A Fresh Look at Bolted End-Plate Behavior and Design

N. KRISHNAMURTHY

End-plate connections of the typical configuration shown in Fig. 1 are increasingly used as moment-resistant connections in framed structures. However, end plates designed by the prying force formulas in the AISC *Manual of Steel Construction*[1] may be unrealistically thick. The prying force formulas were proposed by Nair *et al.*,[2] based on their work on tee hangers. Previously, Douty and McGuire[3] and later Agerskov[4,5] have presented other versions of the same basic model, and/or suggested adjusted coefficients to reflect test results. The research in the U.S.A. and abroad on this topic has been summarized by Fisher and Struik.[6]

In the prying force method, the end-plate region around the beam tension flange is considered analogous to a tee hanger, as in Fig. 2. Hence, the terms "tee flange" and "plate" or "end plate" will be used interchangeably in this paper; "tee stem" will likewise correspond to the "beam flange". Figure 3 illustrates the dimensions and forces involved in the application of the prying force method. The section at or near the face of the tee stem at which the applied force is transferred to the tee flange will be designated the "load line", L. (All the notation used in this paper is listed in Appendix A.)

The major assumption of all the analytical models proposed thus far is that a concentrated prying force Q is developed at or near the edge of the tee flange in response to the load on the tee stem. The moment diagram resulting from the action of Q and the bolt force T along the bolt center line (also assumed concentrated) is therefore linear. The critical values of the plate moment at the bolt line and at the load line are given by

$$M_2 = Qa \tag{1a}$$

and

$$M_1 = Qa - Fb \tag{1b}$$

The prying force is computed from the formula

$$Q = F\left(\frac{c_1 b d_b{}^2 - c_2 w t_p{}^2}{c_3 a d_b{}^2 + c_4 w t_p{}^2}\right) \tag{2}$$

N. Krishnamurthy is Research Professor of Civil Engineering, Vanderbilt University, Nashville, Tenn.

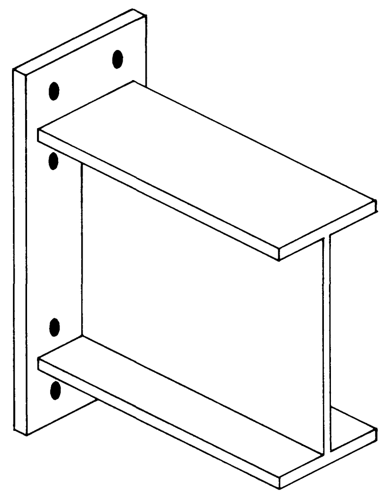


Fig. 1. Typical configuration of end-plate connection

in which the coefficients c_1, c_2, c_3, and c_4 are specified separately for A325 and A490 bolts. If the calculated Q is negative, it is to be taken as zero. The larger of the two moments M_1 and M_2 from Eqs. (1a) and (1b) is the design moment for the end plate.

In recent years, a considerable amount of continuum mechanics and yield line theory has been brought to bear on the problem. Efforts to adjust the plate design procedure to reflect test data have been focused on the modifications of the prying force formulas and of the dimensions a and b. These efforts have indeed provided analysis and design tools where none existed. The theories and formulas are being continually modified to reduce their observed conservatism. But each time a theory is modified, a new set of assumptions must be invoked; every time a coefficient is adjusted, one more theoretical refinement is effectively nullified.

The variations in the bolt forces have been accurately measured in many tests, as reported in the published literature,[3,4] but the magnitude and distribution of the prying force itself are only qualitatively known, either from visual observation or by pressure marks on a paper sheet with

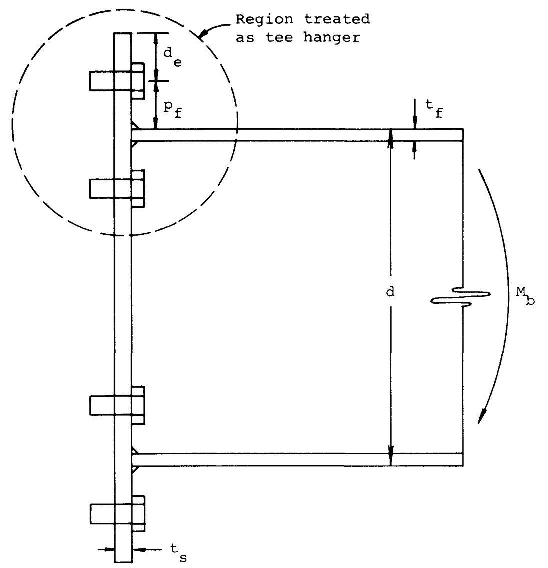


Fig. 2. End-plate connection and region treated as tee hanger

carbon backing. In extenuation of this limited experimental evidence, it must be admitted that the location of the prying forces (at the back of the plate), and of the maximum bending stresses (around the bolt hole and at the junction of the tee flange and stem), are inaccessible to routine instrumentation.

AUTHOR'S RESEARCH

To generate more information on this complicated problem, the author and his research assistants have investigated extensively various aspects of bolted tee-hanger and end-plate connections for the last six years. The research, sponsored by AISC and the Metal Building Manufacturers Association (MBMA), covers the following:

1. Feasibility, sensitivity, convergence, and parameter studies of entire connections and specific components, by the finite element method, including:
   (a) Two-dimensional (2D) analysis of nearly two hundred end-plate connections and more than one hundred tee hangers.
   (b) Three-dimensional (3D) analysis of many bench mark cases and test specimens of end-plate connections and tee hangers.
2. Tests on specimens, namely:
   (a) Twenty-four steel end-plate connections.
   (b) Fourteen steel tee-hangers.
   (c) Eighteen photoelastic models of tee hangers.

The author has published material on the correlation between the 2D and 3D analyses,[7] and on the general methodology of his solution,[8] in addition to discussions.[9,10]

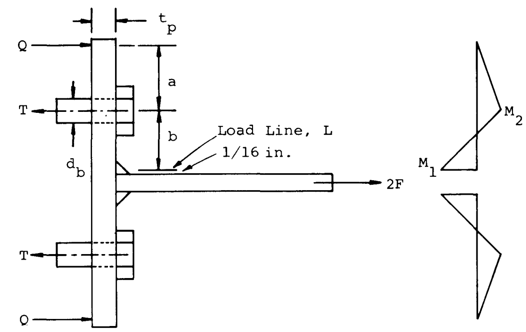


Fig. 3. Assumed forces and bending moment diagram for tee hanger

All the research tasks and the findings therefrom have been reported to the sponsors; most of the information has also been documented as master's degree theses at Auburn University and Vanderbilt University. Papers are under preparation on various details of the project.

The finite element analyses model the geometry, material properties, bolt pretensioning, and the subsequent loadings. The unknown support conditions corresponding to the deformed back of the plate are determined by an iterative process of analysis and support checks. In each cycle, nodes that tend to separate from the support are released, and previously released nodes that tend to migrate into the support are resupported. The 2D programs also incorporate the idealized elastic-perfectly-plastic behavior of the steel.

The tests were aimed at exploring and documenting the behavior of end plates and tee-hanger flanges under the interaction of bolt pretension and externally applied loadings. Deflections (and hence rotations for moment connections), as well as strains (and thus stresses) and bolt forces, were measured; in many of the steel tests, brittle coatings were used to reveal surface yielding. Test findings were used to check the computed results, and improve the analytical models as necessary and to the extent possible.

With so much new information available, it is now possible for a fresh look to be taken at end-plate behavior, and a new procedure to be proposed for its design.

END-PLATE BEHAVIOR

Let us reexamine the basic problem: The end-plate connection represents a highly nonlinear, indeterminate, and complex situation of great practical significance. By the vary nature of the problem, attempts to resolve it by classical theories or rationally reduce it to simple yet familiar formulas would be frustrating and approximate at best. In particular, in the situation where the bending spans are of the same order of magnitude as (and often less than) the plate thickness itself, the simple theory of bending will not apply; it may still be used for convenience and psychological advantage.

The applied force F is delivered as a line load across the width of the tee stem and is dispersed through the plate thickness. With a fillet (weld) at the junction of the beam flange and end plate, the dispersion would start at or close to the toe of the fillet.

The bolt force T is transferred to the plate over the annular area of the bolt head projection, and then begins to disperse through the plate thickness.

The "prying force" Q is actually the action of the pressure bulb developed by the bolt pretension, shifting away from the bolt line in response to the external load. Except for very thick plates and at or near failure loads, the reactive pressures are distributed over extensive areas between the plate edges and the bolts. Moreover, the pretensioning of the bolts tends to "quilt" the material, that is, to lift the plate away from the support at the outer edges; consequently, the lever action on which the prying force concept is based is considerably different from what is assumed.

Thus, all three forces acting on the plate are far from being concentrated. Because of this distributed nature of the forces, the bending moment diagram for the plate is curved, rather than linear as assumed. Hence, for the same force resultants, the actual peak moments at the bolt line and the load line are smaller than their theoretical values computed on the basis of assumed concentrated forces.

Further, the relative magnitudes of the two peak moments are also fairly predictable. Because of the statically redundant state of the plate, stiffnesses are critical in the distribution of bending moments. The plate cross section is reduced at the bolt holes; the bolts restrain the plate only at isolated points along the bolt line; the bending of the plate is biaxial around the bolts. All these factors combine to reduce the plate stiffness at the bolt line considerably below its stiffness at the load line. Thus, the bending moment at the load line is always larger than the moment at the bolt line. In other words, the bolt line moment M_2 (Fig. 2) does not govern the plate design in practice.

BASIC FORMULATION

Based on the postulates presented—every one of which has been amply confirmed by the computer analyses and laboratory tests—the following ultimate simplification can be proposed:

In the final analysis, the end plate must be sized for some shear F_1 and moment M_1 at its junction with the beam flange, as indicated in Fig. 4. If the point of contraflexure is located at a distance s from the load line, then,

$$M_1 = F_1 s \tag{3}$$

Conventionally, the nominal force F_f in the beam flange is taken as

$$F_f = M_b/(d - t_f) \tag{4}$$

In the familiar procedure known as the "split-tee" method, the plate projection is assumed to receive half of this flange force by implied symmetry; further, complete

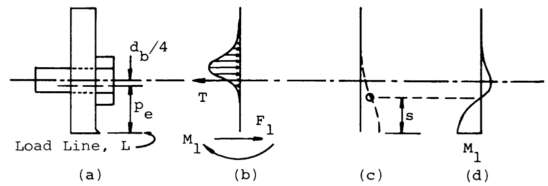


Fig. 4. Modified split-tee method: (a) Geometry, (b) Forces and pressure bulb, (c) Deflected shape, (d) Actual bending moment diagram

fixity is assumed at the bolt line. Under these circumstances, F_1 is $(F_f/2)$ and s is one-half the bolt distance.

The bolt distance itself bears some scrutiny. Again traditionally (in the split-tee method), the bending span is taken as the bolt distance (p_f in Fig. 2) from the outer face of the beam flange to the center of the bolt line. However, as other investigators[2,6] have also noticed, the actual bending span p_e will be less. Correlations of the author's finite element analyses and test results have indicated that the effective span may be safely taken as

$$p_e = p_f - 0.25d_b - w_t \tag{5}$$

Apart from an intuitive appreciation of this reduced value, a similar value for the bolt reduction has also been recommended by Fisher and Struik.[6] In Eq. (5), w_t, the throat size of the fillet weld between the beam flange and end plate, is zero for an unreinforced groove weld.

The benefit of this effective bolt distance p_e will be incorporated in all further discussions in this paper. Thus the moment at the bolt line by the split-tee method is

$$\begin{aligned} M_t &= (F_f/2)(p_e/2) \\ &= 0.25F_f p_e \end{aligned} \tag{6}$$

This value will be used as the base or reference value for the proposed design procedure.

MODIFIED SPLIT-TEE METHOD

Equation (6) represents a very reasonable idealized situation. But the real behavior is considerably different, both in terms of the shear force F_1 and the arm s.

In a tee hanger, F_1 is indeed one-half the applied force on the tee stem. But in end-plate connections where the beam web is welded to the plate, firstly the longitudinal stresses due to the beam moment are delivered to the plate partly down the web; secondly, the plate region between the beam flanges, with part of the beam web acting as a rib on the plate, is much stiffer than the plate projection beyond the beam flanges. Both these effects result in a transfer of less force to the plate projection and more to the plate region between the beam flanges.

The author's finite element analyses have shown that the ratio (F_1/F_f) varies from 0.3 to 0.5. Thus we may write,

$$F_1 = C_1 F_f \qquad C_1 \leqslant 0.5 \tag{7}$$

Again, the point of contraflexure would be at mid-height of the bolt distance p_e under ideal fixity conditions at the bolt. But in reality, except possibly at very low load levels, the applied force overcomes the bolt clamping effects, and the bolts stretch and bend, permitting some rotation of the plate. This, combined with the reduced stiffness at the bolt line, as explained earlier, results in a shift of the inflection point towards the bolt, thus increasing the value of s. The computer analyses confirm that the ratio (s/p_e) can vary from 0.5 to 1.0. Hence,

$$s = C_2 p_e \qquad 0.5 \leqslant C_2 \leqslant 1.0 \tag{8}$$

Thus, the theoretical design moment by the simple bending theory would be

$$M_s = F_1 s = C_1 C_2 F_f p_e \tag{9}$$

Because of the force dispersions and deep beam effects referred to earlier, the actual critical moment is likely to be less, say,

$$M_d = C_3 M_s = C_1 C_2 C_3 F_f p_e \tag{10}$$

Combining Eqs. (6) and (10) gives

$$M_d = \alpha_m M_t \tag{11}$$

where

$$\alpha_m = 4 C_1 C_2 C_3 \tag{11a}$$

The coefficient α_m may thus be considered a moment modification factor to compensate for the many assumptions made in the development of M_t by Eq. (6).

MODIFICATION FACTOR

From the regression analyses of the results of numerous finite element studies (whose formulations have been verified or adjusted by tests), the prediction equation for the plate moment M_d was developed as follows:

$$M_d = 1.29 \left(\frac{F_y}{F_{bu}}\right)^{0.4} \left(\frac{F_{bt}}{F_p}\right)^{0.5} \left(\frac{b_f}{b_s}\right)^{0.5} \left(\frac{A_f}{A_w}\right)^{0.32} \times \left(\frac{p_e}{d_b}\right)^{0.25} M_t \tag{12}$$

From Eqs. (11) and (12),

$$\alpha_m = C_a C_b (A_f / A_w)^{0.32} (p_e / d_b)^{0.25} \tag{13}$$

where

$$C_a = 1.29 (F_y / F_{bu})^{0.4} (F_{bt} / F_p)^{0.5} \tag{13a}$$

and

$$C_b = (b_f / b_s)^{0.5} \tag{13b}$$

Appendix B presents details of the development of Eq. (12). Appendices C and D give values of α_m and C_a.

BOLT SELECTION

The bolt diameter d_b in the preceding development is selected as follows:

The theoretical bolt area a_t per row required is determined from

$$a_t = 0.5 F_f / F_{bt} \tag{14}$$

Two bolts (or more, if necessary) are chosen to provide an actual area a_b not less than a_t.

At this point, one question must be examined. If less than half the longitudinal force in the beam is transferred to the plate projection, as suggested earlier, more than half must remain in the plate region between the beam flanges. Theoretically, then, there must be more bolt area between the beam flanges than in the plate projections. (In practice, both the bolt rows at the tension flange would have to be increased to meet the inner row requirement.) On the other hand, the actual bolt area provided will generally be more than the theoretical area required. Under service loads, at worst, the inner row will be somewhat overstressed and the outer row understressed by the same amount. It is also possible that the excess area in the outer row will help resist any increase in the bolt force there, due to possible development of prying action under high loads. Based on these considerations, additional bolt area or unequal bolt area distribution is not recommended.

PHYSICAL INTERPRETATION

The modification factor α_m , as proposed in Eq. (13), is basically the result of regression analysis of computer results. The statistical analysis could only identify the dominant trends, with no concern for any physical basis. However, the parameters in the factor can be physically interpreted to a certain extent, in the light of the hypotheses presented.

The coefficient C_a lumps all material interactions together. For A36 steel and A325 bolts, F_y is 36.0 ksi, F_p (at $0.75F_y$) is 27.0 ksi, F_{bu} is 93.0 ksi, and F_{bt} is 44.0 ksi, leading to a C_a-value of 1.13. For 90.0 ksi steel and A490 bolts, C_a becomes 1.04. This material coefficient can be tabulated as in Appendix D, for various common combinations of materials and bolts. As all the analyses were based on the assumption that the plate and beam materials were the same for any one connection, Eq. (13a) is not directly applicable to cases where the beam and plate are from different grades of steel. It would be consistent with the original formulation to use the average yield stress for F_y and the plate yield stress in setting F_p , with the restriction that C_a must not be smaller than the single material value based on average F_y. However, on the basis of results of tests which covered variations in the plate, beam flange, and beam web yield stresses, it is considered adequate to use the smallest of the yield stresses (and the *corresponding* value of F_p) in the determination of C_a. In any case, the actual value of F_p for the plate must be used in the subsequent computation of the end-plate thickness.

The coefficient C_b is a plate width correction, amounting to 0.95 for b_f/b_s ratio of 0.9. To avoid any adverse consequence of the end plate being too much wider than the beam flange, an effective maximum plate width is recommended as follows:

$$b_e = b_f + 2w_s + t_s \tag{15}$$

thus allowing for a 45 degree dispersion from the weld toe at the edge of the beam flange. For unreinforced groove welds, w_s is taken as zero.

The (A_f/A_w) ratio quantifies the distribution of the beam material, and determines the fraction of the applied longitudinal force that is transferred to the plate projection. For the majority of rolled and built-up wide-flange or I-shaped sections, A_f/A_w is between 0.25 and 2.50, leading to a variation of 0.63 to 1.36 in the α_m result.

Finally, the p_e/d_b term appears to represent the complex influence of the bolt size and clamping force, in relation to the bolt distance. The ratio generally lies between 0.75 and 2.50, for which the influence on α_m varies from 0.93 to 1.26. *The term p_e/d_b is a very dominant parameter,* and clearly the bolt distance must be kept as small as feasible, for maximum economy.

DESIGN PROCEDURE

The procedure for end-plate design is formulated as follows:

1. Find the nominal flange force F_f from Eq. (4).
2. Find the required bolt area a_t per row from Eq. (14), and hence determine the bolt size d_b.
3. Find the effective bolt distance p_e from Eq. (5).
4. Find the split-tee moment M_t from Eq. (6).
5. Find the moment modification factor α_m from Eqs. (13), (13a), and (13b).
6. Find the design moment M_d for the end plate from Eq. (11).
7. Find the end-plate thickness t_s by the simple theory of bending from the expression

$$t_s = \sqrt{\frac{6M_d}{b_s F_p}} \tag{16}$$

8. Check the effective plate width b_e by Eq. (15). If b_e is less than the actual plate width b_s, repeat steps 5 through 8 with b_e in place of b_s.
9. Check the maximum shear stress f_s in the plate from

$$f_s = F_f/(2b_s t_s) \tag{17}$$

If f_s exceeds the allowable value $0.4F_y$ for the plate material, increase t_s to satisfy Eq. (17).

Implied in the application of the proposed procedure are the other conditions assumed in the entire analysis: The bolts must be pretensioned to the AISC recommended value of 0.7 times their ultimate strength. The vertical edge distance, although apparently of secondary significance in a certain range, must be kept at about 1.75 times (not less than 1.5 times) the bolt diameter. The beam end must be welded to the end plate, not only at the flanges (all around if by fillet welds), but also down the web.

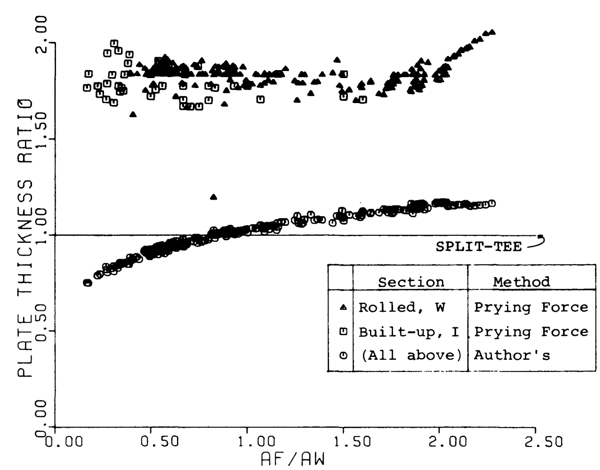


Fig. 5. Comparison of end-plate designs by prying force method and by modified split-tee method, for 36.0 ksi steel, A325 bolts, p_f/d_b = 1.5

Routine design may be simplified in a number of ways. The A_f/A_w ratio, or even its value raised to the 0.32 power as needed in Eq. (13), may be tabulated for standard rolled and built-up sections. Charts may be prepared for α_m as a function of the A_f/A_w ratio, for standard combinations of materials, bolt sizes and bolt distances. Appendix D for the material coefficient is a design aid. Appendix C represents another design aid for the quick determination of α_m for known values of C_aC_b, A_f/A_w, and p_e/d_b.

Appendix E demonstrates the application of the design procedure to the same beam example as in Ref. 1, 1st Printing. The resulting plate thickness is $^{13}\!/_{16}$-in., as against the value of $1^{7}\!/_{16}$ in. by the prying force method.

The proposed procedure results in savings of 25 to 50 percent below the prying force method in most practical situations. Figures 5 and 6 depict comparisons for two specific combinations where beams are attached to the end plates by fillet welds, applied to all the 192 standard rolled wide-flange sections listed in the AISC Manual, and 44 typical built-up sections used by MBMA member companies. In Figs. 5 and 6 the reference split-tee thickness is the value required for the split-tee moment, from Eq. (16), with M_t by Eq. (6) used in place of M_d.

TEST FINDINGS

Notwithstanding the very high correlation obtained in the regression analyses, the real test of the proposed design procedure was whether the thinner plates were strong

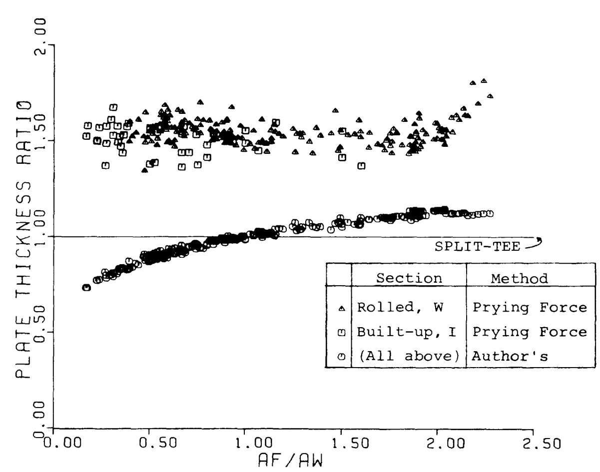


Fig. 6. Comparison of end-plate designs by prying force method and by modified split-tee method, for 50.0 ksi steel, A325 bolts, p_f/d_b = 1.5

enough to develop the ultimate capacities of the beams connected.

To demonstrate this, a series of nine connections, whose end plates were designed according to the proposed procedure, were tested to failure. The beams covered A_f/A_w ratios from 0.5 to 2.0, and the bolt sizes and locations spanned p_e/d_b ratios from 0.8 to 1.4. Table 1 presents significant details of the specimens and the test results.

The first specimen failed by torsional twisting of the beam prematurely; extensive lateral support was provided subsequently to prevent such failure. In all other cases, failure was by local buckling of the beam compression flange and web. In no case was the end plate itself under visible or measured overall distress. In all but three cases (including the aborted first test), the beams developed or exceeded their ultimate strength. The last two, which reached only 80% and 83% of their ultimate strength, were unusually deep beams with very thin webs that buckled between the stiffeners.

CONCLUSION

Admittedly, the proposed design procedure breaks with tradition by deriving empirical relationships based upon statistical analysis of the results from parameter studies on the computer. But the entire process essentially reduces to two concepts: (1) application of simple classical theory to a basic model, and (2) modification by a factor which incorporates dominant influences in relatively compact, experimentally validated proportions. Thus, it accomplishes the same ends as most of the more complicated theoretical methods attempt, namely the development of simple design expressions for complicated phenomena, and their adjustment to reflect observed behavior.

In fact, the design formulas are the direct outcome of analysis; test findings have been incorporated in them only to the extent of refining the computer models to include details such as the bolt heads, and for simplifying the C_a computation.

The proposed procedure is recommended as economical and safe for end-plate connections used as longitudinal splices between beams or frame members, and for beam-to-column connections where the column flanges either are stiffened at the levels of the beam flanges or are otherwise adequate.

The decrease in end-plate thickness by the proposed method will frequently be accompanied by some reduction in connection rigidity. Under circumstances where deformations can be critical, the influence of such increased connection flexibility on the overall behavior of the structure may need review. (The same analysis that led to the proposed procedure for design based on strength have also generated data that quantify the connection stiffness as moment-rotation relationships, which may in turn be used in this suggested reanalysis process.)

It is highly reassuring that in the validation tests none of the designed plates failed or showed any signs of distress, nor did any bolt fracture, even when the attached beams had failed. But could the plates be even thinner, in the typical configuration studied or in modified configurations

Table 1. Details of End-Plate Connection Tests

No.	d	b_f	t_f	t_w	d_b	p_f	b_s	t_s	d_s	w_s	F_y[b]	$\frac{M_m}{M_p}$	Failure Mode[c]
1[a]	15.65	5.5	0.345	0.250	0.750	1.313	6.0	0.625	21.00	0.438	44.2	0.58	T
2[a]	11.96	6.5	0.400	0.237	0.875	1.438	7.0	0.750	18.25	0.375	37.6	0.99	T
3[a]	16.00	7.0	0.503	0.307	1.000	1.500	8.0	0.760	24.00	0.438	39.7	1.01	F
4	13.00	6.0	0.500	0.250	0.875	1.344	7.0	0.622	18.75	0.438	47.4	0.99	F
5[a]	15.65	5.5	0.345	0.250	0.750	1.063	6.0	0.500	20.50	0.313	44.2	1.07	F
6	9.00	6.0	0.500	0.250	0.875	1.313	7.0	0.688	14.75	0.500	47.4	1.03	F,T
7[a]	16.00	7.0	0.503	0.307	1.000	1.563	8.0	0.750	22.50	0.500	39.7	1.01	F
8	24.75	6.0	0.375	0.188	0.875	1.250	7.0	0.500	30.50	0.375	52.1	0.80	F,W
9	24.75	6.0	0.375	0.188	0.875	1.750	7.0	0.750	31.34	0.375	52.1	0.83	F,W

[a] Rolled sections: Nos. 1 and 5, W16X26; No. 2, W12X27; Nos. 3 and 7, W16X40. All dimensions actual and inch units.
[b] Yield stress of beam flange, used in computation of M_p.
[c] Failure modes: F — Flange buckling; T — Torsional twisting; W — Web buckling.

such as with stiffened plates or multiple bolt rows between the beam flanges? To explore this question, additional research sponsored by MBMA is under way.

ACKNOWLEDGMENTS

The research alluded to in the paper was financially supported by the American Institute of Steel Construction and the Metal Building Manufacturers Association since 1971. The valuable counsel of the associated task committees is gratefully acknowledged. The test specimens were donated by American Buildings Company of Eufaula, Alabama, and McMurray Steel Company and Englert Engineering Company, both of Nashville, Tennessee. The massive computational services needed were provided by the computer centers of Auburn University, University of Alabama in Birmingham, and Vanderbilt University. Many of the research tasks referred to were carried out by the author's graduate students Graddy, Huang, Jeffrey, Daniel, Avery, Saxena, Oswalt, and Byers.

REFERENCES

1. Manual of Steel Construction *Seventh Edition, American Institute of Steel Construction, New York, N.Y., 1970.*
2. *Nair, R. S., R. C. Birkemoe, and W. H. Munse* High Strength Bolts Subject to Tension and Prying *Journal of the Structural Division, ASCE, Vol. 100, No. ST2, pp. 351–372, Feb. 1974.*
3. *Douty, R. T. and W. McGuire* High Strength Bolted Moment Connections *Journal of the Structural Division, ASCE, Vol. 91, No. ST2, pp. 101–128, April 1965.*
4. *Agerskov, H.* High-Strength Bolted Connections Subject to Prying *Journal of the Structural Division, ASCE, Vol. 102, No. ST1, pp. 161–175, Jan. 1976.*
5. *Agerskov, H.* Analysis of Bolted Connections Subject to Prying *Journal of the Structural Division, ASCE, Vol. 103, No. ST11, pp. 2145–2163, Nov. 1977.*
6. *Fisher, J. W. and J. H. A. Struik* Guide to Design Criteria for Bolted and Riveted Joints *John Wiley & Sons, New York, N.Y., Chapters 16–18, 1974.*
7. *Krishnamurthy, N. and D. E. Graddy, Jr.* Correlation between 2- and 3-Dimensional Finite Element Analysis of Steel Bolted End-Plate Connections *Computers and Structures—An International Journal, Vol. 6, Nos. 4/5, pp. 381–389, Aug./Oct. 1976.*
8. *Krishnamurthy, N.* Steel Bolted End-Plate Connections *Proceedings of the International Conference on Finite Element Methods in Engineering, Adelaide, Australia, pp. 23.1–23.16, Dec. 1976.*
9. *Krishnamurthy, N.* Discussion of "High Strength Bolts Subject to Tension and Prying" (Ref. 2 above) *Journal of the Structural Division, ASCE, Vol. 101, No. ST1, pp. 335–337, Jan. 1975.*
10. *Krishnamurthy, N.* Discussion of "High-Strength Bolted Connections Subject to Prying" (Ref. 4 above) *Journal of the Structural Division, ASCE, Vol. 103, No. ST1, pp. 299–300, Jan. 1977.*

APPENDIX A — NOTATION

a = Lever arm for prying force; the smaller of d_e and $2t_p$
a_b = Actual bolt area per row
a_t = Theoretical bolt area per row
A = Cross-sectional area of beam
A_f = Area of tension flange of beam
A_w = Area of beam web
b = Bolt distance from section of maximum moment at face of tee stem, in prying force formula
b_e = Effective width of end plate
b_f = Width of beam flange
b_s = Width of end plate
c_i = Coefficients in prying force formula ($i = 1, 2, 3, 4$)
C_a = Material coefficient
C_b = Plate width correction factor
C_i = Coefficients in author's formulas ($i = 1, 2, 3$)
d = Beam depth
d_b = Nominal bolt diameter
d_e = Vertical edge distance of plate beyond outer bolt row
d_s = Depth of end plate
f_b = Extreme fiber bending stress in beam
f_s = Maximum shear stress in end plate
F = Half the force applied in the tee stem, assumed transferred to tee flange
F_b = Allowable extreme fiber bending stress in beam
F_{bt} = Allowable tensile stress in bolt
F_{bu} = Ultimate tensile stress of bolt
F_f = Nominal force in beam flange
F_p = Allowable bending stress in end plate
F_y = Yield stress of beam and plate material; average value if different
F_1 = Shear in plate projection; total force beyond beam tension flange
M_b = Beam bending moment
M_d = Design moment for end plate
M_m = Maximum moment developed by beam in test
M_p = Ultimate moment capacity of beam
M_s = Theoretical design moment for plate
M_t = Plate moment by split-tee method
M_1 = Plate moment at load line
M_2 = Plate moment at bolt line
p_f = Bolt distance from face of beam flange
p_e = Effective bolt distance
s = Distance from load line to point of contraflexure in end plate
S = Section modulus of beam
t_f = Thickness of beam flange
t_p = Thickness of tee flange or end plate in prying force method
t_s = End-plate thickness by author's proposed method
t_w = Thickness of beam web

T = Bolt force
w = Tributary width of tee flange or end plate per bolt
w_s = Size of fillet weld (taken as zero for unreinforced groove welds)
w_t = Throat size of fillet weld (= $0.707w_s$ if 45-degree weld)
α_m = Modification factor for plate moment (= M_d/M_t)
β_i = Beam coefficients in prediction equations ($i = d, m, t$)
μ_i = Material coefficients in prediction equations ($i = d, m, t$)

APPENDIX B

DEVELOPMENT OF PREDICTION EQUATION FOR PLATE MOMENT

By regression analysis of finite element results from 168 end-plate connections for 559 load cases, the prediction equation for plate moment M_d was found to be:

$$M_d = 0.124\beta_m \mu_m t_s^{0.671} p_e^{0.829} f_b^{1.040}/a_b^{0.406} \quad (A.1)$$

in which

$$\beta_m = \frac{b_f^{1.198} t_f^{0.673} A^{0.406}}{d^{0.183} t_w^{0.188}}$$

$$\mu_m = F_y^{0.216}/F_{bu}^{0.256}$$

Also, for design,

$$M_d = F_p(b_s t_s^2/6) \quad (A.2)$$

Substituting for t_s from Eq. (A.2) into Eq. (A.1), and collecting terms in M_d:

$$M_d = 0.106\beta_d \mu_d p_e^{1.247} f_b^{1.565}/(a_b^{0.611} b_s^{0.505}) \quad (A.3)$$

in which

$$\beta_d = \frac{b_f^{1.803} t_f^{1.012} A^{0.611}}{d^{0.275} t_w^{0.282}}$$

and

$$\mu_d = \frac{F_y^{0.325}}{F_{bu}^{0.385} F_p^{0.505}}$$

Let the moment modification factor α_m be defined as follows:

$$M_d = \alpha_m M_t \quad (A.4)$$

in which M_t is the split-tee moment given by

$$M_t = 0.25 F_f p_e \quad (6)$$

as explained in the text.

From Eqs. (A.3), (A.4), and (6),

$$\alpha_m = M_d/M_t = 0.426\beta_d \mu_d p_e^{0.247} f_b^{0.565}/(a_b^{0.611} b_s^{0.505} F_f) \quad (A.5)$$

From other finite element and regression analyses, it was determined that (p_e/d_b) was a dominant parameter. To reduce all the variables to dimensionless parameters, the bolt area term was selectively substituted as follows:

To satisfy the allowable stress requirement,

$$a_b = 0.5F_f/F_{bt} \quad (A.6)$$

Also, for two bolts per row,

$$a_b = 2(\pi d_b^2/4) = 0.5\pi d_b^2 \quad (A.7)$$

Thus, from Eqs. (A.6) and (A.7),

$$a_b^{0.611} = a_b^{0.1235} a_b^{0.4865} = (0.5\pi d_b^2)^{0.1235}(0.5F_f/F_{bt})^{0.4865} = 0.755 d_b^{0.247} F_f^{0.487}/F_{bt}^{0.487} \quad (A.8)$$

In Eqs. (A.5) and (A.8), the term F_f may be written as

$$F_f = f_b S/(d - t_f) \quad (A.9)$$

Substituting Eq. (A.8) into Eq. (A.5), and Eq. (A.9) into the result, and grouping the non-dimensional terms,

$$\alpha_m = 0.564\beta_t \mu_t (f_b/F_y)^{0.078}(p_e/d_b)^{0.247}(b_f/b_s)^{0.505} \quad (A.10)$$

in which

$$\beta_t = \frac{b_f^{1.298} t_f^{1.102} A^{0.611}(d - t_f)^{1.487}}{d^{0.275} t_w^{0.282} S^{1.487}} \quad (A.10a)$$

and

$$\mu_t = \frac{F_y^{0.403} F_{bt}^{0.487}}{F_{bu}^{0.385} F_p^{0.505}} \quad (A.10b)$$

By regression analysis of 192 standard rolled wide-flange sections and 44 typical built-up I sections, it was determined that the beam coefficient Eq. (A.10a), reduced to:

$$\beta_t = 2.387(A_f/A_w)^{0.322} \quad (A.11)$$

Further, with a maximum extreme fiber stress about 0.6 times the yield stress, in Eq. (A.10),

$$(f_b/F_y)^{0.078} = 0.961 \quad (A.12)$$

By rounding exponents in Eq. (A.10b) and combining terms, the following simplified expressions were adopted:

$$\alpha_m = C_a C_b (A_f/A_w)^{0.32}(p_e/d_b)^{0.25} \quad (13)$$

in which

$$C_a = 1.29(F_y/F_{bu})^{0.4}(F_{bt}/F_p)^{0.5} \quad (13a)$$

and

$$C_b = (b_f/b_s)^{0.5} \quad (13b)$$

APPENDIX C—MOMENT MODIFICATION FACTOR

Moment Modification Factor (α_m)

A_f/A_w \ p_e/d_b	C_aC_b = 0.95							
	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.25	0.57	0.61	0.64	0.67	0.70	0.72	0.75	0.77
0.50	0.71	0.76	0.80	0.84	0.88	0.91	0.93	0.96
0.75	0.81	0.87	0.92	0.96	1.00	1.03	1.06	1.09
1.00	0.88	0.95	1.00	1.05	1.09	1.13	1.16	1.19
1.25	0.95	1.02	1.08	1.13	1.17	1.21	1.25	1.28
1.50	1.01	1.08	1.14	1.20	1.24	1.29	1.32	1.36
1.75	1.06	1.14	1.20	1.26	1.31	1.35	1.39	1.43
2.00	1.10	1.19	1.25	1.31	1.36	1.41	1.45	1.49
2.25	1.15	1.23	1.30	1.36	1.42	1.46	1.51	1.55
2.50	1.19	1.27	1.35	1.41	1.46	1.51	1.56	1.60

A_f/A_w \ p_e/d_b	C_aC_b = 1.00							
	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.25	0.60	0.64	0.68	0.71	0.74	0.76	0.79	0.81
0.50	0.75	0.80	0.85	0.89	0.92	0.95	0.98	1.01
0.75	0.85	0.91	0.96	1.01	1.05	1.08	1.12	1.15
1.00	0.93	1.00	1.06	1.11	1.15	1.19	1.22	1.26
1.25	1.00	1.07	1.14	1.19	1.24	1.28	1.32	1.35
1.50	1.06	1.14	1.20	1.26	1.31	1.35	1.39	1.43
1.75	1.11	1.20	1.26	1.32	1.38	1.42	1.46	1.50
2.00	1.16	1.25	1.32	1.38	1.44	1.48	1.53	1.57
2.25	1.21	1.30	1.37	1.43	1.49	1.54	1.59	1.63
2.50	1.25	1.34	1.42	1.48	1.54	1.59	1.64	1.69

A_f/A_w \ p_e/d_b	C_aC_b = 1.05							
	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.25	0.63	0.67	0.71	0.75	0.77	0.80	0.83	0.85
0.50	0.78	0.84	0.89	0.93	0.97	1.00	1.03	1.06
0.75	0.89	0.96	1.01	1.06	1.10	1.14	1.17	1.20
1.00	0.98	1.05	1.11	1.16	1.21	1.25	1.29	1.32
1.25	1.05	1.13	1.19	1.25	1.30	1.34	1.38	1.42
1.50	1.11	1.20	1.26	1.32	1.37	1.42	1.46	1.50
1.75	1.17	1.26	1.33	1.39	1.44	1.49	1.54	1.58
2.00	1.22	1.31	1.39	1.45	1.51	1.56	1.61	1.65
2.25	1.27	1.36	1.44	1.51	1.57	1.62	1.67	1.71
2.50	1.31	1.41	1.49	1.56	1.62	1.67	1.72	1.77

A_f/A_w \ p_e/d_b	C_aC_b = 1.10							
	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.25	0.66	0.71	0.75	0.78	0.81	0.84	0.86	0.89
0.50	0.82	0.88	0.93	0.98	1.01	1.05	1.08	1.11
0.75	0.93	1.00	1.06	1.11	1.15	1.19	1.23	1.26
1.00	1.02	1.10	1.16	1.22	1.27	1.31	1.35	1.38
1.25	1.10	1.18	1.25	1.31	1.36	1.40	1.45	1.49
1.50	1.17	1.25	1.32	1.39	1.44	1.49	1.53	1.57
1.75	1.22	1.32	1.39	1.46	1.51	1.56	1.61	1.65
2.00	1.28	1.37	1.45	1.52	1.58	1.63	1.68	1.73
2.25	1.33	1.43	1.51	1.58	1.64	1.70	1.75	1.79
2.50	1.37	1.47	1.56	1.63	1.70	1.75	1.81	1.85

APPENDIX D—MATERIAL COEFFICIENTS FOR COMMON COMBINATIONS

The general expression for C_a (Eq. 13a) is:

$$C_a = 1.29\,(F_y/F_{bu})^{0.4}(F_{bt}/F_p)^{0.5}$$

For $F_p = 0.75F_y$:

$$C_a = 1.49F_{bt}{}^{0.5}/(F_{bu}{}^{0.4}F_y{}^{0.1})$$

For A325 bolts:

$$F_{bt} = 44.0 \text{ ksi and } F_{bu} = 93.0 \text{ ksi}$$

For A490 bolts:

$$F_{bt} = 54.0 \text{ ksi and } F_{bu} = 116.0 \text{ ksi}$$

F_y (ksi)	A325	A490
36.0	1.13	1.14
42.0	1.11	1.13
45.0	1.10	1.12
50.0	1.09	1.11
55.0	1.08	1.10
60.0	1.07	1.09
65.0	1.06	1.08
90.0	1.03	1.04

APPENDIX E—EXAMPLE OF END-PLATE CONNECTION DESIGN

Given:

W16 × 45 section:
$d = 16.12$ in., $b_f = 7.039$ in., $t_f = 0.563$ in., $t_w = 0.346$ in., $S = 72.5$ in.^{3}

A36 steel: $F_y = 36.0$ ksi and $F_p = 27.0$ ksi

A325 bolts: $F_{bt} = 44.0$ ksi and $F_{bu} = 93.0$ ksi

Maximum bending moment for maximum bending stress of 0.66 F_y:

$$M_b = (72.5)(0.66)(36.0) = 1722.6 \text{ kip-in.}$$

Design:

1. Nominal flange force, by Eq. (4):
   $F_f = 1722.6/(16.12 - 0.563) = 110.7$ kips
2. Bolt area per row, by Eq. (14):
   $a_t = (0.5)\,(110.7)/44.0 = 1.26$ sq. in.

   Use two 1-in. diameter bolts, to provide 1.57 sq. in.
3. Set edge distance:
   $d_e = (1.75)\,(1.0) = 1.75$ in.

   Set bolt distance at (say) 1.5 diameters:
   $p_f = (1.5)\,(1.0) = 1.5$ in.

   Set weld size w_s to transfer F_f to the end plate:
   Use ½-in. fillet welds with E70 electrodes.

   Effective bolt distance, by Eq. (5):
   $p_e = 1.5 - (0.25)(1.0) - (0.707)\,(0.5) = 0.897$ in.
4. Split-tee moment, by Eq. (6):
   $M_t = (0.25)(110.7)(0.897) = 24.83$ kip-in.
5. Material coefficient, by Eq. (13a):
   $C_a = (1.29)\,(3.60/93.0)^{0.4}(44.0/27.0)^{0.5} = 1.127$

   Minimum plate width:
   $b_f + (2 \times \text{weld size}) = 7.039 + (2 \times 0.5) = 8.039$ in.

   Set plate width b_s at 8.5 in.

   Width correction factor, by Eq. (13b):
   $C_b = (7.039/8.5)^{0.5} = 0.910$

   Area of beam tension flange:
   $A_f = (7.039)(0.563) = 3.963$ sq. in.

   Area of beam web (between the two flanges):
   $A_w = (0.346)\,[16.12 - (2 \times 0.563)] = 5.188$ sq. in.

   Hence, $A_f/A_w = 3.963/5.188 = 0.764$

   $(p_e/d_b) = 0.897/1.0 = 0.897$

   Moment modification factor, by Eq. (13):
   $\alpha_m = (1.127)(0.910)(0.764)^{0.32}(0.897)^{0.25} = 0.916$

   (Note: From Appendix D, C_a could have been read off as 1.13. From Appendix C, for C_aC_b of (1.127) (0.910) or 1.03, α_m could have been estimated at about 0.92.)
6. Design moment, by Eq. (11):
   $M_d = (0.916)\,(24.83) = 22.74$ kip-in.
7. End-plate thickness, by Eq. (16):

   $$t_s = \sqrt{\frac{(6)\,(22.74)}{(8.5)\,(27.0)}} = 0.771 \text{ in.}$$

   Try 13/16-in. plate ($t_s = 0.8125$ in.)
8. Check effective plate width, by Eq. (15):
   $b_e = 7.039 + (2 \times 0.5) + 0.8125 = 8.852 \text{ in.} > b_s = 8.5$ in. **o.k.**
9. Check maximum shear stress, by Eq. (17):
   $f_s = 110.7/[(2)(8.5)(0.8125)] = 8.02$ ksi

   Allowable shear stress $= 0.4F_y = (0.4)\,(36.0) = 14.4 \text{ ksi} > f_s = 8.02$ ksi **o.k.**

Use 13/16-in. thick plate, 8.5 in. wide, and 22.5 in. deep.

Design for the same situation by the prying force method requires 1-in. diameter bolts, but a $1\frac{7}{16}$-in. thick plate.

Note: Figure 5 happens to be based on the same materials and p_f/d_b ratio as used in this problem, and hence its use as a design chart may be demonstrated as follows:

Thickness of end plate for the split-tee moment, by the simple bending theory, is:

$$\sqrt{\frac{(6)(24.83)}{(8.5)\,(27.0)}} = 0.806 \text{ in.}$$

The value of the thickness ratio for the modified split-tee method, corresponding to A_f/A_w of 0.764, as read off from Fig. 5, is 0.97.

Hence, the thickness by the proposed procedure is

$$t_s = (0.806)(0.97) = 0.78 \text{ in.}$$

(In Figs. 5 and 6, the plate widths were set to the whole inch above the beam flange width; in this problem, b_s would be 8.0 in.)

APPENDIX F—ADDITIONAL TEST RESULTS

After the rest of the paper had been set up for publication, a tenth specimen was tested in April 1978, with results that reinforce the statements made in the paper. The beam was of W16 × 45 section, with d of 16.12 in., b_f of 7.039 in., t_f of 0.563 in., and t_w of 0.346 in.; d_f was 1.0 in. and p_f was 1.75 in.; the plate had b_s of 8.75 in., t_s of 0.875 in., and d_s of 20.0 in., projecting beyond the beam tension flange only and cut off nearly flush on the compression flange side. The weld size w_s was 0.5 in. and the average flange yield stress F_y was 39.54 ksi. The (M_m/M_p) ratio at failure was 1.04, the specimen failing by torsional twisting. The brittle coating indicated considerable yielding of the beam flanges and web, but no yielding of the plate. The strain gages on the end plate, next to the weld attaching the plate to the junction of beam tension flange and web, indicated very localized yielding, but strain gages elsewhere read low strains. The specimen was supplied by Butler Manufacturing Company of Grandview, Missouri.

Discussion

A Fresh Look at Bolted End-Plate Behavior and Design

Paper presented by N. KRISHNAMURTHY
(2nd Quarter, 1978 issue)

Discussion by **Henning Agerskov**

The author must be congratulated for his investigations on the behavior of bolted tee-hanger and end-plate connections, which have resulted in interesting findings.

During recent years the writer has carried out an investigation on the strength and stiffness characteristics of these types of connections at the Technical University of Denmark. The main results obtained in this investigation have recently been reported in two ASCE Journal of the Structural Division papers (Refs. 4 and 5), and the writer wishes to comment briefly on some of the author's findings, in the light of the results of his own research.

The author has done valuable service in emphasizing that, when considering the tee-hanger model, the bending moment, M_1, at the load line is always larger than the moment, M_2, at the bolt line. Many previous approaches presuming values of $M_2 > M_1$ have led to quite unrealistic results.

The writer does not agree with the author when stating that "except for very thick plates and at or near failure loads, the reactive pressures are distributed over extensive areas between the plate edges and the bolts." The writer finds the assumption that the resultant of the prying forces is *at the yield load* of the connection acting at the edge of the plate to be justified by his own and other investigations. However, the question has been discussed in detail in Ref. 11, and this discussion shall not be repeated here.

A number of details in the results presented might be discussed. However, the writer would like to concentrate on two items: the question of the magnitude of the resulting bolt forces in the connection and the deformation characteristics of the connection.

In the proposed design method, the theoretical bolt area a_t per row required is determined from Eq. (14): $a_t = 0.5F_f/F_{bt}$, i.e., the bolt area is calculated as half the nominal force in the beam flange divided by the allowable tensile stress in the bolt. In Eq. (14) the increase in bolt forces due to the prying action in the connection is quite neglected. This will only be acceptable in case of a connection with a heavy end-plate (e.g., see Figs. 7–9 of Ref. 5). The analytical and experimental investigations carried out at the Technical University of Denmark showed that the increase in bolt forces due to prying action may amount to about 50 percent, depending on the material properties and the geometry of the connection, and this increase may not in general be neglected. Further, the writer finds the statements concerning the distribution of the tensile force in the beam flange on the inner and outer rows of bolts questionable. It should not be accepted that, in some cases, "the inner row will be somewhat overstressed and the outer row understressed by the same amount." The author found the ratio C_1 to vary from 0.3 to 0.5. If a value of 0.3 is supposed, the outer row of bolts carries 30 percent and the inner row 70 percent of the tensile force, while both are designed to take 50 percent of the force. Thus, according to the author's findings, the overstressing of the inner row of bolts due to uneven distribution of the flange force may be up to 40 percent of the bolt design force.

Henning Agerskov is Associate Professor of Civil Engineering, Technical University of Denmark, Lyngby, Denmark, and is currently Visiting Professor, Dept. of Civil Engineering, University of Waterloo, Waterloo, Ontario, Canada.

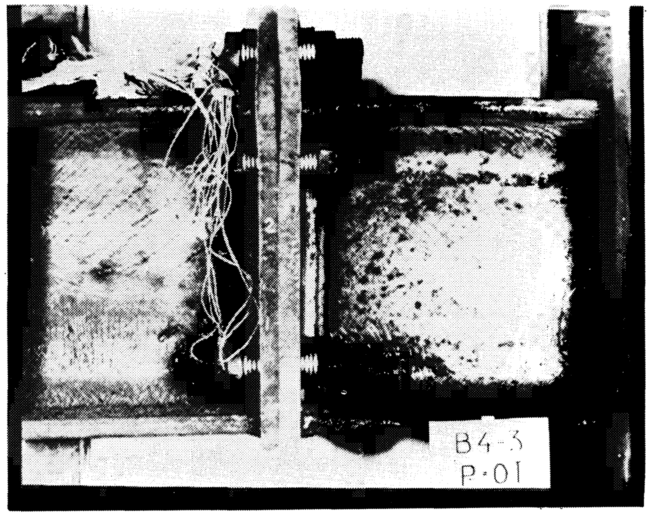


Fig. 7. End-plate connection from test series B4, after unloading (Ref. 4)

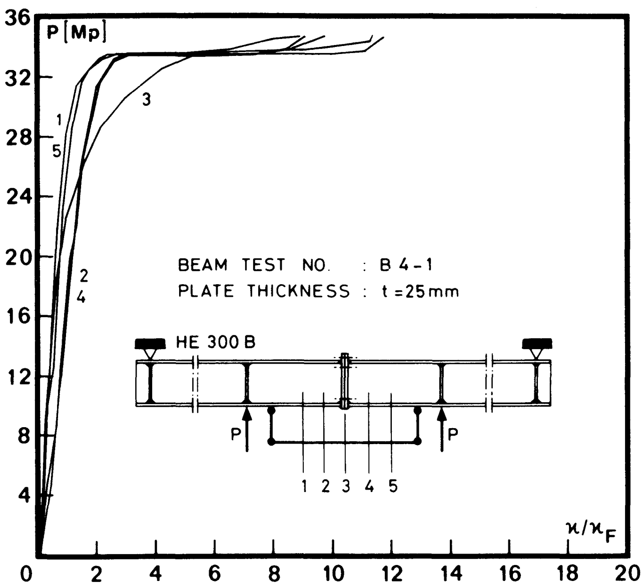


Fig. 8. Applied load vs. curvature in test B4-1 (Ref. 4)

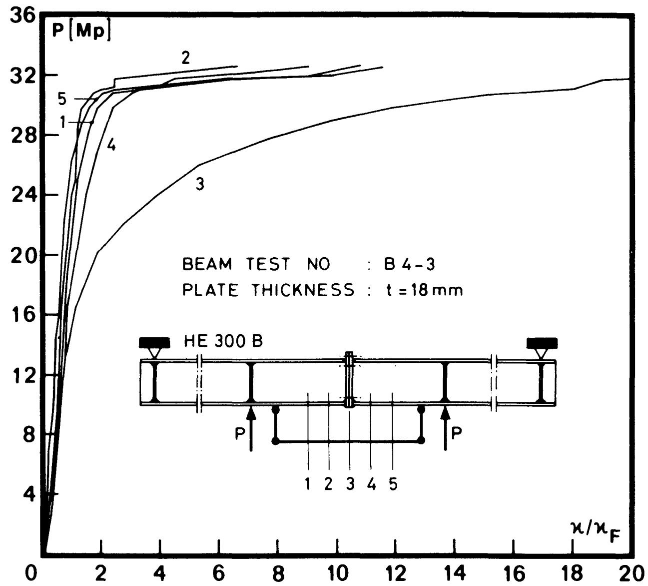


Fig. 9. Applied load vs. curvature in test B4-3 (Ref. 4)

The design method proposed by the author results in a considerable reduction in end-plate thickness. The author states that "the decrease in end-plate thickness by the proposed method will frequently be accompanied by some reduction in connection rigidity". The writer finds that this will in reality always be the case and that this reduction in many cases may be considerable, and thus critical to the structure. In the experimental investigation carried out at the Technical University of Denmark, the influence of the end-plate thickness on the deformation characteristics of the connection was studied. It was found in the tests with relatively thin end-plates that the load could be increased beyond the yield load as defined in the theory proposed by the writer,[4,5,12] but this increase was accompanied by heavy plastic deformations arising after the yield load of the connection was reached and while strain hardening was taking place in the end-plate. Figure 7 shows the end-plate connection from one of the tests, after unloading. Considerable plastic deformations may be observed. It must be emphasized that this connection did not have an extremely thin end-plate. In the connection shown in Fig. 7, the end-plate thickness was equal to the thickness of the beam flange, approx. 18 mm (0.71 in.). More details on the test specimens may be found in Ref. 4.

The reduction in the connection rigidity due to the thinner end-plates further appears from Figs. 8 and 9. The diagrams show the relationship between the applied load, P, i.e., the moment in the central part of the test beam and the curvature, κ (a dimensionless ratio κ/κ_F is used). Curves 1 and 5 are by and large unaffected by the introduction of the connection in the beam, while the deviation between these curves and curve 3 shows the influence of the end-plate connection on the deformations. The test specimens of tests B4-1 and B4-3 were practically identical, with the exception of a thinner end-plate used in test B4-3.

On the basis of the results of the experimental investigation at the Technical University of Denmark, the writer wishes to emphasize that a significant reduction in connection rigidity may be the result of a considerable reduction in end-plate thickness. This reduction in rigidity may be unacceptable in many structures, e.g., high-rise buildings.

REFERENCES

11. *Agerskov, H.* Closure of "High-Strength Bolted Connections Subject to Prying" *(Ref. 4), Journal of the Structural Division, ASCE, Vol. 103, No. ST 10, pp. 2066–2068, Oct. 1977.*
12. *Agerskov, H.* Analysis of High-Strength Bolted End Plate Connections *Bygningsstatiske Meddelelser, Vol. 48, No. 3, pp. 63–93, Oct. 1977.*

Discussion by W. McGuire

The end-plate connection procedure proposed by Professor Krishnamurthy could be of major significance in design. Other investigators who have worked on this problem will probably agree that existing methods may be conservative. Any proposal that can justify more liberal procedures deserves careful attention. The purpose of this discussion is to make some observations and raise some questions that

W. McGuire is Professor of Structural Engineering, School of Civil and Environmental Engineering, Cornell University, Ithaca, New York.

require satisfactory answers before the proposed procedure can gain unqualified acceptance.

Dr. Krishnamurthy notes that, "in reality, except possibly at very low load levels, the applied force overcomes the bolt clamping effects, and the bolts stretch and bend, permitting some rotation of the plates". Since, after initial tightening, the bolts may be at or above the proof load, any significant stretching or bending would appear to result in permanent fastener deformation and the possibility of loosening of the nuts under repeated loads. The writer wonders whether this could happen and, if so, how it is prevented in the proposed method. Some of the existing methods contain implicit provisions intended to retain the clamping effect up to working loads. The author's views on the importance of this aim would be helpful.

Essentially the same questions are raised by the author's statement that use of his method may result in some reduction in connection rigidity. Is there any possibility that this could be the result of bolt yielding and loosening? If so, what would be the consequences of such action under the number of repetitions of live and wind loads that might be expected in a building? Can the designer view connections designed by the author's method as AISC Type 1 connections, or should he be prepared to treat them as Type 3 or Type 2 and satisfy the AISC requirements for such types?

Dr. Krishnamurthy concludes that prying action is of limited importance. In the simplified split-tee model, however, any reduction of his term s/p_e below the value of unity results in the equilibrium requirement of either a prying effect or bending of the bolts. It is recognized that he views the split-tee model as mainly a convenience for routine design. Nevertheless, if one is to deny a requirement of statics, one should have a good reason for doing so. There does not appear to be one in the paper.

The author's coefficients C_1, C_2, and C_3, along with possible beneficial strain hardening, seem to categorize the major behavioral phenomena in end-plate connections. Unfortunately, no attempt is made to explain behavior in terms of these coefficients. Rather, after introducing them and combining them as α_m in Eq. (11a), they immediately disappear from the paper as α_m is redefined in terms of physically less meaningful coefficients in Eq. (13). If, from Dr. Krishnamurthy's extensive analyses and tests, he can explain how each of the factors C_1, C_2, and C_3 enters into the reduction of the connection's actual complex behavior to the simplified split-tee model, and if he can do this quantitatively, he will have made an outstanding contribution to the understanding of an important and difficult problem.

Lastly, the work of Grundy, Thomas, and Bennetts on the same problem deserves attention (Ref. 13). Based on their studies, they suggest assuming full double-curvature plastic moment capacity in the design of end plates. For bolt design they suggest a flat 20% allowance for prying. Dr. Krishnamurthy's comments on Ref. 13 would be useful.

REFERENCE

13. *Grundy, Paul, Ian R. Thomas, and Ian D. Bennetts* The Design of Beam-to-Column Moment Connections Using End Plates and High Strength Bolts *Second Conference on Steel Developments, Aust. Inst. of Steel Construction and Monash University, Melbourne, May 1977.*

Discussion by **John D. Griffiths** and **J. M. Wooten**

Dr. Krishnamurthy's paper provides structural engineers with valuable new information on the behavior of end-plate connections. He is to be congratulated for his tireless energy in the completion of a most arduous task.

His work is comforting to those engineers of several decades past who have designed split-tee connections (Fig. 10) for use in tier buildings, powerhouses, and other large structures. This is so even though the author's study is not directly applicable to split-tee connections *per se.* His work is also comforting to those associated more recently with the design of end-plate moment connections for similar large structures and for most of the single-span rigid frames produced by the Metal Building manufacturers. Many thousands of these assemblies designed by the simple split-tee analogy have given a good account of themselves, even though theoretical questions have been raised as to their design. Thanks to Dr. Krishnamurthy's work, we now realize more clearly why all these connections have performed so well for so many years.

During several years of research, Dr. Krishnamurthy has progressively rationalized and simplified explanations of behavior and recommendations for design. His paper presents a design procedure that is convenient, in its present form, for use with a programmable computer. His simplified method can also be applied, albeit laboriously, with most pocket calculators.

The purpose of this discussion is to present a procedure based on the author's paper which requires little computation and a knowledge of only simple statics, so that the designer can retain a "feel" for the problem.

Dr. Krishnamurthy's work covers a wide range of variables associated with different beam and end-plate materials, bolt properties, and geometry. For the typical case, designs can be simplified if specific values are assigned for end-plate material and bolt pitch (see Fig. 11).

Unlike main material, plate material is usually taken from stock and custom fabricators ordinarily stock only A36 with thickness increments of $^1/_8$-in., except for thin plates.

John D. Griffiths is Vice President — Engineering, Paxton & Vierling Steel Co., Omaha, Nebraska.

J. M. Wooten is Chief — Structural Design, AFCO Steel, Little Rock, Arkansas.

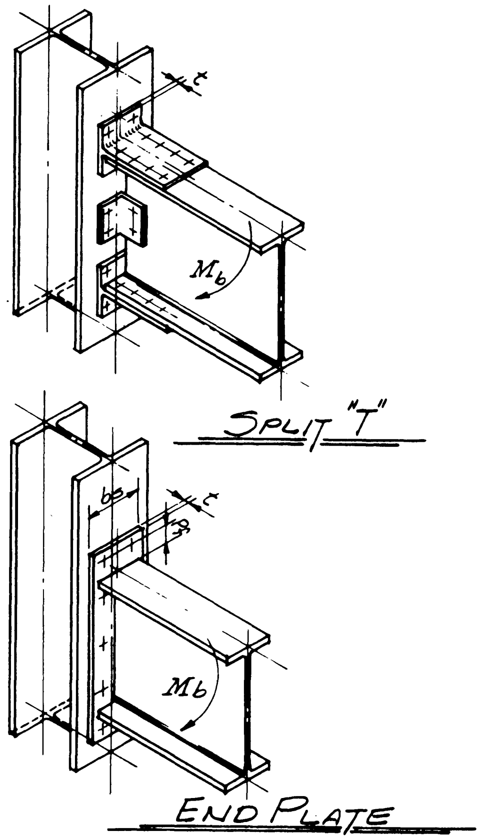
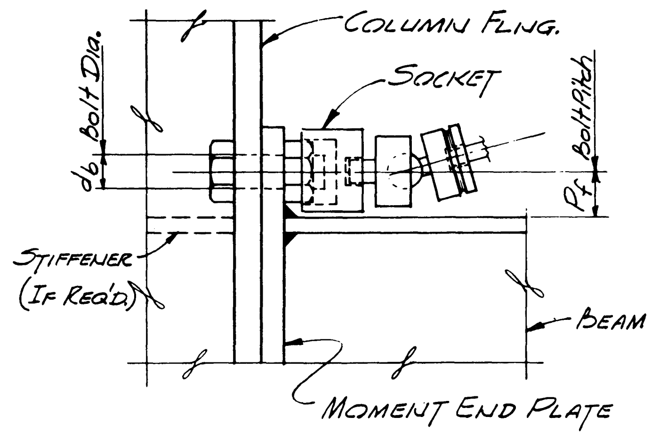


Figure 10



d_b	P_f Based On ¼" Drive CL.	P_f Based On $d_b + ½"$	Maximum Fillet Weld
¾"φ	1 5/16"	1¼"	½"
⅞"φ	1 5/16"	1⅜"	½"
1"φ	1 7/16"	1½"	½"
1⅛"φ	1 9/16"	1⅝"	½"
1¼"φ	1¾"	1¾"	½"
1⅜"φ	1⅞"	1⅞"	½"
1½"φ	2	2	½"

Figure 11

End-plate design is very sensitive to bolt pitch, with the minimum pitch providing greatest economy. As shown in Fig. 11, the minimum pitch (with a reasonable allowance for standardization and shop-field tolerance) can be taken as the bolt diameter plus ½-in. Therefore, for this discussion, it will be assumed that:

1. End-plate material is A36.
2. Beam material is A36 or 50 ksi yield.
3. Bolts are ¾-in. or larger.
4. Bolt pitch $p_f = d_b + ½$-in.
5. Minimum actual plate width and maximum effective plate width $b_e = b_f \times 1.15$.

One word of warning! As stated before, the end-plate design is most sensitive to the bolt pitch p_f. A change from the stated assumption that $p_f = d_b + ½$ cannot be tolerated by the design recommendations of this discussion. Any increase in p_f might result in an unconservative plate thickness; a decrease in p_f would likely result in unacceptable difficulties in field assembly. In fact, no change in the above stated assumptions should be made if the recommendations of this discussion are to remain valid.

In this paper, the author refers to a "modified split-tee analogy" and provides a formula for a preliminary plate moment M_t:

$$M_t = \frac{F_f}{2} \times \frac{P_e}{2}$$

where F_f = total flange force
p_e = effective bolt pitch $(p_f - d_b/4)$

He then multiplies the value of M_t by a coefficient α_m to obtain a final plate design moment M_d:

$$M_d = \alpha_m M_t$$

where:

$$\alpha_m = 1.29 \left[\frac{F_y}{F_{bu}}\right]^{0.4} \left[\frac{F_{bt}}{F_p}\right]^{0.5} \left[\frac{b_f}{b_s}\right]^{0.5} \left[\frac{A_f}{A_w}\right]^{0.32} \left[\frac{p_e}{d_b}\right]^{0.25}$$

It was previously noted that calculations for the above could be made on a non-programmable calculator. The writers have done so many times, but have not been able to establish confidence in their results because of errors they have often made, due to the number of variables and the complexity of the procedure.

The "non-modified split-tee analogy" of past years directly computed a plate design moment M based on a lever arm p_f instead of $(p_f - d_b/4)$:

$$M = \frac{F_f}{2} \times \frac{p_f}{2}$$

Table 1

Section	A_f/A_w	R
W36 × 300	0.887	0.98
× 280	0.882	0.98
× 260	0.850	0.97
× 245	0.835	0.97
× 230	0.818	0.96
× 210	0.588	0.92
× 194	0.587	0.92
× 182	0.579	0.91
× 170	0.573	0.91
× 160	0.554	0.91
× 150	0.530	0.90
× 135	0.463	0.88
W33 × 241	0.853	0.97
× 221	0.829	0.97
× 201	0.807	0.96
× 152	0.612	0.92
× 141	0.583	0.91
× 130	0.541	0.90
× 118	0.492	0.89
W30 × 211	0.905	0.98
× 191	0.887	0.98
× 173	0.861	0.97
× 132	0.606	0.92
× 124	0.590	0.92
× 116	0.558	0.91
× 108	0.516	0.90
× 99	0.476	0.88
W27 × 178	0.909	0.98
× 161	0.902	0.98
× 146	0.885	0.98
× 114	0.646	0.93
× 102	0.635	0.93
× 94	0.597	0.92
× 84	0.545	0.90
W24 × 162	0.994	1.00
× 146	0.959	0.99
× 131	0.904	0.98
× 117	0.877	0.98
× 104	0.848	0.97
× 94	0.683	0.94
× 84	0.655	0.93
× 76	0.616	0.92
× 68	0.560	0.91
× 62	0.428	0.87
× 55	0.397	0.86
W21 × 147	1.011	1.00
× 132	1.002	1.00
× 122	1.003	1.00
× 111	0.994	1.00

Section	A_f/A_w	R
× 101	0.995	1.00
× 93	0.683	0.94
× 83	0.686	0.94
× 73	0.683	0.94
× 68	0.667	0.93
× 62	0.641	0.93
× 57	0.532	0.90
× 50	0.465	0.88
× 44	0.423	0.87
W18 × 119	1.082	1.01
× 106	1.059	1.01
× 97	1.076	1.01
× 86	1.056	1.01
× 76	1.048	1.00
× 71	0.741	0.95
× 65	0.751	0.95
× 60	0.751	0.95
× 55	0.722	0.95
× 50	0.714	0.94
× 46	0.604	0.92
× 40	0.595	0.92
× 35	0.504	0.89
W16 × 100	1.170	1.02
× 89	1.152	1.02
× 77	1.146	1.02
× 67	1.149	1.02
× 57	0.789	0.96
× 50	0.781	0.96
× 45	0.768	0.96
× 40	0.772	0.96
× 36	0.679	0.94
× 31	0.589	0.92
× 26	0.506	0.89
W14 × 120	1.855	1.10
× 109	1.899	1.10
× 99	1.859	1.10
× 90	1.860	1.10
× 82	1.348	1.05
× 74	1.394	1.05
× 68	1.382	1.05
× 61	1.364	1.05
× 53	1.141	1.02
× 48	1.115	1.01
× 43	1.103	1.01
× 38	0.861	0.97
× 34	0.824	0.97
× 30	0.734	0.95
× 26	0.633	0.93
× 22	0.557	0.91

Section	A_f/A_w	R
W12 × 87	1.748	1.09
× 79	1.732	1.09
× 72	1.720	1.09
× 65	1.706	1.09
× 58	1.631	1.08
× 53	1.527	1.07
× 50	1.281	1.04
× 45	1.266	1.03
× 40	1.281	1.04
× 35	0.992	1.00
× 30	0.963	0.99
× 26	0.936	0.99
× 22	0.575	0.91
× 19	0.520	0.90
× 16	0.419	0.87
× 14	0.390	0.86
W10 × 60	1.842	1.10
× 54	1.882	1.10
× 49	1.859	1.10
× 45	1.603	1.07
× 39	1.516	1.07
× 33	1.348	1.05
× 30	1.045	1.00
× 26	1.033	1.00
× 22	0.913	0.98
× 19	0.672	0.94
× 17	0.583	0.91
× 15	0.497	0.89
× 12	0.463	0.88
W8 × 35	1.796	1.09
× 31	1.711	1.09
× 28	1.495	1.06
× 24	1.487	1.06
× 21	1.127	1.02
× 18	1.007	1.00
× 15	0.690	0.94
× 13	0.593	0.92
× 10	0.635	0.93
W6 × 25	1.580	1.07
× 20	1.545	1.07
× 15	1.238	1.03
× 16	1.148	1.02
× 12	0.890	0.98
× 9	0.911	0.98
W5 × 19	1.867	1.10
× 16	1.748	1.09
W4 × 13	1.442	1.06

It can be shown that this moment M closely approximates M_d in the case of (A_f/A_w) values for most wide-flange sections used as beams.

In the case of fillet welds, the author further modifies the lever arm p_f by subtracting w_t from the value $(p_f - d_b/4)$. A similar procedure can be used for the value of M, except that the weld leg size may be deducted instead of the weld throat size, i.e., $M = [F_f(p_f - w_s)]/4$, with the added limitation that the weld size not exceed ½-in. When fillet welds would exceed ½-in., many fabricators consider it economical to change to groove welds. In the case where fillet weld reinforcing is used for such groove welds, the vertical leg dimensions of the fillet weld may be deducted from p_f.

Table 1 lists the W sections to be included in Part 2 (Beam and Girder Design) of the 8th Edition of the AISC Manual. The values of A_f/A_w and R are shown for each section.

$$R = t_s/t \text{ or } t_s = Rt$$

where t_s = plate thickness based on M_d (the basic paper).
t = plate thickness based on M (the discussion).

The values of R are maximum values based on the least favorable combinations of variables.

Thus, where the value of R is 1 or less, the plate thickness based on M is conservative. These R-values are not presented with the thought thay they will be used as factors to achieve a more accurate result. Rather, the listing was developed to show that the "split-tee analogy" (not "modified" except for welds) can be applied with safety over a wide range of sections. Note that for all beams with depths from 16 through 36-in., the maximum value of R exceeds unity by only 2 percent. Also, for at least the lightest series of sections in the 6 through 14-in. depths, R-values are within this 2 percent range. These lighter sections are the ones most often used for end-plate designs.

So much for the conservative aspect of this procedure. How about economy? Although variables have been reduced to be in line with the practice of custom fabricators, some variables, such as bolt materials and weld and bolt sizes, remain. It was mentioned previously that R was chosen as a maximum. If the true value of R is less than the maximum value recorded in Table 1, the plate thickness will be greater than required. The greatest difference between maximum and minimum values of R for any individual wide-flange section shown in Table 1 is 7%.

Design examples for end plates, developed by the authors, by AISC, and by others, are consistently based on the case where full design moment of the beam must be accommodated. In practice, design for full beam moment capacity is often not required. It should be realized that Dr. Krishnamurthy's procedure and the procedure herein discussed are applicable with complete safety for any moment equal to or less than the maximum beam capacity.

This suggested design method will be of slight interest to engineers with Metal Building manufacturers. These firms are computer-equipped to a high degree. Also, for good reasons, they are extremely weight conscious, whereas custom fabricators often experience increased costs if too much emphasis is placed on saving the last pound of steel. Likewise, engineers with large design firms will have little interest, provided their firms use enough end-plates to warrant computer availability on this one item to all hands at all times.

We believe that many professionals would prefer a less complex method for design and would also like to have a procedure for a quick check of proposed details prepared by others. The simple, easily remembered method presented above retains most of the benefits of Dr. Krishnamurthy's investigation, but eliminates the necessity for charts and tables. Of greatest importance, it reduces the possibility of errors inherent in the calculation of lengthy formulas.

DESIGN EXAMPLE*

Given:

W16×45 section
Applied moment: 1723 kip-in.
$\frac{1}{2}$-in. welds

Solution:

$$F_f = \frac{1723}{(16.12 - 0.563)} = 111 \text{ kips}$$

$111/4 = 27.8$ kips

Use 4 - 1-in. A325 bolts

$p_f = d_b + 0.5 - w_s = 1.0 + 0.5 - 0.5 = 1$ in.

$M = 0.25 \times 111 \times 1 = 27.8$ kip-in.

Maximum $b_e = 7.039 \times 1.15 = 8.09$ in.

Use 8-in. plate.

$$t = \sqrt{\frac{6 \times 27.8}{8 \times 27}} = 0.879 \text{ in.}$$

Use 8 x $\frac{7}{8}$-in. plate.

The original design used an $8\frac{1}{2}$ x $\frac{13}{16}$-in. plate. On the basis that plates are stocked in $\frac{1}{8}$-in. thickness increments, most fabricators would have furnished an $8\frac{1}{2}$ x $\frac{7}{8}$-in. plate.

Discussion by **Norman W. Rimmer**

Dr. Krishnamurthy is to be congratulated for the in-depth study on the bolted end-plate moment connection and for the relatively simple design formula that he has developed for a very complex design problem.

I have reviewed the final report written by Dr. Krishnamurthy, which reports the findings of the testing of 9 test specimens that were fabricated with end-plates based on the newly developed end-plate design procedure. It is quite clear from this report that, when this method of design is used, the connection has a capacity to develop the full moment strength of the beams they are connecting.

Norman W. Rimmer is Senior Product Engineer, Buildings Division, Butler Manufacturing Company, Kansas City, Missouri.

* *Same example as Dr. Krishnamurthy's Appendix E.*

I thought it would be of interest to compare the end-plate thickness required for these 9 specimens to that which would be required utilizing the design method developed by Butler Manufacturing Company in 1954. This comparison indicates that the two methods give results within 10 percent of each other. The two largest differences between the two procedures had a magnitude of 7.47 percent and 10.11 percent, and in both cases the Butler procedure indicated that a thinner plate could be used. Where the new method developed by Dr. Krishnamurthy indicated the thinner of the two procedures, the maximum difference was 6.6 percent.

The method developed by Butler Manufacturing Company was first used in 1954 for the development of the new line of buildings that went into production in 1955. All Butler metal buildings have been designed since that time utilizing this procedure. We have load tested to failure many rigid frames at our research center. In all of the tests that have been conducted, we have never had a failure of a bolted end-plate moment connection. The failures have always occurred in the beam section.

Since the introduction of this type of bolted end-plate connection beginning with our production in 1955, we have produced literally thousands of buildings each year during the last 24 years. In all these years, with these many thousands of buildings, we have never to our knowledge experienced a bolt or an end-plate failure in a bolted end-plate splice connection. We believe this speaks very well for the use of this type of connection in our structures.

Discussion by **N. Krishnamurthy**

The author thanks the discussers for their many useful comments, and will attempt to clarify and amplify his statements, supplying supportive information wherever possible. He will first respond to the discussions by Professors Agerskov and McGuire, because they have overlapping concerns and they raise questions requiring answers.

To Professor Agerskov's comment on the reactive pressure distribution, the author concedes that his statement regarding its nature was oversimplified. Not only the plate thickness and load level, but the bolt distance, pretension force, and many other parameters also affect the reactive pressures in a highly nonlinear fashion. The point that the author was making was that except under certain unusual combinations of these various parameters, and only *at the yield load* (emphasis Agerskov's), the prying force was a distributed pressure bulb, and not critical as far as the plate moment was concerned.

N. Krishnamurthy is Research Professor of Civil Engineering, Vanderbilt University, Nashville, Tennessee.

The author agrees with Professor McGuire that any plate moment at the bolt line (corresponding to an s/p_e value less than unity) must imply the presence of a reaction to satisfy static equilibrium. But he also wishes to point out that this moment of resistance can be offered by a combination, in varying degrees, of: (a) unsymmetric clamping force around the bolt head, (b) bending of the bolt shank, and (c) eccentric reactive pressure or force at the back of the plate.

In a photoelastic study[14] that the author directed — which to his knowledge was the first time the pressure distributions at the back of the plate under various applied loads were directly observed and recorded — the reactive pressures in the plate projection and the bending stresses at the bolt line were found to be so small that they could not be easily quantified. The author's numerous finite element analyses and many strain measurements on the plates of his steel test specimens also overwhelmingly support this conclusion.

In any case, the author believes he has demonstrated conclusively his contention that, where strength is concerned, his proposed plate thickness, completely devoid of any prying force consideration in its design, is quite adequate. In particular, as was mentioned in Appendix F of the paper and described in greater detail in the author's discussion[15] of Professor Agerskov's paper,[5] the ⅞-in. plate designed by the author's procedure (and predicted by Agerskov's method to develop only 62% of the capacity of the attached W16 ×45 beam), developed the full capacity of the beam with no overall distress to the plate.

On this basis, the author would prefer to drop the concept of "prying force" as the prime consideration covering both the plate thickeness and bolt size as assumed at present, and instead discuss the bolt force increase as a separate phenomenon not directly or necessarily related to prying.

That the bolt force increases under applied load is a fact, but it in itself should not be cause for concern. The magnitude of the increase of course merits further discussion.

As the author has pointed out in his discussion,[15] the bolt force increases reported by Professor Agerskov himself in his paper[5] are of a low order, mostly less than 10%, at the maximum applied loads reported. Without more detailed information to the contrary, it can only be surmised that the 50% bolt force increases that he refers to in his present discussion of the author's paper could happen only under extreme (and unlikely?) circumstances. (Incidentally, it may be noted that a 50% increase over the recommended pretension of $0.7F_u$ would put the final bolt force at 5% over its nominal ultimate capacity, implying that the actual bolt strength was at least that much higher than nominal.)

The author's analytical and experimental findings also confirm that at beam service load levels, except in rare instances, the force in the bolts designed by his procedure increases by about 5% at service load levels. Beyond service loads, the bolt force increases at a faster rate. Close to the beam yield, the force increases sharply, reaching or occasionally passing the *nominal* yield of the bolt. But even at beam failure, there is generally a 5 to 10% reserve bolt capacity below ultimate strength. In the W16 ×45 beam test referred to earlier, the bolt force increased only by about 15% at beam failure. In all the tests that the author has conducted on connections designed by his procedure, no bolt has completely failed.

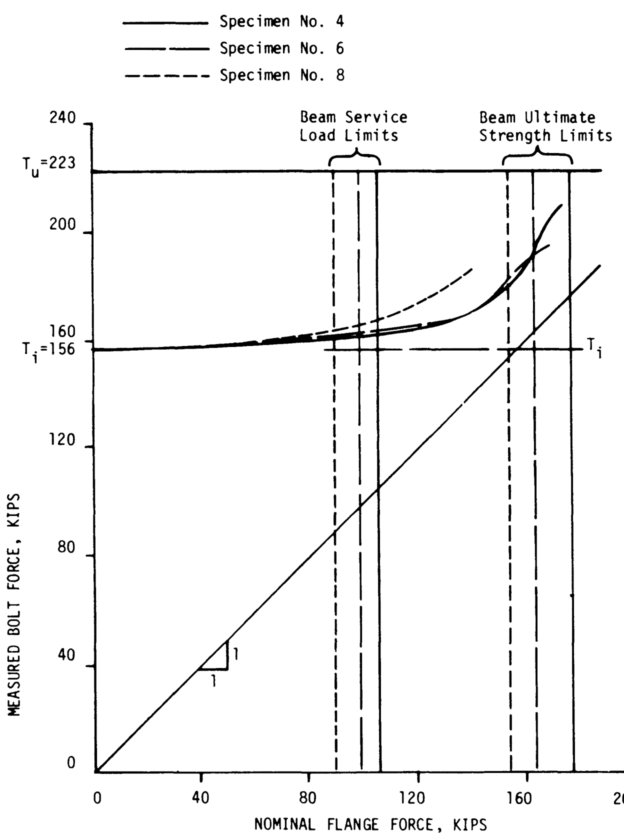


Fig. 12. Variations of measured bolt force with increasing beam loading, for author's specimens, with 7/8-in. bolts.

Figure 12 depicts the bolt force variations from three of the author's tests, illustrating the general behavior described in the preceding paragraph. The nominal flange force is calculated from Eq. (4). The bolt force is calculated as the nominal pretension plus the average increase determined from the strain gages attached to all four bolts around the tension flange. As all plotted quantities for the bolt are based on nominal values, and actual strengths are invariably higher than nominal, additional reserves may be anticipated in a practical situation.

The test connections were designed on the basis of the mill-sheet values of the yield stress F_y for the beam and plate materials. The service load and ultimate strength limits for beam capacity marked in Fig. 12 were also computed for the appropriate mill-sheet values. In the cases plotted, the actual F_y values from coupon pull tests were not significantly different from the mill-sheet values.

The author is familiar with the work of Grundy, Thomas and Bennetts, and is pleased to note their independent confirmation of many of his findings. In the paper mentioned by Professor McGuire, they state, referring to British practice, "The bolt sizes appear to be less than those required by statics. Nevertheless, tests have shown the connections to be satisfactory . . ." They also (again qualitatively) hypothesize reasons for the prying force "being much less in practice than predicted by simplistic theories," endorsing indirectly the author's general lack of concern about prying force in end-plate connections. They quantify the prying force at 10 to 15% of the greatest bolt force, placing the total force well below the bolt capacity.

In regard to Professor Agerskov's comment on the distribution of the tension flange force to the inner and outer rows, two factors must be borne in mind: (1) The author's lower limit of 0.3 for C_1 corresponding to the outer row force was reached only at low loads in a few cases among the hundreds of connections analyzed. (2) Based on the recommended bolt pretension of 0.7 times the ultimate strength, even the 40% increase that Professor Agerskov computes would not be catastrophic in theory, as the maximum force would be still below the ultimate strength.

In reality however, the concern about reaching so close to the ultimate strength due to unbalanced distribution of the bolt forces is unfounded. For, by the time the beam loading exceeds service load levels, the beam web generally yields at the corner between the beam flange and end plate (as the author has observed in this computer analyses and some tests), redistributing the subsequent beam flange forces more evenly to the outer and inner rows.

On the basis of the foregoing, the author reiterates his conviction that his proposed bolts, sized only for the nominal flange force, would be safe against failure prior to beam failure.

The author can respond to Professor McGuire's comments about the coefficients C_1, C_2, and C_3 more in philosophical than in mathematical terms. The coefficients were intended for convenience of conceptualization, rather than as steps in the development of governing equations. It was hoped that the reader would thus identify the many complex phenomena that would modify the idealized value of the split-tee moment, as described in the paper. Once these concepts were lumped into the single modifier α_m, the original coefficients had served their purpose and were allowed to "immediately disappear."

In fact, the situation is similar to Professor McGuire's development of the prying force expression in his book,[16] wherein he introduces the two stiffness coefficients k_b and

k_p symbolically for purposes of discussion, and subsequently replaces them with empirically developed expressions.

The author, too, went through numerous theoretical derivations to develop an expression for C_1 on the basis of a "stepped-beam analogy" for the end plate, with a larger stiffness on the beam web side compared to the plate projection beyond the beam, and another similar expression for C_2 on the basis of the smaller stiffness along the bolt line as compared to the load line. He could also have followed through with a numerical example, very much like the one McGuire used in the prying force section mentioned earlier, (in his words) "to illustrate a principle rather than as a quantitative calculation of representative relative stiffness." Because such derivations and examples cannot be used for practical applications, and are generally supplanted, or at least considerably modified, by other considerations, the author prefers not to have to go through them at this time.

The massive computer output from the finite element analysis would, of course, have been a better source of valuable and precise information, and the author considered quantifying C_1 and C_2 from that data. At best the author could have determined two more expressions of the same general form as Eq. (13), and possibly even more complicated. This would only have resulted in the proliferation and inbreeding of regression equations, without the redeeming suitability for plate design that α_m had.

These difficulties, accentuated by the limitations of time and scope, compelled the author to stop with stating the ranges of variation of C_1 and C_2. Even had they been determined more precisely, there would have been no reasonable way to define C_3, the catch-all coefficient to explain the force dispersions, deep-beam action, and many other effects.

Further, as the author has emphasized in the paper, the redefinition of α_m by Eq. (13) was a gratuitous fallout from a purely mathematical exercise. Since no physical basis was imposed on (or could be expected from) the statistical regression analysis, it was, at least to the author, a pleasant surprise that the computed data revealed groupings of dominant parameters with such recognizable physical significance. Far from being "physically less meaningful" than C_1, C_2, and C_3 as Professor McGuire states, the parameters in α_m, determined to a high degree of statistical correlation, are more meaningful than the originally assumed coefficients. The entire development leading to Eq. (11a) could have been omitted, and the validity of the remaining expressions and interpretations would not have been altered.

With reference to Professor McGuire's interest in the double curvature assumption and plastic moment approach of Grundy *et al.*,[13] the author can point out that his procedure also involves both concepts in some form.

The split-tee moment M_t by the author's Eq. (6) is the double-curvature moment based on an effective bolt distance p_e, instead of the actual bolt distance p_f. Further, the author's prediction equation (A.2) for M_d covers the entire range of elastic-plastic behavior. In any case, if Eq. (A.2) is rewritten with F_p of $(0.75F_y)$, we get

$$M_d = F_y(b_s t_s{}^2/4.5) \tag{18}$$

which would be very close to the plastic design capacity $(F_y Z)$, Z being the plastic section modulus $(b_s t_s{}^2/4)$. The author and his assistants have also examined applications of yield line theories to end-plate connections, and have resolved the complex mechanisms that Grundy *et al.* disregarded in their paper. Some of these developments will be submitted for publication shortly.

There is no question that thinner end plates *generally* result in decreased connection rigidity. In his paper, the author has not only pointed this out, but warned designers against using thinner plates where deformations could be critical. Professor Agerskov's comments reinforce this point.

It must be pointed out, however, that Professor Agerskov's Figs. 8 and 9 must not be interpreted as an illustration of the larger flexibility of connections designed by the author's procedure, compared to Agerskov's or other methods. In fact, to develop the service capacity of the test beam HE300B, the plate thickness by the author's procedure would be 35 mm, considerably thicker than the two plate thicknesses compared, namely 24 mm and 18 mm. With such thicker plates, the loss of rigidity would be of less significance than in the cases represented by Figs. 8 and 9. Situations also occur when neither of two plates of different thicknesses cause any rotation up to working load levels — this being the author's reason for admitting to reduction of connection rigidity with thinner plates "generally" rather than "always."

Incidentally, the test specimen of Figs. 8 and 9 had four bolts per row, and the author took the following approach in applying his procedure to this case:

With n bolts per row, Eq. (A.7) becomes

$$a_b = (n/4)\pi d_b{}^2 \tag{A.7a}$$

and it may be shown that consequently α_m would be *reduced,* by the factor $(2/n)^{0.1235}$. The important point to note is that more than two bolts per row would improve the connection, which may also be intuitively appreciated. However, without further validation tests, the author would not recommend the designer taking advantage of this reduction, which would in any event be only 4% in plate thickness with four bolts per row.

Thus, without arguing against the general validity of statements concerning the increased flexibility of thinner plates, the author would still maintain that, in many cases, such increased flexibility may be insignificant and/or not intolerable. Further, as the author commented on "excessive" deformation in his discussion[15] of Agerskov's paper, the limiting values of deformations are a very subjective topic. As long as the designer is assured of equal strength of two connections with different stiffnesses, he should have the option to choose the one that would suit his circumstances, very much as he has alternatives in other structural details such as with friction-type versus bearing-type connections.

Professor McGuire raises questions regarding the performance of the proposed connections under repeated loads. Because all of the author's tests were run under monotonically increasing loads (except for rare unloading and reloading for some adjustment), his procedure is not applicable to dynamic loading.

On the other hand, the repeated loads that Professor McGuire mentions as examples, namely live and wind loads, are usually considered as static equivalents. For these and all other loads which would be treated as static loads in conventional analysis and design, the author's procedure is equally applicable, in his opinion.

Certainly the proposed connections would be much more rigid than a Type 2 simple connection. But whether a connection with the proposed plate is Type 1 or Type 3 may be a question of terminology. Many of the proposed connections would hold the original angles virtually unchanged, within the working load levels; many would not. But as more and more data are being gathered, the moment-rotation characteristics of various connections can be documented and in due course predicted, and the computational skills and tools are now sufficiently sophisticated for the actual connection behavior to be incorporated into the structural design. Reference may be made to the author's recent paper[17] on the prediction of moment-rotation characteristics of end-plate connections, in which prediction equations and general methods of designing connections with desired rigidity ratios are outlined, some permitting frames to be analyzed as conventional rigid frames.

While hoping that by the preceding explanations the author has allayed the concerns occasioned by his proposed liberalization of the current end-plate design procedure, he would also offer the following as heuristic arguments:

The proposed procedure is based on the findings from investigations as logical and precise as those underlying many other semi-empirical formulas by which engineers design complicated components today. The common denominators to this evolutionary process are: (a) formulation of an idealized model (rigid frame, finite element, etc.), (b) development of governing equations for general behavior (by structural mechanics, regression analysis of parametric data etc.), (c) validation by (and/or adjustment of the prediction equations to reflect) test results, and (d) simplification to suit the user's convenience and accepted practice.

If the proof of the rightness of the conclusions and the efficiency of the recommendations from such a process lies in their practical demonstration, the author's proposals have already been vindicated. Apart from the many controlled tests conducted by the author to validate his procedure, millions of end-plate connections with plates and bolts smaller than by the AISC recommended procedure, have performed satisfactorily in service (mainly in low-rise structures) for decades now, with no loosening of bolts or other deterioration.

The author appreciates the complimentary remarks by Messrs. Griffiths and Wooten. In turn he is happy to acknowledge the constant guidance and support he has received from them, as well as from other members of the AISC/MBMA Task Committee on End-Plate Connections, in the long and complex investigation culminating in the proposed design procedure.

Griffiths and Wooten have presented a further simplification of the author's procedure, applicable to the specific set of material and dimensional parameters detailed in their discussion.

Stated briefly, for the particular assumptions they have listed, and for sections with (A_f/A_w) ratio smaller than about 1.15, the split-tee method (modified only for fillet weld size), would give from 14 percent overdesign to 2 percent underdesign of the end plate, in comparison with the author's procedure. Under these conditions, when simplicity and speed are more important than maximum economy, their approach would indeed be more convenient.

As they have carefully pointed out, their suggestion is *not* a substitute for the author's proposed method in the general case. Any and every deviation from their assumed limits would affect the outcome. Moreover, the influence of (A_f/A_w) ratio has been omitted, conservatively, for the limits they have defined; to appreciate the significance of this omission, it may be noted that a plot of the variation of the discussers' R values with (A_f/A_w) ratio would be very similar to the author's Fig. 5 or 6.

To extend the benefit of the "feel" Messrs. Griffiths and Wooten cite for the split-tee method to his procedure also, the author would point out that the proposal is still the split-tee, with two simple modifications:

(a) Use of an effective bolt distance, increasing the economy, but not essential to the method
(b) Use of a moment modification factor to account for the numerous complex interactions of material properties, beam proportions, etc.

While the discussers' simplification (or similar upper-bound techniques) are valuable short cuts for specific situations, other design aids may also be developed, retaining the generality and economy of the basic procedure. The author's Appendices C and D were intended to be such aids. Appendix C may also be presented as a series of charts of (A_f/A_w) versus α_m curves for various values of (p_e/d_b), one chart for each value of C_aC_b. It is feasible to prepare tables of minimum bolt sizes and plate thicknesses to develop the full capacity of standard rolled sections for general use, and of typical built-up sections for in-house use by pre-engineered metal-building companies.

REFERENCES

14. *Krishnamurthy, N.* Photoelastic and Finite Element Investigation of Steel Bolted Tee Hangers *Report No. CE-MBMA-1903-1, Vanderbilt University, March 1978.*
15. *Krishnamurthy, N.* Discussion of "Analysis of Bolted Connections Subject to Prying" (Ref. 5 of paper) *Journal of the Structural Division, ASCE, Vol. 104, No. ST12, pp. 1928–1930, Dec. 1978.*
16. *McGuire, W.* Steel Structures *Prentice-Hall, Inc., Englewood Cliffs, N.J., 1968.*
17. *Krishnamurthy, N., Horng-Te Huang, P. K. Jeffrey, and L. K. Avery* Analytical M-θ Curves for End-Plate Connections, *Journal of the Structural Division, ASCE, Vol. 105, No. ST1, pp. 133–145, Jan. 1979.*