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Cases of criticality with Plexos Project

Introduction

Critical activities are those that must adhere strictly to their scheduled start and finish times to prevent project delays. These activities form one or more critical paths, which dictate the overall project duration.

The concept of criticality was introduced with the Critical Path Method (CPM) in the 1950s (Kelley & Walker, 1959). However, not all critical activities are equal, and understanding their specific nature is crucial for effective schedule management and production planning.

This article presents an in-depth analysis of criticality cases within Plexos Project, examining how the different types of relationships (*Finish-to-Start*, *Start-to-Start*, *Finish-to-Finish*, and *Start-to-Finish*) influence the determination of critical paths, considering different continuity conditions and interactions between activities and sub-activities.

The example used to expose the cases of criticality can be downloaded on the examples section named Cases of Criticality.

For Finish-to-Start relationships (Totally critical).

The *Finish-to-Start* ($FS_{ij}(z)$) precedence relationship between activities represents the minimum number of z time-periods, or delay, that must elapse between the completion of the predecessor activity i , and the start of the follower activity j .

Let's consider two cases:

- **Case 1.** Predecessor (Act-2) and successor (Act-3) activities are critical (Figure 1, Figure 2, and Figure 3).
- **Case 2.** The successor (Act-5) activity is critical, but not the predecessor one (Act-4) (Figure 1, Figure 2, and Figure 3).

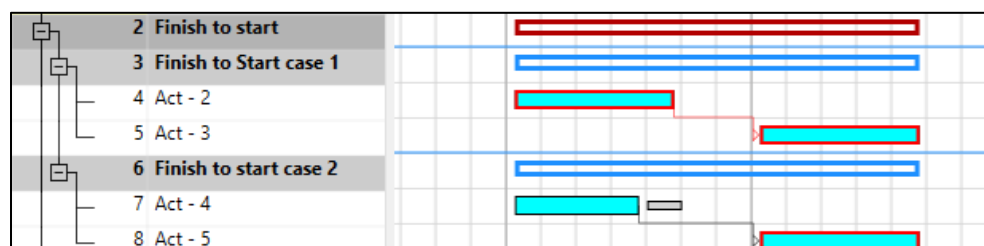


Figure 1 Criticality by Finish-to-Start relationship Cases 1 and 2 (Gantt Chart)



Figure 2 Criticality by Finish-to-Start relationship Cases 1 and 2 (Extended Graph).

Figure 3 Finish-to-start additional of 1 day in case 2

The difference between cases 1 and 2 arises from the calendar considerations in time calculations. Although both cases have a 1-day (Additional Delay z) for the *Finish-to-Start* relationship, the difference lies in how Act-5's start date is handled. In case 2, when Act-5 tries to start on Saturday, it is delayed until Monday, resulting in a float in Act-4.

This effect is produced because Plexos considers that the delay **uses the natural calendar** to compute the times. An example of a delay in the construction industry is the hardening of concrete.

For Finish-to-Finish relationships (Finishing critical).

The *Finish-to-Finish* ($FF_{ij}(w_j, p_j, z)$) relationship between activities represents the minimum percentage of production p_j , or effective workdays w_j required on the follower activity j , after the completion of its predecessor i , with an additional lag of z time-periods.

If the successor activity j is splittable, Plexos will compute the near-optimal splits based on the percentage of production quantity p_j or effective workdays w_j required (Figure 4 and Figure 5).

Let's examine the following situation, where Activity 14 is critical by its finishing split.

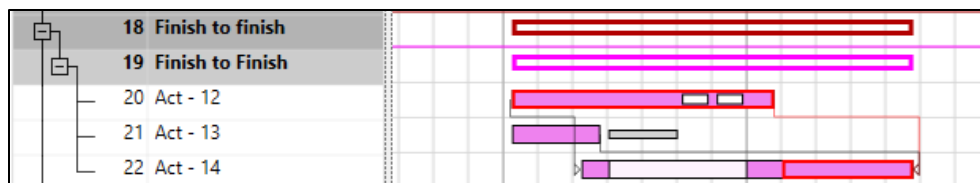


Figure 4 Finishing Critical activity by Finish-to-Finish relationship (Gantt Chart)

When the most restrictive condition is due to a *Finish-to-Finish* relationship, the successor

activity can be critical by the workdays, or production level, established in the relationship. As in the *Start-to-Start* relationship, if you increase the additional delay z , the successor activity is delayed, but the nature of the criticality remains unchanged.

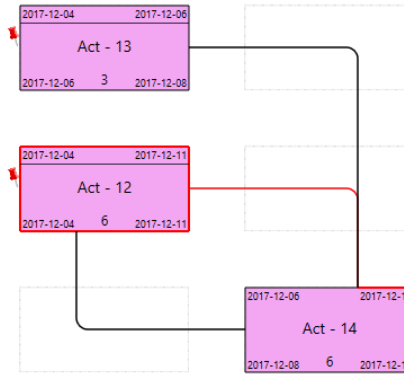


Figure 5 Finishing Critical activity by Finish-to-Finish relationship (Extended Project)

Note that the **finish-to-finish relationships** use the calendar of the successor activity for the workdays and production level and the "natural calendar" for the "Additional Delay" to compute the times.

For Start-to-start relationships (Starting critical).

The *Start-to-Start* ($SS_{ij}(w_i, p_i, z)$) precedence relationship represents the minimum percentage p_i , or w_i effective work-periods, required on the predecessor activity i , prior to the start of the successor activity j , with an additional lag of z time-periods

Let us consider the following scenarios:

- **Case 1.** The predecessor activities (Act-6 and Act-7) are critical by its initial split (Figure 6 and Figure 7).
- **Case 2.** The predecessor activity (Act-10) becomes finishing critical by its finishing split (Figure 6 and Figure 7).

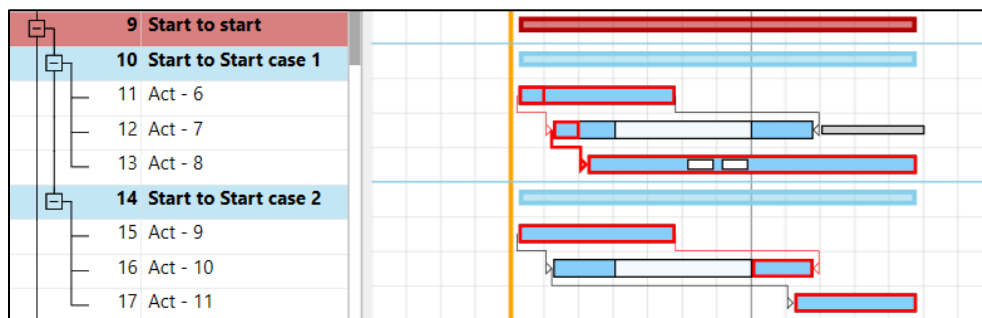


Figure 6 Criticality by Start-to-Start relationship for cases 1 and 2 (Gantt Chart)

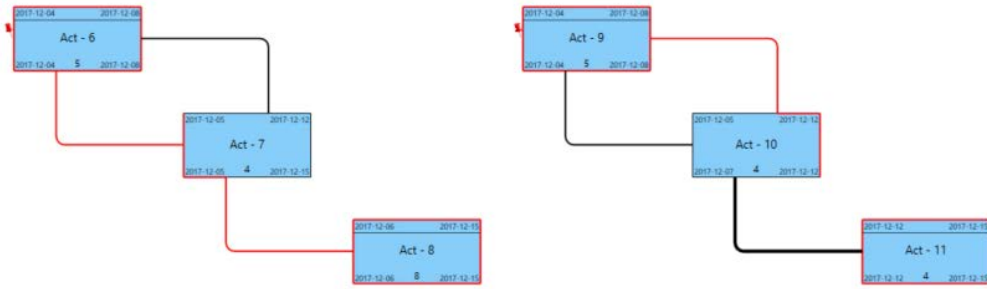


Figure 7 Criticality by Start-to-Start relationship for cases 1 and 2 (Extended Graph)

In both cases, Act-7 is splittable (continuity restriction unchecked in Figure 8).

Figure 8 Act-7 continuity condition

The difference between case 1 and case 2 is that the Start-to-Start relationship is 1 workday in case 1 (Figure 9) and 3 workdays in case 2 (Figure 10).

Figure 9 Start-to-Start relationship of 1 workday between Act-7 and Act-8

Figure 10 Start-to-Start relationship of 3 workdays between Act-10 and Act-11

You can note that if you increase the "Additional Delay", the successor activity is delayed, but the nature of the criticality remains unchanged.

Note that **the start-to-start relationships use the calendar of the predecessor activity** for the workdays and production level and the "natural calendar" for the "Additional Delay" to compute the times.

For Start-to-finish relationships.

The *Start-to-Finish* ($SF_{ij}(w_i, p_i, w_j, p_j, z)$) precedence relationship represents the minimum p_j or/and w_j effective work-periods required on the follower activity j after the minimum number of p_i or/and w_i work-periods on the predecessor activity i has been completed, with an additional lag of z time-periods.

Let's consider the following cases:

- **Case 1.** The activity (Act-19) is critical by its finishing split (Figure 12 and Figure 11).
- **Case 2.** The activity (Act-24) is critical because of its initial split (Figure 12 and Figure 11).
- **Case 3.** The activity (Act-29) is critical by its initial and finishing split, but not in the middle (Figure 12 and Figure 11).

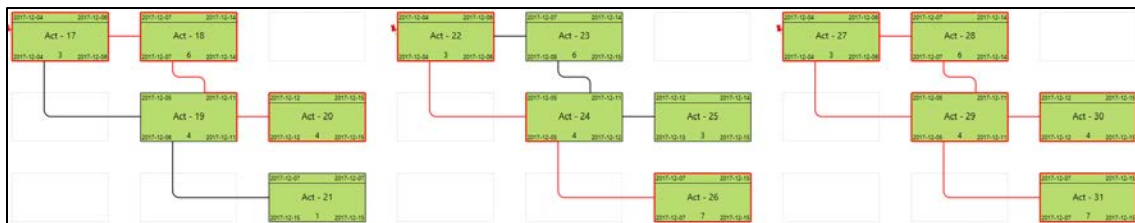


Figure 11 Criticality by Start-to-Finish Cases 1 and 2 (Extended Graph)

The criticality established in Case 1 by the start-to-finish relationship is a special case in which it goes from the starting of the predecessor activity (Act-18) to the finishing of the successor one (Act-18).

Cases 2 and 3 do not directly constitute a case of criticality by the *Start-to-Finish* relationship, but by the interaction between several relationships. Case 3 is especially interesting, as Act-29 is start and finish critical, but with internal float.

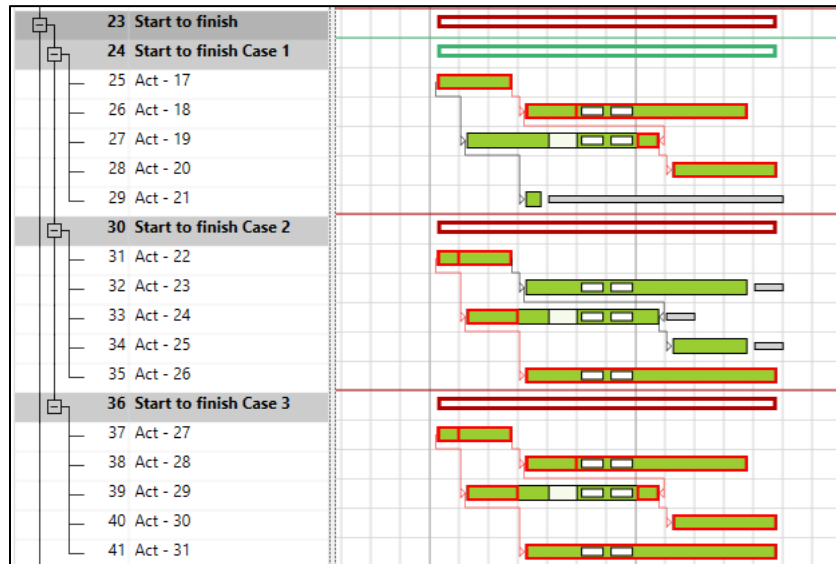


Figure 12 Criticality by Start-to-Finish Cases 1 and 2 (Gantt Chart)

Note that for the **Start-to-finish relationships**, the activity start is computed using the predecessor's calendar, and the finish date is computed with the successor's calendar. As in the previous examples, the natural calendar is used for the "Additional Delay" to compute the times.

Between sub-activities.

Activities with sub-activities offer a powerful tool for fine-tuning workflow control. By dividing production within the activities into smaller packages, takts, or sub-activities, we can achieve a more precise and realistic model of the production process.

When sub-activities are included, the same critical evaluation process applies as for regular activities. However, sub-activities are inherently non-splitting allowed, meaning they cannot be further divided, but their continuity within the parent activity is discretionary.

In Case 1, the successor activity (Act-33) cannot be interrupted along with its execution, and consequently, all its sub-activities are critical (Figure 14), but in Case 2, Act-35 is splitting-allowed, so the criticality starts in sub-activity 3 (Figure 15).

Case 3 illustrates the scenario where a parent activity, containing multiple sub-activities (Act-37), interacts with a regular activity (Act-38) (Figure 16). This interaction can result in a specific sub-activity within the parent activity being identified as critical.



Figure 13 Criticality between sub-activities (Gantt Chart)

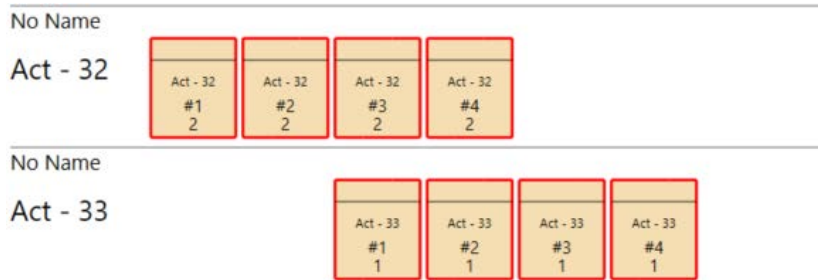


Figure 14 Criticality between sub-activities Case 1 (Takt Graph)

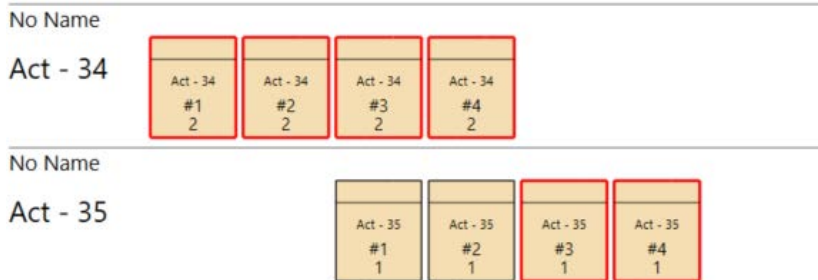


Figure 15 Criticality between sub-activities Case 2 (Takt Graph)

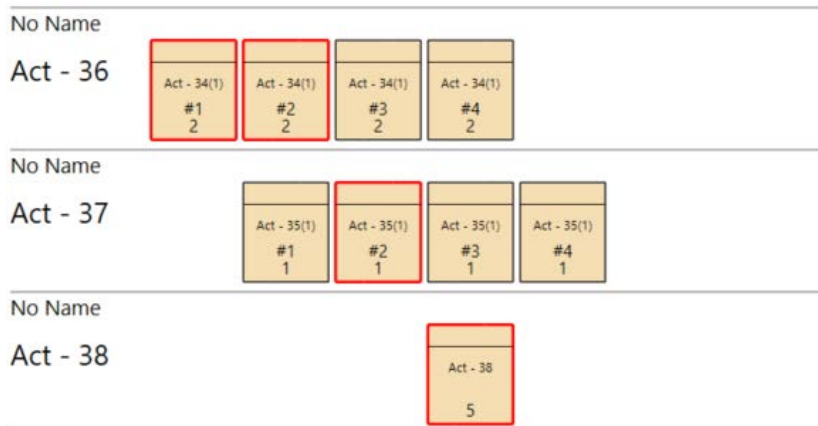


Figure 16 Criticality between sub-activities Case 3 (Takt Graph)

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