Emotional Valence Tracking and Classification via State-Space Analysis of Facial Electromyography

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***Abstract*— Tracking the emotional valence state of an individual can serve as an important marker of personal health and well-being. Through automatic detection of emotional valence, timely intervention can be provided in the events of long periods of negative valence, such as anxiety, particularly for people prone to cardiovascular diseases. Our goal here is to use facial electromyogram (EMG) signal to estimate one's hidden self-labelled emotional valence (EV) state during presentation of emotion eliciting music videos via a state-space approach. We present a novel technique to extract binary and continuous features from EMG signals. We then present a state-space model of valence in which the observation process includes both the continuous and binary extracted features. We use these features simultaneously to estimate the model parameters and unobserved EV state via an expectation maximization algorithm. Using experimental data, we illustrate that the estimated EV State of the subject matches the music video stimuli through different trials. Using three different classifiers: support vector machine, linear discriminant analysis, and k-nearest neighbors, a maximum classification accuracy of 89% was achieved for valence prediction based on the estimated emotional valance state. The results illustrate our system's ability to track valence for personal well-being monitoring.**

I. Introduction

Emotion recognition is one of the important areas of research aimed at recognizing and interpreting different human emotions from facial and/or verbal expressions [1]. In this direction, computer vision has been widely used to recognize patterns in facial expressions using machine learning methods such as support vector machine (SVM) [2], [3], [**?**] and deep learning based convolutional neural networks [4], [5], [6], [7], [8]. While video based emotion recognition techniques have been shown to perform well, they require powerful video analysis as well as huge training data to train the network. Alternately, facial Electromyography (EMG), electroencephalography, electrocardiography, hear rate [9], respiration [10], and skin conductance [11], [12], [13], [14], [15], [16] have been shown to reflect one's underlying emotion such as happiness, stress and anger [17]. Previous researches have classified facial EMG activity into different emotions using artificial neural networks, naive bayes classifier [18], linear discriminant analysis (LDA), linear and structural modeling, and state-space models with 89% classification accuracy [19].

Most of the above studies have used multiple channels of facial EMG to predict facial expression [20]. In addition, these studies aimed to classify expression but not emotional state as the subjects were told to imitate smile or other expressions on their faces which were not involuntary responses corresponding to their actual feelings of valence [21]. Since one's facial expression is a natural response to the external emotional stimuli, it could be beneficial to have real-time emotion detection based on facial EMG. In this research, we present a state-space approach to estimate the emotional valance state from recorded EMG signal. We first present a novel technique to extract binary and continuous features from EMG signal. Then, we present a state-space approach to model emotional valence in which the observation process includes both the continuous and binary features. Employing these features simultaneously, we estimated the model parameters and unobserved emotional valence state via an expectation maximization (EM) algorithm. Next, the estimated emotional valence (EV) state was used to predict one's valence state via three different classification methods: SVM, LDA, and k-nearest neighbors (KNN) in conjunction with cross-validation. Further direction of this research could be employing the results in controlling the external environment depending on the subject's emotional state [11].

II. Methods

A. Experimental Data

We analyzed facial EMG activity from Database for Emotion Analysis using Physiological Signals (DEAP). It is a publicly available multimodal dataset for emotion analysis [22]. We analyzed the facial EMG signal recorded at a sampling rate of 512 Hz from the Zygomaticus major muscle. We used data from 23 participants (11- females and 12- males, mean age 26.5 years). Each participant completed 40, 1-minute trials during which an emotional stimuli, music video (varying on dimensions of arousal and valence) was presented. At the end of each trial, the participants performed self assessment during which they reported their emotional arousal and valence on a scale of 1 to 9. The value on

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this scale served as the ground truth in our analysis. All participants were presented the same set of music videos but in a randomized order.

B. Feature Extraction

We used simultaneously extracted binary and continuous features from the EMG signal to predict the emotional state of the subject. We began our analysis by pre-processing the raw EMG signal (Zygomaticus major). First, the signal was band-pass filtered from 10Hz - 250Hz to remove motion artifacts in the lower frequency bands. To remove the alternating current coupling noise, we also applied notch filters at 50Hz and its harmonic frequencies. Next, the signal was segmented into 1-minute trials which were further binned at 0.5 seconds (no overlap between consecutive bins, to avoid information leaking between trials) as described in the following subsections.

a) Binary feature: The following process was followed to extract the binary observations. We first computed the absolute value of the filtered EMG and then binned it at 0.5 second bin. Next, the binned EMG is convoluted with a Gaussian kernel to smooth it [23]. The smoothed signal is then min-max normalized; this signal is defined as x_{sd}. Using a Bernoulli distribution, an amplitude dependent frequency modulated binary feature was generated from the smoothed EMG. The probability of having a one in the binary feature was dependent on the exponential of the smoothed normalized EMG. The distribution we used was as follows,

$$Pr(n_j|x_{sd(j)}) = p_j^{n_j}(1-p_j)^{1-n_j} \quad (1)$$

where p_j is the probability of a binary event and $x_{sd}(j)$ is the smoothed normalized absolute EMG after binning for j^{th} time bin. The probability was calculated as the following equation,

$$p_j = \frac{ae^{x_{sd(j)}}}{e_{max}^{x_{sd}}} \quad (2)$$

where $e_{max}^{x_{sd}}$ is the maximum value of $e^{x_{sd}}$ and a is the intensity coefficient of the binary sequence. This coefficient was selected heuristically based on the observations. Binary feature extraction for a subject is shown in Figure 1.

b) Continuous feature: The following steps were followed to get the continuous feature. We first estimated the power spectral density (PSD) of the filtered signal in each time bin (0.5 seconds) using Welch periodogram estimation method with 75% overlapping windows (to get a smoother estimate of PSD). Next, we computed the band power in the frequency band 10Hz - 250 Hz and used it as a continuous feature in the model. We later normalized the continuous feature across all trials for each subject to get a relative measure of band power between trials of varying valence. This extracted feature was then input to the model.

c) Observation models: To estimate the hidden emotional state of the subject, we used the state-space model proposed in [24] defined by (3) while using simultaneously extracted continuous and binary features of the EMG signal.

$$x_k = \rho x_{k-1} + \mu_k \quad (3)$$

where μ_k is an independent, zero mean Gaussian random variable $\mathcal{N}(0\ ,\ \sigma_\mu^2)$, and ρ is the correlation coefficient relating subject's emotional state in the current and previous time bins. We assume that the subject's facial EMG response (observed in each time bin k, 1 to K, where K= 120 for a trial and K= 4800 bins for 40 trials) is governed by a hidden emotional valence state x_k and is defined by first order auto-regressive model in (3).

Let n_k denote the binary observations in time bin k. For each k, we assume that n_k can be 0 or 1. the observation model for this feature is as follows:

$$Pr(n_k|x_k) = P_k^{n_k}(1-P_k)^{1-n_k} \quad (4)$$

where $p_k$is the probability of binary event in bin k and is defined by:

$$P_k = \frac{e^{(\epsilon+x_k)}}{1+e^{(\epsilon+x_k)}} \quad (5)$$

where ϵ is estimated from the probability of a binary event by chance as described in [24].

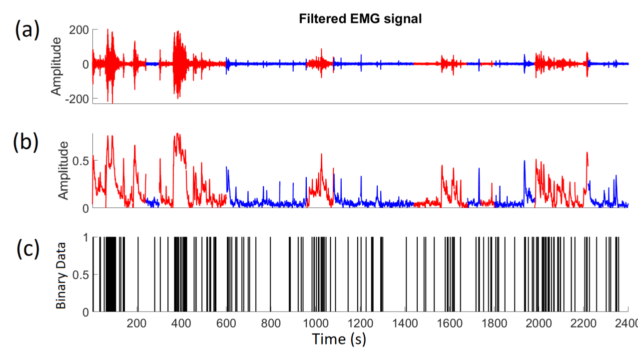

Fig. 1: **Binary feature extraction from EMG**: (a) represents the filtered EMG signal for all the trials for a subject, (b) shows the smoothed value of the absolute EMG after binning and (c) shows the generated binary sequence using (b). Red traces represent high valence trials and blue traces represent low valence trials.

Let z_k represent the continuous observations such that $z_k \in(\infty,\ \infty)$. The observational model for this feature is as follows,

$$z_k = \alpha + \beta x_k + \omega_k \quad (6)$$

where ω_k is an independent zero mean Gaussian random variable $\mathcal{N}(0\ ,\ \sigma_\omega^2)$. Here, $z_k = log(pow_k)$ represents the total power in a frequency band of interest in log scale (db); α represents baseline power and β represents the rate at which power content in a subjects EMG changes as a function of his/her emotional state.

C. State Estimation

Model parameters $\Theta = [\alpha,\ \beta,\ \rho,\ \sigma_\mu^2,\ \sigma_\omega^2]$ and state are estimated using the EM algorithm presented in [24]. In what follows, detailed description of this algorithm is presented.

1) Expectation Step: In this step, the filter algorithm computes the state estimate of the subject $x_{k|k}$ at each bin k. The smoothing algorithm (backward filter) calculates the estimate of the ideal observer.

a) Forward filter: At iteration $(i+1)$, state variable $x_{k|k}$ and variance $\sigma^2_{k|k}$ are estimated using a recursive non-linear filter algorithm (Equations (7) - (11)) given the parameter estimates from iteration i ($\sigma_\mu^{2(i)}$ and $x_0^{(i)}$):

$$x_{k|k-1} = \rho^{(i)} x_{k-1|k-1} \tag{7}$$

$$\sigma^2_{k|k-1} = \rho^{(i)2}\sigma^2_{k-1|k-1} + \sigma_\mu^{2(i)} \tag{8}$$

$$C_k = (\beta^{(i)2}\sigma^2_{k|k-1} + \sigma_\omega^{2(i)})^{-1}\sigma^2_{k|k-1} \tag{9}$$

$$\begin{aligned} x_{k|k} = x_{k|k-1} + C_k\Big[&\beta^{(i)}(z_k - \alpha^{(i)} \\ &- \beta^{(i)}x_{k|k-1}) + \sigma_\omega^{2(i)}(n_k - p_{k|k})\Big] \end{aligned} \tag{10}$$

$$\sigma^2_{k|k} = \Big[(\sigma^2_{k|k-1})^{-1} + p_{k|k}(1-p_{k|k}) + (\sigma_\omega^{2(i)})^{-1}\beta^{(i)2}\Big]^{-1} \tag{11}$$

for $k = 1, \cdots, K$.

b) Backward filter: Using the posterior mode estimates $x_{k|k}$ and its variance $\sigma^2_{k|k}$, fixed-interval smoothing algorithm was used to compute $x_{k|K}$ and $\sigma^2_{k|K}$. This algorithm is given as follows:

$$x_{k|K} = x_{k|k} + A_k(x_{k+1|K} - x_{k+1|k}) \tag{12}$$

$$A_k = \sigma^2_{k|k}(\sigma^2_{k+1|k})^{-1} \tag{13}$$

$$\sigma^2_{k|K} = \sigma^2_{k|k} + A_k^2(\sigma^2_{k+1|k} - \sigma^2_{k+1|K}) \tag{14}$$

for $k = K-1, \cdots, 1$ and initial conditions $x_{K|K}$ and $\sigma^2_{K|K}$.

c) State-Space Covariance Algorithm: This algorithm is used to estimate the covariance $\sigma_{k,u|k}$ is given by equation (15). The variance and covariance terms, $W_{k|K}$ and $W_{k-1,k|K}$ are computed using equations (16) and (17) as follows:

$$\sigma_{k,u|k} = A_k\sigma_{k+1,u|k} \tag{15}$$

$$W_{k|K} = \sigma^2_{k|K} + x^2_{k|K} \tag{16}$$

$$W_{k-1,k|K} = \sigma_{k-1,k|K} + x_{k-1|K}x_{k|K} \tag{17}$$

for $1 \le k \le u \le K$.

2) Maximization Step: The expected value of data log likelihood is maximized with respect to $\theta^{(i+1)}$ as follows:

$$\rho^{(i+1)} = \sum_{k=1}^{K} W_{k-1,k|K}\Big[\sum_{k=1}^{K} W_{k-1|K}\Big]^{-1} \tag{18}$$

$$x_0^{(i+1)} = \rho x_{1|k} \tag{19}$$

$$\begin{aligned} \sigma_\varepsilon^{2(i+1)} = {} & K^{-1}\sum_{k=1}^{K} z_k^2 + K\alpha^{2(i+1)} \\ & + \beta^{2(i+1)}\sum_{k=1}^{K} W_{k|K} - 2\alpha^{(i+1)}\sum_{k=1}^{K} z_k \\ & - 2\beta^{(i+1)}\sum_{k=1}^{K} x_{k|K}z_k + 2\alpha^{(i+1)}\beta^{(i+1)}\sum_{k=1}^{K} x_{k|K} \end{aligned} \tag{20}$$

$$\begin{bmatrix} \alpha^{(i+1)} \\ \beta^{(i+1)} \end{bmatrix} = \begin{bmatrix} K & \sum_{k=1}^{K} x_{k|K} \\ \sum_{k=1}^{K} x_{k|K} & \sum_{k=1}^{K} W_{k|K} \end{bmatrix}^{-1} \begin{bmatrix} \sum_{k=1}^{K} z_k \\ \sum_{k=1}^{K} x_{k|K}z_k \end{bmatrix} \tag{21}$$

$$\sigma_\mu^{2(l+1)} = K^{-1} \sum_{k=1}^{K}\Big[W_{k|K} - 2\rho^{(l+1)}W_{k-1,k|K} + \rho^{2(l+1)}W_{k-1|K}\Big] \tag{22}$$

The algorithm is iterated between the E-step and M-step using the filter algorithm until convergence.

D. Classification

Based on the obtained state estimates, we tested our model's performance to predict low valence (LV) vs. high valence (HV). Following the paper [22], the trials for which the self-reported valence rating was less than 5 were considered LV while the trials with valence rating greater than 5 were considered as HV trials. Then, we used cross-validation method to classify high vs low valence using three different classifiers namely SVM, LDA and KNN.

III. Results

A. State Estimation

For estimation of state-space model parameters described above, we used all 40 trials for each subject. In order to track the subject's emotional state from the beginning of the experiment to the end, we the order of the trials input to the model was the same as the order of presentation in the experiment.

Figures 2 and 3 show the binary and continuous (power spectral density) features and EV state for subjects 10 and 18, respectively. As seen from the figures, the emotional state's amplitude closely captures the valence of the trial such that it takes lower values for low valence trials and higher values for higher valence trials.

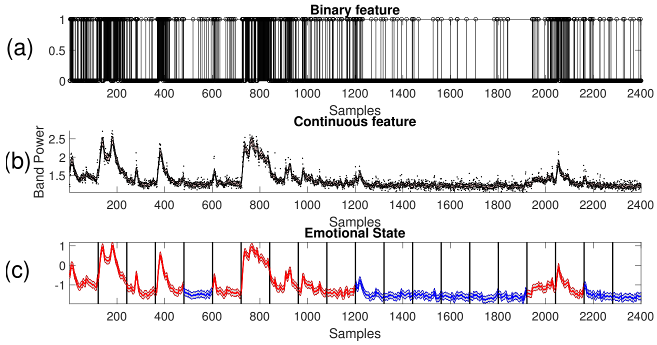

Fig. 2: **Emotional Valence State Estimation for Subject 10:** (a) and (b) show the extracted binary and continuous features from the EMG signal and (c) represents the estimated state for low valence (blue trace) and high valence (red trace) trials. A black vertical line in (c) marks the end of a trial.

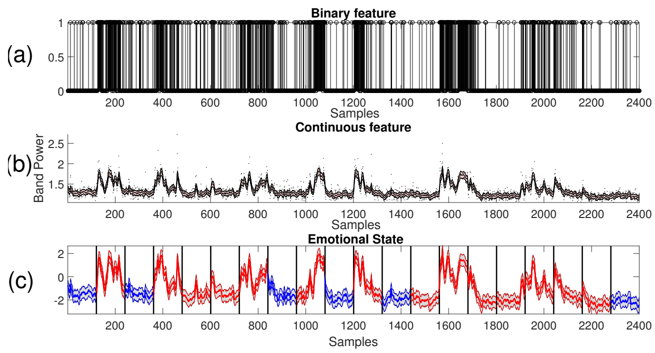

Fig. 3: **Emotional Valence State Estimation for Subject 18:** (a) and (b) show the extracted binary and continuous features from the EMG signal and (c) represents the estimated state for low valence (blue trace) and high valence (red trace) trials. A black vertical line in (c) marks the end of a trial.

B. Valence Classification

Using estimated state from the model, we performed a binary classification between high vs low valence emotion. The state variable for classification was estimated using all 40 trials in the order of their presentation. The EM algorithm converged for 22 of the 23 subjects. For these 22 subjects, the state variable was binned at 5 seconds and replaced by its mean value leading to 12 bins per trial. Binning here was necessary since using all 120 values (per trial) as predictors would create a predictor size larger than the training sample size. We used cross-validation method to classify high vs low valence using three different classifiers namely SVM, LDA and KNN. We used 75% of the trials (30) for training the classifier and rest 25% (10) for testing. We ran 30 iterations such that training and testing sets were randomly chosen from available 40 trials. The mean accuracy of 30 iterations was calculated for all 22 subjects where Figure 4 shows the mean accuracy for valence classification and chance level for all subjects.

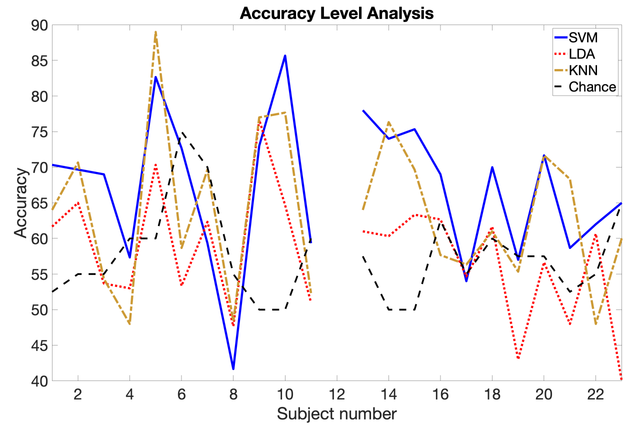

Fig. 4: **Classification Accuracy for Valence Prediction:** Dotted line represents the chance level for all the subjects and other three traces represent the accuracy achieved by the three classifiers.

We found that the mean accuracy (Figure 4) for 16 out of 22 subjects is above the chance level. A maximum mean accuracy of 89% was achieved in valence classification (chance level - 60%) (for subject 5). Among 30 iterations, the highest accuracy achieved in a single iteration was 100% for 12 of the 22 subjects. As seen from the figure, the mean accuracy is not reported for subject 12 since the EM algorithm didn't converge for this subject.

IV. Discussion

In this research, we showed how continuous and binary features extracted from the facial EMG signal can be used to estimate one's hidden emotional state. We used a dataset where the emotion level of subject was responsible for their facial expression compared to other studies where the subject was instructed to perform various expressions. Emotions are best described through the dimensions of valence and arousal. In the present research, we approached the state estimation problem as a way to distinguish between high and low levels of valence. Availability of EMG signal from one facial muscle (zygomaticus major) alone wasn't sufficient for tracking a variety of emotions; although, binary classification of valence was possible. In order to perform classification, extracting binary and continuous features from one signal was critical. In present work, we processed EMG signal to extract maximum information without redundancy. It is well known that EMG signal carries information in both its amplitude and frequency. Therefore, using the EMG's amplitude, we generated the binary features. Total power in different frequency bands was used as a continuous feature. To our knowledge, our method of generating a binary feature from the amplitude of EMG is novel and hasn't been applied to EMG signal processing for emotion analysis before. Using the EM algorithm with extracted features, we presented the plots for two example subjects, both of which showed visually distinguishable estimated state between low and high valence trials.

To extend our work and test our model's performance, we used the estimated state variable as a predictor of high vs low valence emotion for each subject. High classification accuracy in valence prediction shows considerable usefulness of the extracted features in reliably estimating the hidden state variable with 72.7% of the subjects showing above chance accuracy.

One of the limitations of our proposed approach is that we rely on the self assessment ratings of the participant as a way to know their true feelings due to unavailability of any expert ratings. Therefore, we expect that possible inaccuracy in reporting one's true emotions could be a reason for low classification accuracy for some subjects. Since the signal being analyzed here primarily depends on the muscle activity, there is a possibility that our method may not accurately estimate the true emotional state for those subjects who generally don't express their emotions strongly. In order to estimate both valence and arousal states of a person, EMG signals from more than one facial muscles may be needed. Nonetheless, based on the present results of classification accuracy, our approach is useful in extracting good features from limited number of recording channels and can be applied to analyze multi-channel EMG signals as well.

V. Conclusion

We developed a state-space model from EMG signal capable of distinguishing between low and high valence emotions. By adding data from multiple EMG channels, we can eventually build a complete emotion detection system which detects an individual's emotion on the dimensions of both valence and arousal. Further, this system can be used to control external environment depending on the subject's emotional state e.g. a room's light or music system can be controlled by an individual's emotions such that after the changes in lights or in the type of music being played, the subject feels more relaxed. We can also use this system as an intervention indicator which can send an alert to an individual's family when long periods of certain emotions (i.e. anger) are detected. This can potentially be helpful in monitoring a person's health especially when they are prone to cardiovascular diseases.

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