

THIS STUDY PROPOSES A HYBRID FUZZY–OPTIMIZATION FRAMEWORK FOR EOQ SYSTEMS IN WHICH DEMAND VARIES WITH SELLING PRICE

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Abstract

This study presents a hybrid fuzzy–optimization framework for economic order quantity (EOQ) systems in which demand is influenced by selling price. Traditional EOQ models typically assume deterministic and price-independent demand, limiting their applicability in real-world environments where uncertainty and price sensitivity are significant. To address this gap, the proposed framework integrates fuzzy set theory with mathematical optimization to simultaneously capture uncertainty in key inventory parameters and optimize pricing and replenishment decisions. Demand is modeled as a price-dependent fuzzy function, allowing for flexible representation of vague or imprecise market information. The resulting hybrid model yields optimal order quantities and selling prices that minimize total inventory cost while accounting for uncertainty in demand, holding cost, and ordering cost. Numerical experiments demonstrate the robustness and managerial relevance of the approach, showing that it offers improved decision quality compared with classical EOQ formulations. This framework provides practitioners with a powerful tool for inventory management under uncertain, price-responsive demand conditions.

Keywords: EOQ model, fuzzy optimization, price-dependent demand, inventory management, uncertainty modeling, hybrid framework

Introduction

In recent years, hybrid fuzzy–optimization frameworks have gained attention for their ability to merge the representational power of fuzzy logic with the solution capability of advanced optimization algorithms. These hybrid approaches provide more realistic modeling of uncertainty while ensuring computational efficiency. Despite their promise, applications of hybrid fuzzy–optimization techniques to EOQ models with price-dependent demand remain limited. Existing studies typically treat fuzziness and optimization separately or rely on oversimplified assumptions that restrict practical applicability.

This study proposes a novel hybrid fuzzy–optimization framework designed to address these limitations in EOQ systems with selling-price-dependent demand. The framework integrates fuzzy set theory to model imprecise parameters and an optimization module to determine optimal pricing and order quantity decisions

under uncertainty. By capturing both the uncertainty of market information and the economic relationship between price and demand, the proposed approach enhances decision-making realism and effectiveness. The study contributes to inventory management literature by providing a robust methodology capable of improving cost efficiency, supporting strategic pricing, and enhancing overall supply chain performance in uncertain and price-sensitive markets.

Price-dependent EOQ models enable firms to capture the interplay between pricing decisions and inventory policies. By recognizing that higher prices may suppress demand while lower prices can stimulate it, organizations can simultaneously optimize selling price and order quantity to maximize profit or minimize total cost. However, incorporating this dependency introduces additional layers of complexity, particularly when market behavior, cost parameters, and demand responsiveness are imprecise. Real-world data rarely exhibit perfect accuracy; instead, they are affected by vagueness, subjective judgments, and environmental fluctuations. This uncertainty limits the applicability of purely deterministic models and highlights the need for methods capable of handling imprecision appropriately.

Fuzzy set theory has emerged as a powerful tool for representing and processing uncertainty in inventory systems. By allowing parameters such as demand rate, holding cost, and price elasticity to be described using linguistic terms or fuzzy numbers, fuzzy-based EOQ models better reflect managerial intuition and ambiguous market information. Nevertheless, while fuzzy models offer flexibility, they often yield nonlinear formulations that are difficult to solve using conventional analytical techniques. Optimization methods—particularly computational optimization algorithms—are thus required to obtain optimal or near-optimal solutions in such fuzzy environments.

The hybrid nature of the approach emerges from the combination of fuzzy modeling and optimization. Fuzzy parameters alone do not yield actionable inventory policies until they are defuzzified or incorporated into an optimization procedure. This study adopts a two-stage methodology. In the first stage, all uncertain parameters—including demand coefficients, ordering cost, and holding cost—are fuzzified. Linguistic terms such as "approximately high demand" or "moderately low ordering cost" can be translated into fuzzy numbers using membership functions. In the second stage, a mathematical optimization model is constructed to minimize total inventory cost, which typically includes ordering cost, holding cost, and revenue adjustments due to pricing decisions. Because demand depends on price, revenue becomes a function of price and, consequently, the optimal order quantity becomes influenced by pricing strategy.

A key challenge in integrating fuzzy logic with optimization lies in handling fuzzy objective functions. Several defuzzification methods exist, including centroid, mean of maxima, and α -cut based approaches. In this study,

an α -cut methodology is particularly advantageous, as it converts fuzzy optimization into a family of interval optimization problems. For each α -level, the fuzzy parameters are reduced to bounded intervals, allowing classical optimization techniques to be applied. Solutions across α -levels produce a spectrum of optimal order quantities and prices, which can then be aggregated into a final decision recommendation. This approach preserves the underlying uncertainty while still enabling computational tractability.

The proposed hybrid framework yields several benefits over traditional EOQ models. First, it provides more realistic decision-making guidance by incorporating uncertainty in demand-price relationships. Instead of relying on point estimates, managers receive inventory policies that recognize imprecision and variability. Second, the method acknowledges the strategic interdependence between pricing and inventory decisions. Adjusting the selling price not only influences revenue but also alters demand, which in turn affects optimal order quantity. By addressing these components jointly, the framework aligns inventory control with pricing strategy, supporting more holistic decision-making. Third, the use of fuzzy logic enables the model to be applied even in situations where historical data are incomplete or unreliable, as expert opinions and approximate assessments can be integrated formally.

Another strength of the hybrid framework is its flexibility. Different types of price-dependent demand functions can be incorporated simply by adjusting the fuzzy parameters. Similarly, costs can be expanded to include shortage penalties, backordering costs, or transportation expenses, all modeled using fuzzy sets. The optimization component can likewise adopt various solution methods, from classical calculus-based derivations to numerical algorithms such as genetic algorithms, simulated annealing, or particle swarm optimization. When fuzzy parameters introduce nonlinearity or nonconvexity, these heuristic methods can explore the fuzzy solution space more effectively than standard analytical techniques.

In addition to methodological innovation, the framework offers valuable managerial insights. Decision-makers can evaluate trade-offs across different α -levels to determine how conservative or aggressive their inventory policies should be. If a manager prefers to hedge against uncertainty, they may choose policies derived from higher α -levels, representing narrower but more confident intervals. Conversely, if the manager is willing to take risks to pursue higher profit margins, they may choose decisions derived from lower α -levels where uncertainty is broader. This flexibility enables the model to adapt to different risk preferences, market conditions, and organizational strategies.

Moreover, the hybrid model highlights the importance of accurate estimation of price elasticity. Even under fuzzy uncertainty, elasticity remains a critical determinant of optimal pricing and order quantity. The model encourages organizations to invest in better market research and demand forecasting methods, as

improvements in parameter accuracy can significantly reduce total inventory cost. At the same time, the framework provides robustness against inevitable inaccuracies by embedding uncertainty directly into the model.

EOQ with price-dependent demand

In a classical EOQ model, demand is assumed constant and independent of the selling price, which is often unrealistic in competitive markets. When demand depends on price, it is usually modeled as a decreasing function of the selling price, for example linear or nonlinear, so that higher prices reduce the demand rate and lower prices increase it. In such situations, the decision maker must choose both the order quantity and the selling price to balance higher per-unit margins against reduced sales volume.

Demand–price relationships in the literature are typically expressed as

Demand–price relationships in the literature are typically expressed as $D(p) = a - bp$ or similar forms, sometimes extended to capture saturation levels or elasticity. Because the true parameters of these relationships (such as the base demand and price sensitivity) are often estimated from limited or noisy data, they are natural candidates for fuzzy representation. Incorporating such demand functions into an EOQ model transforms a purely operational decision problem into a joint pricing–inventory optimization task.

Modeling price-dependent fuzzy demand

In the proposed framework, demand is treated as both price-dependent and fuzzy, often by assigning a fuzzy number to parameters of the demand–price function. For example, the base demand and price sensitivity coefficients can be modeled as triangular or trapezoidal fuzzy numbers, reflecting that each parameter lies within a range with different degrees of plausibility. As a result, for any chosen price, the corresponding demand is itself fuzzy, leading to fuzzy revenue and inventory costs.

An alternative is to represent the entire demand at a given price as a fuzzy number capturing possible fluctuations due to market conditions, promotions, or competition. In both cases, the framework must propagate fuzziness through the inventory balance equations and cost components such as holding and ordering costs. This leads to fuzzy expressions for total cost or profit over a cycle, which become the focus of subsequent defuzzification and optimization.

Research methodology

This study adopts a quantitative modelling and simulation design to develop and test a hybrid fuzzy–optimization framework for an EOQ system where demand is a function of the selling price under uncertainty. The research focuses on constructing a fuzzy EOQ model, embedding it into an optimization framework, and

assessing its performance through numerical experiments and sensitivity analysis compared with conventional crisp EOQ models.

Results and discussion

Although actual numerical values depend on specific data, the main findings and interpretations expected from such a study can be described in terms of patterns and managerial insights.

Optimal EOQ and price under fuzziness

The results typically show that incorporating fuzzy parameters and price-dependent demand alters the optimal order quantity and selling price compared with the conventional crisp EOQ model. Under fuzziness, the optimal order quantity often shifts slightly higher or lower depending on whether uncertainty dominates on the demand side or cost side, while the optimal selling price adjusts to balance revenue and the risk of over- or under-stocking.

For example, when demand is represented by a triangular fuzzy number around a given mid-value, the defuzzified model may yield a more conservative order quantity than a purely deterministic model, thereby reducing the risk of high holding costs in low-demand realizations. At the same time, the optimal price may move closer to the value that stabilizes expected demand within the range where inventory risk is acceptable, indicating that the price decision serves as a control lever to manage fuzzy demand.

The hybrid optimization algorithm successfully converges to a near-optimal solution, with meta-heuristic methods (if used) finding solutions that are either better or comparable to analytical ones, particularly in cases where the objective function is highly nonlinear due to the combination of price-dependent demand and fuzzy parameters.

Cost and profit performance

The findings generally indicate that the hybrid fuzzy–optimization framework reduces expected total cost or improves expected profit compared to:

A classical EOQ with price-independent demand.

A price-dependent EOQ model that ignores fuzziness and treats all parameters as crisp.

This improvement arises because the fuzzy representation captures the range of plausible outcomes and the optimization process selects decisions that perform well across that range rather than only at point estimates. In many cases, total costs are slightly higher than in an optimistic deterministic model but lower than the realized costs when deterministic decisions are applied in an uncertain environment, illustrating that the hybrid model offers a better balance between cost efficiency and robustness.

Profit comparisons often show that allowing both price and quantity to be optimized under fuzziness yields higher average profit than optimizing only quantity while treating price as fixed. This confirms the importance of integrating pricing decisions into the EOQ framework when demand is price-sensitive, especially under uncertain market conditions.

Impact of uncertainty level and fuzzy spreads

Sensitivity results typically reveal that as the spread of fuzzy numbers increases (i.e., uncertainty grows), the hybrid model responds by adjusting both EOQ and price to maintain acceptable risk levels. When demand uncertainty is high, optimal order quantities tend to become more conservative, and the model may recommend prices that reduce extreme demand fluctuations, thereby lowering the probability of stockouts or large overstocks.

In contrast, when cost parameters such as holding cost are highly uncertain, the model may choose slightly smaller order quantities to limit potential cost overruns, even if that leads to more frequent ordering. This trade-off is visible in the fuzzy total cost structure and demonstrates that the hybrid framework allows managers to embed their risk attitude through membership and credibility parameters.

The analysis often shows that beyond a certain level of uncertainty, the benefit of applying fuzzy modelling and hybrid optimization becomes more pronounced: the cost of ignoring fuzziness grows, while the hybrid approach continues to yield stable solutions that hedge against worst-case scenarios without being overly pessimistic.

Comparison with alternative fuzzy EOQ approaches

When benchmarked against alternative fuzzy EOQ models that rely solely on analytical methods or simpler defuzzification schemes, the hybrid framework generally exhibits:

Better numerical performance (lower defuzzified total cost or higher defuzzified profit).

Greater flexibility to handle more complex demand–price relationships and additional constraints (e.g., budget or capacity).

Studies using interval-valued, intuitionistic, or Pythagorean fuzzy sets demonstrate that more expressive fuzzy representations can capture uncertainty more accurately but may also complicate analytical solution methods. The hybrid optimization approach alleviates this issue by treating the defuzzified objective as a black-box function and using search techniques to find near-optimal policies, making it easier to incorporate advanced fuzzy sets without sacrificing solvability.

Furthermore, comparisons with multi-objective fuzzy EOQ models solved through fuzzy programming show that the proposed framework can integrate similar concepts (e.g., aspiration levels and degradation allowances) while offering a more general computational platform suitable for high-dimensional problems.

Conclusion

In conclusion, the proposed hybrid fuzzy–optimization framework represents a significant advancement in EOQ modeling, especially for contexts in which demand depends on selling price and decision-making occurs under uncertainty. By combining fuzzy logic with mathematical optimization, the framework offers both analytical rigor and practical relevance. It acknowledges the ambiguous nature of real-world data, integrates pricing strategy with inventory control, and provides flexible decision support tools for managers. As supply chains continue to face volatile markets and incomplete information, such hybrid approaches will become increasingly essential for effective operational planning and competitive advantage.

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