

Fetal-Maternal ECG Signal Separation from Two Channels based on the Continuous Wavelet Transform and the JADE Algorithm

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Abstract— this paper presents a new approach based on the continuous wavelet transform CWT and JADE algorithm for the blind source separation. The JADE algorithm has been widely used to separate the fECG and mECG signals from 8 recordings or channels, while not all of this number is needed. The present work will show that it isn't the number of channels that counts but rather the quality of the channel used in terms of the energy or the information that it carries. That's why before introducing the 8 channels to the JADE algorithm we will make a selection by the mathematical microscope Continuous Wavelet Transform CWT and we will show that the number of channels will be reduced to 5 or 3 or even to 2 channels. This algorithm has been validated on several real data.

Keywords—*blind source separation BSS. continuous wavelet transform. Independent Component Analysis ICA. JADE.*

I. INTRODUCTION

Fetal electrocardiogram signal, non-invasively taken from the abdominal of a pregnant woman is an efficient diagnostic tool for evaluating the health status of fetus. Heart defects are being the most common birth defects and the leading cause of sudden prenatal death [2]. The cardiac defect may be very slight so that the baby appears healthy and normal for many years after birth, but suddenly becomes so severe due to that its life is in immediate danger. Since heart defects originate in the early weeks of pregnancy when the heart is forming [3], the regular monitoring of the fetal heart and the early detection of cardiac abnormalities may help obstetrics and pediatric cardiologist to prescribe proper medications in time, or to consider the necessary precautions during delivery. The fECG can be measured by placing electrodes on the mother's abdomen. However, this signal has very low power and is mixed with several sources of noise and interference. These include fetal brain activity, and power line interference.

Moreover, its variability is increased by factors related to gestational age, position of the electrodes, skin impedance, etc. Nevertheless, the main contamination is the maternal ECG [3], since its amplitude is much higher. The basic problem is to extract the fECG from the mixture of mECG and fECG signals, where the interfering mECG is a much stronger signal. Since 1960, many different methods [9, 10, 11, 12, 13] have been developed for detecting the fetal ECG. Most of these methods focus on multi-channel mixtures of signals [12]. One direct method subtracts a thoracic maternal ECG from the abdominal composite ECG. A more recent approach [13] employs Independent Component Analysis (ICA), which extracts the sources from their mixtures by assuming the sources are statistically independent. Relatively few works address the problem separating ECG signals recorded on a single-channel. Kanjilal et. al. [4,9,10,14] developed a method for single-channel signals by first detecting both the maternal and fetal heart beats. Next the signal is "cut" into pieces. These pieces are aligned (to form a matrix) and SVD is then performed to obtain the ECG complex. In this paper we will first give a detailed overview of the theory of independent component analysis ICA, JADE algorithm and the continuous wavelet transform CWT, then we will present our new method and finally a simulation Matlab for the validation of the methods.

II. PRELIMINARY RESULTS

A-Blind Sources Separation

1-Description of the problem

BSS [1, 8, 9, 10, 29], consist of recovering unobserved sources from several observed mixture. Typically the observations are obtained at the output of a set of sensors, where each sensor receives a different combination of the sources signals. The adjective "blind" stresses the fact that:

-The sources signal are not observed

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- No information is available about the mixture

This is a sound approach when modeling the transfer from the sources to the sensors is too difficult; it is unavoidable when no prior information is available about the transfer. The lack of prior knowledge about the mixture is compensated by a statistically strong but often physically plausible assumption of independence between the source signals. The simplest BSS model assumes the existence of n independent signals $s_1(t), \dots, s_n(t)$ and the observation of as many mixtures $x_1(t), \dots, x_m(t)$, these mixtures being linear and instantaneous, i.e., $x_i = \sum_{j=1}^m a_{ij} s_j$ for each $i=1, \dots, n$

This is represented compactly by the mixing equation

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{n} \quad (1)$$

Where:

- $\mathbf{x} = [x_1, \dots, x_m]^T$ is an $m \times 1$ column vector collecting the observed signals.

- $\mathbf{s} = [s_1, \dots, s_n]^T$ is an $n \times 1$ column vector collecting the source signals.

- $\mathbf{n} = [n_1, \dots, n_m]^T$ is an $m \times 1$ column vector collecting the noise signals.

- $\mathbf{A}$ is the square $n \times n$ mixing matrix

It is assumed that:

- $\mathbf{m} \geq \mathbf{n}$

- $\text{rang}(\mathbf{A}) = \mathbf{n}$

- $\mathbf{s}$ is the realization of a centered vector random sequence of non-zero components.

- $\mathbf{n}$ is the realization of a centered vector random sequence of covariance $\sigma^2 \mathbf{I}_n$ independent of $\mathbf{s}$

2-Fundamental assumptions:

↪ The sources are sequences **identically and independently distributed iid**

$$\forall i \in [1, n], \quad p(s_i) = \prod_{t=0}^{T-1} p(s_i[t]) \quad \text{and}$$

$$\forall (t, t'), \quad p(s_i[t]) = p(s_i[t'])$$

↪ Sources are mutually independent non-Gaussian stochastic processes with zero mean.

$$p(s_i) = \prod_{i=1}^n p(s_i)$$

↪ One source at most is Gaussian

3-Fundamental theorem

Let $\mathbf{x}$ be a vector with independent components [1,8,9,10,29], of which at most one is Gaussian, and whose densities are not reduced to a point-like mass. Let $\mathbf{B}$ be an orthogonal $n \times n$ matrix and $\mathbf{y}$ the vector $\mathbf{y} = \mathbf{B}\mathbf{x}$. Then the following three properties are equivalent:

i) The components y_i are pairwise independent.

ii) The components y_i are mutually independent.

iii) $\mathbf{B} = \mathbf{D}\mathbf{P}$, $\mathbf{D}$ diagonal, $\mathbf{P}$ permutation.

In short,

We will therefore seek the separation matrix $\mathbf{B}$ such that the components of $\mathbf{y}$ are the most independent possible, in the sense of a certain measure of independence between the components of $\mathbf{y}$, and the problem of the BSS becomes then that of the Independent Components Analysis **ICA**. We will assume $\mathbf{n} = \mathbf{0}$ and $\mathbf{m} = \mathbf{n}$

The problem can be summarized in the following figure:

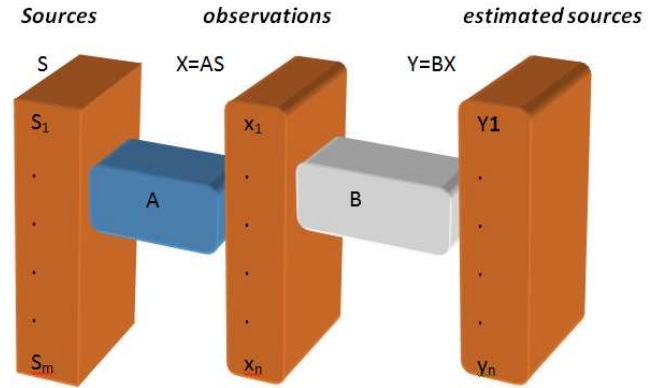


Fig.1. Mixing and separating sources

B-Independent Component Analysis ICA

ICA is a method of Blind Source Separation (BSS) [4]. The assumption underlying ICA is that each row of the data matrix is weighted sum of pure source signals, the weights being proportional to the contribution of the corresponding pure signals to that particular mixture. The original source signals and their proportions in the analyzed mixtures are unknown. In ICA, $\mathbf{x}$ is not seen as a collection of points in a multidimensional space, but rather as a collection of signals with a certain number of common sources. ICA aims to extract these pure sources, underlying the observed signals, as well as their concentration in each mixture. ICA finds the independent component by maximizing the statistical independence of the estimated components. The two broadest definitions of independence for ICA are minimization of mutual information and maximization of non-Gaussianity. Typical algorithms for ICA use the centering and whitening as preprocessing steps in order to simplify and reduce the complexity of the problem. The theory of ICA is in several steps:

1-Contrast function

A contrast function $\mathbf{C}\{\mathbf{v}\}$ is a function of probability density of random sequence $\mathbf{v} \in \mathbb{R}^{n \times T}$

$$\left\{ \begin{array}{l} \forall \mathbf{M} \in \mathbb{R}^{n \times n}, \quad \mathbf{C}\{\mathbf{M}\mathbf{s}\} \geq \mathbf{C}\{\mathbf{s}\}, \\ \mathbf{C}\{\mathbf{M}\mathbf{s}\} = \mathbf{C}\{\mathbf{s}\} \quad \text{if and only if} \quad \mathbf{M} \sim \mathbf{I}_n \end{array} \right. \quad (2)$$

If $\mathbf{B}$ is a matrix minimizes $\mathbf{C}\{\mathbf{B}\mathbf{x}\} = \mathbf{C}\{\mathbf{B}\mathbf{A}\mathbf{s}\}$

So $BA \sim I_n$

And consequently $B = A^{-1}$

Let us denote that $y = Bx$

It is important to remember that $C\{y\}$ is a function of probability density following the assumptions in (A-2)

2- Orthonormal contrast functions

To the extent that the exact independence can not be recovered the mutual decorrelation of the components of y is desirable, so it must explicitly be taken into account in the minimization of the contrast function. We shall therefore speak of minimization of $C\{y\}$ under the whiteness constraint. This could correspond to two options:

Or a prior spatial whitening of the observations has been carried out so that the estimate of B has been reduced to the estimate of an orthonormal separation matrix V such that:

$$C\{y\} = C\{Vz\}$$

Or the minimization of a contrast $C\{y\} = C\{Bx\}$ is carried out under the constraint: $R_{yy} = I_n$

Where R_{yy} is a Correlation matrix

3- Maximum likelihood ML approach

Either q any hypothetical distribution of any sample s [...] of the sources. The log-likelihood is thus written:

$$L(B, q) = \log |\det(B)| + \frac{1}{T} \sum_{t=0}^{T-1} \log q(Bx[t]) \quad (3)$$

Asymptotically if $T \rightarrow +\infty$ the criterion of ML Can be formulated as a KULLBACK DIVERGENCE between the two probability densities functions, $p_v(u)$ and $p_w(u)$, such that:

$$K\{v\|w\} \stackrel{\text{def}}{=} \int p_v(u) \ln \frac{p_v(u)}{p_w(u)} du \quad (4)$$

An important property of the Kullback divergence is that it is zero if and only if v and w have the same distribution. It is thus possible to associate the ML with the following contrast:

$$C_{ML}\{y\} = K\{Bx\|s'\} \quad (5)$$

In this respect, the ml estimator offers a new interpretation: It consists in finding the separation matrix B which minimizes the kullback divergence between the density of $y = Bx$ and the density of the sources.

4- ICA in practice : Approximation of the kulback divergence by cumulant

Let a, b, c and d be four real centered random variables. The fourth-order cumulant of these random variables is defined by:

$$c_{abcd} \stackrel{\text{def}}{=} E\{abcd\} - E\{ab\}E\{cd\} - E\{ac\}E\{bd\} - E\{ad\}E\{bc} \quad (6)$$

If the random variables a, b, c and d can be separated into two sets of mutually independent random variables then the fourth-order cumulant is canceled [1,8,9,10,29].

The fourth-order cumulant of a real centered random variable is given by:

$$k_a \stackrel{\text{def}}{=} c_{aaaa} \quad (7)$$

k_a is called Kurtosis.

By using the Edgeworth expansion the Kulback divergence of two random vectors v and w is given by:

$$K\{v\|w\} \approx \frac{1}{4} \sum_{ij} (r_{v_i v_j} - r_{w_i w_j})^2 + \frac{1}{48} \sum_{ijkl} (c_{v_i v_j v_k v_l} - c_{w_i w_j w_k w_l})^2 \quad (8)$$

Where $v = [v_1, \dots, v_n]^T$ and $w = [w_1, \dots, w_n]^T$

Having this approximation it is possible to approximate the contrast function defined by (5)

So:

$$C_{ML}\{y\} \approx C_{24}\{y\} \quad (9)$$

Minimize $C_{24}\{y\}$ under constrained whiteness allows to write:

$$C_{ML}^{\circ}\{y\} \approx C_4^{\circ}\{y\} \quad (10)$$

With:

$$C_4^{\circ}\{y\} \stackrel{\text{def}}{=} \frac{1}{48} \sum_{ijkl} (c_{y_i y_j y_k y_l} - k_{s'_i} \delta_{ijkl})^2 \quad (11)$$

It can still be shown that:

$$C_{kurt}^{\circ}\{y\} \stackrel{\text{def}}{=} \frac{1}{48} \sum_{ijkl \neq iiii} c_{y_i y_j y_k y_l}^2 \quad (12)$$

And if $E\{yy^T\} = I_n$

We will have

$$C_{kurt}^{\circ}\{y\} = -\frac{1}{48} \sum_{i=1}^n k_{y_i}^2 \quad (13)$$

Since the kurtosis of a Gaussian random variable is zero, maximizing the sum of the kurtosis to the square of y returns once again to exclude as much as possible the densities of the components of y from a Gaussian density. The maximization of the sum of kurtosis squared amounts to canceling the crossed cumulants of y . The **JADE** method that we will now propose proposes to minimize only certain cumulant slices, this leading to an efficient minimization procedure by simultaneous diagonalization.

C-JADE algorithm

The jade algorithm [1,8] designed by Cardoso & Souloumiac consists in minimizing the following contrast function:

$$C_{JADE}^{\circ}\{y\} \stackrel{\text{def}}{=} \sum_{ijk} c_{y_i y_j y_k y_l}^2 \quad (14)$$

And this minimization will be reduced to the simultaneous diagonalization of a matrix set of cumulants.

Definition 1

Let $v = [v_1, \dots, v_n]^T$ be a random vector of $\mathbb{R}^{n \times 1}$ and M a matrix $\in \mathbb{R}^{n \times n}$. We define the matrix of cumulants by:

$$\forall (i, j) \in \llbracket 1, n \rrbracket^2, |Q_v(M)|_{ij} = \sum_{kl} c_{v_i v_j v_k v_l} m_{kl} \quad (15)$$

The matrices of cumulants thus defined will allow us to reduce some slices of the tensor structure of cumulants of order 4 to a simpler matrix structure.

Definition 2

Let M a matrix $\in \mathbb{R}^{n \times n}$, We define the diagonal measure by:

$$\text{Off}(M) = \sum_{i \neq j} m_{ij}^2 \quad (16)$$

Result:

Let N be a base of the matrix set $\in \mathbb{R}^{n \times n}$

Then:

$$C_{JADE}^{\circ}\{VZ\} \stackrel{\text{def}}{=} \sum_{M \in N} \text{off}(V^T Q_v(M) V) \quad (17)$$

It can thus be seen that the minimization of the contrast function $C_{JADE}^{\circ}\{VZ\}$ with respect to the orthonormal matrix V can be achieved by the simultaneous diagonalization of the matrix assembly

$$\{Q_v(M_i), M_i \in N\}$$

Remark

The simultaneous diagonalization algorithm used in JADE is inspired by the JACOBI method for the calculation of the eigenvalues of a symmetric real matrix. The algorithm proposed by Cardoso and Souloumiac makes it possible to carry out an approximate simultaneous diagonalization because in practice the cumulant matrices are not rigorously diagonalisable simultaneously

III. THE CONTINUOUS WAVELETS TRANSFORM: CWT

A –Introduction

This part covers the basic theory of the continuous wavelet transform. We will first determine what constitutes a wavelet, how it is used in the transformation of a signal and what it can tell us about the signal. We will look at the energy-preserving features of the wavelet transform and how it may be used to produce scalogram.

B –The wavelet

The wavelet transform is a method of converting a signal into another form which either makes certain features of the original signal more amenable to study or enables the original data set to be described more succinctly. To perform a wavelet transform we need a wavelet which is localized waveform. In fact, a wavelet is a function $\varphi(t)$ which satisfies certain mathematical criteria. A selection of wavelets commonly used in practice is shown in figure (2).

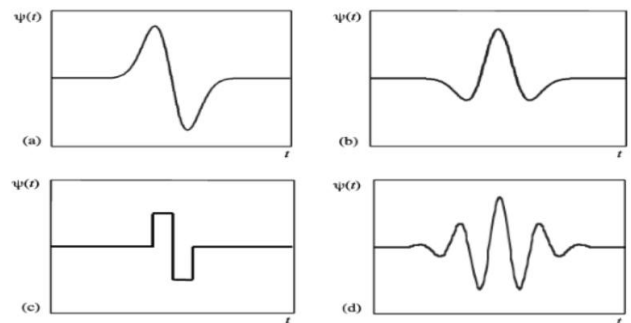


Fig.2. (a) Gaussian wave. (b) Mexican hat. (c) Haar. (d) Morlet [17, 18]

C-Requirements for the wavelet

In order to be classified as a wavelet, a function must satisfy certain mathematical criteria. These are:

1. Wavelet must have finite energy:

$$E = \int_{-\infty}^{+\infty} |\psi(t)|^2 dt < \infty \quad (18)$$

Where E is the energy of a function equal to the integral of its Squared magnitude and the vertical brackets $|\cdot|$ represent the modulus operator which gives the magnitude of $\psi(t)$. If (t) is a complex function the magnitude must be found using both its real and its complex parts.

2. If $\widehat{\psi}(f)$ is the Fourier transform of $\psi(t)$, i.e.

$$\widehat{\psi}(f) = \int_{-\infty}^{+\infty} \psi(t) e^{-i2\pi ft} dt \quad (19)$$

Then the following condition must hold:

$$C_g = \int_0^{+\infty} \frac{|\widehat{\psi}(f)|^2}{f} df \quad (20)$$

This implies that the wavelet has no zero frequency components, $\widehat{\psi}(0) = 0$ or, to put it another way, the wavelet (t) must have a zero mean. Equation (3) is known as the *admissibility condition* and C_g is called the *admissibility constant*. The value of C_g depends on the chosen wavelet and is equal to π for the Mexican hat wavelet.

C-The energy spectrum of the wavelet

Wavelets satisfying the admissibility condition are in fact *bandpass filters*. This means in simple terms that they let through only those signals components within a finite range of frequencies (the pass band) and in proportions characterized by the energy spectrum of the wavelet. A plot of the squared magnitude of the Fourier transform against frequency for the wavelet gives its *energy spectrum*.

D-The wavelet transforms

1-Preliminary

Now we have chosen a mother wavelet, how do we put it to good use in a signal analysis capacity? First we require our wavelet to be more flexible than that defined earlier, i.e. (t) . We can perform two basic manipulations to make or wavelet more flexible: we can stretch and squeeze it (dilation) or we

can move it (translation). Figure III-2(a) shows the Mexican wavelet stretched and squeezed to respectively double and half its original width on the time axis. This dilation and contraction of the wavelet is governed by the dilation parameter a which, for the Mexican hat wavelet, is the distance between the centre of the wavelet and its crossing of the time axis. The movement of the wavelet is governed by the translation parameter b . Figure III-2(b) shows the movement of a wavelet along the time axis from b_1 via b_2 to b_3 . These shifted and dilated versions of the mother wavelet are denoted $\psi(\frac{t-b}{a})$

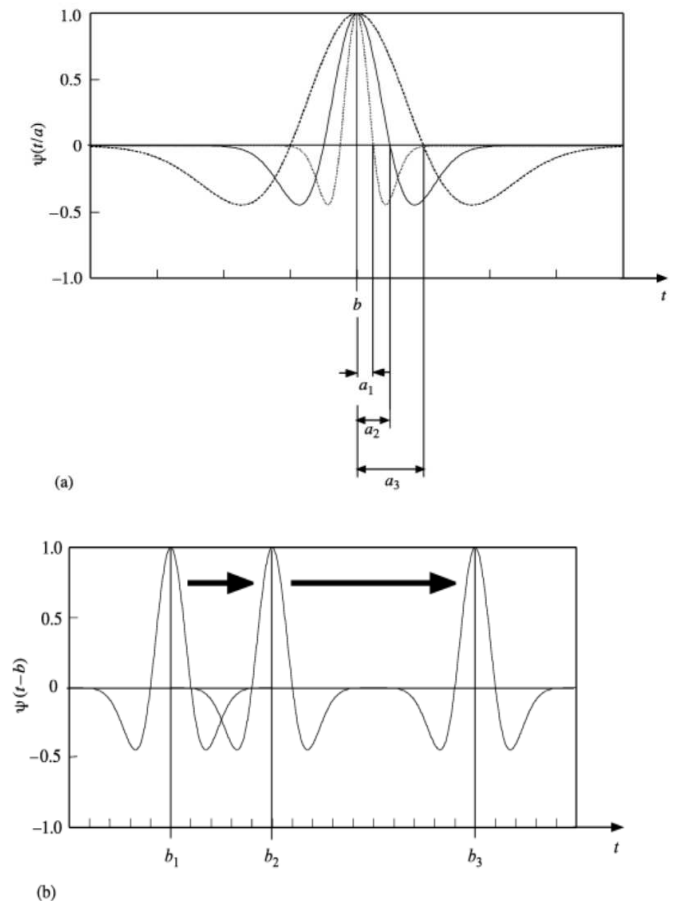


Fig.3 .Dilation (a) and(b) translation of a wavelet [17, 18]

2-Definition

The wavelet transform of a continuous signal with respect to the wavelet function is defined as

$$T(a, b) = w(a) \int_{-\infty}^{+\infty} x(t) \psi^*\left(\frac{t-b}{a}\right) dt \quad (21)$$

Where $w(a)$ is a weighting function. The asterisk indicates that the complex conjugate of the wavelet function is used in the transform. The wavelet transform can be thought of as the cross-correlation of a signal with a set of wavelets of various

'width'. Typically $w(a)$ is set to $\frac{1}{\sqrt{a}}$ for reasons of energy conservation. Thus the wavelet transform is written

$$T(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi^*\left(\frac{t-b}{a}\right) dt \quad (22)$$

This is the continuous wavelet transform CWT. Take a closer Look at this equation. It contains both dilated and translated wavelet $\left(\frac{t-b}{a}\right)$ and the $x(t)$ signal, where $x(t)$ could be a beating heart. In mathematical terms this is called a convolution. The normalized wavelet is often written more compactly as

$$\psi_{a,b} = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \quad (23)$$

We can express the wavelet transform in even more compact form as an inner product:

$$T(a, b) = \langle x, \psi_{a,b} \rangle \quad (24)$$

3-The inverse wavelet transforms

As with its Fourier counterpart, there is an *inverse wavelet transform*, defined as

$$x(t) = \frac{1}{C_g} \int_{-\infty}^{+\infty} \int_0^{+\infty} T(a, b) \psi_{a,b}(t) \frac{da db}{a^2} \quad (25)$$

This allows the original signal to be recovered from its wavelet transform by integrating over all scales and location, a and b . Note that for the inverse transform, the original wavelet function is used, rather than its conjugate which is used in the forward transformation. If we limit the integration over a range of a scale rather than all a scales, we can perform a basic filtering of the original signal.

4-The signal energy: wavelet-based energy

The total energy contained in a signal, $x(t)$, is defined as its integrated squared magnitude

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \|x(t)\|^2 \quad (26)$$

For this equation to be useful the signal must contain finite energy. The relative contribution of the signal energy contained at a specific a scale and b location is given by the two-dimensional wavelet energy density function:

$$E(a, b) = |T(a, b)|^2 \quad (27)$$

A plot of $E(a, b)$ is known as a **scalogram**. In practice, all functions which differ from $|T(a, b)|^2$ by only a constant multiplicative factor are also called scalogram. The scalogram can be integrated across a and b to recover the total energy in

the signal using the admissibility constant, C_g , as follows:

$$E = \frac{1}{C_g} \int_{-\infty}^{+\infty} \int_0^{+\infty} |T(a, b)|^2 \psi_{a,b}(t) \frac{da db}{a^2} \quad (28)$$

The relative contribution to the total energy contained within the signal at a specific a scale is given by the scale dependent energy distribution:

$$E = \frac{1}{C_g} \int_{-\infty}^{+\infty} |T(a, b)|^2 db \quad (29)$$

Peaks in $E(a)$ highlight dominant energetic scales within the signal.

5-The wavelet transform in terms of the Fourier transform

As we saw in equation (7), the wavelet transform is the convolution of the signal with wavelet function. Hence we can employ the convolution theorem to express the wavelet transform in terms of products of the Fourier transforms of the signal, $\widehat{x(f)}$, and wavelet, $\widehat{\psi_{a,b}(f)}$, as follows:

$$T(a, b) = \int_{-\infty}^{+\infty} \widehat{x(f)} \widehat{\psi_{a,b}(f)}^* df \quad (30)$$

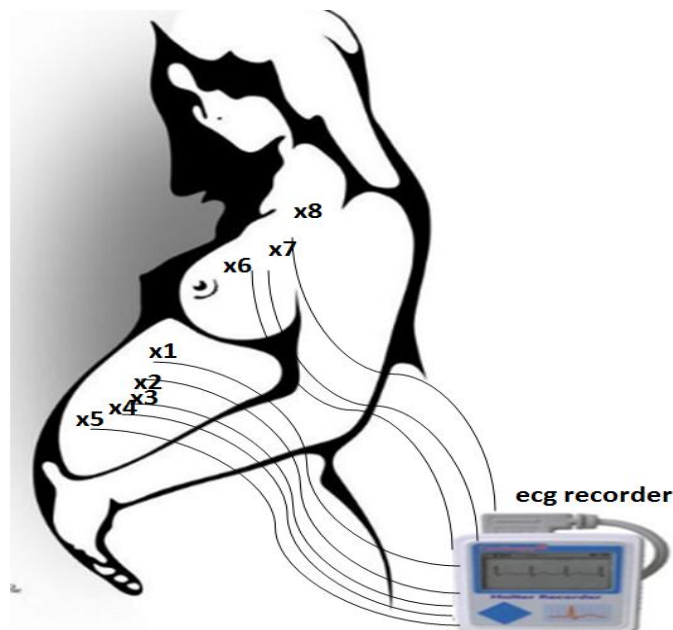
Where we note that the conjugate of the wavelet transform is used. The Fourier transform of the dilated and translated wavelet is

$$\widehat{\psi_{a,b}(f)} = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \psi\left(\frac{t-b}{a}\right) e^{-2\pi i f t} dt \quad (31)$$

The equivalence between the time convolution and Fourier integrals for determining $T(a, b)$ is depicted in figure III-3. This is a particularly useful result when using discretized approximations of the continuous wavelet transform in practice with large large signal data sets, as the fast Fourier transform (FFT) algorithm may be employed to facilitate rapid calculation of the wavelet transform and its inverse. In addition, the Fourier transform of the wavelet function, $\widehat{\psi_{a,b}(f)}$, is usually known in analytic form and hence need not be computed using an FFT.

IV. METHODS AND RESULTS

A-ECG recordings



The data is extracted from the DaISy database (Database for the Identification of Systems). And corresponding to ECG records collected simultaneously by eight electrodes Figure IV-2. The sampling frequency is 250 Hz. Duration $T=10s$. We used the MATLAB 2015a on Windows 7.



Fig.6. 8 Channel set of cutaneous data recordings.

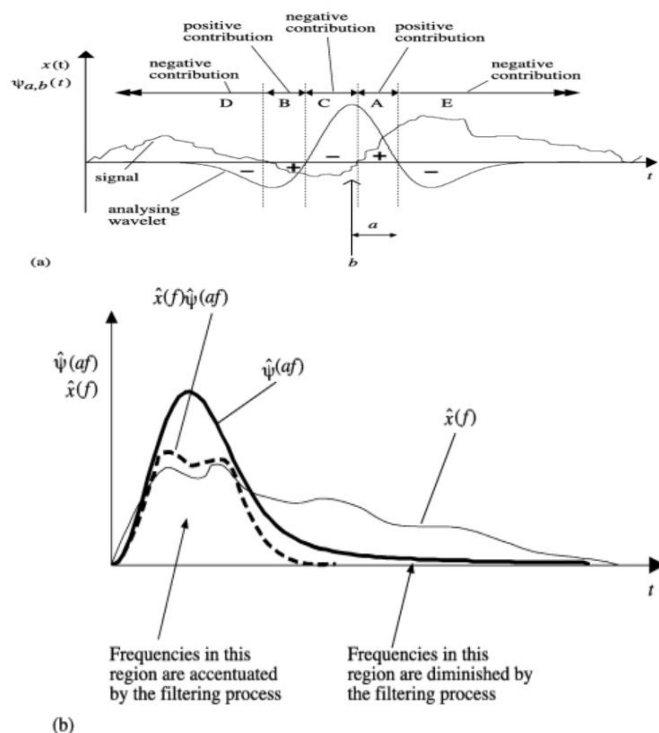


Fig.4. (a and b) Schematic representation of the wavelet transform in its time and frequency representations[17, 18].

6-Heisenberg boxes

This mechanism represents the richness of time-scale analysis. It allows a small scale to carry out a precise temporal examination of the signal corresponding to the high frequencies. On the contrary, the low frequencies are examined with a very high frequency resolution:

$$\Delta t \Delta f \geq 1/4\pi \quad (\text{III-15})$$

This is illustrated in figure III-4:

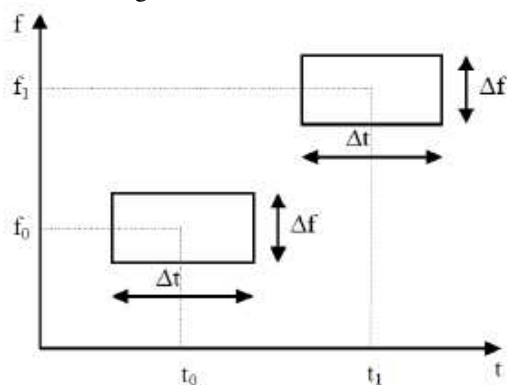


Fig.5. Heisenberg boxes in the time –frequency plane

B-Method 1-Algorithm

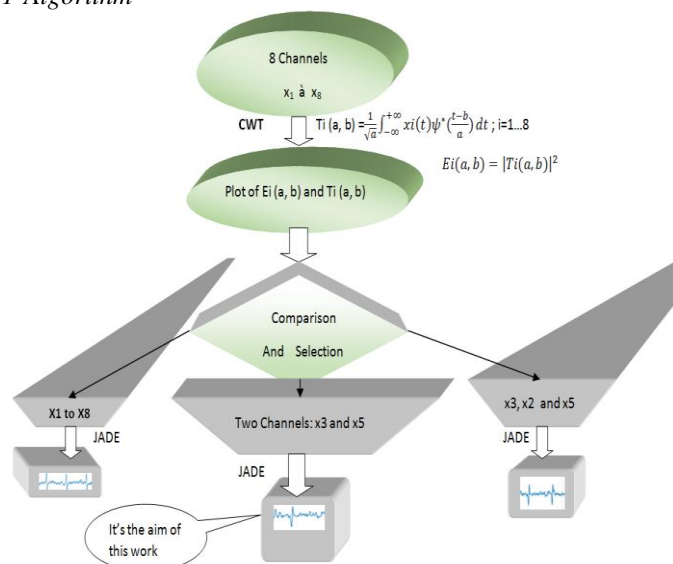
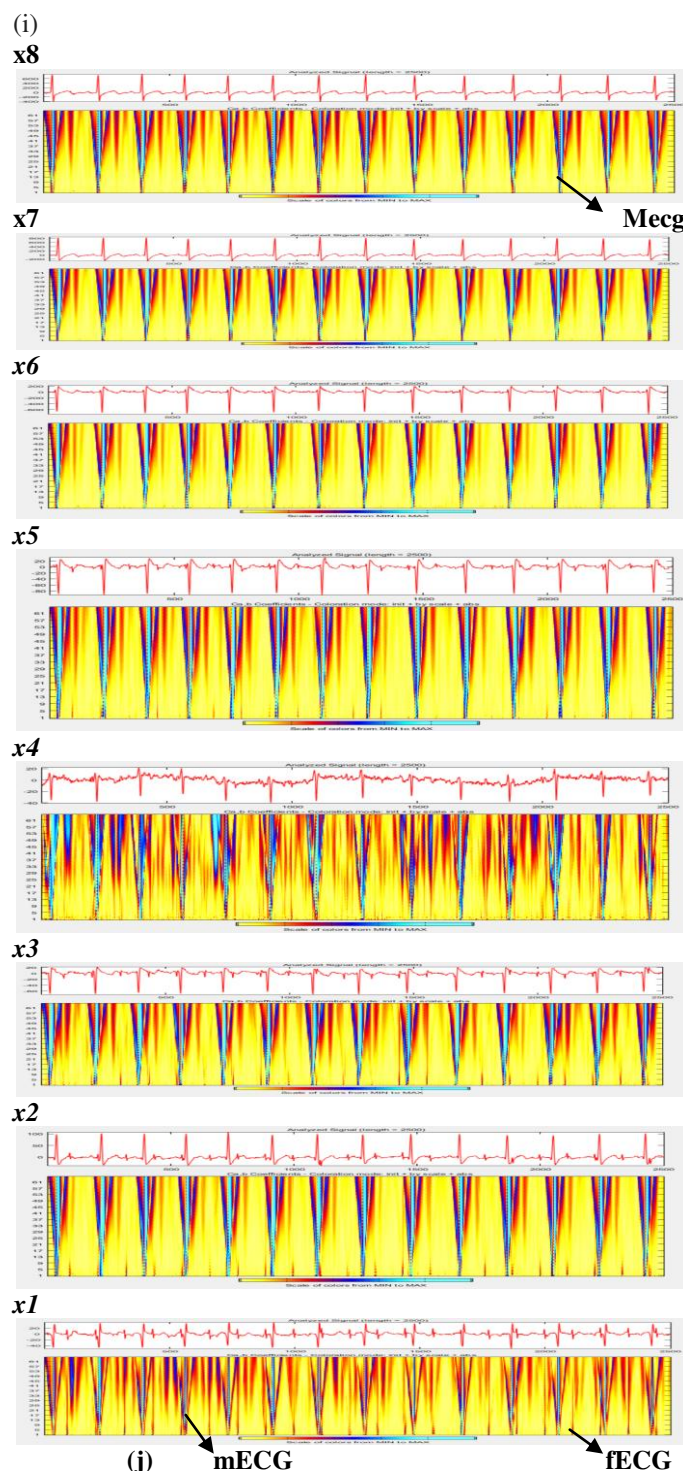
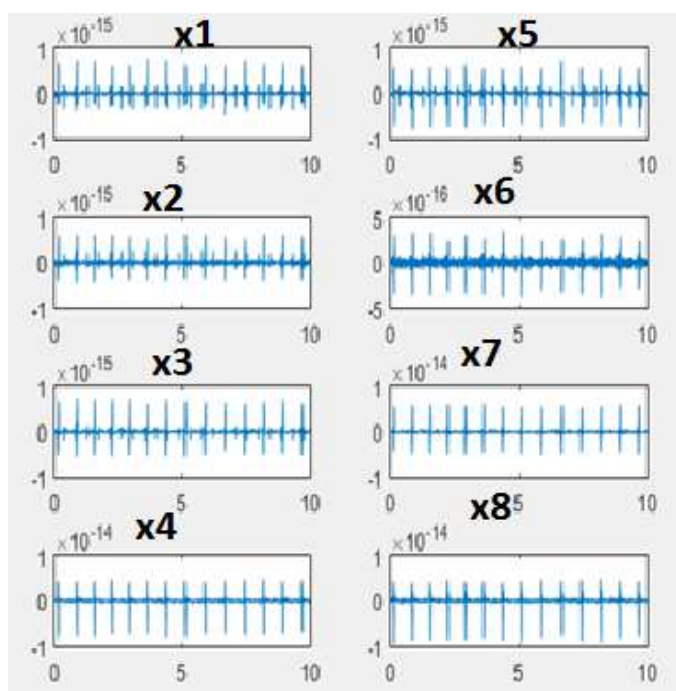


Fig.8. Algorithm

2-Continuous Wavelet Transform CWT

In order to make a good selection of the useful signals. Our approach consists to compute at a first time the continuous wavelet transform CWT of x_1 to x_8 signals and in second time to plot $T_i(a, b)$ as a function of time and $E_i(a, b)$ in the time-scale space. Finally we will calculate the mean energy for a low scale a , in order to be able to make the comparison and the selection of the useful signals for JADE algorithm **Fig (4)**. Simulations are made by the Morley wavelet. The results found are:



(j) mECG fECG

(k)

Signal	X1	X2	X3	X4	X5
energy	$E1=1.13 \cdot 10^{-32}$	$E2=2.48 \cdot 10^{-33}$	$E3=5.51 \cdot 10^{-33}$	$E4=2.99 \cdot 10^{-33}$	$E5=8.87 \cdot 10^{-33}$

Fig.9.

- (i) Representation of the temporal evolution of $T_i(a, b)$
 (j) Plotting of $E_i(a, b)$ in the time-scale space (Scalogram)
 (k) Mean energy for the scale $a=0.1$

C- Results:**1- Results 1:**

We see that the average energy of x_1 is large, but unfortunately it is richer in information on the cardiac activity of the mother than that of the fetus so it is not a good candidate for JADE. Because it mustn't be forgotten that the real problem is the fetal electrocardiogram extraction fECG.

2- Results 2:

The electrical activity of the fetus is absolutely absent in the signals x_6 to x_8 : Are useless for JADE.

3- Results 3:

The energy classification of the four remaining signals

$$E(x_5) \geq E(x_3) \geq E(x_2) \geq E(x_4)$$

The first finding is that it is possible to reduce the number of channels from 8 to 4.

4- Results 4:

The two channels 2 and 4 are a comparable energy, therefore only one of the two is considered in the simulation.

5- Results 5:

There are three possibilities (cases):

- a-Four channels: x_2 to x_5
- b-Three channels: x_3 , x_2 and x_5 ;
- c-Two channels x_3 and x_5 ;

6- Results 6:

The application of the JADE function under Matlab in the three cases gives the following results:

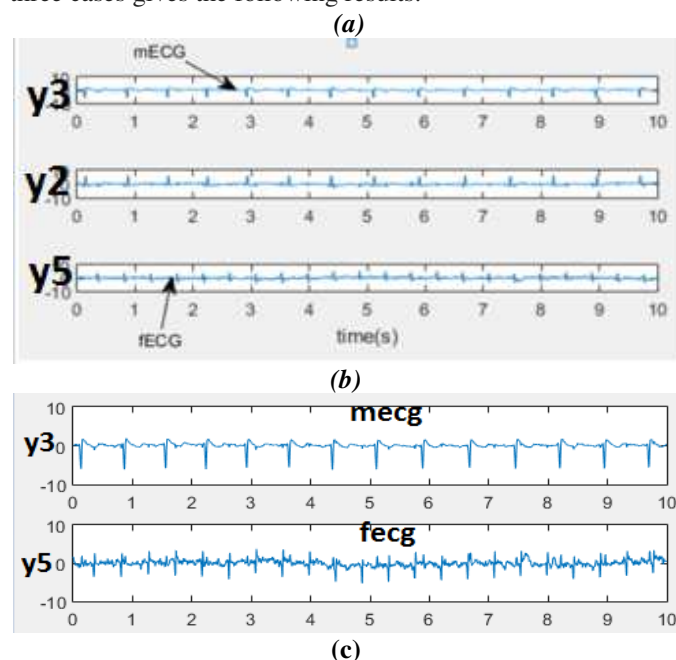


Fig.10.Sources estimated via JADE Algorithm:

(a) Tree channels: x_3 , x_2 and x_5

(b) Two channels: x_3 and x_5

(c) Four channels x_2 to x_5

V. DISCUSSION AND CONCLUSION

A-Discussion

The blind source separation was bieeb successful in the three cases. More, the case of the **two channels** gave a surprising result because the fECG signal is extracted with well distinguished complex QRS.

In order to measure the quality of separation we are computing the fourth-order cumulant **table1** defined by:

$$CUM = C_{y_i y_j y_k y_l}$$

Cases	Two channels	Three channels	Four channels
CUM	0.0019	0.0025	0.0063

Table1: Values of fourth order cumulant

** These values indicate a very great independence between the extracted components and therefore a high quality of separation.

B- Conclusion:

Indeed the use of JADE as a blind source separation algorithm is not a new approach, but what is new in this article is the fact of combining the JADE algorithm and the continuous wavelets transform CWT in order to reduce the number of channels from eight to two .More, this combination led us to three very important and originals results:

- 1- JADE is not limited and doesn't require multiple recordings as reported in several articles [12].
- 2- The energy criterion is more efficient than the statistical criterion of independence.
- 3- A careful modification of JADE in the formation of cumulant matrices can lead to important results and even to solve the case of a single channel.

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