INTRODUCTION TO MULTICRITERIA DECISION ANALYSIS (MCDA)

06 – ELECTRE

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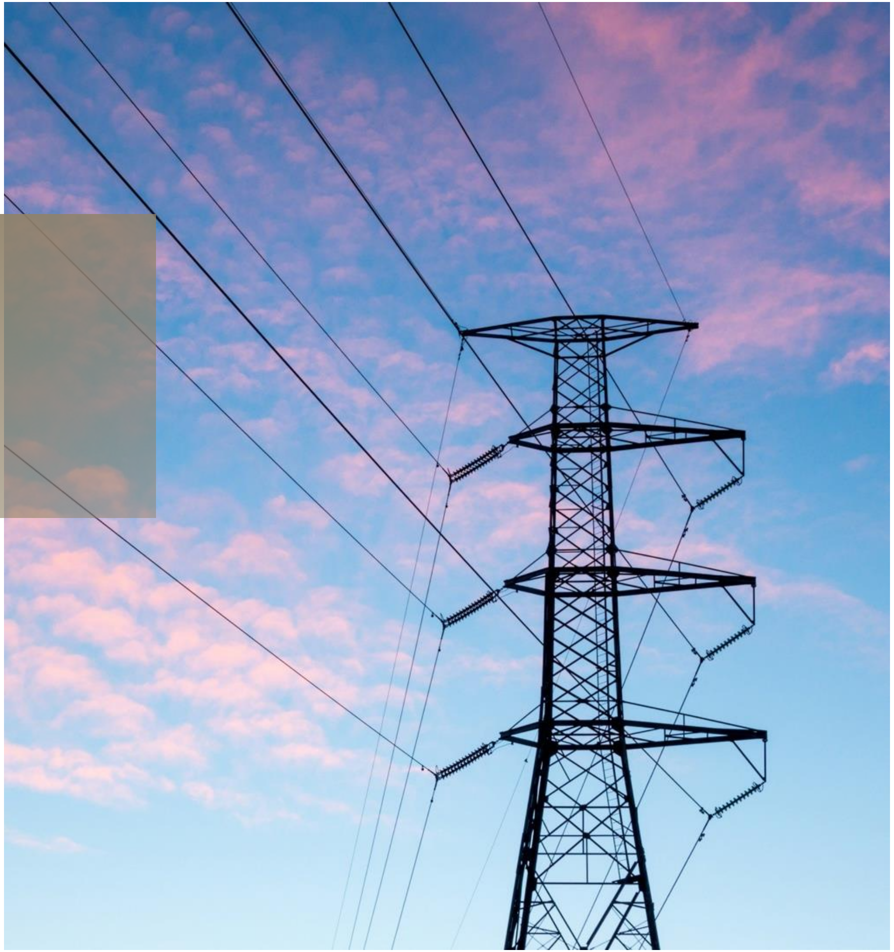


ELECTREE

The famous, yet excentric one

What does ELECTRE stand for?

ELECTRE method (short for *ELimination Et Choix Traduisant la REalité*, which translates to *Elimination and Choice Expressing the Reality*) was developed in the 1960s by Bernard Roy and is one of the earliest **outranking multicriteria decision-making methods.**

ELECTRE I is designed for **choice problems**, where the goal is to **select one or more alternatives** from a set, rather than rank or sort them. It works best when there is **conflict** among criteria and **compensatory methods** (like AHP or simple weighted sums) are inappropriate.

Key concepts

• **Alternatives**: The different options being considered (e.g., A, B, C).

• **Criteria**: Dimensions used to evaluate alternatives (e.g., cost, quality, delivery time).

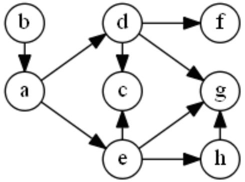
• **Weights**: Importance of each criterion.

• **Outranking**: Instead of comparing scores, ELECTRE checks whether one alternative is at least as good as another, taking into account **agreement and disagreement** (more about that later).

Versions

ELECTRE is actually a family of methods.

The first iteration (ELECTRE) aims at proposing an ouranking graph



It provides:

• A partial solution (some alternatives can’t be compared).

• Iterative filtering: you can repeat the process removing dominated options to reveal a “refined” kernel.

Versions

This version is quite complex, and not applicable to every problem.

Also, it may work OK for recommendations but we cannot really use them to separate criteria / alternatives.

It is, though, a true outranking method (if compared with other methods we have seen before).

ELECTRE II

This version slowly goes from an outranking method to a ranking one.

Its goal is generating a ranking (not just a kernel), by considering the strength of outranking and opposition.

What you do:

- Define two thresholds for each matrix:
- Concordance
- Discordance

Ranking from the Net superior value (NSV) (and negative from the the Net inferior Value).

➔ Compare to TOPSIS (NIS / PIS)

OTHER VERSIONS

These extend ELECTRE I & II by adding:

• **ELECTRE III**: fuzzy preferences, thresholds for indifference/preference/veto — for ranking.

• **ELECTRE IV**: no weights used — for ordinal data or voting situations.

• **ELECTRE IS**: simplified ELECTRE III.

• **ELECTRE TRI**: sorts alternatives into predefined categories (e.g., “acceptable,” “unacceptable”).

ELECTRE II – STEP 1

Build a matrix Ai x Ci (as before)

Normalize the matrix (same procedure as in TOPSIS);

$$x_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{M} a_{ij}^2}}$$

This converts the raw values to **dimensionless scores** between 0 and 1, allowing comparisons across criteria with different units.

If you have any "negative" or "cost" criteria, remember to invert *after* normalization ($1 - n$, with n being the number).

ELECTRE II – STEP 1

Build a matrix Ai x Ci (as before)

A company wants to choose among 3 suppliers based on the following **criteria / alternatives**:

Criteria	**Type**	**Weight**
Price (C1)	Cost ↓	0.4
Quality (C2)	Benefit ↑	0.3
Delivery (C3)	Benefit ↑	0.3

Alternative	**Price**	**Quality**	**Delivery**
A1	100	80	70
A2	120	90	60
A3	110	75	8

ELECTRE II – STEP 1

Normalize each column:

$$x_{ij} = \frac{1/a_{ij}}{\sqrt{\sum_{i=1}^{m} (1/a_{ij})^2}}$$

For Price (C1 – cost → use 1/value):

- A1: 1/100 = 0.01
- A2: 1/120 = 0.00833
- A3: 1/110 = 0.00909

→ Norms: sqrt($0.01^2 + 0.00833^2 + 0.00909^2$) ≈ 0.0154

Do the same for the others

ELECTRE II – STEP 1

Normalize each column:

For Quality (C2):

- Values: 80, 90, 75
- Norm: sqrt(80² + 90² + 75²) = sqrt(6400 + 8100 + 5625) ≈ 139.82

For Delivery (C3):

- Values: 70, 60, 80 → Norm ≈ 122.47

$$x_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{M} a_{ij}^2}}$$

ELECTRE II – STEP 2

Same as before:

$$y_{ij} = x_{ij} \times w_j$$

So far, we are quite similar to TOPSIS, but now things begin to change.

ELECTRE II – STEP 2

Multiply each normalized value by its criterion weight.

Alt	**w*C1**	**w*C2**	**w*C3**
A1	0.649×0.4 = 0.260	0.572×0.3 = 0.172	0.572×0.3 = 0.172
A2	0.541×0.4 = 0.216	0.644×0.3 = 0.193	0.490×0.3 = 0.147
A3	0.590×0.4 = 0.236	0.537×0.3 = 0.161	0.653×0.3 = 0.196

ELECTRE II – STEP 3

Determine Concordance and Discordance Sets

Formulas:

$$C_{kl} = \{j \mid y_{kj} \geq y_{lj}\}$$

$$D_{kl} = \{j \mid y_{kj} < y_{lj}\}$$

For each pair of alternatives A_k and A_l :

- Concordance set: criteria where A_k performs better or equal to A_l
- Discordance set: criteria where A_k performs worse than A_l

Alt	C1	C2	C3
A1	0.260	0.172	0.172
A2	0.216	0.193	0.147
A3	0.236	0.161	0.196

ELECTRE II – STEP 3

Concordance

$$C_{kl} = \{j \mid y_{kj} \geq y_{lj}\}$$

Example 1: A1 vs A2

Criterion	A1	A2	A1 ≥ A2?	Belongs to Concordance?
C1	0.260	0.216	Yes	(weight = 0.4)
C2	0.172	0.193	No	NO
C3	0.172	0.147	Yes	(weight = 0.3)

Alt	C1	C2	C3
A1	0.260	0.172	0.172
A2	0.216	0.193	0.147
A3	0.236	0.161	0.196

ELECTRE II – STEP 3

Concordance

$$C_{kl} = \{j \mid y_{kj} \geq y_{lj}\}$$

Example 1: A1 vs A3

Criterion	A1	A3	A1 ≥ A3?	Concordance?
C1	0.260	0.236	Yes	(weight = 0.4)
C2	0.172	0.161	Yes	(weight = 0.3)
C3	0.172	0.196	No	NO

Alt	C1	C2	C3
A1	0.260	0.172	0.172
A2	0.216	0.193	0.147
A3	0.236	0.161	0.196

ELECTRE II – STEP 3

Concordance

$$C_{kl} = \{j \mid y_{kj} \geq y_{lj}\} \qquad D_{kl} = \{j \mid y_{kj} < y_{lj}\}$$

Pair	Concordance Set	Concordance Weight	Discordance Set
A1 vs A2	{C1, C3}	0.7	{C2}
A1 vs A3	{C1, C2}	0.7	{C3}
A2 vs A3	{C2}	0.3	{C1, C3}
A3 vs A2	{C1, C3}	0.7	{C2}

ELECTRE II – STEP 4

Construct Concordance and Discordance Matrices

For concordance:

$$C_{kl} = \sum_{j \in C_{kl}} w_j$$

- Add up the **weights** of the criteria where $A_k \geq A_l$.

ELECTRE II – STEP 4

For discordance it is a bit more complicated:

$$D_{kl} = \frac{\max_{j \in D_{kl}} |y_{kj} - y_{lj}|}{\max_j |y_{kj} - y_{lj}|}$$

- Numerator: largest difference where $A_k \geq$ is worse than A_l
- Denominator: largest difference across all criteria for this pair

Alt	C1	C2	C3
A1	0.260	0.172	0.172
A2	0.216	0.193	0.147
A3	0.236	0.161	0.196

ELECTRE II – STEP 4

$$C_{kl} = \sum_{j \in C_{kl}} w_j$$

Pair	Concordance Set	Concordance Weight	Discordance Set
A1 vs A2	{C1, C3}	0.7	{C2}
A1 vs A3	{C1, C2}	0.7	{C3}
A2 vs A3	{C2}	0.3	{C1, C3}
A3 vs A2	{C1, C3}	0.7	{C2}

A1 vs A2

- From Step 3:
- Concordance Set = {C1, C3} → C_{12} = **0.4 + 0.3 = 0.7**
- Discordance Set = {C2}

Alt	C1	C2	C3
A1	0.260	0.172	0.172
A2	0.216	0.193	0.147
A3	0.236	0.161	0.196

ELECTRE II – STEP 4

$$D_{kl} = \frac{\max_{j \in D_{kl}} |y_{kj} - y_{lj}|}{\max_j |y_{kj} - y_{lj}|}$$

Pair	Concordance Set	Concordance Weight	Discordance Set
A1 vs A2	{C1, C3}	0.7	{C2}
A1 vs A3	{C1, C2}	0.7	{C3}
A2 vs A3	{C2}	0.3	{C1, C3}
A3 vs A2	{C1, C3}	0.7	{C2}

A1 vs A2

$y_{A2,C2} = 0.193, y_{A1,C2} = 0.172$

Numerator: **0.193 - 0.172 = 0.021**

- Denominator: max absolute difference across all 3:
- **C1: |0.260 - 0.216| = 0.044**
- **C2: |0.172 - 0.193| = 0.021**
- **C3: |0.172 - 0.147| = 0.025**

→ **Max = 0.044**

ELECTRE II – STEP 5

Calculate Dominance Thresholds

$$\bar{c} = \frac{1}{M(M-1)} \sum_{k \neq l} C_{kl} \quad \text{(average of all C values)}$$

$$\bar{d} = \frac{1}{M(M-1)} \sum_{k \neq l} D_{kl} \quad \text{(average of all D values)}$$

ELECTRE II – STEP 4

Concordance

From \ To	A1	A2	A3
A1	–	0.7	0.7
A2	–	–	0.3
A3	–	0.7	–

Discordance

From \ To	A1	A2	A3
A1	–	0.477	1.000
A2	–	–	1.000
A3	–	0.653	–

$$c^* = \frac{0.7 + 0.7 + 0.3 + 0.7}{4} = 0.6$$

$$d^* = \frac{0.477 + 1.000 + 1.000 + 0.653}{4} = 0.7825$$

ELECTRE II – STEP 6

$$c^* = \frac{0.7 + 0.7 + 0.3 + 0.7}{4} = 0.6$$

$$d^* = \frac{0.477 + 1.000 + 1.000 + 0.653}{4} = 0.7825$$

Apply Outranking Conditions:

Concordance test:

$$f_{kl} = \begin{cases} 1, & \text{if } C_{kl} \geq \bar{c} \\ 0, & \text{otherwise} \end{cases}$$

Discordance test:

$$g_{kl} = \begin{cases} 1, & \text{if } D_{kl} \leq \bar{d} \\ 0, & \text{otherwise} \end{cases}$$

Pair	C	D	C ≥ 0.6?	D ≤ 0.7825?	Outranks?
A1 → A2	0.7	0.477	Yes	Yes	Yes
A1 → A3	0.7	1.000	Yes	No	No
A2 → A3	0.3	1.000	No	No	No
A3 → A2	0.7	0.653	Yes	Yes	Yes

ELECTRE II – STEP 7

Extract Net Superiority and Inferiority Values (NSV and NIV)

NSV:

$$c_k = \sum_l C_{kl} - \sum_l C_{lk}$$

NIV:

$$d_k = \sum_l D_{kl} - \sum_l D_{lk}$$

Why Concordance and Discordance?

Real-World Decision-Making Is Not Always Compensatory

ELECTRE was designed with **non-compensatory logic** in mind.

In many decisions**, a strong performance on one criterion cannot make up for a very weak one on another** (e.g., a cheap product that’s extremely unsafe isn’t acceptable).

Thus, ELECTRE separates the evaluation into two distinct lenses:

- Concordance (support): To what extent is A better than B across criteria?
- Discordance (opposition): Is there any strong reason (criterion) to reject A over B?

Only when there’s enough agreement (concordance) and no strong opposition (discordance) do we say “A outranks B”.

What if rankings from Net Superiority and Inferiority don't match?

Sometimes, the ranking based on Outflow (how much an option dominates) differs from the ranking based on Inflow (how much an option is dominated by others).

This happens when:

• An alternative dominates many weak options but is heavily dominated by one or two strong ones.

• There is intransitivity or cycles in the outranking relations.

• Trade-offs across criteria create conflicting signals.

What if rankings from Net Superiority and Inferiority don't match?

OPTION 1: You can combine Outflow and Inflow into a net score:

$$\phi_i = \mathrm{Outflow}(A_i) - \mathrm{Inflow}(A_i)$$

(Outflow = NSV; Inflow = NIV)

Then rank alternatives by ϕ_i .

What if rankings from Net Superiority and Inferiority don’t match?

OPTION 2: Interpret Dual Rankings Separately

Sometimes, the Outflow ranking and Inflow ranking are both shown:

- **High Outflow = proactive or strong alternative**
- **Low Inflow = resilient or not easily dominated**

This dual view helps in strategic decision-making, e.g., in portfolio analysis.

Hands-on approach

- Let’s try this on Google Sheets (same spreadsheet as before):

TOPSIS (Hwang & Yoon 1981)

1 Create a decision matrix Ai x Ci

1.1 Remeber to mark well the cost (negative) and benefit (positive) criteria:

Cost criteria impact negatively your model (risk, cost, danger, etc), benefit criteria impact positively (c

	Cost Criteria		Benefit Criteria		
Alternatives	C1 Cost	C2 Risk	C3 Flexibility	C4 Performance	C5 Reliability
A1	375	59	9	100	92
A2	354	64	2	84	89
A3	338	77	11	89	93
A4	217	67	20	85	84
W	0.3	0.2	0.05	0.3	0.15

1.2 Normalize the matrix using r_ij

	Cost Criteria		Benefit Criteria		
r_ij	C1 Cost	C2 Risk	C3 Flexibility	C4 Performance	C5 Reliability
A1	0.574	0.440	0.366	0.557	0.514
A2	0.542	0.477	0.081	0.468	0.497
A3	0.517	0.574	0.447	0.496	0.519
A4	0.332	0.499	0.812	0.474	0.469

2 Apply weights

	Cost Criteria		Benefit Criteria		
v_ij	C1 Cost	C2 Risk	C3 Flexibility	C4 Performance	C5 Reliability
A1	0.172	0.088	0.018	0.167	0.077
A2	0.162	0.095	0.004	0.140	0.075
A3	0.155	0.115	0.022	0.149	0.078
A4	0.100	0.100	0.041	0.142	0.070

Hands-on approach

- This is a free online TOPSIS tool
- **https://decision-radar.com/electre**

TIMELINE

Subject to change

03
Analytical Hierarchical Process (AHP)

04
Fuzzy AHP

05
TOPSIS & Fuzzy TOPSIS

06
ELECTRE

07
PAPRIKA / MACBETH?

REFERENCES

Today’s content was mainly based on

- **Goodwin, P., & Wright, G. (2014). Decision analysis for management judgment. John Wiley & Sons.**
  Belton, V., & Stewart, T. (2012). Multiple criteria decision analysis: an integrated approach. Springer Science & Business Media.
- **Greco, S., Figueira, J., & Ehrgott, M. (Eds.). (2016). Multiple criteria decision analysis: state of the art surveys. New York, Springer.**
- **Shih, H. S., & Olson, D. L. (2022). TOPSIS and its extensions: A distance-based MCDM approach (Vol. 447). Springer Nature.**

REFERENCES

REFERENCES

IMAGES

The image for PIS/NIS (TOPSIS Step 4) is from:

- **Chauhan, A., & Vaish, R. (2014). A comparative study on decision making methods with interval data. Journal of Computational Engineering, 2014(1), 793074.**

The images for Classical Visualization / Choice Behavior is from:

- **Shih, H. S., & Olson, D. L. (2022). TOPSIS and its extensions: A distance-based MCDM approach (Vol. 447). Springer Nature.**

THANKS

Does anyone have any questions?
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