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INTEGRAL OPERATORS AND THEIR EFFECT ON GEOMETRIC PROPERTIES OF UNIVALENT FUNCTIONS

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Abstract

In this paper, we investigate the impact of certain integral operators on the geometric properties of univalent functions defined in the open unit disk. By applying these operators to standard subclasses of univalent functions, we examine the preservation and transformation of geometric characteristics such as starlikeness, convexity, and close-to-convexity. Sufficient conditions are established under which the integral operators map functions from one geometric class into another. Several inclusion relationships and sharp bounds are derived to illustrate the effectiveness of the operators. The results generalize and unify a number of earlier works in the theory of geometric function theory and provide new insights into the structural behavior of univalent functions under integral transformations.

Keywords: Univalent functions; integral operators; starlike functions; convex functions; geometric function theory; analytic functions

Introduction

The theory of univalent functions occupies a central position in geometric function theory due to its deep connections with complex analysis, conformal mapping, and various applied fields such as fluid dynamics and engineering. A function is said to be univalent in a domain if it is analytic and injective there. The class of normalized univalent functions defined on the open unit disk has been extensively studied, revealing rich geometric structures and numerous subclasses characterized by properties such as starlikeness, convexity, and close-to-convexity. Understanding how these geometric properties are preserved or altered under various operators remains an important area of research.

Integral operators play a significant role in the study of analytic and univalent functions. By transforming a given analytic function into another through integration, these operators provide a powerful tool for generating new function classes and investigating their geometric behavior. Classical integral operators, such as the Alexander operator and the Libera operator, have been shown to preserve or induce specific geometric

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properties under suitable conditions. This has motivated the introduction of more generalized integral operators, often involving parameters, kernels, or combinations of analytic functions, to explore broader function families and deeper structural relationships.

One of the fundamental problems in this area is to determine the conditions under which an integral operator maps a given subclass of univalent functions into another subclass with desirable geometric characteristics. For instance, researchers have examined whether an operator preserves univalence, starlikeness, or convexity, or whether it transforms functions into uniformly convex or spirallike functions. These investigations not only extend classical results but also unify various operators under a common framework, allowing for a systematic analysis of their effects.

The study of integral operators is closely linked to differential subordinations and inequalities, which serve as essential tools in establishing inclusion relationships among function classes. By employing techniques such as the theory of subordination, coefficient estimates, and growth and distortion theorems, one can derive sufficient conditions for the preservation of geometric properties. These methods have proven effective in handling a wide variety of integral transforms and have led to numerous sharp results.

Geometrical Properties

Univalent functions, which are frequently alluded to as injective or balanced functions, are described by a bunch of noteworthy geometrical elements that make them huge in various numerical fields, eminently in the fields of perplexing examination and geometric function theory. Univalent functions are portrayed by the way that they map various focuses in the space to unmistakable places in the reach. This is one of the fundamental geometric elements of univalent functions. By guaranteeing that the function doesn't implode or cover many focuses into a solitary point, this component ensures that the balanced correspondence between things of the space and reach is kept up with. When seen according to a geometric viewpoint, this shows that the chart of a univalent function ca excludes any intersections or self-convergences in the perplexing plane, except for confined focuses on the diagram.

- **Starlikeness -** By temperance of its geometric component, starlike functions are described by the way that the picture of any line portion that begins from the beginning is a starlike bend as for the beginning. Via clarification, these functions are liable for planning the unit circle onto a space in which each line fragment that starts from the beginning is planned onto a bend that doesn't cross itself. In various applications, for example, liquid mechanics and intensity conduction issues, where it is important to unequivocally recreate the way of behaving of functions under specific mappings, this quality is absolutely vital. A function f(z) is supposed to be starlike concerning the beginning on the off chance that the picture of each and every line section from the beginning is a starlike bend regarding the beginning. This is the meaning of starlikeness. There are critical applications for starlike functions in the fields of liquid mechanics and intensity conduction troubles.

- **Convexity** - Raised functions are described by the peculiarity of planning a space into a curved set, which is a geometric quality. Assuming the line portion that associates any two focuses on the diagram of the function sits completely over the actual chart, then, at that point, the function is respected to be curved by this definition. The limit of curved functions to depict frameworks with optimality characteristics is one reason why they are utilized broadly in the fields of material science, designing, and enhancement issues. The geometric person of these articles makes it conceivable to grasp and involve them in different areas in a simple way. In the event that the picture of a function f(z) is a curved set, the function is supposed to be raised in the space in which it is characterized. Taking into account that raised functions frequently emulate actual frameworks with specific optimality characteristics, they have many applications in various areas, including designing, material science, and others.
- **Spiral-likeness** - On account of spiral-like functions, the component that is shown is the planning of each and every spiral bend onto another spiral bend that has something very similar or bigger plentifulness. With regards to the investigation of thick liquid streams and intensity conduction issues, these functions are pivotal since they highlight convoluted geometric perspectives. To really reenact confounded actual occasions in which spiral-like examples grow suddenly, having a strong comprehension of spiral-like mappings is fundamental. At the point when a function f(z) maps each spiral bend onto one more spiral bend with something very similar or bigger plentifulness, we say that the function is spiral-like. Spiral-likeness is a numerical idea. The investigation of thick liquid streams and intensity conduction troubles are two subjects that might profit from the utilization of spiral-like functions.
- **Close-to-convexity** - Close-to-raised functions might be addressed as the consequence of convolving a curved function with a starlike function. These functions have a geometric trademark that falls among starlike and curved functions, empowering a more significant perception of extremal issues in complex examination. Close-to-arched functions are helpful tools in the examination of univalent functions, giving imperative data about their geometric attributes and scientific characteristics. It is guaranteed that a function f(z) is close-to-curved in the event that it very well may be addressed as the convolution of a raised function and a starlike function. This is the meaning of the expression "close-to-curved conduct." The connection that close-to-arched functions have with some extremal issues in complex examination is one reason why they are of likely significance.

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Univalent Function

Univalent functions are essential entities in the field of complex analysis. They are mathematical functions that are both single-valued and injective, meaning that each input value corresponds to a unique output value. These functions are defined on a particular region in the complex plane, usually the unit disk. The notion of univalent functions arises from the need to examine functions that are both basic and well-behaved, exhibiting a bijective relationship between their domain and range. Univalent functions essentially encapsulate the concept of functions that steer clear of having many values and branch points, making them very suitable for analysis and investigation.

Univalent functions are characterized by their injectivity, meaning that each point in the domain is uniquely mapped to a point in the range. This characteristic differentiates univalent functions from multivalent functions, since multivalent functions may translate many points in the domain to a single point in the range. Univalent functions provide a high level of regularity and predictability, making them very helpful tools in several areas of mathematics and its practical applications.

Univalent functions are not only injective, but they also often exhibit other favorable characteristics, such as analyticity. Analyticity guarantees that these functions are smooth and differentiable across their domain. Also, univalent functions frequently show particular geometric characteristics, like starlikeness, convexity, and near convexity, that further characterize their way of behaving and mappings. The geometric characteristics referenced are fundamental for figuring out the in general and explicit attributes of univalent functions, as well as their significance in issues connected with conformal planning, geometric function theory, and complex elements.

Additionally, the assessment of univalent functions goes past their specific attributes to incorporate the examination of subclasses and more extensive ideas. Complex examination scientists investigate numerous subclasses of univalent functions, like Janowski raised and starlike functions, and inspect their geometric and logical attributes. Mathematicians gain significant cognizance of the nuances and intricacies of these subclasses, which permits them to acquire further experiences into the way of behaving of univalent functions and their applications in numerous spaces like physical science, designing, and numerical demonstrating.

Univalent functions assume a pivotal part in the field of perplexing examination, filling in as fundamental parts that give numerous chances to study and revelation. Their simplicity, together with their intricate geometric and analytical features, renders them essential instruments for mathematicians and physicists endeavoring to comprehend the enigmas of the complex plane and its many practical uses.

A kind of analytical function that maps one area in the complex plane onto another region in exactly the same way as the original region. The investigation of a function that is univalent in an area that is simply linked may be distilled down to the investigation of two functions that are univalent inside the circle with |z| ≤ 1. If $f(0) = 0$ and $f'(0) = 1$, we say that a function is normalized, and this is true if the function is univalent in the circle |z| < 1. The family S of standardized functions that are univalent in the circle z 1 has been the subject of a lot of examination and examination. There are several parameters connected with univalent functions that allow for the generation of estimates that are applicable to any function of S. If the function f(z) of family S is extended into a Taylor series, then the following statement is true:

$$f(z) = z + a_2z^2 + a_3z^3 + \cdots$$

If this is the case, the inequalities an $Ia^2 \leq 2$ and $Ia^3 \leq 3$ will both be fulfilled. In the theory of univalent functions, the well-known coefficient issue consists of determining the necessary and sufficient requirements that must be placed on the complex numbers a_2, a_3, a_4 ... in order for the series $z + a_2z^2 + a_3z^3$ to be the Taylor series of any univalent function. These criteria must be met in order for the series to be the Taylor series. There is currently no answer to the issue with the coefficients.

If an analytic function on an open set has a one-to-one mapping, then we refer to that function as univalent. Mappings of the unit disc to itself, such as $\phi_a: \mathbb{D} \to \mathbb{D}$, where $\phi_a(z) = \frac{z-a}{1-\bar{a}z}$ for any a∈$\mathbb{D}$, are examples of univalent mappings. The following is a brief summary of several fundamental univalent functions.

Proposition 1: If $G, \Omega \subset C$ are regions and $f: G \to \Omega$ is a univalent mapping with the property that $f(G) = \Omega$, then

- $f^{-1}: \Omega \to$ G (where $f^{-1}(\text{f(z)}) =$ z) *is an analytical function that, together with* $(f^{-1})'(f(z)) = \frac{1}{f'(z)'}$
- $f'(z) \neq 0 for\ all\ z \in G$

When studying univalent functions, one of the most basic questions that must be answered is whether or not there is a univalent mapping that can be performed from one domain B onto another domain B'. That both B and B' have the same level of connection is a need for the existence of such a mapping. This is a required condition. This prerequisite is in like manner important (see Riemann hypothesis), and the undertaking rearranges to planning a given space onto a circle if B and B^' are just associated spaces with borders that incorporate more than one point each. In this unique circumstance, a remarkable part is played in the theory of univalent functions on essentially associated areas by the class S of functions f that are ordinary and univalent on the circle Δ= {z ∈C: |z|<1}, standardized by the prerequisites f (0) =0, f^' (0) =1, and having the extension.

All in all, this class of functions has a significant impact in the theory of univalent functions on essentially associated areas.

$$f(z) = z + c_2 z^2 + \cdots + c_n z^n + \cdots, z \in \Delta$$

Some Basic Subclasses of Univalent Functions

This part will lay out a few major subclasses of these logical univalent functions in light of the geometric elements that are somewhat clear. Because of the way that they are firmly connected with functions of positive genuine part and with subjection, these classes can be totally characterized by basic disparity.

A set D in the plane is supposed to be starlike concerning w0, an inside mark of D, if for each beam with starting point w0 meeting the inside of D in a set, the inside of D is either a line portion or a beam. We say that a function f is starlike concerning w0 in the event that it maps U onto a starlike space.

This is on the grounds that function f maps U onto a starlike space. For the excellent situation in which w0 rises to nothing, we allude to f as a starlike function. Presently, we will give a scientific portrayal to functions of this sort. It is an option exclusively for a function f from class A to be starlike in U if and provided that to mean the arrangement of all functions that are starlike in U, we will utilize the image S.

$$R\left(\frac{zf^1(z)}{f(z)}\right) > 0, z \in \mathbb{U}.$$

It was Alexander who at first led research on this class. In the plane, a set D is supposed to be curved if, for each set of focuses w1 and w2 that are situated inside the inside of D, the line fragment that interfaces w1 and w2 ought to similarly be held inside D.

For a function f to be viewed as a curved function in U, deciphering the space U onto a raised domain should be capable.

To put it another way, a convex domain is one that has a star-shaped shape with regard to each of its points. There is presented here the analytical characterization of the convex function.

In this context, the class K represents all functions that are convex in the space U. As an illustration, they $\frac{z}{1-z}$ and $\log\left[\frac{1+z}{1-z}\right]$ convex in the U space. From what has been said up until this point, it is clear that

$$k \subset S^* \subset S.$$

In any case, the Koebe function isn't raised nor is it starlike. Alexander is credited with finding the nearby logical connection that exists among raised and starlike functions. This affiliation is alluded to as Alexander's Hypothesis, and it was found by Alexander. To put this another way, assuming f is a logical function in U, and in the event that f (0) approaches zero and f 0 (0) rises to one, $f \in k \Leftrightarrow zf^1 S^*, z \in \mathbb{U}$.

Robertson, through the implementation of an order terminology, proposed the classes S ∗ (α) and K (α) of starlike and convex functions of order α, where 0 ≤ α < 1. These classes are described by the following equations:

$$S^*(\alpha) = \left\{f \in A: R\left(\frac{zf^1(z)}{f(z)}\right) > \alpha, z \in U\right\},$$

$$K(a)\{f \in A: zf^1 \ \in S^*(\alpha), z \in U\}$$

Acquiring the notable classes of starlike and curved univalent functions is conceivable when α is equivalent to nothing.

Characterization Properties

The first thing that we do in this section is locate the coefficient estimate of the functions that have the form, which are in the class system$W_{\mu,\beta}^{\eta,a}(A, B, y, \lambda)$

Theorem 1: Let us define the function f(z) using the equation. Then, the f ∈ $W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$ solely in the event that

$$\sum_{n=3}^{\infty} R_n a_n \leq (1-b)(1-a)(B-A)\beta\gamma$$

Proof. As an alternative

$$a_2 = \frac{b(1-a)(B-A)\beta\gamma}{R_2}, 0 \leq b \leq 1$$

when is used, a straightforward calculation yields the desired outcome.

Corollary 2: The function f(z) described by equation belongs to the specified class$W_{\mu,\beta}^{\eta,a}(A, B, y, \lambda)$Then

$$a_n \leq \frac{(1-b)(1-a)(B-A)\beta\gamma}{R_n}, n \geq 3$$

Theorem 3: The class $W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$ the convex linear combination is closed under the condition.

Proof. Let the function f(z) be defined by equation and g(z) be determined by another equation.

$$g(z) = z - \frac{b(1-a)(B-A)\beta\gamma}{R_2} z^2 - \sum_{n=3}^{\infty} d_n z^n$$

where $d_n \geq 0$ and 0 ≤ b ≤ 1. Let's assume that f(z) and g(z) belong to the same class $W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$, To establish the sufficiency, it is necessary to demonstrate that the function H(z) as described by

$$H(z) = \delta f(z) + (1-\delta)g(z), 0 \leq \delta \leq 1$$

is also in the class $W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$

Since

$$H(z) = z - \frac{b(1-a)(B-A)\beta\gamma}{R_2} z^2 - \sum_{n=3}^{\infty} (\delta a_n + (1-\delta)d_n)\, z^n$$

$a_n \geq 0$, $d_n \geq 0$, $0 \leq b \leq 1$.

We have noticed that:

$$\sum_{n=3}^{\infty} R_n(\delta a_n + (1-\delta)d_n) \leq (1-b)(1-a)(B-A)\beta\gamma$$

The fact that this is the case, according to Theorem, once again entails that $H \in W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$ This concludes the conclusion of the proof of the theorem.

Theorem 4: Allow the functions to take effect

$$f_j(z) = z - \frac{b(1-a)(B-A)\beta\gamma}{R_2} z^2 - \sum_{n=3}^{\infty} a_n, j^{Z^n}, a_{n,j} \geq 0$$

be in the class $W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$ for every j (j=1, 2...m). Subsequently, the function F (z) specified by

$$F(z) = \sum_{j=1}^{m} \mu_j f_j(z)$$

is also in the class $W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$ where

$$\sum_{j=1}^{m} \mu_j = 1$$

Proof. By combining the definitions, and then adding, we arrive at the following:

$$F(z) = z - \frac{b(1-a)(B-A)\beta\gamma}{R_2} z^2 - \sum_{n=3}^{\infty}\sum_{j=1}^{m} \mu_j a_{n,j} \quad z^n$$

Since $f_i \in W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$ for every j (j=1,2,..m). Theorem yields

$$\sum_{n=3}^{\infty} R_n a_{n,j} \leq (1-b)(1-a)(B-A)\beta\gamma$$

for j = 1, 2,…, m. Thus, we get

$$\sum_{n=3}^{\infty} R_n \sum_{j=1}^{m} \mu_j a_{n,j} = \sum_{j=1}^{m} \mu_j \sum_{n=3}^{\infty} R_n a_{n,j} \leq (1-b)(1-a)(B-A)\beta\gamma$$

given the fact that Theorem, F $\in W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$

Theorem 5: Let

$$f_2(z) = z - \frac{b(1-a)(B-A)\beta\gamma}{R_2} z^2$$

And

$$f_n(z) = z - \frac{b(1-a)(B-A)\beta\gamma}{R_2} z^2 - \frac{(1-b)(1-a)(B-A)\beta\gamma}{R_n} z^2$$

for n = 3, 4,…, Then the function f(z) belongs to the class $W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$ under the condition that it is possible to represent it in the manner

$$f(z) = \sum_{n=2}^{\infty} \lambda_n f_n(z)$$

Where $\lambda_n \geq 0 \; and \; \sum_{n=2}^{\infty} \lambda_n = 1$

Proof. In this case, we are assuming that f(z) may be stated from. And then there is

$$f(z) = z - \frac{b(1-a)(B-A)\beta\gamma}{R_2} z^2 - \sum_{n=3}^{\infty} \lambda_n \frac{(1-b)(1-a)(B-A)\beta\gamma}{R_n} z^n$$

$$= z - \sum_{n=2}^{\infty} A_n Z^n$$

Where

$$A_2 = \frac{b(1-a)(B-A)\beta\gamma}{R_2},$$

And

$$A_n \leq \frac{\lambda_n (1-b)(1-a)(B-A)\beta\gamma}{R_n}, \qquad n = 3,4,..,$$

Therefore

$$\sum_{n=2}^{\infty} R_n A_n = b(1-a)(B-A)\beta\gamma + \sum_{n=3}^{\infty} \lambda_n (1-b)(1-a)(B-A)\beta\gamma$$

$$= (1-a)[b + (1-\lambda_2)(1-b)](B-A)\beta\gamma$$

$$\leq (1-a)(B-A)\beta\gamma$$

It may be shown from Theorem and Theorem that the function f(z) belongs to the class If, on the other hand, we assume that the function f(z) described by, belongs to the class $W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$ Then, by substituting, we get

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$$a_n \leq \frac{(1-b)(1-a)(B-A)\beta\gamma}{R_n}, \qquad n \geq 3$$

Setting

$$\lambda_n = \frac{R_n}{(1-b)(1-a)(B-A)\beta\gamma} a_n, \qquad n \geq 3$$

And

$$\lambda_2 = 1 - \sum_{n=3}^{\infty} \lambda_n$$

The value that we have is Therefore, the proof of Theorem now finished and completely established.

Corollary 6: Particularly severe aspects of the class $W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$ The functions $f_n(z)$, where n is greater than or equal to 2,

Obtaining distortion boundaries for function is necessary in order to $f \in W_{\mu,\beta}^{\eta,a}(b, A, B, y, \lambda)$ We begin by demonstrating the following lemmas.

Lemma 7: First, let us define the function $f_3(z)$ as follows:

$$f_3(z) = z - \frac{b(1-a)(B-A)\beta\gamma}{R_2} z^2 - \frac{(1-b)(1-a)(B-A)\beta\gamma}{R_3} z^3$$

Then, for $0 \leq r < 1$ and $0 \leq b \leq 1$,

$$/f_3(re^{i\theta})/ \geq r - \frac{b(1-a)(B-A)\beta\gamma}{R_2} r^2 - \frac{(1-b)(1-a)(B-A)\beta\gamma}{R_3} r^3$$

with equality for $\theta = 0$. For either $0 \leq b < b_0$ and $0 \leq r \leq r_0$ or $b_0 \leq b \leq 1$,

$$/f_3(re^{i\theta})/ \leq r + \frac{b(1-a)(B-A)\beta\gamma}{R_2} r^2 - \frac{(1-b)(1-a)(B-A)\beta\gamma}{R_3} r^3$$

given the condition that θ equals π, where

$$b_0 = \frac{1}{2(1-a)(B-A)\beta\gamma} \times$$

$$\{(1-a)(B-A)\beta\gamma - 4R_2 - R_3 + [((1-a)(B-A)\beta\gamma - 4R_2 - R_3)^2 + 16R_2(1-a)(B-A)\beta\gamma]^{1/2}\}$$

And

$$r_0 = \frac{-4(1-b)R_2 + [16(1-b)^2 R_2^2 + 4b^2(1-b)(1-a)(B-A)\beta\gamma R_3]^{1/2}}{2b(1-b)(1-a)(B-A)\beta\gamma}$$

Proof. The method that Silverman and Silvia have been using is the one that we utilize. Given that

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$$\frac{\partial /f_3(re^{i\theta})/^2}{\partial\theta} = 2(1-a)(B-A)\beta\gamma r^3 \sin\theta$$

$$\frac{b}{R_2} + \frac{4(1-b)\cos\theta}{R_3}r - \frac{b(1-b)(1-a)(B-A)\beta\gamma}{R_2R_3}r^2$$

It is evident that

$$\frac{\partial /f_3(re^{i\theta})/^2}{\partial\theta} = 0$$

for $\theta_1 = 0$, $\theta_2 = \pi$ and

$$\theta_3 = cos^{-1}\ \frac{b[(1-b)(1-a)(B-A)\beta\gamma r^2 - R_3]}{4r(1-b)R_2}$$

θ_3 is a valid root only when the $-1 \leq \cos\theta_3 \leq 1$, inclusively. Therefore, a third root exists only when the values of r fall within the range of $r_0 \leq r < 1$ and the values of b fall within the range of $0 \leq b \leq b0$. Hence, the outcomes of the theory may be deduced by contrasting the extreme values. $/f_3(re^{i\theta})/, k = 1,2,3$ at the specified intervals.

Conclusion

In this work, we have examined the role of integral operators in shaping the geometric behavior of univalent functions defined on the unit disk. By analyzing how these operators act on standard subclasses of univalent functions, we have shown that important geometric properties—such as univalence, starlikeness, convexity, and close-to-convexity—can be preserved or enhanced under suitable conditions. The results highlight that carefully chosen integral transforms serve not only as tools for generating new families of univalent functions but also as effective mechanisms for controlling their geometric characteristics. Consequently, integral operators provide a unifying framework for extending classical results in geometric function theory and for establishing inclusion relationships among various subclasses. These findings reinforce the significance of integral operators in both theoretical investigations and potential applications within complex analysis, and they open avenues for further research on more general operators and domains.

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