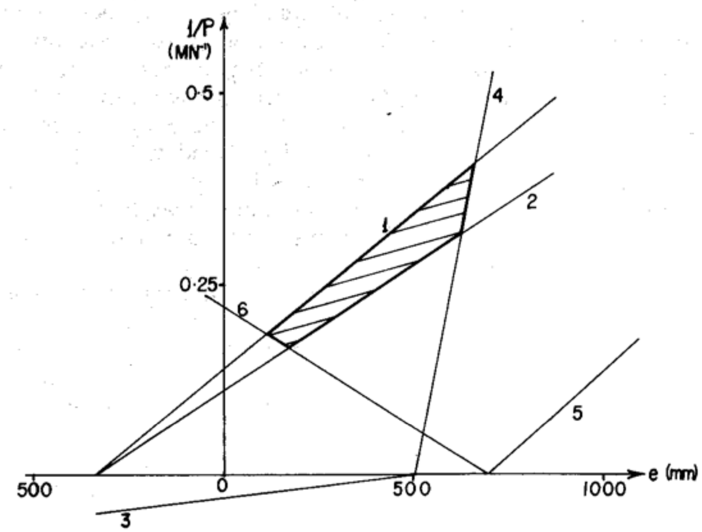

FIG. 8.—Computer-Generated Magnel Diagram for Composite Section Example

and the designer can interact with the computer to modify the dimensions of the cross section until a satisfactory result is obtained. If required, a hard copy, including all pertinent calculated quantities, may then be produced.

Closure by Natarajan Krishnamurthy,[6] F. ASCE

The writer thanks the discussers for their interest in the paper and their constructive comments.

Most of Castella's comments reiterate or reinforce the writer's statements. But his statement that the minimum eccentricity could be determined as the intersection of lines 2 and 3, corresponding to Eqs. 3*b* and 3*c* of the paper, must be qualified with the possibility that occasionally it can be the intersection of lines 3 and 4, corresponding to Eqs. 3*c* and 3*d*. Further, with a programmable calculator available as he suggests, one might as well locate all four corners to define the safe range completely.

In response to the discussion by Orr and O'Mahony, the writer cannot agree more that computer graphics can considerably enhance the writer's (or for that matter, anyone else's) analytical methods based on geometrical representations of governing equations. It must be admitted however, that almost every designer these days has (or has ready access to) a calculator, while computer graphics hardware and software are still not so accessible.

As the paper did not intend to cover composite sections, the writer will not go beyond acknowledging the applicability of his simplification procedures to those and other similar cases.

Boczkaj's comments need a more detailed response because some of them imply a difference of opinion where none need exist, and others imply a misinterpretation of the writer's intent, if not his statements.

The writer appreciates Boczkaj's concern about what the writer refers to as the fifth constraint (in addition to the four limiting stress conditions), i.e., the need for the tendon to be placed within the concrete section with adequate cover. But perhaps his apprehension about the writer's lack of its specific mention in the paper may be unfounded.

First, the writer intended and wrote his paper to address one and only one aspect of prestressed concrete design for flexure, i.e., the modifications to the Magnel diagram to determine the zone of safe combinations of prestressing force P and eccentricity e. By the term "safe zone," the writer has consistently meant the domain for combinations of P and e for which none of the four permissible extreme fiber stresses will be exceeded. He considers the feasibility of locating the tendon inside the concrete section as a practical requirement. Nowhere in his paper has the writer claimed that the determination of the safe zone for tendon force and location would automatically ensure a feasible design.

On the other hand, as the discusser himself has noted, the writer made a point of this very situation early in his paper, stating "the concrete section may be adequate to take the stresses and yet the steel tendon cannot be positioned within the section." Although limitations of space and scope prevented the writer from expanding on the implications of this situation, that he has not been unaware or negligent of them may be inferred from the following quote from his earlier paper (4):

> If no portion of the safe range of eccentricity overlaps the concrete section sufficiently to accommodate the tendon with adequate cover, the section needs to be revised. The interesting implication here is that although theoretically the concrete section could be adequate to take the given moments with the specified permissible stresses, the fact that the corresponding tendon cannot be located inside this section would force a revision, often an increase, of the concrete section.

Actually, whether this practicality condition is expressed in words or as an equation, its consideration becomes inescapable in any real design. Once the safe zone is drawn, or equivalently the range of safe eccentricities is determined from the computed coordinates of the four corners of the safe zone, the check whether any part of it overlaps into the concrete section with adequate cover, and if not, the revision of the section until the check is satisfied, is the logical next step.

Boczkaj states that the writer's comment, that "it would be extremely difficult, and (if R is also fixed) generally impossible for doubly symmetric sections to develop a beam with the precise dimensions that would provide the exact moduli required" is false. In this the writer feels that further explanation might clear the air.

The "difficulty" that the writer refers to is two-fold: First, as the number of unknowns for a section would be more than the number of equations available, assumptions of the excess quantities would be required. As the section moduli involve the ratios of polynomials in the unknowns, closed form solutions would be impossible, requiring an iterative solution. Of course, charts and modern programmable calculators reduce and camouflage this difficulty, but they cannot eliminate it.

Second, even conceding that by trial and error a section exactly satisfying the section requirements can be determined, invariably one or more dimensions of the section would be in odd figures, meaning not in round numbers or convenient fractions suitable for economical production. Normally these values would be rounded off upwards, immediately making the section not fully stressed.

This should explain the part about the difficulty of developing a fully stressed beam in practice.

The writer's claim of the impossibility of developing fully stressed doubly symmetric sections is based on the following: A doubly symmetrical beam will have both top and bottom section moduli equal; equating S_t and $(-S_b)$ in Eq. 5*a* and Eq. 5*b* gives:

$$f_{tf} - f_{bf} = R(f_{bi} - f_{ti}) \quad (10)$$

As all the limiting stresses are specified in the codes, if R is also fixed, the chances of Eq. 10 being satisfied exactly are very, very remote, and

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hence the development of a fully stressed doubly symmetric section is generally (and practically) impossible.

The writer hopes that the preceding would suffice to set the record straight.

In regard to the statement by Boczkaj that "the problem of a section design lies in finding an adequate section which satisfies all five constraints, not in how to find the dimensions of a section which exactly satisfies section moduli required by Eqs. 5*a*–*b*," the writer can only point out that: (1) Satisfaction of Eqs. 5*a* and 5*b* automatically will satisfy the four stress constraints (—and enough has already been said about the fifth); and (2) a realistic design will probably not satisfy the moduli requirements exactly, but exceed them at top and/or bottom.

Finally, although it might be quibbling about a detail, Boczkaj's P-e lines are admittedly curved, while $(1/P)$ versus e lines are straight lines; in any case, the determination of reciprocals these days is just a flick of a finger!

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