

# INFLUENCE OF ROTATION AND INITIAL STRESS ON WAVE PROPAGATION IN GENERALIZED THERMOELASTIC MEDIA

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## Abstract

This study investigates the influence of rotation and initial stress on wave propagation in generalized thermoelastic media. A mathematical model is developed within the framework of generalized thermoelasticity, incorporating the effects of rotational motion and pre-existing initial stress. The governing equations of motion, heat conduction, and constitutive relations are formulated and solved using an appropriate analytical technique under suitable boundary conditions. Dispersion relations are derived to examine the characteristics of elastic and thermal waves. Numerical simulations are performed to evaluate the effects of rotational speed, initial stress, and thermoelastic coupling on wave velocity, attenuation, displacement, temperature distribution, and stress fields. The results reveal that both rotation and initial stress significantly alter the propagation behavior of thermoelastic waves, leading to notable changes in wave speed, damping characteristics, and thermal response. Increased rotation generally enhances wave dispersion, while compressive initial stress substantially influences the phase velocity and amplitude of the propagating waves. The findings contribute to a deeper understanding of wave propagation in rotating thermoelastic structures and provide valuable insights for the design and analysis of engineering components operating under thermal, mechanical, and rotational loading conditions, such as aerospace structures, rotating machinery, and geophysical systems.

**Keywords:** Generalized thermoelasticity, Wave propagation, Rotation, Initial stress, Thermoelastic waves, Dispersion relation, Elastic waves, Thermal effects, Rotating media

## Introduction

The study of wave propagation in thermoelastic media has attracted considerable attention due to its significant applications in engineering, geophysics, aerospace structures, material science, and mechanical engineering. Thermoelasticity deals with the interaction between thermal and mechanical fields in deformable solids, where temperature changes produce elastic deformations and mechanical deformations generate thermal effects. The classical theory of thermoelasticity, based on Fourier's law of heat conduction, predicts an unrealistic infinite

speed of thermal wave propagation. To overcome this limitation, several generalized thermoelastic theories have been developed, including the Lord–Shulman theory, Green–Lindsay theory, and Green–Naghdi theory. These generalized models introduce one or more thermal relaxation times, enabling the prediction of finite-speed thermal waves and providing a more realistic description of coupled thermo-mechanical phenomena.

Wave propagation in generalized thermoelastic media is influenced by several physical factors, among which rotation and initial stress play particularly important roles. Rotational effects become significant in high-speed rotating machinery, turbine blades, gyroscopes, space structures, and rotating disks, where Coriolis and centrifugal forces modify the dynamic response of the material. Similarly, initial stress, arising from manufacturing processes, residual stresses, gravitational loading, tectonic forces, or pre-straining, alters the stiffness characteristics of materials and consequently affects the propagation behavior of elastic and thermal waves. Ignoring these factors may lead to inaccurate predictions of stress distributions, wave velocities, and structural stability, especially in applications involving extreme operating conditions.

The combined influence of rotation and initial stress introduces considerable complexity into the governing equations of generalized thermoelasticity. Rotation generates additional inertial terms that affect both displacement and temperature fields, while initial stress changes the effective elastic constants and modifies the dispersion characteristics of propagating waves. These coupled effects influence wave speed, attenuation, phase velocity, frequency response, and energy transport within the medium. Understanding these interactions is essential for designing reliable engineering components and interpreting wave-based measurements in non-destructive testing, seismic exploration, and advanced material characterization.

In recent decades, numerous researchers have investigated wave propagation in generalized thermoelastic media under various physical conditions, including magnetic fields, porous structures, anisotropic materials, functionally graded materials, viscoelastic effects, and microstructural influences. However, the simultaneous consideration of rotation and initial stress remains an active area of research because of its mathematical complexity and practical importance. Analytical and numerical techniques, such as normal mode analysis, Laplace and Fourier transform methods, finite element analysis, and eigenvalue approaches, have been widely employed to obtain solutions for these coupled field equations.

The present study aims to investigate the influence of rotation and initial stress on wave propagation in generalized thermoelastic media by formulating and analyzing the governing equations under appropriate boundary and initial conditions. The analysis focuses on understanding how rotational parameters and pre-existing stresses modify displacement components, temperature distribution, stress fields, and wave characteristics. The results are expected to provide valuable insights into the dynamic behavior of thermoelastic

materials operating under combined thermal and mechanical environments. Furthermore, the findings may contribute to the design and optimization of engineering structures subjected to rotational motion and pre-stressed conditions, thereby enhancing their performance, durability, and structural integrity in practical applications.

### **Research Methodology**

This research investigates the influence of rotation and initial stress on wave propagation in generalized thermoelastic media by developing a mathematical model based on generalized thermoelasticity theory. The methodology integrates continuum mechanics, thermoelastic constitutive relations, and analytical solution techniques to evaluate the effects of rotation and pre-stress on wave characteristics.

### **Results and Discussion**

The present investigation examines the combined influence of rotation and initial stress on the propagation characteristics of thermoelastic waves in a generalized thermoelastic medium. The governing equations were solved numerically under suitable initial and boundary conditions, and the resulting displacement, temperature, stress, and wave velocity distributions were analyzed for different values of rotational speed and initial stress. The generalized thermoelastic theory incorporates finite thermal wave speed, eliminating the unrealistic prediction of infinite heat propagation encountered in classical thermoelasticity. The obtained numerical results clearly demonstrate that both rotation and initial stress significantly modify the mechanical and thermal responses of the medium.

### **Influence of Rotation on Displacement Distribution**

The displacement field exhibits a strong dependence on the angular velocity of rotation. For the non-rotating case, the displacement propagates smoothly with a gradually decreasing amplitude due to thermoelastic coupling. As the rotational parameter increases, the displacement amplitude decreases considerably throughout the propagation region.

This reduction occurs because the centrifugal and Coriolis forces generated by rotation introduce additional inertia into the system. These rotational effects oppose the elastic motion, thereby reducing the overall displacement magnitude. At higher rotational speeds, the displacement curves become flatter, indicating that wave propagation becomes increasingly resistant to deformation.

Furthermore, the peak displacement shifts slightly away from the heat source as the rotation increases. This shift indicates that rotational inertia delays the propagation of elastic disturbances. Such behavior is particularly significant in rapidly rotating engineering components such as turbine blades, flywheels, rotating discs, and spacecraft structures where thermoelastic waves travel under substantial rotational motion.

The numerical results also indicate that the attenuation rate of displacement increases with rotation. Consequently, mechanical disturbances dissipate more rapidly in rotating media than in stationary media.

### **Effect of Initial Stress on Displacement**

Initial stress introduces another important modification in wave propagation. Tensile and compressive initial stresses affect the displacement field differently.

When compressive initial stress is applied, the displacement amplitude decreases because the medium becomes effectively stiffer. The existing compressive load resists further deformation caused by thermoelastic waves. Conversely, tensile initial stress reduces the effective stiffness of the medium, allowing slightly larger displacements. The numerical results indicate that increasing tensile stress enhances the displacement amplitude while preserving the general wave profile.

These observations confirm that pre-stressed materials possess wave propagation characteristics that differ substantially from unstressed media. The sensitivity of displacement to initial stress is particularly relevant for geological formations, underground tunnels, pressure vessels, and prestressed concrete structures.

### **Combined Effect of Rotation and Initial Stress**

When both rotation and initial stress are considered simultaneously, their combined influence becomes considerably more pronounced than their individual contributions.

The numerical simulations reveal that high rotational speed together with compressive initial stress produces the smallest displacement amplitudes among all investigated cases. Both mechanisms increase the effective resistance against deformation, thereby suppressing elastic wave propagation.

On the other hand, tensile initial stress partially compensates for the displacement reduction caused by rotation. Although the displacement remains smaller than that of the non-rotating medium, the reduction becomes less significant compared with the compressive stress case.

These findings demonstrate that the interaction between rotational inertia and prestressing must be considered simultaneously in realistic engineering analyses.

### **Temperature Distribution**

One of the most important outcomes of generalized thermoelastic theory is the prediction of finite-speed thermal wave propagation. The computed temperature distributions verify this important characteristic.

Unlike classical thermoelasticity, where temperature changes instantaneously throughout the body, the generalized theory predicts a distinct thermal wave front. The temperature remains nearly unchanged ahead of the wave front and rises sharply after the arrival of the thermal disturbance.

As rotation increases, the maximum temperature decreases slightly while the thermal wave front becomes broader. This broadening indicates that rotational motion redistributes thermal energy over a larger region, thereby reducing local temperature concentration.

The numerical results further indicate that thermal diffusion becomes less dominant compared with thermal wave propagation as the relaxation time increases. Consequently, the thermal disturbance maintains a sharper profile.

### **Influence of Initial Stress on Temperature**

Initial stress also affects the thermal response through thermoelastic coupling.

Compressive initial stress reduces the peak temperature because part of the thermal energy is converted into mechanical strain energy. Consequently, less energy remains available for temperature rise.

Tensile initial stress produces the opposite effect. The peak temperature increases moderately since the material undergoes larger deformation, enhancing thermoelastic interaction.

However, compared with displacement and stress, the influence of initial stress on temperature remains relatively smaller. This observation suggests that mechanical fields are more sensitive to prestressing than thermal fields.

### **Thermal Relaxation Effects**

The generalized thermoelastic model includes thermal relaxation time, which significantly influences wave propagation.

Increasing the relaxation time causes the thermal wave to travel with a well-defined finite speed while reducing excessive thermal diffusion.

The numerical solutions indicate that larger relaxation times produce sharper temperature fronts and more localized heating regions. The corresponding displacement and stress waves also become more localized due to stronger thermoelastic coupling.

These observations agree well with Lord-Shulman and Green-Lindsay generalized thermoelastic theories reported in previous investigations.

### **Stress Distribution**

The computed stress distributions exhibit behavior similar to the displacement field but with larger sensitivity to both rotation and initial stress.

Near the heat source, compressive stresses attain their maximum magnitude because of rapid thermal expansion. As the distance increases, the stress gradually decreases and eventually approaches zero.

Rotation significantly reduces the peak stress values. The centrifugal force redistributes internal forces throughout the medium, thereby lowering stress concentration near the heated region.

The stress profiles also become smoother with increasing rotational speed, indicating enhanced mechanical stability.

### **Initial Stress Effects on Stress Distribution**

Initial stress directly modifies the total stress field.

Compressive prestressing increases the overall compressive stress magnitude, producing larger stress concentrations near the loading region.

In contrast, tensile prestressing reduces compressive stress and may even generate tensile regions depending on the loading conditions.

These numerical observations indicate that residual stresses existing before thermal loading strongly influence subsequent thermoelastic behavior.

Therefore, neglecting initial stress may result in significant errors in predicting stress concentration and structural integrity.

### **Wave Velocity**

Wave velocity represents one of the most important characteristics of thermoelastic wave propagation.

The numerical calculations show that increasing rotation decreases the effective propagation velocity of both elastic and thermal waves.

This reduction is attributed to additional inertial effects generated by rotation. Part of the wave energy is continuously consumed in overcoming rotational resistance, resulting in slower propagation.

Initial compressive stress slightly increases wave velocity because of increased effective stiffness.

Conversely, tensile initial stress reduces wave speed owing to decreased elastic rigidity.

The combined influence of rotation and prestressing therefore determines the final propagation speed observed in practical structures.

### **Stability of Numerical Solution**

The numerical computations remained stable throughout the investigated parameter range.

No oscillatory instability or divergence was observed, confirming the effectiveness of the adopted numerical algorithm.

The displacement, temperature, and stress curves satisfy expected physical continuity conditions and converge smoothly toward steady-state values at larger distances.

The obtained solutions also satisfy boundary conditions accurately, confirming the correctness of the computational implementation.

### **Comparison with Previous Studies**

The present numerical results agree well with earlier investigations on generalized thermoelasticity.

Several previous researchers reported that rotation reduces displacement amplitudes because of centrifugal effects. The present computations confirm this trend quantitatively.

Similarly, earlier studies demonstrated that compressive initial stress increases effective material stiffness and reduces deformation. The current numerical results fully support this observation.

The finite thermal wave speed predicted by the generalized thermoelastic theory is also consistent with the Lord-Shulman and Green-Lindsay formulations.

Minor quantitative differences between the present results and previous studies are primarily attributed to differences in material parameters, boundary conditions, and numerical techniques.

Overall, the present analysis validates existing theoretical predictions while providing additional insight into the simultaneous interaction between rotation and initial stress.

### **Physical Interpretation**

From a physical viewpoint, rotation introduces additional inertial resistance against elastic deformation. This resistance suppresses displacement and stress amplitudes while slowing wave propagation.

Initial stress modifies the internal energy state of the material before wave propagation begins. Compressive stress increases stiffness, whereas tensile stress reduces stiffness.

The interaction between these two mechanisms determines the overall wave characteristics.

Thermal relaxation further modifies energy transport by replacing unrealistic instantaneous heat conduction with finite-speed thermal waves.

The combined formulation therefore provides a much more realistic description of thermoelastic wave propagation in modern engineering materials.

### **Overall Findings**

The numerical investigation demonstrates that both rotation and initial stress exert significant influence on generalized thermoelastic wave propagation.

Increasing rotational speed consistently reduces displacement, stress, temperature, and wave velocity while increasing attenuation.

Compressive initial stress decreases displacement but increases overall stress concentration because of enhanced material stiffness.

Tensile initial stress produces larger deformation while reducing compressive stress levels.

The generalized thermoelastic theory successfully predicts finite thermal wave speed and provides physically realistic temperature distributions.

The simultaneous consideration of rotation, prestressing, and thermal relaxation yields a comprehensive mathematical model capable of accurately describing wave propagation in advanced engineering structures.

In summary, the present study establishes that rotational motion and initial stress cannot be neglected in thermoelastic analyses involving high-speed rotating components or prestressed materials. Their combined influence substantially alters mechanical and thermal responses, making generalized thermoelastic theory an effective framework for predicting the dynamic behavior of such systems. The results contribute to the improved design and reliability assessment of thermo-mechanical systems subjected to complex loading environments and provide a useful theoretical basis for future investigations on anisotropic, layered, functionally graded, and magneto-thermoelastic media.

### **Conclusion**

This study examined the combined influence of rotation and initial stress on wave propagation in generalized thermoelastic media. The governing equations of generalized thermoelasticity were formulated by incorporating rotational effects and pre-existing initial stress, and analytical solutions were obtained to investigate the propagation characteristics of thermoelastic waves.

Overall, the findings contribute to a better understanding of wave propagation in rotating, prestressed thermoelastic solids and may be useful in the design and analysis of engineering structures subjected to thermal and mechanical loading, such as rotating machinery, aerospace components, geophysical materials, and advanced composite structures. Future research may extend the present work by considering anisotropic or functionally graded materials, nonlinear thermoelastic effects, magneto-thermoelastic coupling, or numerical simulations for more complex geometries and boundary conditions.

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