SIXTEENTH EDITION

Test of Mathematics at the 10 + 2 Level

INDIAN STATISTICAL INSTITUTE

EWP EAST-WEST PRESS

Sixteenth Edition

TEST OF MATHEMATICS AT THE 10+2 LEVEL

Indian Statistical Institute

The Indian Statistical Institute is a pioneer in objective type testing in India, and, for almost seven decades, has used this medium for selecting candidates for admission to its courses at various levels. The B.Stat. (Hons.) course, for which the admission requirement is successful completion of Higher Secondary (10+2) examinations, attracts thousands of applicants, from whom a few are selected on the basis of written and oral tests on Mathematics. In response to growing demand from students and teachers for access to these tests, the Institute published for the first time, early in 1992, a collection of questions of the B.Stat. (Hons.) admission tests of the preceding sixteen years. In succeeding editions, tests given in the later years were added and the material reorganised. In this sixteenth edition, question papers of the B.Stat. (Hons.) and B.Math. (Hons.) admission tests of 2007 to 2011, and those of the common B.Stat.(Hons.) and B.Math.(Hons.) admission tests of 2012 to 2016 are included.

The questions in this collection cover a wide range of difficulty levels — from the very simple to challenging levels. They contain both Multiple-Choice and Short-Answer types of questions. They cover the range of topics found in the curricula of most Boards. The topics covered are:

* Arithmetic
* Set Theory
* Interpretation of Tables and Charts
* Logical Reasoning
* Algebra (Number Theory, Permutations, Combinations and the Binomial Theorem, Logarithms, Simultaneous Linear Equations, Inequalities, Solution of Equations, Series, Complex Numbers)
* Geometry (Euclidean Geometry, Coordinate Geometry — lines, circles, conic sections, polar coordinates, — Locus of a Point, Three-Dimensional Geometry)
* Trigonometry (Identities, Solution of Trigonometric Equations, Heights and Distances, Sine & Cosine Formulæ for Triangles)
* Calculus (Sequences, Functions, Limits, Continuity, Derivative, Maxima & Minima, Definite Integrals, Primitives, Methods of Integration, Evaluation of Areas and Volumes using Integration)
* Mensuration

These questions can be used by Class XI and XII students to test their comprehension and skills in Mathematics and as a help in the process of learning Mathematics. It is expected that they will be better prepared to face the innumerable competitive and other examinations they have to take during and after school/junior college.

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Sixteenth Edition

INDIAN STATISTICAL INSTITUTE

AFFILIATED EAST-WEST PRESS PVT LTD
NEW DELHI

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FOREWORD

The Indian Statistical Institute, founded by Professor Prasanta Chandra Mahalanobis, grew out of the Statistical Laboratory, set up by him in the Presidency College in Calcutta. In 1932 the Institute was registered as a learned society for the advancement of Statistics in India. The Institute has been offering formal courses in Statistics leading to certificates and diplomas since the late thirties. Post-M.Sc. advanced courses in Statistics were started in the late forties. In 1959, in recognition of the role of Statistics as a key technology of the modern times and the importance of the Institute in the development and application of Statistics, the Parliament of India enacted the Indian Statistical Institute Act, declaring the Institute to be an Institution of National Importance and empowering it to award degrees and diplomas in Statistics. The B.Stat.(Hons.) and the M.Stat. degree programmes in Statistics, and research programmes leading to the Ph.D. degree were introduced in the Institute following the enactment of this Act. Subsequently M.Tech. degree courses in Computer Science, and Quality, Reliability and Operations Research were introduced. The Institute also conducts a variety of courses for Diplomas and Certificates.

Admissions to all these courses are based on academic records, performance in a test and, for most of the courses, also on an interview. The admission test forms an important component of the selection procedure. The courses of the Institute, especially the B.Stat.(Hons.) course, attract a large number of applicants from all over the country and admission tests are held every year in about 20 centres across the country, involving thousands of applicants. A majority of these tests are of the Multiple-Choice type, which makes it possible for the Institute to complete the task of selection of candidates in a short time. Short-Answer type of questions are also used, especially for courses, where the number of applicants is few, and conventional "essay type" of tests are sometimes used. The Institute is a pioneer in India in Multiple-Choice type of testing and has been engaged in the development of such tests and their use, and in research in Psychometry since the fifties.

Professor P.C.Mahalanobis, who had a major role in the Institute's pioneering work in objective testing and psychometric research in India, had desired that a question bank consisting of a large number of questions on each topic for each major examination be organised, from which questions for a given test could be drawn according to the requirements of difficulty level and coverage. He believed that such a bank would also help in standardising tests and scores thereof. Although this booklet is neither intended to be such an item bank nor large enough to be one such, it fulfils the late Professor Mahalanobis' desire to a certain extent. It is a happy coincidence that this booklet is published on the eve of his birth centenary, which the Institute will start celebrating from December 1992.

Over the years, there has been a demand from candidates, students and teachers for access to these tests, since, generally, the test booklets are taken back from the candidates at the end of the tests. In response to this growing demand, the Institute has now decided to publish a collection of questions from the past tests. This booklet contains the questions given at the B.Stat.(Hons.) admission tests in the last sixteen years. Besides being useful for candidates aspiring to join the B.Stat.(Hons.) course, we believe that they can be used by Class XI and XII students to test their skills in Mathematics. I do hope that this booklet will contribute to the process of learning Mathematics at this level.

These questions have been collated, checked, edited and prepared in camera-ready form by a team of the Institute's staff consisting of

- **A.K.Adhikari**
- **T.Krishnan**
- **J.Mathew**
- **D.Roy**
- **K.K.Roy**
- **S.M.Srivastava**

I would like to record my appreciation to them, on behalf of the Institute.

Calcutta
December 1991

J.K.Ghosh
Director
Indian Statistical Institute

PREFACE TO THE FIRST EDITION

The questions in this booklet cover a wide range of difficulty levels— from the very simple to challenging levels. They cover the range of topics found in the curricula of most Boards. The topics covered are: Arithmetic, Set Theory, Interpretation of Tables and Charts, Logical Reasoning, Algebra (Number Theory, Permutations, Combinations and the Binomial Theorem, Logarithms, Linear Equations, Matrices, Solution of Equations, Series, Inequalities, Complex Numbers), Geometry (Euclidean Geometry, Coordinate Geometry— lines, circles, conic sections, polar coordinates— Locus of a Point, Three-Dimensional Geometry), Trigonometry (Identities, Solution of Trigonometric Equations, Heights and Distances, Sine & Cosine Formulæ for Triangles), Calculus (Sequences, Functions, Limits, Continuity, Derivative, Maxima & Minima, Definite Integrals, Primitives, Methods of Integration, Evaluation of Areas and Volumes using Integration) and Mensuration. The questions are arranged topic-wise (approximately) and within a topic arranged in order of difficulty from easy to hard, according to our judgement of difficulty. Of course, a particular candidate may find a 'hard' question easy and may struggle over what we thought was an easy question! Also, unique classification of a question under a topic may not always be feasible. Many questions involve ideas and terms from more than one topic; for instance, a question on maxima using Calculus techniques, may relate to trigonometric functions. Such questions have been classified under the topic in which the central idea and the method of solution rest. Further, it is quite possible that a student solves a question posed under one topic, using the techniques of another; for instance, one may solve a problem posed in the language of geometry using calculus. Thus both the ways of arrangement— topic-wise and difficulty-wise—are subject to these limitations. The first section presents the multiple-choice questions. The next section provides the correct alternatives (key) to the multiple-choice questions. Candidates are advised not to use this key until after they have made all attempts to work out the correct alternative to a question. The last section presents the Short-Answer type of questions.

A great deal of care has been taken to eliminate mistakes in the questions and in the alternatives provided. However, a few mistakes may still remain. We hope that we shall discover them before the next edition.

We would like to express our appreciation to Shri Ranjan Bhattacharyya and Shri Subhasish Kumar Pal for help in transcription of the material for computer processing and to Shri R.N.Kar for help with his expertise in LaTeX.

Editors

Calcutta
December 1991

PREFACE TO THE SIXTEENTH EDITION

The multiple-choice and short-answer type tests for B.Stat. (Hons.) and B.Math. (Hons.) admission in 2007-16 have been included in the sixteenth edition.

The Editorial Committee consists of the following members:

- Pradipta Bandyopadhyay
- Rana Barua
- Mausumi Bose
- Goutam Mukherjee
- Sumitra Purkayastha
- Tapas Samanta
- Debasis Sengupta.

Kolkata
February 2017

Editors

Multiple-Choice Questions

1. A worker suffers a 20% cut in wages. He regains his original pay by obtaining a rise of

(A) 20%; (B) $22\frac{1}{2}$%; (C) 25%; (D) $27\frac{1}{2}$%.

2. If m men can do a job in d days, then the number of days in which $m+r$ men can do the job is

(A) $d+r$; (B) $\frac{d}{m}(m+r)$; (C) $\frac{d}{m+r}$; (D) $\frac{md}{m+r}$.

3. A boy walks from his home to school at 6 km per hour (kmph). He walks back at 2 kmph. His average speed, in kmph, is

(A) 3; (B) 4; (C) 5; (D) $\sqrt{12}$.

4. A car travels from P to Q at 30 kilometres per hour (kmph) and returns from Q to P at 40 kmph by the same route. Its average speed, in kmph, is nearest to

(A) 33; (B) 34; (C) 35; (D) 36.

5. A man invests Rs. 10,000 for a year. Of this Rs. 4,000 is invested at the interest rate of 5% per year, Rs. 3,500 at 4% per year and the rest at α% per year. His total interest for the year is Rs. 500. Then α equals

(A) 6.2; (B) 6.3; (C) 6.4; (D) 6.5.

6. Let $x_1, x_2, \ldots, x_{100}$ be positive integers such that $x_i + x_{i+1} = k$ for all i, where k is a constant. If $x_{10} = 1$, then the value of x_1 is

(A) k; (B) $k-1$; (C) $k+1$; (D) 1.

7. If $a_0 = 1$, $a_1 = 1$ and $a_n = a_{n-1}a_{n-2} + 1$ for $n > 1$, then

(A) a_{465} is odd and a_{466} is even; (B) a_{465} is odd and a_{466} is odd;
(C) a_{465} is even and a_{466} is even; (D) a_{465} is even and a_{466} is odd.

8. Two trains of equal length L, travelling at speeds V_1 and V_2 miles per hour in opposite directions, take T seconds to cross each other. Then L in feet (1 mile = 5280 feet) is

(A) $\frac{11T}{15(V_1+V_2)}$; (B) $\frac{15T}{11(V_1+V_2)}$; (C) $\frac{11(V_1+V_2)T}{15}$; (D) $\frac{11(V_1+V_2)}{15T}$.

9. A salesman sold two pipes at Rs. 12 each. His profit on one was 20% and the loss on the other was 20%. Then on the whole, he

(A) lost Re. 1; (B) gained Re. 1;
(C) neither gained nor lost; (D) lost Rs. 2.

10. The value of $(256)^{0.16}(16)^{0.18}$ is

(A) 4; (B) 16; (C) 64; (D) 256.25.

11. The digit in the unit position of the integer

$$1! + 2! + 3! + \ldots + 99!$$

is

(A) 3; (B) 0; (C) 1; (D) 7.

12. July 3, 1977, was a SUNDAY. Then July 3, 1970, was a

(A) Wednesday; (B) Friday; (C) Sunday; (D) Tuesday.

13. June 10, 1979, was a SUNDAY. Then May 10, 1972, was a

(A) Wednesday; (B) Thursday; (C) Tuesday; (D) Friday.

14. A man started from home at 14:30 hours and drove to a village, arriving there when the village clock indicated 15:15 hours. After staying for 25 minutes (min), he drove back by a different route of length (5/4) times the first route at a rate twice as fast, reaching home at 16:00 hours. As compared to the clock at home, the village clock is

(A) 10 min slow; (B) 5 min slow; (C) 5 min fast; (D) 20 min fast.

15. If $\frac{a+b}{b+c} = \frac{c+d}{d+a}$, then

(A) $a = c$; (B) either $a = c$ or $a + b + c + d = 0$;
(C) $a + b + c + d = 0$; (D) $a = c$ and $b = d$.

16. The expression

$$(1+q)(1+q^2)(1+q^4)(1+q^8)(1+q^{16})(1+q^{32})(1+q^{64}),$$

where $q \neq 1$, equals

(A) $\frac{1-q^{128}}{1-q}$; (B) $\frac{1-q^{64}}{1-q}$; (C) $\frac{1-q^{2^{1+2+\ldots+6}}}{1-q}$;
(D) none of the foregoing expressions.

17. In an election 10% of the voters on the voters' list did not cast their votes and 60 voters cast their ballot papers blank. There were only two candidates. The winner was supported by 47% of all voters in the list and he got 308 votes more than his rival. The number of voters on the list was

(A) 3600; (B) 6200; (C) 4575; (D) 6028.

18. A student took five papers in an examination, where the full marks were the same for each paper. His marks in these papers were in the proportion of 6:7:8:9:10. He obtained (3/5) part of the total full marks. Then the number of papers in which he got more than 50% marks is

(A) 2; (B) 3; (C) 4; (D) 5.

19. Two contestants run a 3-kilometre race along a circular course of length 300 metres. If their speeds are in the ratio of 4:3, how often and where would the winner pass the other? (The initial start-off is not counted as passing.)

(A) 4 times; at the starting point.
(B) Twice; at the starting point.
(C) Twice; at a distance of 225 metres from the starting point.
(D) Twice; once at 75 metres and again at 225 metres from the starting point.

20. If a, b, c and d satisfy the equations

$$\begin{aligned} a+7b+3c+5d &= 0, \\ 8a+4b+6c+2d &= -16, \\ 2a+6b+4c+8d &= 16, \\ 5a+3b+7c+d &= -16, \end{aligned}$$

then $(a+d)(b+c)$ equals

(A) 16; (B) -16; (C) 0; ; (D) none of the foregoing numbers.

21. Suppose x and y are positive integers, $x > y$, and $3x+2y$ and $2x+3y$ when divided by 5, leave remainders 2 and 3 respectively. It follows that when $x-y$ is divided by 5, the remainder necessarily equals

(A) 2; (B) 1; (C) 4; (D) none of the foregoing numbers.

22. The number of different solutions (x, y, z) of the equation $x+y+z=10$, where each of x, y and z is a positive integer, is

(A) 36; (B) 121; (C) 10^3-10; (D) $\binom{10}{3}-\binom{10}{2}$.

23. The hands of a clock are observed continuously from 12:45 p.m. onwards. They will be observed to point in the same direction some time between

(A) 1:03 p.m. and 1:04 p.m.; (B) 1:04 p.m. and 1:05 p.m.;
(C) 1:05 p.m. and 1:06 p.m.; (D) 1:06 p.m. and 1:07 p.m.

24. A, B and C are three commodities. A packet containing 5 pieces of A, 3 of B and 7 of C costs Rs. 24.50. A packet containing 2, 1 and 3 of A, B and C respectively, costs Rs. 17.00. The cost of a packet containing 16, 9 and 23 items of A, B and C respectively

(A) is Rs. 55.00; (B) is Rs. 75.50; (C) is Rs. 100.00;
(D) cannot be determined from the given information.

25. Four statements are given below regarding elements and subsets of the set $\{1, 2, \{1, 2, 3\}\}$. Only one of them is correct. Which one is it?

(A) $\{1, 2\} \in \{1, 2, \{1, 2, 3\}\}$. (B) $\{1, 2\} \subseteq \{1, 2, \{1, 2, 3\}\}$.
(C) $\{1, 2, 3\} \subseteq \{1, 2, \{1, 2, 3\}\}$. (D) $3 \in \{1, 2, \{1, 2, 3\}\}$.

26. A collection of non-empty subsets of the set $\{1, 2, \ldots, n\}$ is called a *simplex* if, whenever a subset S is included in the collection, any non-empty subset T of S is also included in the collection. Only one of the following collections of subsets of $\{1, 2, \ldots, n\}$ is a simplex. Which one is it?

(A) The collection of all subsets S with the property that 1 belongs to S.
(B) The collection of all subsets having exactly 4 elements.
(C) The collection of all non-empty subsets which do not contain any even number.
(D) The collection of all non-empty subsets except for the subset $\{1\}$.

27. S is the set whose elements are zero and all even integers, positive and negative. Consider the five operations: [1] addition; [2] subtraction; [3] multiplication; [4] division; and [5] finding the arithmetic mean. Which of these operations when applied to any pair of elements of S, yield only elements of S?

(A) [1], [2], [3], [4]. (B) [1], [2], [3], [5].
(C) [1], [3], [5]. (D) [1], [2], [3].

. . . .

Directions for Items 28 to 36:

For sets P, Q of numbers, define
$P \cup Q$: *the set of all numbers which belong to at least one of P and Q;*
$P \cap Q$: *the set of all numbers which belong to both P and Q;*
$P - Q$: *the set of all numbers which belong to P but not to Q;*
$P \bigtriangleup Q = (P - Q) \cup (Q - P)$: *the set of all numbers which belong to set P alone or set Q alone, but not to both at the same time. For example, if $P = \{1, 2, 3\}$, $Q = \{2, 3, 4\}$, then $P \cup Q = \{1, 2, 3, 4\}$, $P \cap Q = \{2, 3\}$, $P - Q = \{1\}$, $P \bigtriangleup Q = \{1, 4\}$.*

28. If $X = \{1,2,3,4\}, Y = \{2,3,5,7\}, Z = \{3,6,8,9\}, W = \{2,4,8,10\}$, then $(X \triangle Y) \triangle (Z \triangle W)$ is

(A) $\{4,8\}$; (B) $\{1,5,6,10\}$; (C) $\{1,2,3,5,6,7,9,10\}$;
(D) none of the foregoing sets.

29. If X, Y, Z are any three sets of numbers, then the set of all numbers which belong to exactly two of the sets X, Y, Z is

(A) $(X \cap Y) \cup (Y \cap Z) \cup (Z \cap X)$;
(B) $[(X \cup Y) \cup Z] - [(X \triangle Y) \triangle Z]$;
(C) $(X \triangle Y) \cup (Y \triangle Z) \cup (Z \triangle X)$;
(D) not necessarily any of (A) to (C).

30. For any three sets P, Q and R, s is an element of $(P \triangle Q) \triangle R$ if s is in

(A) exactly one of P, Q and R;
(B) at least one of P,Q and R, but not in all three of them at the same time;
(C) exactly two of P,Q and R;
(D) exactly one of P, Q and R or in all the three of them.

31. Let $X = \{1,2,3,\ldots,10\}$ and $P = \{1,2,3,4,5\}$. The number of subsets Q of X such that $P \triangle Q = \{3\}$ is

(A) $2^4 - 1$; (B) 2^4; (C) 2^5; (D) 1.

32. For each positive integer n, consider the set $P_n = \{1,2,3,\ldots,n\}$. Let $Q_1 = P_1$, $Q_2 = P_2 \triangle Q_1 = \{2\}$, and, in general, $Q_{n+1} = P_{n+1} \triangle Q_n$ for $n \geq 1$. Then the number of elements in Q_{2k} is

(A) 1; (B) $2k-2$; (C) $2k-3$; (D) k.

33. For any two sets S and T, $S\Delta T$ is defined as the set of all elements that belong to either S or T but not both, that is, $S\Delta T = (S \cup T) - (S \cap T)$. Let A, B and C be sets such that $A \cap B \cap C = \phi$, and the number of elements in each of $A\Delta B$, $B\Delta C$ and $C\Delta A$ equals 100. Then the number of elements in $A \cup B \cup C$ equals
(A) 150; (B) 300; (C) 230; (D) 210.

34. Let A, B, C and D be finite sets such that $|A| < |C|$ and $|B| = |D|$, where $|A|$ stands for the number of elements in the set A. Then

(A) $|A \cup B| < |C \cup D|$;
(B) $|A \cup B| \leq |C \cup D|$ but $|A \cup B| < |C \cup D|$ need not always be true;
(C) $|A \cup B| < 2|C \cup D|$ but $|A \cup B| \leq |C \cup D|$ need not always be true;
(D) none of the foregoing statements is true.

35. For subsets A and B of a set X, define the set $A * B$ as

$$A * B = (A \cap B) \cup ((X - A) \cap (X - B)).$$

Then only one of the following statements is true. Which one is it?

(A) $A * (X - B) \subset A * B$ and $A * (X - B) \neq A * B$.
(B) $A * B = A * (X - B)$.
(C) $A * B \subset A * (X - B)$ and $A * B \neq A * (X - B)$.
(D) $X - (A * B) = A * (X - B)$.

36. Suppose that A, B and C are sets satisfying $(A - B)\Delta(B - C) = A\Delta B$. Which of the following statements must be true?

(A) $A = C$; (B) $A \cap B = B \cap C$; (C) $A \cup B = B \cup C$;
(D) none of the foregoing statements necessarily follows.

Directions for Items 37 to 39:

A word *is a finite string of the two symbols α and β. (An empty string, that is, a string containing no symbols at all, is also considered a word.) Any collection of words is called a* language. *If P and Q are words, then by $P \cdot Q$ is meant the word formed by first writing the string of symbols in P and then following it by that of Q. For example, $P = \alpha\beta\alpha\alpha$ and $Q = \beta\beta$ are words and $P \cdot Q = \alpha\beta\alpha\alpha\beta\beta$. For two languages L_1 and L_2, $L_1 \cdot L_2$ denotes the language consisting of all words of the form $P \cdot Q$ with the word P coming from L_1 and Q from L_2. We also use abbreviations like α^3 for the word $\alpha\alpha\alpha$, $\alpha\beta^3\alpha^2$ for $\alpha\beta\beta\beta\alpha\alpha$, $(\alpha^2\beta\alpha)^2$ for $\alpha^2\beta\alpha\alpha^2\beta\alpha (= \alpha^2\beta\alpha^3\beta\alpha)$ and α^0 or β^0 for the empty word.*

37. If $L_1 = \{\alpha^n : n = 0, 1, 2, \ldots\ldots\}$ and $L_2 = \{\beta^n : n = 0, 1, 2, \cdots\cdots\}$, then $L_1 \cdot L_2$ is

(A) $L_1 \bigcup L_2$;
(B) the language consisting of all words;
(C) $\{\alpha^n\beta^m : n = 0, 1, 2, \cdots\cdots\ m = 0, 1, 2, \cdots\cdots\}$;
(D) $\{\alpha^n\beta^n : n = 0, 1, 2, \cdots\cdots\}$.

38. Suppose L is a language which contains the empty word and has the property that whenever P is in L, the word $\alpha \cdot P \cdot \beta$ is also in L. The smallest such L is

(A) $\{\alpha^n\beta^m : n = 0, 1, 2, \cdots\cdots\ m = 0, 1, 2, \cdots\cdots\}$;
(B) $\{\alpha^n\beta^n : n = 0, 1, 2, \cdots\cdots\}$;
(C) $\{(\alpha\beta)^n : n = 0, 1, 2, \cdots\cdots\}$;
(D) the language consisting of all possible words.

39. Suppose L is a language which contains the empty word, the word α and the word β, and has the property that whenever P and Q are in L, the word $P \cdot Q$ is also in L. The smallest such L is

(A) the language consisting of all possible words;
(B) $\{\alpha^n \beta^n : n = 0, 1, 2, \cdots\cdots\}$;
(C) the language containing precisely the words of the form

$$\alpha^{n_1}\beta^{n_1}\alpha^{n_2}\beta^{n_2}\cdots\alpha^{n_k}\beta^{n_k},$$

where k is any positive integer and $n_1, n_2, \ldots, n_k$ are nonnegative integers;
(D) none of the foregoing languages.

. . . .

40. A relation denoted by $\leftarrow$ is defined as follows: For real numbers x, y, z and w, say that "$(x,y) \leftarrow (z,w)$" if *either* (i) $x < z$ *or* (ii) $x = z$ and $y > w$. If $(x,y) \leftarrow (z,w)$ and $(z,w) \leftarrow (r,s)$ then which one of the following is always true?

(A) $(y,x) \leftarrow (r,s)$;
(B) $(y,x) \leftarrow (s,r)$;
(C) $(x,y) \leftarrow (s,r)$;
(D) $(x,y) \leftarrow (r,s)$.

41. A subset W of the set of all real numbers is called a *ring* if the following two conditions are satisfied:

(i) $1 \in W$ and

(ii) if $a, b \in W$ then $a - b \in W$ and $ab \in W$.

Let

$$S = \{\tfrac{m}{2^n} \mid m \text{ and } n \text{ are integers}\}$$

and

$$T = \{\tfrac{p}{q} \mid p \text{ and } q \text{ are integers and } q \text{ is odd}\}$$

Then

(A) neither S nor T is a ring;
(B) S is a ring and T is not;
(C) T is a ring and S is not;
(D) both S and T are rings.

42. For a real number a, define $a^+ = \max\{a, 0\}$. For example, $2^+ = 2, (-3)^+ = 0$. Then, for two real numbers a and b, the equality $(ab)^+ = (a^+)(b^+)$ holds *if and only if*

(A) both a and b are positive;
(B) a and b have the same sign;
(C) $a = b = 0$;
(D) at least one of a and b is greater than or equal to 0.

43. For any real number x, let $[x]$ denote the largest integer less than or equal to x and $< x > = x - [x]$, that is, the fractional part of x. For arbitrary real numbers x, y and z, only one of the following statements is correct. Which one is it?

(A) $[x+y+z] = [x] + [y] + [z]$.
(B) $[x+y+z] = [x+y] + [z] = [x] + [y+z] = [x+z] + [y]$.
(C) $< x+y+z > = y + z - [y+z] + < x >$.
(D) $[x+y+z] = [x+y] + [z + < y+x >]$.

44. Suppose that $x_1, \ldots, x_n$ $(n > 2)$ are real numbers such that $x_i = -x_{n-i+1}$ for $1 \leq i \leq n$. Consider the sum $S = \sum\sum\sum x_i x_j x_k$, where the summations are taken over all $i, j, k : 1 \leq i, j, k \leq n$ and i, j, k are all distinct. Then S equals

(A) $n! x_1 x_2 \cdots x_n$; (B) $(n-3)(n-4)$;
(C) $(n-3)(n-4)(n-5)$; (D) none of the foregoing expressions.

45. By an *upper bound* for a set A of real numbers, we mean any real number x such that every number a in A is smaller than or equal to x. If x is an upper bound for a set A and no number strictly smaller than x is an upper bound for A, then x is called $\sup A$.

Let A and B be two sets of real numbers with $x = \sup A$ and $y = \sup B$. Let C be the set of all real numbers of the form $a + b$ where a is in A and b is in B. If $z = \sup C$, then

(A) $z > x + y$; (B) $z < x + y$; (C) $z = x + y$;
(D) nothing can be said in general about the relation between x, y and z.

46. There are 100 students in a class. In an examination, 50 of them failed in Mathematics, 45 failed in Physics and 40 failed in Statistics; and 32 failed in exactly two of these three subjects. Only one student passed in all the three subjects. The number of students failing in all the three subjects

(A) is 12; (B) is 4; (C) is 2;
(D) cannot be determined from the given information.

47. A television station telecasts three types of programs X, Y and Z. A survey gives the following data on television viewing. Among the people interviewed 60% watch program X, 50% watch program Y, 50% watch program Z, 30% watch programs X and Y, 20% watch programs Y and Z, 30% watch programs X and Z while 10% do not watch any television program. The percentage of people watching all the three programs X, Y and Z is

(A) 90; (B) 50; (C) 10; (D) 20.

48. In a survey of 100 families, the number of families that read the most recent issues of various magazines was found to be: *India Today* 42, *Sunday* 30, *New Delhi* 28, *India Today* and *Sunday* 10, *India Today* and *New Delhi* 5, *Sunday* and *New Delhi* 8, all three magazines 3. Then the number of families that read none of the three magazines is

(A) 30; (B) 26; (C) 23; (D) 20.

49. In a survey of 100 families, the number of families that read the most recent issues of various magazines was found to be: *India Today* 42, *Sunday* 30, *New Delhi* 28, *India Today* and *Sunday* 10, *India Today* and *New Delhi* 5, *Sunday* and *New Delhi* 8, all three magazines 3. Then the number of families that read either both or none of the two magazines *Sunday* and *India Today* is

(A) 48; (B) 38; (C) 72; (D) 58.

50. In a village of 1000 inhabitants, there are three newspapers P, Q and R in circulation. Each of these papers is read by 500 persons. Papers P and Q are read by 250 persons, papers Q and R are read by 250 persons, papers R and P are read by 250 persons. All the three papers are read by 250 persons. Then the number of persons who read no newspaper at all

(A) is 500; (B) is 250; (C) is 0;
(D) cannot be determined from the given information.

51. Sixty (60) students appeared in a test consisting of three Papers I, II and III. Of these students, 25 passed in Paper I, 20 in Paper II and 8 in Paper III. Further, 42 students passed in at least one of Papers I and II, 30 in at least one of Papers I and III, 25 in at least one of Papers II and III. Only one student passed in all the three papers. Then the number of students who failed in all the papers is

(A) 15; (B) 17; (C) 45; (D) 33.

52. A student studying the weather for d days observed that (i) it rained on 7 days, morning or afternoon; (ii) when it rained in the afternoon, it was clear in the morning; (iii) there were five clear afternoons; and (iv) there were six clear mornings. Then d equals

(A) 7; (B) 11; (C) 10; (D) 9.

53. A club with x members is organized into four committees according to the following rules:
(i) Each member belongs to exactly two committees.
(ii) Each pair of committees has exactly one member in common.
Then

(A) $x = 4$; (B) $x = 6$; (C) $x = 8$;
(D) x cannot be determined from the given information.

54. There were 41 candidates in an examination and each candidate was examined in Algebra, Geometry and Calculus. It was found that 12 candidates failed in Algebra, 7 failed in Geometry and 8 failed in Calculus, 2 in Geometry and Calculus, 3 in Calculus and Algebra, 6 in Algebra and Geometry, whereas only 1 failed in all three subjects. Then the number of candidates who passed in all three subjects

(A) is 24; (B) is 2; (C) is 14;
(D) cannot be determined from the given information.

55. In a group of 120 persons there are 80 Bengalis and 40 Gujaratis. Further, 70 persons in the group are Muslims and the remaining Hindus. Then the number of Bengali Muslims in the group is

(A) 30 or more; (B) exactly 20;
(C) between 15 and 25; (D) between 20 and 25.

56. In a group of 120 persons there are 70 Bengalis, 35 Gujaratis and 15 Maharashtrians. Further, 75 persons in the group are Muslims and the remaining are Hindus. Then the number of Bengali Muslims in the group is

(A) between 10 and 14; (B) between 15 and 19;
(C) exactly 20; (D) 25 or more.

57. Four passengers in a compartment of the Delhi-Howrah Rajdhani Express discover that they form an interesting group. Two are lawyers and two are doctors. Two of them speak Bengali and the other two Hindi and no two of the same profession speak the same language. They also discover that two of them are Christians and two Muslims, no two of the same religion are of the same profession and no two of the same religion speak the same language. The Hindi-speaking doctor is a Christian. Then only one of the statements below logically follows. Which one is it?

(A) The Bengali-speaking lawyer is a Muslim.
(B) The Christian lawyer speaks Bengali.
(C) The Bengali-speaking doctor is a Christian.
(D) The Bengali-speaking doctor is a Hindu.

58. In a football league, a particular team played 60 games in a season. The team never lost three games consecutively and never won five games consecutively in that season. If N is the number of games the team won in that season, then N satisfies

(A) $24 \leq N \leq 50$; (B) $20 \leq N \leq 48$;
(C) $12 \leq N \leq 40$; (D) $18 \leq N \leq 42$.

59. A box contains 100 balls of different colours: 28 red, 17 blue, 21 green, 10 white, 12 yellow and 12 black. The smallest number n such that any n balls drawn from the box will contain at least 15 balls of the same colour, is

(A) 73;
(B) 77;
(C) 81;
(D) 85.

60. Let x, y, z, w be positive real numbers, which satisfy the two conditions that
(i) if $x > y$ then $z > w$; and
(ii) if $x > z$ then $y < w$.
Then one of the statements given below is a valid conclusion. Which one is it?

(A) If $x < y$ then $z < w$.
(B) If $x < z$ then $y > w$.
(C) If $x > y + z$ then $z < y$.
(D) If $x > y + z$ then $z > y$.

61. Consider the statement:

$$x(\alpha - x) < y(\alpha - y) \text{ for all } x, y \text{ with } 0 < x < y < 1.$$

The statement is true

(A) if and only if $\alpha \geq 2$;
(B) if and only if $\alpha > 2$;
(C) if and only if $\alpha < -1$;
(D) for no values of α.

62. In a village, at least 50% of the people read a newspaper. Among those who read a newspaper at the most 25% read more than one paper. Only one of the following statements follows from the statements we have given. Which one is it?

(A) At the most 25% read exactly one newspaper.
(B) At least 25% read all the newspapers.
(C) At the most $37\frac{1}{2}\%$ read exactly one newspaper.
(D) At least $37\frac{1}{2}\%$ read exactly one newspaper.

63. We consider the relation "a person x shakes hand with a person y". Obviously, if x shakes hand with y, then y shakes hand with x. In a gathering of 99 persons, one of the following statements is *always* true, considering 0 to be an even number. Which one is it?

(A) There is at least one person who shakes hand exactly with an odd number of persons.
(B) There is at least one person who shakes hand exactly with an even number of persons.
(C) There are even number of persons who shake hand exactly with an even number of persons.
(D) None of the foregoing statements.

64. Let P, Q, R, S and T be statements such that if P is true then both Q and S are true, and if both R and S are true then T is false. We then have:

(A) If T is true then both P and R must be true.
(B) If T is true then both P and R must be false.
(C) If T is true then at least one of P and R must be true.
(D) If T is true then at least one of P and R must be false.

65. Let P, Q, R and S be four statements such that if P is true then Q is true, if Q is true then R is true and if S is true then at least one of Q and R is false. Then it follows that

(A) if S is false then both Q and R are true;
(B) if at least one of Q and R is true then S is false;
(C) if P is true then S is false;
(D) if Q is true then S is true.

66. If A, B, C and D are statements such that if at least one of A and B is true, then at least one of C and D must be true. Further, both A and C are false. Then

(A) if D is false then B is false
(B) both B and D are false
(C) both B and D are true
(D) if D is true then B is true.

67. P, Q and R are statements such that if P is true then at least one of the following is correct: (i) Q is true, (ii) R is not true. Then

(A) if both P and Q are true then R is true;
(B) if both Q and R are true then P is true;
(C) if both P and R are true then Q is true;
(D) none of the foregoing statements is correct.

68. It was a hot day and four couples drank together 44 bottles of cold drink. Anita had 2, Biva 3, Chanchala 4, and Dipti 5 bottles. Mr. Panikkar drank just as many bottles as his wife, but each of the other men drank more than his wife— Mr.Dubé twice, Mr.Narayan three times and Mr.Rao four times as many bottles. Then only one of the following statements is correct. Which one is it?

(A) Mrs.Panikkar is Chanchala.
(B) Anita's husband had 8 bottles.
(C) Mr.Narayan had 12 bottles.
(D) Mrs.Rao is Dipti.

69. Every integer of the form $(n^3 - n)(n - 2)$, (for $n = 3, 4, \ldots$) is

(A) divisible by 6 but not always divisible by 12;
(B) divisible by 12 but not always divisible by 24;
(C) divisible by 24 but not always divisible by 48;
(D) divisible by 9.

70. The number of integers $n > 1$, such that $n, n+2, n+4$ are all prime numbers, is

(A) zero; (B) one; (C) infinite; (D) more than one, but finite.

71. The number of *ordered* pairs of integers (x, y) satisfying the equation

$$x^2 + 6x + y^2 = 4$$

is

(A) 2; (B) 4; (C) 6; (D) 8.

72. The number of integer (positive, negative or zero) solutions of

$$xy - 6(x + y) = 0$$

with $x \leq y$ is

(A) 5; (B) 10; (C) 12; (D) 9.

73. Let P denote the set of all positive integers and

$$S = \{(x, y) : x \in P, y \in P \text{ and } x^2 - y^2 = 666\}.$$

The number of distinct elements in the set S is

(A) 0; (B) 1; (C) 2; (D) more than 2.

74. If numbers of the form $3^{4n-2} + 2^{6n-3} + 1$, where n is a positive integer, are divided by 17, the set of all possible remainders is

(A) $\{1\}$; (B) $\{0, 1\}$; (C) $\{0, 1, 7\}$; (D) $\{1, 7\}$.

75. Consider the sequence: $a_1 = 101, a_2 = 10101, a_3 = 1010101$, and so on. Then a_k is a composite number (that is, not a prime number)

(A) if and only if $k \geq 2$ and 11 divides $10^{k+1} + 1$;
(B) if and only if $k \geq 2$ and 11 divides $10^{k+1} - 1$;
(C) if and only if $k \geq 2$ and $k - 2$ is divisible by 3;
(D) if and only if $k \geq 2$.

76. Let n be a *positive* integer. Now consider all numbers of the form $3^{2n+1} + 2^{2n+1}$. Only one of the following statements is true regarding the *last digit* of these numbers. Which one is it?

(A) It is 5 for some of these numbers but not for all.
(B) It is 5 for all these numbers.
(C) It is always 5 for $n \leq 10$ and it is 5 for some $n > 10$.
(D) It is odd for all of these numbers but not necessarily 5.

77. Which of the following numbers can be expressed as the sum of squares of two integers?

(A) 1995; (B) 1999; (C) 2003; (D) none of these integers.

78. If the product of an odd number of odd integers is of the form $4n + 1$, then

(A) an even number of them must always be of the form $4n + 1$;
(B) an odd number of them must always be of the form $4n + 3$;
(C) an odd number of them must always be of the form $4n + 1$;
(D) none of the above statements is true.

79. The two sequences of numbers $\{1, 4, 16, 64, \ldots\}$ and $\{3, 12, 48, 192, \ldots\}$ are mixed as follows: $\{1, 3, 4, 12, 16, 48, 64, 192, \ldots\}$. One of the numbers in the mixed series is 1048576. Then the number immediately preceding it is

(A) 786432; (B) 262144; (C) 814572; (D) 786516.

80. Let $(a_1, a_2, a_3, \ldots)$ be a sequence such that $a_1 = 2$ and $a_n - a_{n-1} = 2n$ for all $n \geq 2$. Then $a_1 + a_2 + \cdots + a_{20}$ is

(A) 420; (B) 1750; (C) 3080; (D) 3500.

81. The value of $\sum ij$, where the summation is over all i and j such that $1 \leq i < j \leq 10$, is

(A) 1320; (B) 2640; (C) 3025; (D) none of the foregoing numbers.

82. Let $x_1, x_2, \ldots, x_{100}$ be hundred integers such that the sum of any five of them is 20. Then

(A) the largest x_i equals 5; (B) the smallest x_i equals 3;
(C) $x_{17} = x_{83}$; (D) none of the foregoing statements is true.

83. The smallest positive integer n with 24 divisors (where 1 and n are also considered as divisors of n) is

(A) 420; (B) 240; (C) 360; (D) 480.

84. The last digit of $(2137)^{754}$ is

(A) 1; (B) 3; (C) 7; (D) 9.

85. The smallest integer that produces remainders of 2, 4, 6 and 1 when divided by 3, 5, 7 and 11 respectively, is

(A) 104; (B) 1154; (C) 419; (D) none of the foregoing numbers.

86. How many integers n are there such that $2 \leq n \leq 1000$ and the highest common factor of n and 36 is 1?

(A) 166. (B) 332. (C) 361. (D) 416.

87. The remainder when 3^{37} is divided by 79 is

(A) 78; (B) 1; (C) 2; (D) 35.

88. The remainder when 4^{101} is divided by 101 is

(A) 4; (B) 64; (C) 84; (D) 36.

89. The 300-digit number with all digits equal to 1 is

(A) divisible by neither 37 nor 101; (B) divisible by 37 but not by 101;
(C) divisible by 101 but not by 37; (D) divisible by both 37 and 101.

90. The remainder when $3^{12} + 5^{12}$ is divided by 13 is

(A) 1; (B) 2; (C) 3; (D) 4.

91. When $3^{2002} + 7^{2002} + 2002$ is divided by 29 the remainder is

(A) 0; (B) 1; (C) 2; (D) 7.

92. Let $x = 0.101001000100001\ldots + 0.272727\ldots$. Then x

(A) is irrational;
(B) is rational but $\sqrt{x}$ is irrational;
(C) is a root of $x^2 + 0.27x + 1 = 0$;
(D) satisfies none of the above properties.

93. The highest power of 18 contained in $\binom{50}{25}$ is
(A) 3; (B) 0; (C) 1; (D) 2.

94. The number of divisors of 2700 including 1 and 2700 equals

(A) 12; (B) 16; (C) 36; (D) 18.

95. The number of different factors of 1800 equals

(A) 12; (B) 210; (C) 36; (D) 18.

96. The number of different factors of 3003 is

(A) 2; (B) 15; (C) 7; (D) 16.

97. The number of divisors of 6000, where 1 and 6000 are also considered as divisors of 6000, is

(A) 40; (B) 50; (C) 60; (D) 30.

98. The number of positive integers which divide 240 (where both 1 and 240 are considered as divisors) is

(A) 18; (B) 20; (C) 30; (D) 24.

99. The sum of all the positive divisors of 1800 (including 1 and 1800) is

(A) 7201; (B) 6045; (C) 5040; (D) 4017.

100. Let $d_1, d_2, \ldots, d_k$ be all the factors of a positive integer n including 1 and n. Suppose $d_1 + d_2 + \ldots + d_k = 72$. Then the value of

$$\frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_k}$$

(A) is $\frac{k^2}{72}$; (B) is $\frac{72}{k}$; (C) is $\frac{72}{n}$;
(D) cannot be computed from the given information.

101. The number of ways of distributing 12 identical oranges among 4 children so that every child gets at least one and no child more than 4 is

(A) 31; (B) 52; (C) 35; (D) 42.

102. The number of terms in the expansion of $[(a+3b)^2(a-3b)^2]^2$, when simplified, is

(A) 4; (B) 5; (C) 6; (D) 7.

103. The number of ways in which 5 persons P, Q, R, S and T can be seated in a ring so that P sits between Q and R is

(A) 120; (B) 4; (C) 24; (D) 9.

104. Four married couples are to be seated in a merry-go-round with 8 identical seats. In how many ways can they be seated so that

(i) males and females seat alternately, and

(ii) no husband seats adjacent to his wife?

(A) 8; (B) 12; (C) 16; (D) 20.

105. For a regular polygon with n sides ($n > 5$), the number of triangles whose vertices are joining non-adjacent vertices of the polygon is

(A) $n(n-4)(n-5)$; (B) $(n-3)(n-4)(n-5)/3$; (C) $2(n-3)(n-4)(n-5)$; (D) $n(n-4)(n-5)/6$.

106. The term that is independent of x in the expansion of $(\frac{3x^2}{2} - \frac{1}{3x})^9$ is

(A) $\binom{9}{6}(\frac{1}{3})^3(\frac{3}{2})^6$; (B) $\binom{9}{5}(\frac{3}{2})^5(-\frac{1}{3})^4$; (C) $\binom{9}{3}(\frac{1}{6})^3$; (D) $\binom{9}{4}(\frac{3}{2})^4(-\frac{1}{3})^5$.

107. The value of

$$\binom{50}{0}\binom{50}{1} + \binom{50}{1}\binom{50}{2} + \ldots + \binom{50}{49}\binom{50}{50}$$

is

(A) $\binom{100}{50}$; (B) $\binom{100}{51}$; (C) $\binom{50}{25}$; (D) $\binom{50}{25}^2$.

108. The value of

$$\binom{50}{0}^2 + \binom{50}{1}^2 + \binom{50}{2}^2 + \ldots + \binom{50}{49}^2 + \binom{50}{50}^2$$

is

(A) $\binom{100}{50}$; (B) $(50)^{50}$; (C) 2^{100}; (D) 2^{50}.

109. The value of

$$\binom{100}{0}\binom{200}{150} + \binom{100}{1}\binom{200}{151} + \ldots + \binom{100}{50}\binom{200}{200}$$

is

(A) $\binom{300}{50}$; (B) $\binom{100}{50} \times \binom{200}{150}$; (C) $[\binom{100}{50}]^2$;
(D) none of the foregoing numbers.

110. The number of four-digit numbers strictly greater than 4321 that can be formed from the digits $0, 1, 2, 3, 4, 5$ allowing for repetition of digits is

(A) 310; (B) 360; (C) 288; (D) 300.

111. The sum of all the distinct four-digit numbers that can be formed using the digits 1, 2, 3, 4 and 5, each digit appearing at most once, is

(A) 399900; (B) 399960; (C) 390000; (D) 360000.

112. The number of integers lying between 3000 and 8000 (including 3000 and 8000) which have at least two digits equal is

(A) 2481; (B) 1977; (C) 4384; (D) 2755.

113. The greatest integer which, when dividing the integers 13511, 13903 and 14593, leaves the same remainder is

(A) 98; (B) 56; (C) 2; (D) 7.

114. An integer n has the property that when divided by $10, 9, 8, \ldots, 2$, it leaves remainders $9, 8, 7, \ldots, 1$ respectively. A possible value of n is

(A) 59; (B) 419; (C) 1259; (D) 2519.

115. If n is a positive integer such that $8n + 1$ is a perfect square, then

(A) n must be odd; (B) n cannot be a perfect square;
(C) n must be a prime number; (D) $2n$ cannot be a perfect square.

116. For any two positive integers a and b, define $a \equiv b$ if $a - b$ is divisible by 7. Then $(1512 + 121) \cdot (356) \cdot (645) \equiv$

(A) 4; (B) 5; (C) 3; (D) 2.

117. The coefficient of x^2 in the binomial expansion of $(1 + x + x^2)^{10}$ is

(A) $\binom{10}{1} + \binom{10}{2}$; (B) $\binom{10}{2}$; (C) $\binom{10}{1}$;
(D) none of the foregoing numbers.

118. The coefficient of x^{17} in the expansion of $\log_e(1 + x + x^2)$, where $|x| < 1$, is

(A) $\frac{1}{17}$; (B) $-\frac{1}{17}$; (C) $\frac{3}{17}$; (D) none of the foregoing quantities.

119. Let $a_1, a_2, \ldots, a_{11}$ be an arbitrary arrangement (i.e., permutation) of the integers $1, 2, \ldots, 11$. Then the number $(a_1 - 1)(a_2 - 2)\ldots(a_{11} - 11)$ is

(A) necessarily ≤ 0; (B) necessarily 0;
(C) necessarily even; (D) not necessarily ≤ 0, 0 or even.

120. Three boys of class I, 4 boys of class II and 5 boys of class III sit in a row. The number of ways they can sit, so that boys of the same class sit together is

(A) $3!4!5!$; (B) $\frac{(12)!}{3!4!5!}$; (C) $(3!)^2 4!5!$; (D) $3 \times 4!5!$.

121. For each positive integer n consider the set S_n defined as follows: $S_1 = \{1\}$, $S_2 = \{2, 3\}, S_3 = \{4, 5, 6\}, \ldots$, and, in general, S_{n+1} consists of $n+1$ consecutive integers the smallest of which is one more than the largest integer in S_n. Then the sum of all the integers in S_{21} equals

(A) 1113; (B) 53361; (C) 5082; (D) 4641.

122. If the constant term in the expansion of $(\sqrt{x} - \frac{k}{x^2})^{10}$ is 405, then k is

(A) $\pm(3)^{\frac{1}{4}}$; (B) ± 2; (C) $\pm(4)^{\frac{1}{3}}$; (D) ± 3.

123. Consider the equation of the form $x^2+bx+c=0$. The number of such equations that have real roots and have coefficients b and c in the set $\{1, 2, 3, 4, 5, 6\}$, (b may be equal to c), is

(A) 20; (B) 18; (C) 17; (D) 19.

124. The number of polynomials of the form $x^3 + ax^2 + bx + c$ which are divisible by $x^2 + 1$ and where a, b and c belong to $\{1, 2, \ldots, 10\}$, is

(A) 1; (B) 10; (C) 11; (D) 100.

125. The number of distinct 6-digit numbers between 1 and 300000 which are divisible by 4 and are obtained by rearranging the digits of 112233, is

(A) 12; (B) 15; (C) 18; (D) 90.

126. The number of odd positive integers smaller than or equal to 10,000 which are divisible neither by 3 nor by 5 is

(A) 3,332; (B) 2,666; (C) 2,999; (D) 3,665.

127. The number of ways one can put three balls numbered 1, 2, 3 in three boxes labelled a, b, c such that at the most one box is empty is equal to

(A) 6; (B) 24; (C) 42; (D) 18.

128. A bag contains coloured balls of which at least 90% are red. Balls are drawn from the bag one by one and their colour noted. It is found that 49 of the first 50 balls drawn are red. Thereafter 7 out of every 8 balls drawn are red. The number of balls in the bag CAN NOT BE

(A) 170; (B) 210; (C) 250; (D) 194.

129. There are N boxes, each containing at most r balls. If the number of boxes containing at least i balls is N_i for $i = 1, 2, \ldots, r$, then the total number of balls contained in these N boxes

(A) cannot be determined from the given information;
(B) is exactly equal to $N_1 + N_2 + \ldots, +N_r$;
(C) is strictly larger that $N_1 + N_2 + \cdots + N_r$;
(D) is strictly smaller than $N_1 + N_2 + \cdots + N_r$.

130. For all n, the value of $\binom{2n}{n}$ is equal to

(A) $\binom{2n}{0} - \binom{2n}{1} + \binom{2n}{2} - \binom{2n}{3} + \ldots + \binom{2n}{2n}$;
(B) $\binom{2n}{0}^2 + \binom{2n}{1}^2 + \binom{2n}{2}^2 + \ldots + \binom{2n}{n}^2$;
(C) $\binom{2n}{0}^2 - \binom{2n}{1}^2 + \binom{2n}{2}^2 - \binom{2n}{3}^2 + \ldots + \binom{2n}{2n}^2$;
(D) none of the foregoing expressions.

131. The coefficients of three consecutive terms in the expansion of $(1+x)^n$ are 165, 330 and 462. Then the value of n is

(A) 10; (B) 12; (C) 13; (D) 11.

132. The number of ways in which 4 persons can be divided into two equal groups is

(A) 3; (B) 12; (C) 6; (D) none of the foregoing numbers.

133. The number of ways in which 8 persons numbered $1, 2, \ldots, 8$ can be seated in a ring so that 1 always sits between 2 and 3 is

(A) 240; (B) 360; (C) 72; (D) 120.

134. There are seven greeting cards, each of a different colour, and seven envelopes of the same seven colours. The number of ways in which the cards can be put in the envelopes, so that exactly four of the cards go into the envelopes of the right colours, is

(A) $\binom{7}{3}$; (B) $2\binom{7}{3}$; (C) $(3!)\binom{4}{3}$; (D) $(3!)\binom{7}{3}\binom{4}{3}$.

135. The number of distinct positive integers that can be formed using 0, 1, 2, 4 where each integer is used at the most once is equal to

(A) 48; (B) 84; (C) 64; (D) 36.

136. A class contains three girls and four boys. Every Saturday five students go on a picnic, a different group being sent each week. During the picnic, each girl in the group is given a doll by the accompanying teacher. After all possible groups of five have gone once, the total number of dolls the girls have got is

(A) 27; (B) 11; (C) 21; (D) 45.

137. From a group of seven persons, seven committees are formed. Any two committees have exactly one member in common. Each person is in exactly three committees. Then

(A) at least one committee must have more than three members;
(B) each committee must have exactly three members;
(C) each committee must have more than three members;
(D) nothing can be said about the sizes of the committees.

138. Three ladies have each brought a child for admission to a school. The head of the school wishes to interview the six people one by one, taking care that no child is interviewed before its mother. In how many different ways can the interviews be arranged?

(A) 6; (B) 36; (C) 72; (D) 90.

139. The coefficient of x^4 in the expansion of $(1 + x - 2x^2)^7$ is

(A) -81; (B) -91; (C) $+81$; (D) $+91$.

140. The coefficient of $a^3b^4c^5$ in the expansion of $(bc + ca + ab)^6$ is

(A) $\frac{12!}{3!4!5!}$; (B) $\binom{6}{3}3!$; (C) 33; (D) $3\binom{6}{3}$.

141. The coefficient of t^3 in the expansion of

$$\left(\frac{1-t^6}{1-t}\right)^3$$

is

(A) 10; (B) 12; (C) 18; (D) 0.

142. The value of

$$\binom{2n}{0}^2 - \binom{2n}{1}^2 + \binom{2n}{2}^2 - \ldots - \binom{2n}{2n-1}^2 + \binom{2n}{2n}^2$$

is

(A) $\binom{4n}{2n}$; (B) $\binom{2n}{n}$; (C) 0; (D) $(-1)^n \binom{2n}{n}$.

143. There are 14 intermediate stations between Dusi and Visakhapatnam on the South Eastern Railway. A train is to be arranged from Dusi to Visakhapatnam so that it halts at exactly three intermediate stations, no two of which are consecutive. Then the number of ways of doing this is

(A) $\binom{14}{3} - \binom{13}{1}\binom{12}{1} + \binom{12}{1}$; (B) $\frac{10\times11\times12}{6}$;
(C) $\binom{14}{3} - \binom{14}{2} - \binom{14}{1}$; (D) $\binom{14}{3} - \binom{14}{2} + \binom{14}{1}$.

144. The letters of the word "MOTHER" are permuted, and all the permutations so formed are arranged in alphabetical order as in a dictionary. Then the number of permutations which come before the word "MOTHER" is

(A) 503; (B) 93; (C) $\frac{6!}{2} - 1$; (D) 308.

145. All the letters of the word **PESSIMISTIC** are to be arranged so that no two **S**'s occur together, no two **I**'s occur together, and **S**, **I** do not occur together. The number of such arrangements is

(A) 2,400; (B) 5,480; (C) 48,000; (D) 50,400.

146. Suppose that x is an irrational number and a, b, c, d are rational numbers such that $\frac{ax+b}{cx+d}$ is rational. Then it follows that

(A) $a = c = 0$; (B) $a = c$ and $b = d$; (C) $a + b = c + d$; (D) $ad = bc$.

147. Let p, q and s be integers such that $p^2 = sq^2$. Then it follows that

(A) p is an even number;
(B) if s divides p, then s is a perfect square;
(C) s divides p;
(D) q^2 divides p.

148. The number of pairs of positive integers (x, y) where x and y are prime numbers and $x^2 - 2y^2 = 1$ is

(A) 0; (B) 1; (C) 2; (D) 8.

149. A point P with coordinates (x, y) is said to be *good* if both x and y are positive integers. The number of good points on the curve $xy = 27027$ is

(A) 8; (B) 16; (C) 32; (D) 64.

150. Let p be an odd prime number. Then the number of positive integers k with $1 < k < p$, for which k^2 leaves a remainder of 1 when divided by p, is

(A) 2; (B) 1; (C) $p - 1$; (D) $\frac{p-1}{2}$.

151. Let $n = 51! + 1$. Then the number of primes among $n + 1, n + 2, \ldots, n + 50$ is

(A) 0; (B) 1; (C) 2; (D) more than 2.

152. If three prime numbers, all greater than 3, are in A.P., then their common difference

(A) must be divisible by 2 but not necessarily by 3;
(B) must be divisible by 3 but not necessarily by 2;
(C) must be divisible by both 2 and 3;
(D) need not be divisible by any of 2 and 3.

153. Let N be a positive integer not equal to 1. Then note that none of the numbers $2, 3, \ldots, N$ is a divisor of $(N! - 1)$. From this, we can conclude that

(A) $(N! - 1)$ is a prime number;
(B) at least one of the numbers $N+1, N+2, \ldots, N! - 2$ is a divisor of $(N! - 1)$;
(C) the smallest number between N and $N!$ which is a divisor of $(N! - 1)$, is a prime number;
(D) none of the foregoing statements is necessarily correct.

154. The number $1000! = 1.2.3.\ldots.1000$ ends exactly with

(A) 249 zeros; (B) 250 zeros; (C) 240 zeros; (D) 200 zeros.

155. Let A denote the set of all prime numbers, B the set of all prime numbers and the number 4, and C the set of all prime numbers and their squares. Let D be the set of positive integers k, for which

$$\frac{(k-1)!}{k}$$

is *not* an integer. Then

(A) $D = A$; (B) $D = B$; (C) $D = C$; (D) $B \subset D \subset C$.

156. Let n be any integer. Then $n(n+1)(2n+1)$

(A) is a perfect square;
(B) is an odd number;
(C) is an integral multiple of 6;
(D) does not necessarily have any of the foregoing properties.

157. The numbers $12n + 1$ and $30n + 2$ are relatively prime for

(A) any positive integer n;
(B) infinitely many, but not all, integers n;
(C) for finitely many integers n;
(D) none of the above.

158. The expression

$$1+\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}+\ldots+\frac{1}{n+1}\binom{n}{n}$$

equals

(A) $\frac{2^{n+1}-1}{n+1}$; (B) $\frac{2(2^n-1)}{n+1}$; (C) $\frac{2^n-1}{n}$; (D) $\frac{2(2^{n+1}-1)}{n+1}$.

159. The value of

$$\frac{{}^{30}C_1}{2}+\frac{{}^{30}C_3}{4}+\frac{{}^{30}C_5}{6}+\cdots+\frac{{}^{30}C_{29}}{30}$$

is

(A) $\frac{2^{31}}{30}$; (B) $\frac{2^{30}}{31}$; (C) $\frac{2^{31}-1}{31}$; (D) $\frac{2^{30}-1}{31}$.

160. The value of $\left\{\sum_{i=0}^{100}\binom{k}{i}\binom{M-k}{100-i}\frac{k-i}{M-100}\right\} \Big/ \binom{M}{100}$, where $M-k>100$, $k>100$ and $\binom{m}{n}=\frac{m!}{n!(m-n)!}$ equals

(A) $\frac{k}{M}$; (B) $\frac{M}{k}$; (C) $\frac{k}{M^2}$; (D) $\frac{M}{k^2}$.

161. The remainder obtained when $1!+2!+\ldots+95!$ is divided by 15 is

(A) 14; (B) 3; (C) 1; (D) none of the foregoing numbers.

162. Let $x_1, x_2, \ldots, x_{50}$ be fifty integers such that the sum of any six of them is 24. Then

(A) the largest of x_i equals 6;
(B) the smallest of x_i equals 3;
(C) $x_{16}=x_{34}$;
(D) none of the foregoing statements is correct.

163. Let $x_1, x_2, \ldots, x_{50}$ be fifty nonzero numbers such that $x_i+x_{i+1}=k$ for all i, $1\le i\le 49$. If $x_{14}=a, x_{27}=b$, then $x_{20}+x_{37}$ equals

(A) $2(a+b)-k$; (B) $k+a$; (C) $k+b$;
(D) none of the foregoing expressions.

164. Let S be the set of all numbers of the form 4^n-3n-1, where $n=1,2,3,\ldots$. Let T be the set of all numbers of the form $9(n-1)$, where $n=1,2,3,\ldots$. Only one of the following statements is correct. Which one is it?

(A) Each number in S is also in T.

(B) Each number in T is also in S.
(C) Every number in S is in T and every number in T is in S.
(D) There are numbers in S which are not in T and there are numbers in T which are not in S.

165. The number of four-digit numbers greater than 5000 that can be formed out of the digits 3, 4, 5, 6 and 7, no digit being repeated, is

(A) 52; (B) 61; (C) 72; (D) 80.

166. The number of positive integers of 5 digits such that each digit is 1, 2 or 3, and all three of the digits appear at least once, is

(A) 243; (B) 150; (C) 147; (D) 193.

167. In a chess tournament, each of the 5 players plays against every other player. No game results in a draw and the winner of each game gets one point and the loser gets zero. Then which one of the following sequences *cannot* represent the scores of the five players?

(A) 3,3,2,1,1. (B) 3,2,2,2,1. (C) 2,2,2,2,2. (D) 4,4,1,1,0.

168. Ten (10) persons numbered $1, 2, \ldots, 10$ play a chess tournament, each player playing against every other player exactly one game. Assume that each game results in a win for one of the players (that is, there is no draw). Let $w_1, w_2, \ldots, w_{10}$ be the number of games won by players $1, 2, \ldots, 10$ respectively. Also, let $l_1, l_2, \ldots, l_{10}$ be the number of games lost by players $1, 2, \ldots, 10$ respectively. Then

(A) $w_1^2 + w_2^2 + \ldots + w_{10}^2 = 81 - (l_1^2 + l_2^2 + \ldots + l_{10}^2)$;
(B) $w_1^2 + w_2^2 + \ldots + w_{10}^2 = 81 + (l_1^2 + l_2^2 + \ldots + l_{10}^2)$;
(C) $w_1^2 + w_2^2 + \ldots + w_{10}^2 = l_1^2 + l_2^2 + \ldots + l_{10}^2$;
(D) none of the foregoing equalities is necessarily true.

169. A game consisting of 10 rounds is played among three players A, B and C as follows: Two players play in each round and the loser is replaced by the third player in the next round. If the only rounds when A played against B are the first, fourth and the tenth rounds, the number of games won by C

(A) is 5; (B) is 6; (C) is 7;
(D) cannot be determined from the above information.

170. An $n \times n$ chess board is a square of side n units which has been sub-divided into n^2 unit squares by equally-spaced straight lines parallel to the sides. The total number of squares of all sizes on an $n \times n$ chess board is

(A) $\frac{n(n+1)}{2}$;

(B) $1^2 + 2^2 + \ldots + n^2$;
(C) $2 \times 1 + 3 \times 2 + 4 \times 3 + \ldots + n \times (n-1)$;
(D) given by none of the foregoing expressions.

171. Given any five points in the square $I^2 = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$, only one of the following statements is true. Which one is it?

(A) The five points lie on a circle.
(B) At least one square can be formed using four of the five points.
(C) At least three of the five points are collinear.
(D) There are at least two points such that the distance between them does not exceed $\frac{1}{\sqrt{2}}$.

172. The quantities ℓ, c, h and m are measured in the units mentioned against each: ℓ: centimetre; c: centimetre per second; h: ergs × second; mc^2: ergs. Of the expressions

$$\alpha = (\frac{ch}{ml})^{\frac{1}{2}}; \quad \beta = (\frac{mc}{hl})^3; \quad \gamma = \frac{h}{mcl},$$

which ones are pure numbers, that is, do not involve any unit?

(A) Only α. (B) Only β. (C) Only γ. (D) None.

173. The number of distinct rearrangements of the letters of the word "MULTIPLE" that can be made preserving the order in which the vowels (U,I,E) occur and not counting the original arrangement is

(A) 6719; (B) 3359; (C) 6720; (D) 3214.

174. The number of terms in the expansion of $(x + y + z + w)^{10}$ is

(A) $\binom{10}{4}$; (B) $\binom{13}{3}$; (C) $\binom{14}{4}$; (D) 11^4.

175. The number of ways in which three non-negative integers n_1, n_2, n_3 can be chosen such that $n_1 + n_2 + n_3 = 10$ is

(A) 66; (B) 55; (C) 10^3; (D) $\frac{10!}{3!2!1!}$.

176. In an examination, the score in each of the four languages—Bengali, Hindi, Urdu and Telugu—can be integers between 0 and 10. Then the number of ways in which a student can secure a total score of 21 is

(A) 880; (B) 760; (C) 450; (D) 1360.

177. The number of ordered pairs (x, y) of positive integers such that $x + y = 90$ and their greatest common divisor is 6 equals
(A) 15; (B) 14; (C) 8; (D) 10.

178. How many pairs of positive integers (m, n) are there satisfying $m^3 - n^3 = 21$?

(A) exactly one; (B) none; (C) exactly three; (D) infinitely many.

179. The number of ways in which three distinct numbers in A.P. can be selected from $1, 2, \ldots, 24$ is
(A) 144; (B) 276; (C) 572; (D) 132.

180. The number of ways you can invite 3 of your friends on 5 consecutive days, exactly one friend a day, such that no friend is invited on more than two days is

(A) 90; (B) 60; (C) 30; (D) 10.

181. Consider three boxes, each containing 10 balls labelled $1, 2, \ldots, 10$. Suppose one ball is drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i-th box, $i = 1, 2, 3$. Then the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$, is

(A) 120; (B) 130; (C) 150; (D) 160.

182. The number of sequences of length five with 0 and 1 as terms which contain at least two consecutive 0's is

(A) 4.2^3; (B) $\binom{5}{2}$; (C) 20; (D) 19.

183. There are 7 identical white balls and 3 identical black balls. The number of distinguishable arrangements in a row of all the balls, so that no two black balls are adjacent, is

(A) 120; (B) 89(8!); (C) 56; (D) 42×5^4.

184. In a multiple-choice test there are 6 questions. Four alternative answers are given for each question, of which only one answer is correct. If a candidate answers all the questions by choosing one answer for each question, then the number of ways to get exactly 4 correct answers is

(A) $4^6 - 4^2$; (B) 135; (C) 9; (D) 120.

185. In a multiple-choice test there are 8 questions. Each question has 4 alternatives, of which only one is correct. If a candidate answers all the questions by choosing one alternative for each, the number of ways of doing it so that exactly 4 answers are correct is

(A) 70; (B) 2835; (C) 5670; (D) none of the foregoing numbers.

186. Among the 8! permutations of the digits $1, 2, 3, \ldots\ldots, 8$, consider those arrangements which have the following property: if you take *any* five consecutive positions, the product of the digits in those positions is divisible by 5. The number of such arrangements is

(A) 7!; (B) $2 \cdot 7!$; (C) $8 \cdot 7!$; (D) $4 \cdot \binom{7}{4} 5!3!4$.

187. A closet has 5 pairs of shoes. The number of ways in which 4 shoes can be chosen from it so that there will be no complete pair is

(A) 80; (B) 160; (C) 200;
(D) none of the foregoing numbers.

188. The number of ways in which 4 distinct balls can be put into 4 boxes labelled a, b, c, d, so that exactly one box remains empty is

(A) 232; (B) 196; (C) 192; (D) 144.

189. The number of permutations of the letters a, b, c, d such that b does not follow a, and c does not follow b, and d does not follow c, is

(A) 12; (B) 11; (C) 14; (D) 13.

190. The number of ways of seating three gentlemen and three ladies in a row, such that each gentleman is adjacent to at least one lady, is

(A) 360; (B) 72; (C) 720;
(D) none of the foregoing numbers.

191. The number of maps f from the set $\{1, 2, 3\}$ into the set $\{1, 2, 3, 4, 5\}$ such that $f(i) \leq f(j)$, whenever $i < j$, is

(A) 30; (B) 35; (C) 50; (D) 60.

192. For each integer i, $1 \leq i \leq 100$; ϵ_i be either $+1$ or -1. Assume that $\epsilon_1 = +1$ and $\epsilon_{100} = -1$. Say that a sign change occurs at $i \geq 2$ if ϵ_i, ϵ_{i-1} are of opposite sign. Then the total number of sign changes

(A) is odd; (B) is even; (C) is at most 50; (D) can have 49 distinct values.

193. Let $S = \{1, 2, \ldots, n\}$. The number of possible pairs of the form (A, B) with $A \subseteq B$ for subsets A and B of S is

(A) 2^n; (B) 3^n; (C) $\sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k}$; (D) $n!$.

194. There are 4 pairs of shoes of different sizes. Each of the 8 shoes can be coloured with one of the four colours: black, brown, white and red. In how many ways can one colour the shoes so that in at least three pairs, the left and the right shoes do not have the same colour?

(A) 12^4; (B) 28×12^3, (C) 16×12^3; (D) 4×12^3.

195. Let $S = \{1, 2, \cdots, 100\}$. The number of nonempty subsets A of S such that the product of elements in A is even is

(A) $2^{50}(2^{50} - 1)$; (B) $2^{100} - 1$; (C) $2^{50} - 1$; (D) none of these numbers.

196. The number of functions f from $\{1, 2, \cdots, 20\}$ onto $\{1, 2, \cdots, 20\}$ such that $f(k)$ is a multiple of 3 whenever k is a multiple of 4 is

(A) $5! \cdot 6! \cdot 9!$; (B) $5^6 \cdot 15!$; (C) $6^5 \cdot 14!$; (D) $15! \cdot 6!$.

197. Let $X = \{a_1, a_2, \ldots, a_7\}$ be a set of seven elements and $Y = \{b_1, b_2, b_3\}$ a set of three elements. The number of functions f from X to Y such that (i) f is onto and (ii) there are exactly three elements x in X such that $f(x) = b_1$, is

(A) 490; (B) 558; (C) 560; (D) 1680.

198. Consider the quadratic equation of the form $x^2 + bx + c = 0$. The number of such equations that have real roots and coefficients b and c from the set $\{1, 2, 3, 4, 5\}$ (b and c may be equal) is

(A) 18; (B) 15; (C) 12; (D) none of the foregoing quantities.

199. Let A_1, A_2, A_3 be three points on a straight line. Let B_1, B_2, B_3, B_4, B_5 be five points on a straight line parallel to the first one. Each of the three points on the first line is joined by a straight line to each of the five points on the second line. Further, no three or more of these joining lines meet at a point except possibly at the A's or the B's. Then the number of points of intersections of the joining lines lying between the two given straight lines is

(A) 30; (B) 25; (C) 35; (D) 20.

200. There are 11 points on a plane with 5 lying on one straight line and another 5 lying on a second straight line which is parallel to the first line. The remaining point is not collinear with any two of the previous 10 points. The number of triangles that can be formed with vertices chosen from these 11 points is

(A) 85; (B) 105; (C) 125; (D) 145.

201. Let $a_1, a_2, a_3, \ldots$ be a sequence of real numbers such that $\lim_{n\to\infty} a_n = \infty$. For any real number x, define an integer-valued function $f(x)$ as the smallest positive integer n for which $a_n \geq x$. Then for any integer $n \geq 1$ and any real number x,

(A) $f(a_n) \leq n$ and $a_{f(x)} \geq x$; (B) $f(a_n) \leq n$ and $a_{f(x)} \leq x$;
(C) $f(a_n) \geq n$ and $a_{f(x)} \geq x$; (D) $f(a_n) \geq n$ and $a_{f(x)} \leq x$.

202. There are 25 points in a plane, of which 10 are on the same line. Of the rest, no three are collinear and no two are collinear with any of the first ten points. The number of different straight lines that can be formed by joining these points is

(A) 256; (B) 106; (C) 255; (D) 105.

203. If $f(x) = \sin(\log_{10} x)$ and $h(x) = \cos(\log_{10} x)$, then

$$f(x)f(y) - \frac{1}{2}\left[h\left(\frac{x}{y}\right) - h(xy)\right]$$

equals

(A) $\sin(\log_{10}(xy))$; (B) $\cos(\log_{10}(xy))$;
(C) $\sin(\log_{10}(\frac{x}{y}))$; (D) none of the foregoing expressions.

204. The value of $\log_5 \frac{(125)(625)}{25}$ is

(A) 725; (B) 6; (C) 3125; (D) 5.

205. The value of $\log_2 10 - \log_8 125$ is

(A) $1 - \log_2 5$; (B) 1; (C) 0; (D) $1 - 2\log_2 5$.

206. If $\log_k x \times \log_5 k = 3$, then x equals

(A) k^5; (B) k^3; (C) 125; (D) 245.

207. If $a > 0, b > 0, a \neq 1, b \neq 1$, then the number of real x satisfying the equation

$$(\log_a x).(\log_b x) = \log_a b$$

is

(A) 0; (B) 1; (C) 2; (D) infinite.

208. If $\log_{10} x = 10^{\log_{100} 4}$, then x equals

(A) 4^{10}; (B) 100; (C) $\log_{10} 4$;
(D) none of the foregoing numbers.

209. If $\log_{12} 27 = a$, then $\log_6 16$ equals

(A) $\frac{1+a}{a}$; (B) $4\left(\frac{3-a}{3+a}\right)$; (C) $\frac{2a}{3-a}$; (D) $5\left(\frac{2-a}{2+a}\right)$.

210. Consider the number $\log_{10}(2)$. It is

(A) a rational number less than 1/3 and greater than 1/4;
(B) a rational number less than 1/4;
(C) an irrational number less than 1/3 and greater than 1/4;
(D) an irrational number less than 1/4.

211. If $y = a + b\log_e x$, then

(A) $\frac{1}{y-a}$ is proportional to x^b;
(B) $\log_e y$ is proportional to x;
(C) e^y is proportional to x^b;
(D) $y - a$ is proportional to x^b.

212. Let $y = \log_a x$ and $a > 1$. Then only one of the following statements is *false*. Which one is it?

(A) If $x = 1$, then $y = 0$.
(B) If $x < 1$, then $y < 0$.
(C) If $x = \frac{1}{2}$, then $y = \frac{1}{2}$.
(D) If $x = a$, then $y = 1$.

213. If $p = \frac{s}{(1+k)^n}$, then n equals

(A) $\log \frac{s}{p(1+k)}$; (B) $\frac{\log(s/p)}{\log(1+k)}$; (C) $\frac{\log s}{\log p(1+k)}$; (D) $\frac{\log(1+k)}{\log(s/p)}$.

214. If $(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$, then y equals

(A) 125; (B) 25; (C) $\frac{5}{3}$; (D) 243.

215. If $(\log_5 k)(\log_3 5)(\log_k x) = k$, then the value of x equals

(A) k^3; (B) 5^k; (C) k^5; (D) 3^k.

216. Given that $\log_p x = \alpha$ and $\log_q x = \beta$, the value of $\log_{p/q} x$ equals

(A) $\frac{\alpha\beta}{\beta-\alpha}$; (B) $\frac{\beta-\alpha}{\alpha\beta}$; (C) $\frac{\alpha-\beta}{\alpha\beta}$; (D) $\frac{\alpha\beta}{\alpha-\beta}$.

217. If $\log_{30} 3 = a$ and $\log_{30} 5 = b$, then $\log_{30} 8$ is equal to

(A) $a + b$ (B) $3(1 - a - b)$ (C) $\frac{8}{3}(1 - a - b)$ (D) $\frac{1}{2}(1 - a - b)$

218. If $\log_a x = 6$, and $\log_{25a}(8x) = 3$, then a is

(A) 8.5; (B) 10; (C) 12; (D) 12.5.

219. Let
$$a = \frac{(\log_{100} 10)(\log_2(\log_4 2))(\log_4(\log_2(256)^2))}{\log_4 8 + \log_8 4}.$$
Then the value of a is

(A) $-\frac{1}{3}$; (B) 2; (C) $-\frac{6}{13}$; (D) $\frac{2}{3}$.

220. If $f(x) = \log(\frac{1+x}{1-x})$, then $f(x) + f(y)$ is

(A) $f(x+y)$; (B) $f(\frac{x+y}{1+xy})$; (C) $(x+y)f(\frac{1}{1+xy})$; (D) $f(x) + \frac{f(y)}{1+xy}$.

221. If $\log_{ab} a = 4$, then the value of $\log_{ab}(\frac{\sqrt[3]{a}}{\sqrt{b}})$ is

(A) $\frac{17}{6}$; (B) 2; (C) 3; (D) $\frac{7}{6}$.

222. The value of $\sqrt{10^{2+\frac{1}{2}\log_{10} 16}}$ is

(A) 80; (B) $20\sqrt{2}$; (C) 40; (D) 20.

223. If $\log_b a = 10$, then $\log_{b^5}(a^3)$ equals

(A) $\frac{50}{3}$; (B) 6; (C) $\frac{5}{3}$; (D) $\frac{3}{5}$.

224. If $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$, then y equals

(A) $\frac{9}{2}$; (B) 9; (C) 18; (D) 27.

225. The number of real roots of the equation
$$\log_{2x}\left(\frac{2}{x}\right)(\log_2(x))^2 + (\log_2(x))^4 = 1,$$
for values of $x > 1$, is

(A) 0; (B) 1; (C) 2; (D) none of the foregoing numbers.

226. The equation $\log_3 x - \log_x 3 = 2$ has

(A) no real solution; (B) exactly one real solution;
(C) exactly two real solutions; (D) infinitely many real solutions.

227. If $(\log_3 x)(\log_4 x)(\log_5 x) = (\log_3 x)(\log_4 x) + (\log_4 x)(\log_5 x) + (\log_5 x)(\log_3 x)$ and $x \neq 1$, then x is

(A) 10; (B) 100; (C) 50; (D) 60.

228. If $\log_2(\log_3(\log_4 x)) = \log_3(\log_4(\log_2 y)) = \log_4(\log_2(\log_3 z)) = 0$ then $x + y + z$ is

(A) 99; (B) 50; (C) 89; (D) 49.

229. If x is a positive number different from 1 such that $\log_a x$, $\log_b x$ and $\log_c x$ are in A.P., then

(A) $c^2 = (a.c)^{\log_a b}$; (B) $b = \frac{a+c}{2}$; (C) $b = \sqrt{a.c}$;
(D) none of the foregoing equations is necessarily true.

230. Given that $\log_{10} 5 = 0.70$ and $\log_{10} 3 = 0.48$, the value of $\log_{30} 8$ (correct upto 2 places of decimal) is

(A) 0.56; (B) 0.61; (C) 0.68; (D) 0.73.

231. If x is a real number and $y = \frac{1}{2}(e^x - e^{-x})$, then

(A) x can be either $\log(y + \sqrt{y^2+1})$ or $\log(y - \sqrt{y^2+1})$;
(B) x can only be $\log(y + \sqrt{y^2+1})$;
(C) x can be either $\log(y + \sqrt{y^2-1})$ or $\log(y - \sqrt{y^2-1})$;
(D) x can only be $\log(y + \sqrt{y^2-1})$.

232. A solution to the system of equations

$$ax + by + cz = 0 \text{ and } a^2x + b^2y + c^2z = 0$$

is

(A) $x = a(b-c), y = b(c-a), z = c(a-b)$;
(B) $x = \frac{k(b-c)}{a^2}, y = \frac{k(c-a)}{b^2}, z = \frac{k(a-b)}{c^2}$, where k is an arbitrary constant;
(C) $x = \frac{b-c}{bc}, y = \frac{c-a}{ca}, z = \frac{a-b}{ab}$;
(D) $x = \frac{k(b-c)}{a}, y = \frac{k(c-a)}{b}, z = \frac{k(a-b)}{c}$, where k is an arbitrary constant.

233. $(x+y+z)(yz+zx+xy) - xyz$ equals

(A) $(y+z)(z+x)(x+y)$; (B) $(y-z)(z-x)(x-y)$;
(C) $(x+y+z)^2$; (D) none of the foregoing expressions, in general.

234. The number of points at which the curve $y = x^6 + x^3 - 2$ cuts the x-axis is

(A) 1; (B) 2; (C) 4; (D) 6.

235. Suppose $a+b+c$ and $a-b+c$ are positive and $c<0$. Then the equation $ax^2+bx+c=0$

(A) has exactly one root lying between -1 and $+1$;
(B) has both the roots lying between -1 and $+1$;
(C) has no root lying between -1 and $+1$;
(D) nothing definite can be said about the roots without knowing the values of a, b, and c.

236. Number of real roots of the equation $8x^3-6x+1=0$ lying between -1 and 1, is

(A) 0; (B) 1; (C) 2; (D) 3.

237. The equation $\frac{x^3+7}{x^2+1}=5$ has

(A) no solution in $[0,2]$; (B) exactly two solutions in $[0,2]$;
(C) exactly one solution in $[0,2]$; (D) exactly three solutions in $[0,2]$.

238. The roots of the equation $2x^2-6x-5\sqrt{x^2-3x-6}=10$ are

(A) $\frac{3}{2}\pm\frac{1}{2}\sqrt{41}$, $\frac{3}{2}\pm\frac{1}{2}\sqrt{35}$; (B) $3\pm\sqrt{41}$, $3\pm\sqrt{35}$;
(C) -2, 5, $\frac{3}{2}\pm\frac{1}{2}\sqrt{34}$; (D) -2, 5, $3\pm\sqrt{34}$.

239. Suppose that the roots of the equation $ax^2+b\lambda x+\lambda=0$ (where a and b are given real numbers) are real for all positive values of λ. Then we must have
(A) $a\geq 0$; (B) $a=0$; (C) $b^2\geq 4a$; (D) $a\leq 0$.

240. The equations $x^2+x+a=0$ and $x^2+ax+1=0$

(A) cannot have a common real root for any value of a;
(B) have a common real root for exactly one value of a;
(C) have a common real root for exactly two values of a;
(D) have a common real root for exactly three values of a.

241. It is given that the expression ax^2+bx+c takes positive values for all x greater than 5. Then

(A) the equation $ax^2+bx+c=0$ has equal roots;
(B) $a>0$ and $b<0$;
(C) $a>0$, but b may or may not be negative;
(D) $c>5$.

242. The roots of the equation $\frac{1}{2}x^2 + bx + c = 0$ are integers if

(A) $b^2 - 2c > 0$; (B) $b^2 - 2c$ is the square of an integer and b is an integer; (C) b and c are integers; (D) b and c are even integers.

243. Consider the quadratic equation $(a + c - b)x^2 + 2cx + (b + c - a) = 0$, where a, b, c are distinct real numbers and $a + c - b \neq 0$. Suppose that both the roots of the equation are rational. Then

(A) a, b and c are rational; (B) $c/(a - b)$ is rational; (C) $b/(c - a)$ is rational; (D) $a/(b - c)$ is rational.

244. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

(A) $x^2 + x + 1 = 0$; (B) $x^2 - x + 1 = 0$; (C) $x^2 - x - 1 = 0$; (D) $x^2 + x - 1 = 0$.

245. If α, β are the roots of $ax^2 + bx + c = 0$, then the equation whose roots are α^2, β^2 is

(A) $a^2x^2 + (b^2 - 2ac)x + c^2 = 0$;
(B) $a^2x^2 - (b^2 + 2ac)x + c^2 = 0$;
(C) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$;
(D) none of the foregoing equations.

246. Suppose that the equation $ax^2 + bx + c = 0$ has roots α and β, both of which are different from $\frac{1}{2}$. Then an equation whose roots are $\frac{1}{2\alpha-1}$ and $\frac{1}{2\beta-1}$ is

(A) $(a + 2b + 4c)x^2 + 2(a + b)x + a = 0$;
(B) $4cx^2 + 2(b - 2c)x + (a - b + c) = 0$;
(C) $cx^2 + 2(a + b)x + (a + 2b + 4c) = 0$;
(D) none of the foregoing equations.

247. If α and β are roots of the equation $x^2 + 5x - 5 = 0$, then $(\frac{1}{\alpha+1})^3 + (\frac{1}{\beta+1})^3$ equals

(A) -322; (B) $\frac{4}{27}$; (C) $-\frac{4}{27}$; (D) $3 + \sqrt{5}$.

248. If α is a positive integer and the roots of the equation $6x^2 - 11x + \alpha = 0$ are rational numbers, then the smallest value of α is

(A) 4; (B) 5; (C) 6; (D) none of the foregoing numbers.

249. $P(x)$ is a quadratic polynomial whose values at $x = 1$ and at $x = 2$ are equal in magnitude but opposite in sign. If -1 is a root of the equation $P(x) = 0$, then the other root is

(A) $\frac{8}{5}$; (B) $\frac{7}{6}$; (C) $\frac{13}{7}$; (D) none of the foregoing numbers.

250. If $4x^{10} - x^9 - 3x^8 + 5x^7 + kx^6 + 2x^5 - x^3 + kx^2 + 5x - 5$, when divided by $(x+1)$ gives a remainder of -14, then the value of k equals

(A) 2; (B) 0; (C) 7; (D) -2.

251. A polynomial $f(x)$ with real coefficients leaves the remainder 15 when divided by $x-3$, and the remainder $2x+1$ when divided by $(x-1)^2$. Then the remainder when $f(x)$ is divided by $(x-3)(x-1)^2$ is

(A) $2x^2 - 2x + 3$; (B) $6x - 3$; (C) $x^2 + 2x$; (D) $3x + 6$.

252. The remainder obtained when the polynomial $x + x^3 + x^9 + x^{27} + x^{81} + x^{2^{243}}$ is divided by $x^2 - 1$ is
(A) $6x + 1$; (B) $5x + 1$; (C) $4x$; (D) $6x$.

253. Let $(1 + x + x^2)^9 = \alpha_0 + \alpha_1 x + \ldots + \alpha_{18} x^{18}$. Then

(A) $\alpha_0 + \alpha_2 + \ldots + \alpha_{18} = \alpha_1 + \alpha_3 + \ldots + \alpha_{17}$;
(B) $\alpha_0 + \alpha_2 + \ldots + \alpha_{18}$ is even;
(C) $\alpha_0 + \alpha_2 + \ldots + \alpha_{18}$ is divisible by 9;
(D) $\alpha_0 + \alpha_2 + \ldots + \alpha_{18}$ is divisible by 3 but not by 9.

254. The minimum value of $x^8 - 8x^6 + 19x^4 - 12x^3 + 14x^2 - 8x + 9$ is

(A) -1; (B) 9; (C) 6; (D) 1.

255. The cubic expression in x, which takes the value zero when $x = 1$ and $x = -2$, and takes values -800 and 28 when $x = -7$ and $x = 2$ respectively, is

(A) $3x^3 + 2x^2 - 7x + 2$; (B) $3x^3 + 4x^2 - 5x - 2$;
(C) $2x^3 + 3x^2 - 3x - 2$; (D) $2x^3 + x^2 - 5x + 2$.

256. If $f(x)$ is a polynomial in x and a, b are distinct real numbers, then the remainder in the division of $f(x)$ by $(x-a)(x-b)$ is

(A) $\frac{(x-a)f(a)-(x-b)f(b)}{a-b}$; (B) $\frac{(x-a)f(b)-(x-b)f(a)}{b-a}$;
(C) $\frac{(x-a)f(b)-(x-b)f(a)}{a-b}$; (D) $\frac{(x-a)f(a)-(x-b)f(b)}{b-a}$.

257. The number of real roots of $x^5 + 2x^3 + x^2 + 2 = 0$ is

(A) 0; (B) 3; (C) 5; (D) 1.

258. Let a, b, c be distinct real numbers. Then the number of real solutions of $(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$ is

(A) 1; (B) 2; (C) 3; (D) depends on a, b, c.

259. Let a, b and c be real numbers. Then the fourth degree polynomial in x

$$acx^4 + b(a+c)x^3 + (a^2+b^2+c^2)x^2 + b(a+c)x + ac$$

(A) has four complex (non-real) roots;
(B) has either four real roots or four complex roots;
(C) has two real roots and two complex roots;
(D) has four real roots.

260. Let $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + bx + c$, where $ac \neq 0$. Consider the polynomial $P(x)Q(x)$.

(A) All its roots are real;
(B) None of its roots is real;
(C) At least two of its roots are real;
(D) Exactly two of its roots are real.

261. For the roots of the quadratic equation $x^2 + bx - 4 = 0$ to be integers

(A) it is sufficient that $b = 0, \pm 3$;
(B) it is sufficient that $b = 0, \pm 2$;
(C) it is sufficient that $b = 0, \pm 4$;
(D) none of the foregoing conditions is sufficient.

262. The smallest positive solution of the equation

$$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$$

is

(A) $\frac{\pi}{12}$; (B) $\frac{\pi}{6}$; (C) $\frac{\pi}{8}$; (D) is none of the foregoing quantities.

263. If α and β are the roots of the equation $x^2 + ax + b = 0$, where $b \neq 0$, then the roots of the equation $bx^2 + ax + 1 = 0$ are

(A) $\frac{1}{\alpha}, \frac{1}{\beta}$; (B) α^2, β^2; (C) $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$; (D) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$.

264. A necessary and sufficient condition for the quadratic function $ax^2 + bx + c$ to take both positive and negative values is

(A) $ab \neq 0$; (B) $b^2 - 4ac > 0$; (C) $b^2 - 4ac \geq 0$;
(D) none of the foregoing statements.

265. The quadratic equation $x^2 + bx + c = 0$ (b, c real numbers) has both roots real and positive, if and only if

(A) $b < 0$ and $c > 0$; (B) $bc < 0$ and $b \geq 2\sqrt{c}$;
(C) $bc < 0$ and $b^2 \geq 4c$; (D) $c > 0$ and $b \leq -2\sqrt{c}$.

266. If the equation $ax^2 + bx + c = 0$ has a root less than -2 and a root greater than 2, and if $a > 0$, then

(A) $4a + 2|b| + c < 0$; (B) $4a + 2|b| + c > 0$; (C) $4a + 2|b| + c = 0$;
(D) none of the foregoing statements need always be true.

267. Which of the following is a square root of $21 - 4\sqrt{5} + 8\sqrt{3} - 4\sqrt{15}$?

(A) $2\sqrt{3} - 2 - \sqrt{5}$; (B) $\sqrt{5} - 3 + 2\sqrt{3}$;
(C) $2\sqrt{3} - 2 + \sqrt{5}$; (D) $2\sqrt{3} + 2 - \sqrt{5}$.

268. If $x > 1$ and $x + x^{-1} < \sqrt{5}$, then

(A) $2x < \sqrt{5} + 1$, $2x^{-1} > \sqrt{5} - 1$;
(B) $2x < \sqrt{5} + 1$, $2x^{-1} < \sqrt{5} - 1$;
(C) $2x > \sqrt{5} + 1$, $2x^{-1} < \sqrt{5} - 1$;
(D) none of the foregoing pair of inequalities hold.

269. If the roots of

$$\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$$

are equal in magnitude but opposite in sign, then the product of the roots is

(A) $-\frac{a^2+b^2}{2}$; (B) $-\frac{a^2+b^2}{4}$; (C) $\frac{a+b}{2}$; (D) $\frac{a^2+b^2}{2}$.

270. If α, β are the roots of $x^2 + x + 1 = 0$, then the equation whose roots are α^k, β^k, where k is a positive integer not divisible by 3, is

(A) $x^2 - x + 1 = 0$; (B) $x^2 + x + 1 = 0$; (C) $x^2 - x - 1 = 0$;
(D) none of the foregoing equations.

271. If α and β are the roots of the quadratic equation $x^2 + x + 1 = 0$, then the equation whose roots are $\alpha^{2000}, \beta^{2000}$ is

(A) $x^2 + x - 1 = 0$; (B) $x^2 + x + 1 = 0$;
(C) $x^2 - x + 1 = 0$; (D) $x^2 - x - 1 = 0$.

272. If α, β, γ are the roots of $x^3 + 2x^2 + 3x + 3 = 0$, then the value of

$$\left(\frac{\alpha}{\alpha+1}\right)^3 + \left(\frac{\beta}{\beta+1}\right)^3 + \left(\frac{\gamma}{\gamma+1}\right)^3$$

is

(A) 18; (B) 44; (C) 13; (D) none of the foregoing numbers.

273. $a \pm bi$ ($b \neq 0$, $i = \sqrt{-1}$) are complex roots of the equation $x^3 + qx + r = 0$, where a, b, q and r are real numbers. Then q in terms of a and b is

(A) $a^2 - b^2$; (B) $b^2 - 3a^2$; (C) $a^2 + b^2$; (D) $b^2 - 2a^2$.

274. Let α, β, γ be the roots of $x^3 - x - 1 = 0$. Then the equation whose roots are

$$\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$$

is given by

(A) $x^3 + 7x^2 - x + 1 = 0$; (B) $x^3 - 7x^2 - x + 1 = 0$;
(C) $x^3 + 7x^2 + x - 1 = 0$; (D) $x^3 + 7x^2 - x - 1 = 0$.

275. Let 1, ω and ω^2 be the cube roots of unity. The least possible degree of a polynomial with real coefficients, having 2ω, $2 + 3\omega$, $2 + 3\omega^2$ and $2 - \omega - \omega^2$ as roots is

(A) 4; (B) 5; (C) 6; (D) 8.

276. Let x_1 and x_2 be the roots of the equation $x^2 - 3x + a = 0$, and let x_3 and x_4 be the roots of the equation $x^2 - 12x + b = 0$. If $x_1 < x_2 < x_3 < x_4$ are in G.P., then $a \cdot b$ equals

(A) 5184; (B) 64; (C) -5184; (D) -64.

277. If $x = \frac{3+5\sqrt{-1}}{2}$ is a root of the equation $2x^3 + ax^2 + bx + 68 = 0$ where a, b are real numbers, then which of the following is also a root?

(A) $\frac{5+3\sqrt{-1}}{2}$; (B) -8; (C) -4;
(D) can not be answered without knowing the values of a and b.

278. If the equation $6x^3 - ax^2 + 6x - 1 = 0$ has three real roots α, β and γ such that $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$ are in Arithmetic Progression, then the value of a is

(A) 9; (B) 10; (C) 11; (D) 12.

279. Let x, y and z be real numbers. Then only one of the following statements is true. Which one is it?

(A) If $x < y$, then $xz < yz$ for all values of z.
(B) If $x < y$, then $\frac{x}{z} < \frac{y}{z}$ for all values of z.
(C) If $x < y$, then $(x + z) < (y + z)$ for all values of z.
(D) If $0 < x < y$, then $xz < yz$ for all values of z.

280. If $x + y + z = 0$ and $x^3 + y^3 + z^3 - kxyz = 0$, then only one of the following is true. Which one is it?

(A) $k = 3$ whatever be x, y and z.
(B) $k = 0$ whatever be x, y and z.
(C) k can be only one of the numbers $+1, -1, 0$.
(D) If none of x, y, z is zero, then $k = 3$.

281. For real numbers x and y, if $x^2 + xy - y^2 + 2x - y + 1 = 0$, then

(A) y cannot be between 0 and $\frac{8}{5}$;
(B) y cannot be between $-\frac{8}{5}$ and $\frac{8}{5}$;
(C) y cannot be between $-\frac{8}{5}$ and 0;
(D) none of the foregoing statements is correct.

282. It is given that the expression $ax^2 + bx + c$ takes negative values for all $x < 7$. Then

(A) the equation $ax^2 + bx + c = 0$ has equal roots;
(B) a is negative;
(C) a and b are both negative;
(D) none of the foregoing statements is correct.

283. The coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are 45, 120 and 210. Then the value of n is

(A) 8; (B) 12; (C) 10; (D) none of the foregoing numbers.

284. The polynomials

$$x^5 - 5x^4 + 7x^3 + ax^2 + bx + c \quad \text{and} \quad 3x^3 - 15x^2 + 18x$$

have three common roots. Then the values of a, b and c are

(A) $c = 0$ and a and b are arbitrary;
(B) $a = -5, b = 6$ and $c = 0$;
(C) $a = -\frac{5}{6}b, b$ arbitrary, $c = 0$;
(D) none of the foregoing statements.

285. The equations $x^3 + 2x^2 + 2x + 1 = 0$ and $x^{200} + x^{130} + 1 = 0$ have

(A) exactly one common root; (B) no common root;
(C) exactly three common roots; (D) exactly two common roots.

286. For any integer $p \geq 3$, the largest integer r, such that $(x-1)^r$ is a factor of the polynomial $2x^{p+1} - p(p+1)x^2 + 2(p^2-1)x - p(p-1)$, is

(A) p; (B) 4; (C) 1; (D) 3.

287. When $4x^{10} - x^9 + 3x^8 - 5x^7 + cx^6 + 2x^5 - x^4 + x^3 - 4x^2 + 6x - 2$ is divided by $(x-1)$, the remainder is $+2$. The value of c is

(A) $+2$; (B) $+1$; (C) 0; (D) -1.

288. The remainder $R(x)$ obtained by dividing the polynomial x^{100} by the polynomial $x^2 - 3x + 2$ is

(A) $2^{100} - 1$; (B) $(2^{100} - 1)x - 2(2^{99} - 1)$
(C) $2^{100}x - 3 \cdot 2^{100}$; (D) $(2^{100} - 1)x + 2(2^{99} - 1)$.

289. If $3x^4 - 6x^3 + kx^2 - 8x - 12$ is divisible by $x - 3$ then it is also divisible by

(A) $3x^2 - 4$ (B) $3x^2 + 4$ (C) $3x^2 + x$ (D) $3x^2 - x$.

290. The number of integers x such that $2^{2x} - 3(2^{x+2}) + 2^5 = 0$ is

(A) 0; (B) 1; (C) 2; (D) none of the foregoing numbers.

291. If the roots of the equation $(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$, (where a, b, c are real numbers) are equal, then

(A) $b^2 - 4ac = 0$; (B) $a = b = c$; (C) $a + b + c = 0$;
(D) none of the foregoing statements is correct.

292. Suppose that a, b, c are three distinct real numbers. The expression
$\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} - 1$
takes the value zero for

(A) no real x;
(B) exactly two distinct real x;
(C) exactly three distinct real x;
(D) more than three real x.

293. If $| x^2 - 7x + 12 | > x^2 - 7x + 12$, then

(A) $x \leq 3$ or $x \geq 4$; (B) $3 \leq x \leq 4$; (C) $3 < x < 4$;
(D) x can take any value except 3 and 4.

294. The real numbers x such that $x^2 + 4|x| - 4 = 0$ are

(A) $-2 \pm \sqrt{8}$; (B) $2 \pm \sqrt{8}$; (C) $-2 \pm \sqrt{8}, 2 \pm \sqrt{8}$; (D) $\pm(\sqrt{8} - 2)$.

295. The number of *distinct* real roots of the equation $|x^2 + x - 6| - 3x + 7 = 0$ is

(A) 0; (B) 2; (C) 3; (D) 4.

296. If a is strictly negative and is not equal to -2, then the equation

$$x^2 + a|x| + 1 = 0$$

(A) cannot have any real roots;
(B) must have either exactly four real roots or no real roots;
(C) must have exactly two real roots;
(D) must have either exactly two real roots or no real roots.

297. The angles of a triangle are in A.P. and the ratio of the greatest to the smallest angle is 3:1. Then the smallest angle is

(A) $\frac{\pi}{6}$; (B) $\frac{\pi}{3}$; (C) $\frac{\pi}{4}$; (D) none of the foregoing angles.

298. Let $x_1, x_2, \ldots$ be positive integers in A.P., such that $x_1 + x_2 + x_3 = 12$ and $x_4 + x_6 = 14$. Then x_5 is

(A) 7; (B) 1; (C) 4; (D) none of the foregoing numbers.

299. The sum of the first m terms of an Arithmetic Progression is n and the sum of the first n terms is m, where $m \neq n$. Then the sum of the first $m + n$ terms is

(A) 0; (B) $m + n$; (C) $-mn$; (D) $-m - n$.

300. In an A.P., suppose that, for some $m \neq n$, the ratio of the sum of the first m terms to the sum of the first n terms is $\frac{m^2}{n^2}$. If the 13th term of the A.P. is 50, then the 26th term of the A.P. is

(A) 75; (B) 76; (C) 100; (D) 102.

301. Let $S_n, n \geq 1$, be the sets defined as follows:

$$S_1 = \{0\}, \quad S_2 = \{\tfrac{3}{2}, \ \tfrac{5}{2}\}, \quad S_3 = \{\tfrac{8}{3}, \tfrac{11}{3}, \tfrac{14}{3}\}, \quad S_4 = \{\tfrac{15}{4}, \tfrac{19}{4}, \tfrac{23}{4}, \tfrac{27}{4}\},$$

and so on. Then, the sum of the elements of S_{20} is

(A) 589; (B) 609; (C) 189; (D) 209.

302. The value of $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + 99 \cdot 100$ equals

(A) 333000; (B) 333300; (C) 30330; (D) 33300.

303. The value of $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \ldots + 20 \cdot 21 \cdot 22$ equals

(A) 51330; (B) 53130; (C) 53310; (D) 35130.

304. Six numbers are in A.P. such that their sum is 3. The first number is four times the third number. The fifth number is equal to

(A) -15; (B) -3; (C) 9; (D) -4.

305. The sum of the first n terms $(n > 1)$ of an A.P. is 153 and the common difference is 2. If the first term is an integer, the number of possible values of n is

(A) 3; (B) 4; (C) 5; (D) 6.

306. Six numbers are in G.P. such that their product is 512. If the fourth number is 4, then the second number is

(A) $\frac{1}{2}$; (B) 1; (C) 2; (D) none of the foregoing numbers.

307. Let a and b be positive integers with no common factors. Then

(A) $a + b$ and $a - b$ have no common factor other than 3, whatever be a and b;
(B) $a + b$ and $a - b$ have no common factor greater than 2, whatever be a and b;
(C) $a + b$ and $a - b$ have a common factor, whatever be a and b;
(D) none of the foregoing statements is correct.

308. If positive numbers a, b, c, d are in harmonic progression and $a \neq b$, then

(A) $a + d > b + c$ is always true;
(B) $a + b > c + d$ is always true;
(C) $a + c > b + d$ is always true;
(D) none of the foregoing statements is always true.

309. The sum of the series $1 + 11 + 111 + \ldots$ to n terms is

(A) $\frac{1}{9}[\frac{10}{9}(10^n - 1) + n]$; (B) $\frac{1}{9}[\frac{10}{9}(10^n - 1) - n]$;
(C) $\frac{10}{9}[\frac{1}{9}(10^n - 1) - n]$; (D) $\frac{10}{9}[\frac{1}{9}(10^n - 1) + n]$.

310. Two men set out at the same time to walk towards each other from points A and B, 72 km apart. The first man walks at the rate of 4 km per hour. The second man walks 2 km the first hour, $2\frac{1}{2}$ km the second hour, 3 km the third hour, and so on. Then the men will meet

(A) in 7 hours; (B) nearer A than B;
(C) nearer B than A; (D) midway between A and B.

311. The second term of a geometric progression (of positive numbers) is 54 and the fourth term is 24. Then the fifth term is

(A) 12; (B) 15; (C) 16; (D) none of the foregoing numbers.

312. Consider an arithmetic progression whose first term is 4 and the common difference is -0.1. Let s_n stand for the sum of the first n terms. Suppose r is a number such that $s_n = r$ for some n. Then the number of *other* values of n for which $s_n = r$ is

(A) 0 or 1; (B) 0; (C) 1; (D) > 1.

313. The three sides of a right-angled triangle are in G.P. The tangents of the two acute angles are

(A) $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$; (B) $\sqrt{\frac{(\sqrt{5}+1)}{2}}$ and $\sqrt{\frac{(\sqrt{5}-1)}{2}}$;
(C) $\sqrt{5}$ and $\frac{1}{\sqrt{5}}$; (D) none of the foregoing pairs of numbers.

314. The m^{th} term of an arithmetic progression is x and the n^{th} term is y. Then the sum of the first $(m + n)$ terms is

(A) $\frac{m+n}{2}[(x + y) + \frac{x-y}{m-n}]$; (B) $\frac{m+n}{2}[(x - y) + \frac{x+y}{m-n}]$;
(C) $\frac{1}{2}[\frac{x+y}{m+n} + \frac{x-y}{m-n}]$; (D) $\frac{1}{2}[\frac{x+y}{m+n} - \frac{x-y}{m-n}]$.

315. The time required for any initial amount of a radioactive substance to decrease to half that amount is called the *half-life* of that substance. For example, radium has a half-life of 1620 years. If 1 gm of radium is taken in a capsule, then after 4860 years, the amount of radium left in the capsule will be, in gm,

(A) $\frac{1}{3}$; (B) $\frac{1}{4}$; (C) $\frac{1}{6}$; (D) $\frac{1}{8}$.

316. The sum of all the numbers between 200 and 400 which are divisible by 7 is

(A) 9872; (B) 7289; (C) 8729; (D) 8279.

317. The sum of the series $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots - 100^2$ is

(A) -10100; (B) -5050; (C) -2525; (D) none of the foregoing numbers.

318. $x_1, x_2, x_3, \ldots$ is an infinite sequence of positive integers in G.P., such that $x_1x_2x_3x_4 = 64$. Then the value of x_5 is

(A) 4; (B) 64; (C) 128; (D) 16.

319. The value of $100[\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{99\cdot 100}]$

(A) is 99; (B) lies between 50 and 98; (C) is 100;
(D) is different from values specified in the foregoing statements.

320. The value of $1 \cdot 3 \cdot 5 + 3 \cdot 5 \cdot 7 + 5 \cdot 7 \cdot 9 + \ldots + 17 \cdot 19 \cdot 21$ equals

(A) 12270; (B) 17220; (C) 12720; (D) 19503.

321. The sum

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + 50 \cdot 50!$$

equals

(A) $51!$; (B) $2 \cdot 51!$; (C) $51! - 1$; (D) $51! + 1$.

322. The value of

$$\frac{1}{1 \cdot 3 \cdot 5} + \frac{1}{3 \cdot 5 \cdot 7} + \frac{1}{5 \cdot 7 \cdot 9} + \frac{1}{7 \cdot 9 \cdot 11} + \frac{1}{9 \cdot 11 \cdot 13}$$

equals

(A) $\frac{70}{249}$; (B) $\frac{53}{249}$; (C) $\frac{35}{429}$; (D) $\frac{35}{249}$.

323. The value of $\frac{1}{1.2.3.4} + \frac{1}{2.3.4.5} + \frac{1}{3.4.5.6} + \ldots + \frac{1}{9.10.11.12}$ is

(A) $\frac{73}{1320}$; (B) $\frac{733}{11880}$; (C) $\frac{73}{440}$; (D) $\frac{1}{18}$.

324. The value of

$$\frac{1}{1 \cdot 3 \cdot 5} + \frac{1}{3 \cdot 5 \cdot 7} + \ldots + \frac{1}{11 \cdot 13 \cdot 15}$$

equals

(A) $\frac{32}{195}$; (B) $\frac{16}{195}$; (C) $\frac{64}{195}$; (D) none of the foregoing numbers.

325. The value of

$$\frac{1\cdot 2}{3!}+\frac{2\cdot 2^2}{4!}+\frac{3\cdot 2^3}{5!}+\cdots\cdots+\frac{15\cdot 2^{15}}{(17)!}$$

equals

(A) $2-\frac{16\cdot 2^{17}}{(17)!}$; (B) $2-\frac{2^{17}}{(17)!}$; (C) $1-\frac{16\cdot 2^{17}}{(17)!}$; (D) $1-\frac{2^{16}}{(17)!}$.

326. The value of $4^2+2\cdot 5^2+3\cdot 6^2+\cdots+27\cdot 30^2$ is

(A) 187854; (B) 187860; (C) 187868; (D) 187866.

327. The distances passed over by a pendulum bob in successive swings are 16, 12, 9, 6.75, ... cm. Then the total distance traversed by the bob before it comes to rest is (in cm)

(A) 60; (B) 64; (C) 65; (D) 67.

328. In a sequence $a_1, a_2, \ldots$ of real numbers it is observed that $a_p=\sqrt{2}, a_q=\sqrt{3}$ and $a_r=\sqrt{5}$, where $1\le p<q<r$ are positive integers. Then a_p, a_q, a_r can be terms of

(A) an arithmetic progression; (B) a harmonic progression;
(C) an arithmetic progression if and only if p, q and r are perfect squares;
(D) neither an arithmetic progression nor an harmonic progression.

329. Suppose a, b, c are in G.P. and $a^p=b^q=c^r$. Then

(A) p, q, r are in G.P.; (B) p, q, r are in A.P.; (C) $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in A.P.;
(D) none of the foregoing statements is true.

330. Three real numbers a, b, c are such that a^2, b^2, c^2 are terms of an arithmetic progression. Then

(A) a, b, c are terms of a geometric progression.
(B) $(b+c), (c+a), (a+b)$ are terms of an arithmetic progression.
(C) $(b+c), (c+a), (a+b)$ are terms of a harmonic progression.
(D) none of the foregoing statements is necessarily true.

331. If a, b, c, d and p are distinct real numbers such that

$$(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\le 0,$$

then
(A) a, b, c and d are in H.P.; (B) ab, bc and cd are in A.P.;
(C) a, b, c and d are in A.P.; (D) a, b, c and d are in G.P.

332. Let n quantities be in A.P., d being the common difference. Let the arithmetic mean of the squares of these quantities exceed the square of the arithmetic mean of these quantities by a quantity p. Then p

(A) is always negative; (B) equals $\frac{n^2-1}{12}d^2$;
(C) equals $\frac{d^2}{12}$; (D) equals $\frac{n^2-1}{12}$.

333. Suppose that $F(n+1) = \frac{2F(n)+1}{2}$ for $n = 1, 2, 3, \ldots$ and $F(1) = 2$. Then $F(101)$ equals

(A) 50; (B) 52; (C) 54; (D) none of the foregoing quantities.

334. Let $\{F_n\}$ be the sequence of numbers defined by $F_1 = 1 = F_2$; $F_{n+1} = F_n + F_{n-1}$ for $n \geq 2$. Let f_n be the remainder left when F_n is divided by 5. Then f_{2000} equals

(A) 0; (B) 1; (C) 2; (D) 3.

335. Consider the two arithmetic progressions 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709. The number of common terms of these two progressions is

(A) 0; (B) 7; (C) 15; (D) 14.

336. The arithmetic mean of two positive numbers is $18\frac{3}{4}$ and their geometric mean is 15. The larger of the two numbers is

(A) 24; (B) 25; (C) 20; (D) 30.

337. The difference between the roots of the equation $6x^2 + \alpha x + 1 = 0$ is $\frac{1}{6}$. Further, α is a positive number. Then the value of α is

(A) 3; (B) 4; (C) 5; (D) $2\frac{1}{3}$.

338. If $4^x - 4^{x-1} = 24$, then $(2x)^x$ equals

(A) $5\sqrt{5}$; (B) $25\sqrt{5}$; (C) 125; (D) 25.

339. The number of solutions of the simultaneous equations

$$y = 3\log_e x, y = \log_e(3x)$$

is

(A) 0; (B) 1; (C) 3; (D) infinite.

340. The number of solutions to the system of simultaneous equations $|z+1-i| = \sqrt{2}$ and $|z| = 3$ is

(A) 0; (B) 1; (C) 2; (D) > 2.

341. The number of pairs (x, y) of real numbers that satisfy

$$2x^2 + y^2 + 2xy - 2y + 2 = 0$$

is

(A) 0; (B) 1; (C) 2; (D) none of the foregoing numbers.

342. Consider the following equation in x and y : $(x - 2y - 1)^2 + (4x + 3y - 4)^2 + (x - 2y - 1)(4x + 3y - 4) = 0$.
How many solutions to (x, y) with x, y real, does the equation have?

(A) none; (B) exactly one; (C) exactly two; (D) more than two.

343. Let x and y be positive numbers and let a and b be real numbers, positive or negative. Suppose that $x^a = y^b$ and $y^a = x^b$. Then we can conclude that

(A) $a = b$ and $x = y$; (B) $a = b$ but x need not be equal to y;
(C) $x = y$ but a need not be equal to b; (D) $a = b$ if $x \neq y^{-1}$.

344. On a straight road XY, 100 metres long, 15 heavy stones are placed one metre apart beginning at the end X. A worker, starting at X, has to transport all the stones to Y, by carrying only one stone at a time. The minimum distance he has to travel is (in km)

(A) 1.395; (B) 2.79; (C) 2.69; (D) 1.495.

345. $\lim_{n\to\infty}[\frac{1}{1\cdot 3} + \frac{1}{2\cdot 4} + \frac{1}{3\cdot 5} + \ldots + \frac{1}{n(n+2)}]$ is

(A) 0; (B) $\frac{3}{2}$; (C) $\frac{1}{2}$; (D) $\frac{3}{4}$.

346. $\lim_{n\to\infty}[\frac{1\cdot 3}{2n^3} + \frac{3\cdot 5}{2n^3} + \ldots + \frac{(2n-1)(2n+1)}{2n^3}]$ is

(A) $\frac{2}{3}$; (B) $\frac{1}{3}$; (C) 0; (D) 2.

347. The coefficient of x^n in the expansion of $\frac{2-3x}{1-3x+2x^2}$ is

(A) $(-3)^n - (2)^{\frac{1}{2}n-1}$; (B) $2^n + 1$; (C) $3(2)^{\frac{1}{2}n-1} - 2(3)^n$;
(D) none of the foregoing numbers.

348. The infinite sum

$$1+\frac{1}{3}+\frac{1\cdot 3}{3\cdot 6}+\frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9}+\frac{1\cdot 3\cdot 5\cdot 7}{3\cdot 6\cdot 9\cdot 12}+\ldots$$

is

(A) $\sqrt{2}$; (B) $\sqrt{3}$; (C) $\sqrt{\frac{3}{2}}$; (D) $\sqrt{\frac{1}{3}}$.

349. The sum of the infinite series

$$1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\frac{1+2+3+4}{4!}+\ldots$$

is

(A) $\frac{3e}{2}$; (B) $\frac{3e}{4}$; (C) $\frac{3(e+e^{-1})}{2}$; (D) e^2-e.

350. For a nonzero number x, if

$$y=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots \text{ and } z=-y-\frac{y^2}{2}-\frac{y^3}{3}-\cdots$$

then the value of $\log_e(\frac{1}{1-e^z})$ is

(A) $1-x$; (B) $\frac{1}{x}$; (C) $1+x$; (D) x.

351. For a given real number $\alpha>0$, define $a_n=(1^\alpha+2^\alpha+\ldots+n^\alpha)^n$ and $b_n=n^n(n!)^\alpha$, for $n=1,2,\ldots$. Then

(A) $a_n<b_n$ for all $n>1$.
(B) there exists an integer $n>1$ such that $a_n<b_n$.
(C) $a_n>b_n$ for all $n>1$.
(D) there exist integers n and m both larger than one such that $a_n>b_n$ and $a_m<b_m$.

352. Let $a_n=\frac{10^{n+1}+1}{10^n+1}$ for $n=1,2,\cdots$. Then

(A) for every n, $a_n\geq a_{n+1}$;
(B) for every n, $a_n\leq a_{n+1}$;
(C) there is an integer k such that $a_{n+k}=a_n$ for all n;
(D) none of the above holds.

353. Let a, b and c be fixed positive real numbers. Let $u_n = \frac{na}{b+nc}$ for $n \geq 1$. Then as n increases,

(A) u_n increases;
(B) u_n decreases;
(C) u_n increases first and then decreases;
(D) none of the foregoing statements is necessarily true.

354. Suppose n is a positive integer. Then the least value of N for which

$$\left| \frac{n^2+n+1}{3n^2+1} - \frac{1}{3} \right| < \frac{1}{10},$$

when $n \geq N$, is

(A) 4; (B) 5; (C) 100; (D) 1000.

355. The maximum value of xyz for positive x, y, z, subject to the condition $xy + yz + zx = 12$ is

(A) 9; (B) 6; (C) 8; (D) 12.

356. If a, b are positive real numbers satisfying $a^2 + b^2 = 1$, then the minimum value of $a + b + \frac{1}{ab}$ is

(A) 2; (B) $2 + \sqrt{2}$; (C) 3; (D) $1 + \sqrt{2}$.

357. Let M and m be, respectively, the maximum and the minimum of n arbitrary real numbers $x_1, x_2, \ldots, x_n$. Further, let M' and m' denote the maximum and the minimum, respectively, of the following numbers:

$$x_1, \frac{x_1+x_2}{2}, \frac{x_1+x_2+x_3}{3}, \ldots, \frac{x_1+x_2+\ldots+x_n}{n}.$$

Then

(A) $m \leq m' \leq M \leq M'$; (B) $m \leq m' \leq M' \leq M$;
(C) $m' \leq m \leq M' \leq M$; (D) $m' \leq m \leq M \leq M'$.

358. A stick of length 20 units is to be divided into n parts so that the product of the lengths of the parts is greater than unity. The maximum possible value of n is

(A) 18; (B) 20; (C) 19; (D) 21.

359. It is given that the numbers $a \geq 0,\ b \geq 0,\ c \geq 0$ are such that

$$a + b + c = 4$$

$$\text{and} \qquad (a+b)(b+c)(c+a) = 24.$$

Then only one of the following statements is correct. Which one is it?

(A) More information is needed to determine the values of a, b and c.
(B) Even when a is given to be 1, more information is needed to determine the values of b and c.
(C) These two equations are inconsistent.
(D) There exist values of a and b from which the value of c could be determined.

360. Let a, b, c be any real numbers such that $a^2 + b^2 + c^2 = 1$. Then the quantity $(ab + bc + ca)$ satisfies the condition(s)

(A) $(ab + bc + ca)$ is constant;
(B) $-\frac{1}{2} \leq (ab + bc + ca) \leq 1$;
(C) $-\frac{1}{4} \leq (ab + bc + ca) \leq 1$;
(D) $-1 \leq (ab + bc + ca) \leq \frac{1}{2}$.

361. Let $x,\ y,\ z$ be positive numbers. The least value of

$$\frac{x(1+y) + y(1+z) + z(1+x)}{\sqrt{xyz}}$$

is

(A) $\frac{9}{\sqrt{2}}$; (B) 6; (C) $\frac{1}{\sqrt{6}}$; (D) none of these numbers.

362. Let $a,\ b$ and c be such that $a + b + c = 0$ and

$$\ell = \frac{a^2}{2a^2 + bc} + \frac{b^2}{2b^2 + ca} + \frac{c^2}{2c^2 + ab}$$

is defined. Then the value of ℓ is

(A) 1; (B) -1; (C) 0; (D) none of the foregoing numbers.

363. Let $a,\ b$ and c be distinct real numbers such that

$$a^2 - b = b^2 - c = c^2 - a.$$

Then $(a+b)(b+c)(c+a)$ equals

(A) 0; (B) 1; (C) -1; (D) none of the foregoing numbers.

364. Let x and y be real numbers such that $x + y \neq 0$. Then there exists an angle θ such that $\sec^2\theta = \frac{4xy}{(x+y)^2}$ if and only if

(A) $x + y > 0$; (B) $x + y > 1$; (C) $xy > 0$; (D) $x = y$.

365. Consider the equation $\sin\theta = \frac{a^2+b^2+c^2}{ab+bc+ca}$, where a, b, c are fixed nonzero real numbers. This equation has a solution for θ

(A) whatever be a, b, c;
(B) if and only if $a^2 + b^2 + c^2 < 1$;
(C) if and only if a, b and c all lie in the interval $(-1, 1)$;
(D) if and only if $a = b = c$.

366. Consider the real-valued function f, defined over the set of real numbers, as $f(x) = e^{\sin(x^2+px+q)}$, $-\infty < x < \infty$, where p, q are arbitrary real numbers. The set of real numbers y for which the equation $f(x) = y$ has a solution depends on

(A) p and not on q; (B) q and not on p,
(C) both p and q; (D) neither p nor q.

367. The equation $x - \log_e(1 + e^x) = c$ has a solution

(A) for every $c \geq 1$; (B) for every $c < 1$;
(C) for every $c < 0$; (D) for every $c > -1$.

368. A real value of $\log_e(6x^2 - 5x + 1)$ can be determined if and only if x lies in the subset of the real numbers defined by

(A) $\{x : \frac{1}{3} < x < \frac{1}{2}\}$; (B) $\{x : x < \frac{1}{3}\} \cup \{x : x > \frac{1}{2}\}$;
(C) $\{x : x \leq \frac{1}{3}\} \cup \{x : x \geq \frac{1}{2}\}$; (D) all the real numbers.

369. The domain of definition of the function $f(x) = \sqrt{\log_{10}\left(\frac{3x-x^2}{2}\right)}$ is

(A) (1,2) (B) $(0, 1] \cup [2, \infty)$ (C) $[1, 2]$ (D) $(0, 3)$.

370. A collection S of points (x, y) of the plane is said to be *convex*, if whenever two points $P = (u, v)$ and $Q = (s, t)$ belong to S, every point on the line segment PQ also belongs to S. Let S_1 be the collection of all points (x, y) for which $1 < x^2 + y^2 < 2$ and let S_2 be the collection of all points (x, y) for which x and y have the same sign. Then

(A) S_1 is convex and S_2 is not convex;
(B) S_1 and S_2 are both convex;
(C) neither S_1 nor S_2 is convex;
(D) S_1 is not convex and S_2 is convex.

371. A function $y = f(x)$ is said to be *convex* if the *line segment* joining any two points $A = (x_1, f(x_1))$ and $B = (x_2, f(x_2))$ on the graph of the function *lies above the graph*. Such a line may also touch the graph at some or all points. Only one of the following four functions is *not* convex. Which one is it?

(A) $f(x) = x^2$. (B) $f(x) = e^x$. (C) $f(x) = \log_e x$. (D) $f(x) = 7 - x$.

372. If S is the set of all real numbers x such that $|1 - x| - x \geq 0$, then

(A) $S = (-\infty, -\frac{1}{2}]$; (B) $S = [-\frac{1}{2}, \frac{1}{2}]$;
(C) $S = (-\infty, 0]$; (D) $S = (-\infty, \frac{1}{2}]$.

373. The inequality $\sqrt{x+2} \geq x$ is satisfied if and only if

(A) $-2 \leq x \leq 2$; (B) $-1 \leq x \leq 2$;
(C) $0 \leq x \leq 2$; (D) none of the foregoing conditions.

374. If $l^2 + m^2 + n^2 = 1$ and $l'^2 + m'^2 + n'^2 = 1$, then the value of $ll' + mm' + nn'$

(A) is always greater than 2;
(B) is always greater than 1, but less than 2;
(C) is always less than or equal to 1;
(D) does not satisfy any of the foregoing conditions.

375. If a and b are positive numbers and c and d are real numbers, positive or negative, then $a^c \leq b^d$

(A) if $a \leq b$ and $c \leq d$;
(B) if either $a \leq b$ or $c \leq d$;
(C) if $a \geq 1, b \geq 1, d \geq c$;
(D) is not implied by any of the foregoing conditions.

376. For all x such that $1 \leq x \leq 3$, the inequality $(x - 3a)(x - a - 3) < 0$ holds for

(A) no value of a; (B) all a satisfying $\frac{2}{3} < a < 1$;
(C) all a satisfying $0 < a < \frac{1}{3}$; (D) all a satisfying $\frac{1}{3} < a < \frac{2}{3}$.

377. Given that x is a real number satisfying

$$(3x^2 - 10x + 3)(2x^2 - 5x + 2) < 0,$$

it follows that

(A) $x < \frac{1}{3}$; (B) $\frac{1}{3} < x < \frac{1}{2}$;
(C) $2 < x < 3$; (D) $\frac{1}{3} < x < \frac{1}{2}$ or $2 < x < 3$.

378. If x, y, z are arbitrary real numbers satisfying the condition $xy + yz + zx < 0$, and if

$$u = \frac{x^2 + y^2 + z^2}{xy + yz + zx},$$

then only one of the following statements is always correct. Which one is it?

(A) $-1 \leq u < 0$.
(B) u takes all negative real values.
(C) $-2 < u \leq -1$.
(D) $u \leq -2$.

379. The inequality

$$\frac{|x|^2 - |x| - 2}{2|x| - |x|^2 - 2} > 2$$

holds if and only if

(A) $-1 < x < -\frac{2}{3}$ or $\frac{2}{3} < x < 1$;
(B) $-1 < x < 1$;
(C) $\frac{2}{3} < x < 1$;
(D) $x > 1$ or $x < -1$ or $-\frac{2}{3} < x < \frac{2}{3}$.

380. The set of all real numbers x satisfying the inequality $|x^2 + 3x| + x^2 - 2 \geq 0$ is

(A) all the real numbers x with either $x \leq -3$ or $x \geq 2$;
(B) all the real numbers x with either $x \leq -\frac{2}{3}$ or $x \geq \frac{1}{2}$;
(C) all the real numbers x with either $x \leq -2$ or $x \geq \frac{1}{2}$;
(D) described by none of the foregoing statements.

381. The least value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ for positive x, y, z satisfying the condition $x+y+z = 9$ is

(A) $\frac{15}{7}$; (B) $\frac{1}{9}$; (C) 3; (D) 1.

382. The smallest value of α satisfying the conditions that α is a positive integer and that $\frac{\alpha}{540}$ is the square of a rational number is

(A) 15; (B) 5; (C) 6; (D) 3.

383. The set of all values of x satisfying the inequality $\frac{6x^2+5x+3}{x^2+2x+3} > 2$ is

(A) $x > \frac{3}{4}$; (B) $|x| > 1$; (C) either $x > \frac{3}{4}$ or $x < -1$; (D) $|x| > \frac{3}{4}$.

384. The set of all x satisfying $|x^2 - 4| > 4x$ is

(A) $x < 2(\sqrt{2} - 1)$ or $x > 2(\sqrt{2} + 1)$;
(B) $x > 2(\sqrt{2} + 1)$;
(C) $x < -2(\sqrt{2} - 1)$ or $x > 2(\sqrt{2} + 1)$;
(D) none of the foregoing sets.

385. If a, b, c are positive real numbers and

$$\alpha = \frac{b^2+c^2}{b+c} + \frac{c^2+a^2}{c+a} + \frac{a^2+b^2}{a+b},$$

then only one of the following statements is *always* true. Which one is it?

(A) $0 \leq \alpha < a$. (B) $a \leq \alpha < a+b$.
(C) $a+b \leq \alpha < a+b+c$. (D) $a+b+c \leq \alpha < 2(a+b+c)$.

386. Suppose a, b, c are real numbers such that $a^2b^2 + b^2c^2 + c^2a^2 = k$, where k is a constant. Then the set of all possible values of $abc(a+b+c)$ is precisely the interval

(A) $[-k, k]$; (B) $[-\frac{k}{2}, \frac{k}{2}]$; (C) $[-\frac{k}{2}, k]$; (D) $[-k, \frac{k}{2}]$.

387. If a, b, c, d are real numbers such that $b > 0, d > 0$ and $\frac{a}{b} < \frac{c}{d}$, then only one of the following statements is *always* true. Which one is it?

(A) $\frac{a}{b} < \frac{a-c}{b-d} < \frac{c}{d}$. (B) $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$. (C) $\frac{a}{b} < \frac{a-c}{b+d} < \frac{c}{d}$. (D) $\frac{a}{b} < \frac{a+c}{b-d} < \frac{c}{d}$.

388. If x, y, z are arbitrary positive real numbers satisfying the equation

$$4xy + 6yz + 8zx = 9,$$

then the maximum possible value of the product xyz is

(A) $\frac{1}{2\sqrt{2}}$; (B) $\frac{\sqrt{3}}{4}$; (C) $\frac{3}{8}$; (D) none of the foregoing values.

389. Let P and Q be the subsets of the X–Y plane defined as:

$$P = \{(x,y) : x > 0,\ y > 0 \text{ and } x^2 + y^2 = 1\},$$
$$\text{and } Q = \{(x,y) : x > 0,\ y > 0 \text{ and } x^8 + y^8 < 1\}.$$

Then, $P \cap Q$ is

(A) the empty set ϕ; (B) P; (C) Q; (D) none of the foregoing sets.

390. The minimum value of the quantity $\frac{(a^2+3a+1)(b^2+3b+1)(c^2+3c+1)}{abc}$, where a, b and c are positive real numbers, is

(A) $\frac{11^3}{2^3}$; (B) 125; (C) 25; (D) 27.

391. The smallest integer greater than the real number $(\sqrt{5}+\sqrt{3})^{2n}$ (for nonnegative integer n) is

(A) 8^n; (B) 4^{2n}; (C) $(\sqrt{5}+\sqrt{3})^{2n}+(\sqrt{5}-\sqrt{3})^{2n}-1$;
(D) $(\sqrt{5}+\sqrt{3})^{2n}+(\sqrt{5}-\sqrt{3})^{2n}$.

392. The set all values of m for which $mx^2-6mx+5m+1>0$ for all real x is

(A) $0 \le m \le \frac{1}{4}$; (B) $m < \frac{1}{4}$; (C) $m \ge 0$; (D) $0 \le m < \frac{1}{4}$.

393. The value of $(1^r+2^r+\ldots+n^r)^n$, where r is a real number, is

(A) greater than or equal to $n^n \cdot (n!)^r$; (B) less than $n^n \cdot (n!)^{2r}$;
(C) less than or equal to $n^{2n} \cdot (n!)^r$; (D) greater than $n^n \cdot (n!)^r$.

394. The value of $(\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2})^{165}$ is

(A) -1; (B) $\frac{\sqrt{3}}{2}-i\cdot\frac{1}{2}$; (C) i; (D) $-i$.

395. The value of the expression

$$\left(\frac{-1+\sqrt{-3}}{2}\right)^n+\left(\frac{-1-\sqrt{-3}}{2}\right)^n$$

is

(A) 3 when n is a positive multiple of 3, and 0 when n is any other positive integer;
(B) 2 when n is a positive multiple of 3, and -1 when n is any other positive integer;
(C) 1 when n is a positive multiple of 3, and -2 when n is any other positive integer;
(D) none of the foregoing numbers.

396. How many integers k are there for which $(1-i)^k=2^k$? (Here $i=\sqrt{-1}$.)

(A) one; (B) none; (C) two; (D) more than two.

397. If n is a multiple of 4, the sum $S=1+2i+3i^2+\cdots+(n+1)i^n$, where $i=\sqrt{-1}$, is

(A) $1-i$; (B) $\frac{n+2}{2}$; (C) $\frac{n^2+8-4ni}{8}$; (D) $\frac{n+2-ni}{2}$.

398. If $a_0, a_1, \ldots, a_n$ are real numbers such that

$$(1+z)^n = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n,$$

for all complex numbers z, then the value of

$$(a_0 - a_2 + a_4 - a_6 + \ldots)^2 + (a_1 - a_3 + a_5 - a_7 + \ldots)^2$$

equals

(A) 2^n; (B) $a_0^2 + a_1^2 + \ldots + a_n^2$; (C) 2^{n^2}; (D) $2n^2$.

399. If $t_k = \binom{100}{k} x^{100-k}$, for $k = 0, 1, \cdots, 100$, then

$$(t_0 - t_2 + t_4 - \cdots + t_{100})^2 + (t_1 - t_3 + t_5 - \cdots - t_{99})^2$$

equals

(A) $(x^2-1)^{100}$; (B) $(x+1)^{100}$; (C) $(x^2+1)^{100}$; (D) $(x-1)^{100}$.

400. The expression $\frac{(1+i)^n}{(1-i)^{n-2}}$ equals

(A) $-i^{n+1}$; (B) i^{n+1}; (C) $-2i^{n+1}$; (D) 1.

401. The value of the sum

$$\cos\frac{\pi}{1000} + \cos\frac{2\pi}{1000} + \cdots + \cos\frac{999\pi}{1000}$$

is

(A) 0; (B) 1; (C) $\frac{1}{1000}$; (D) an irrational number.

402. The sum

$$1 + \binom{n}{1}\cos\theta + \binom{n}{2}\cos 2\theta + \ldots + \binom{n}{n}\cos n\theta$$

equals

(A) $(2\cos\frac{\theta}{2})^n \cos\frac{n\theta}{2}$; (B) $(2\cos^2\frac{\theta}{2})^n$;
(C) $(2\cos^2\frac{n\theta}{2})^n$; (D) none of the foregoing quantities.

403. Let $i = \sqrt{-1}$. Then

(A) i and $-i$ each has exactly one square root;
(B) i has two square roots but $-i$ does not have any;
(C) neither i nor $-i$ has any square root;
(D) i and $-i$ each has exactly two square roots.

404. If the complex numbers w and z represent two diagonally opposite vertices of a square, then the other two vertices are given by the complex numbers

(A) $w + iz$ and $w - iz$;
(B) $\frac{1}{2}(w + z) + \frac{1}{2}i(w + z)$ and $\frac{1}{2}(w + z) - \frac{1}{2}i(w + z)$;
(C) $\frac{1}{2}(w - z) + \frac{1}{2}i(w - z)$ and $\frac{1}{2}(w - z) - \frac{1}{2}i(w - z)$;
(D) $\frac{1}{2}(w + z) + \frac{1}{2}i(w - z)$ and $\frac{1}{2}(w + z) - \frac{1}{2}i(w - z)$.

405. Let

$$A = \{a + b\sqrt{-1} \mid a, b \text{ are integers}\}$$

and

$$U = \{x \in A \mid \tfrac{1}{x} \in A\}.$$

Then the number of elements in U is

(A) 2; (B) 4; (C) 6; (D) 8.

406. Let $i = \sqrt{-1}$. Then the number of distinct elements in the set

$$S = \{i^{\,n} + i^{\,-n} : n \text{ an integer}\}$$

is

(A) 3; (B) 4; (C) greater than 4 but finite; (D) infinite.

407. Let $i = \sqrt{-1}$ and p be a positive integer. A necessary and sufficient condition for $(-i)^p = i$ is

(A) p is one of $3, 11, 19, 27, \ldots$; (B) p is an odd integer;
(C) p is not divisible by 4; (D) none of the foregoing conditions.

408. Recall that for a complex number $z = x + iy$, where $i = \sqrt{-1}$, $\bar{z} = x - iy$ and $|z| = (x^2 + y^2)^{\frac{1}{2}}$.

The set of all pairs of complex numbers (z_1, z_2) which satisfy

$$\left|\frac{z_1 - z_2}{1 - \bar{z}_1 z_2}\right| < 1$$

is

(A) all possible pairs (z_1, z_2) of complex numbers;
(B) all pairs of complex numbers (z_1, z_2) for which $|z_1| < 1$ and $|z_2| < 1$;
(C) all pairs of complex numbers (z_1, z_2) for which at least one of the following statements is true:

(i) $|z_1| < 1$ and $|z_2| > 1$,
(ii) $|z_1| > 1$ and $|z_2| < 1$;

(D) all pairs of complex numbers (z_1, z_2) for which at least one of the following statements is true:
(i) $|z_1| < 1$ and $|z_2| < 1$,
(ii) $|z_1| > 1$ and $|z_2| > 1$.

409. Suppose z_1, z_2 are complex numbers satisfying $z_2 \neq 0$, $z_1 \neq z_2$ and $\left|\dfrac{z_1 + z_2}{z_1 - z_2}\right| = 1$. Then $\dfrac{z_1}{z_2}$ is

(A) real and negative; (B) real and positive;
(C) purely imaginary; (D) not necessarily any of these.

410. The modulus of the complex number

$$\left(\frac{2 + i\sqrt{5}}{2 - i\sqrt{5}}\right)^{10} + \left(\frac{2 - i\sqrt{5}}{2 + i\sqrt{5}}\right)^{10}$$

is

(A) $2\cos(20\cos^{-1}\frac{2}{3})$; (B) $2\sin(10\cos^{-1}\frac{2}{3})$;
(C) $2\cos(10\cos^{-1}\frac{2}{3})$; (D) $2\sin(20\cos^{-1}\frac{2}{3})$.

411. For any complex number $z = x + iy$ with x and y real, define $\langle z\rangle = |x| + |y|$. Let z_1 and z_2 be any two complex numbers. Then

(A) $\langle z_1 + z_2\rangle \leq \langle z_1\rangle + \langle z_2\rangle$;
(B) $\langle z_1 + z_2\rangle = \langle z_1\rangle + \langle z_2\rangle$;
(C) $\langle z_1 + z_2\rangle \geq \langle z_1\rangle + \langle z_2\rangle$;
(D) none of the foregoing statements need always be true.

412. Recall that for a complex number $z = x + iy$, where $i = \sqrt{-1}$, $|z| = (x^2 + y^2)^{\frac{1}{2}}$ and $\arg(z) =$ principal value of $\tan^{-1}(\frac{y}{x})$.

Given complex numbers $z_1 = a + ib$, $z_2 = \frac{a}{\sqrt{2}}(1 - i) + \frac{b}{\sqrt{2}}(1 + i)$, $z_3 = \frac{a}{\sqrt{2}}(i - 1) - \frac{b}{\sqrt{2}}(i + 1)$, where a and b are real numbers, only one of the following statements is true. Which one is it?

(A) $|z_1| = |z_2|$ and $|z_2| > |z_3|$;
(B) $|z_1| = |z_3|$ and $|z_1| < |z_2|$;
(C) $\arg z_1 = \arg z_2$ and $\arg z_1 - \arg z_3 = \frac{\pi}{4}$;
(D) $\arg z_2 - \arg z_1 = -\frac{\pi}{4}$ and $\arg z_3 - \arg z_2 = \pm\pi$.

413. If $a_0, a_1, \ldots, a_{2n}$ are real numbers such that

$$(1+z)^{2n} = a_0 + a_1 z + a_2 z^2 + \ldots + a_{2n} z^{2n},$$

for all complex numbers z, then

(A) $a_0 + a_1 + a_2 + \ldots + a_{2n} = 2^n$;
(B) $(a_0 - a_2 + a_4 - \ldots)^2 + (a_1 - a_3 + a_5 - \ldots)^2 = 2^{2n}$;
(C) $a_0^2 + a_1^2 + a_2^2 + \ldots + a_{2n}^2 = 2^{2n}$;
(D) $(a_0 + a_2 + a_4 + \ldots)^2 + (a_1 + a_3 + a_5 + \ldots)^2 = 2^{2n}$.

414. If z is a nonzero complex number and $\frac{z}{1+z}$ is purely imaginary, then z

(A) can be neither real nor purely imaginary;
(B) is real;
(C) is purely imaginary;
(D) satisfies none of the above properties.

415. Let a and b be any two nonzero real numbers. Then the number of complex numbers z satisfying the equation $|z|^2 + a|z| + b = 0$ is

(A) 0 or 2 and both these values are possible;
(B) 0 or 4 and both these values are possible;
(C) 0, 2 or 4 and all these values are possible;
(D) 0 or infinitely many and both these values are possible.

416. Let C denote the set of complex numbers and define A and B by

$$A = \{(z, w) : z, w \in C \text{ and } |z| = |w|\}$$

$$B = \{(z, w) : z, w \in C \text{ and } z^2 = w^2\}.$$

Then

(A) $A = B$; (B) $A \subset B$; (C) $B \subset A$;
(D) none of the foregoing statements is correct.

417. Among the complex numbers z satisfying $|z - 25i| \leq 15$, the number having the least argument is

(A) $10i$; (B) $-15 + 25i$; (C) $12 + 16i$; (D) $7 + 12i$.

418. The minimum possible value of

$$|z|^2 + |z-3|^2 + |z-6i|^2,$$

where z is a complex number and $i = \sqrt{-1}$, is

(A) 15; (B) 45; (C) 30; (D) 20.

419. The curve in the complex plane given by the equation $Re(\frac{1}{z}) = \frac{1}{4}$ is a

(A) vertical straight line at a distance of 4 from the imaginary axis;
(B) circle with radius unity;
(C) circle with radius 2;
(D) straight line not passing through the origin.

420. The set of all complex numbers z such that $\arg(\frac{z-2}{z+2}) = \frac{\pi}{3}$ represents

(A) part of a circle; (B) a circle; (C) an ellipse; (D) part of an ellipse.

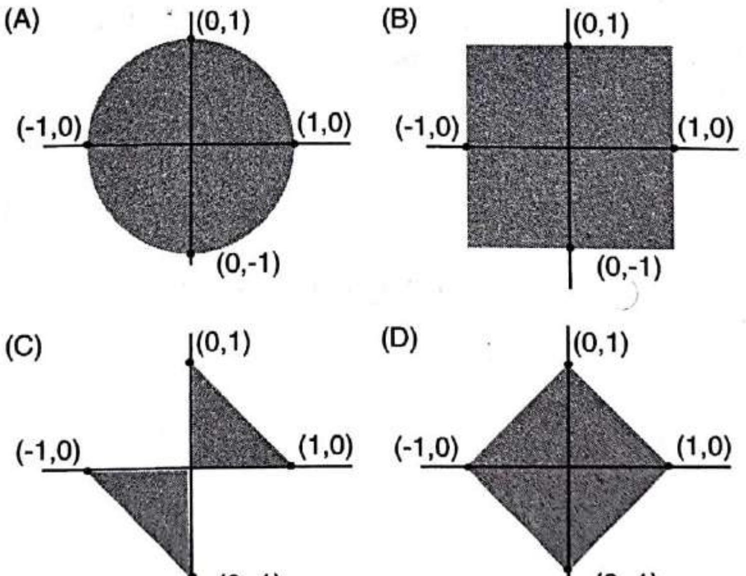
421. Let $z = x + iy$ where x and y are real and $i = \sqrt{-1}$. The points (x, y) in the plane, for which $\frac{z+i}{z-i}$ is purely imaginary (that is, it is of the form ib where b is a real number), lie on

(A) a straight line; (B) a circle; (C) an ellipse; (D) a hyperbola.

422. If the point z in the complex plane describes a circle of radius 2 with centre at the origin, then the point $z + \dfrac{1}{z}$ describe

(A) a circle; (B) a parabola; (C) an ellipse (D) a hyperbola.

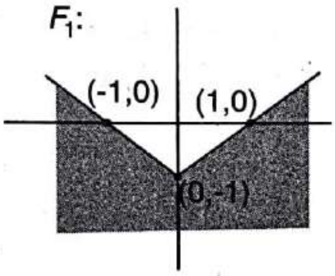
423. The set $\{(x, y) : |x| + |y| \leq 1\}$ is represented by the shaded region in one of the four figures. Which one is it?

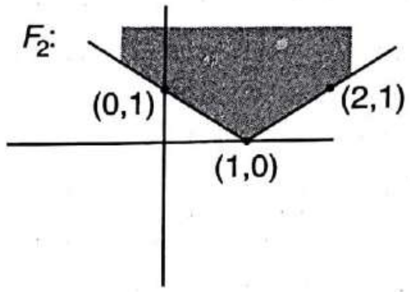


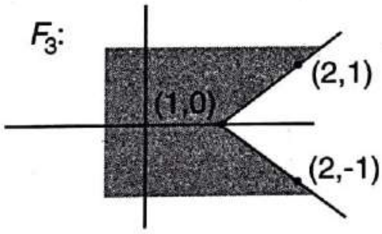
424. The sets

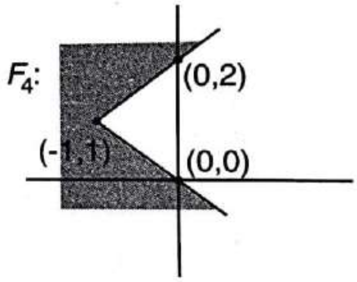
$$\{(x,y) : |y-1| - x \geq 1\}$$
$$\{(x,y) : |x| - y \geq 1\}$$
$$\{(x,y) : x - |y| \leq 1\}$$
$$\{(x,y) : y - |x-1| \geq 0\}$$

are represented by the shaded regions in the figures given below in some order.





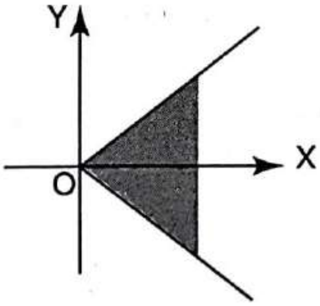




Then the correct order of the figures is

(A) F_4, F_1, F_2, F_3; (B) F_4, F_2, F_3, F_1;
(C) F_1, F_4, F_3, F_2; (D) F_4, F_1, F_3, F_2.

425. The shaded region in the diagram represents the relation



(A) $y \leq x$;
(B) $|\, y \,| \leq |\, x \,|$;
(C) $y \leq |\, x \,|$;
(D) $|\, y \,| \leq x$.

426. The number of points (x, y) in the plane satisfying the two equations $|x|+|y| = 1$ and $\cos\left(2(x+y)\right) = 0$ is

(A) 0; (B) 2; (C) 4; (D) infinitely many.

Directions for Items 427 and 428:

Let the diameter of a subset S of the plane be defined as the maximum of the distances between arbitrary pairs of points of S.

427. Let

$$S = \{(x, y) : (y - x) \leq 0,\ (x + y) \geq 0, x^2 + y^2 \leq 2\}.$$

Then the diameter of S is

(A) 4; (B) 2; (C) $2\sqrt{2}$; (D) $\sqrt{2}$.

428. Let

$$S = \{(x, y) : |x| + |y| = 2\}.$$

Then the diameter of S is

(A) 2; (B) $4\sqrt{2}$; (C) 4; (D) $2\sqrt{2}$.

429. The points (2,1), (8,5) and (x,7) lie on a straight line. The value of x is

(A) 10; (B) 11; (C) 12; (D) $11\frac{2}{3}$.

430. In a parallelogram $PQRS$, P is the point $(-1, -1)$, Q is (8,0) and R is (7,5). Then S is the point

(A) $(-1, 4)$; (B) $(-2, 4)$; (C) $(-2, 3\frac{1}{2})$; (D) $(-1\frac{1}{2}, 4)$.

431. The equation of the line passing through the point of intersection of the lines $x - y + 1 = 0$ and $3x + y - 5 = 0$ and is perpendicular to the line $x + 3y + 1 = 0$ is

(A) $x + 3y - 1 = 0$; (B) $x - 3y + 1 = 0$;
(C) $3x - y + 1 = 0$; (D) $3x - y - 1 = 0$.

432. A rectangle $PQRS$ joins the points $P = (2,3), Q = (x_1, y_1), R = (8,11)$, $S = (x_2, y_2)$. The line QS is known to be parallel to the y-axis. Then the coordinates of Q and S are, respectively,

(A) (0,7) and (10,7); (B) (5,2) and (5,12); (C) (7,6) and (7,10);
(D) none of the foregoing pairs.

433. The sum of the interior angles of a polygon is equal to 56 right angles. Then the number of sides of the polygon is

(A) 12; (B) 15; (C) 30; (D) 25.

434. The ratio of the circumference of a circle to the perimeter of the inscribed regular polygon with n sides is

(A) $2\pi : 2n \sin\frac{\pi}{n}$; (B) $2\pi : n \sin\frac{\pi}{n}$; (C) $2\pi : 2n \sin\frac{2\pi}{n}$; (D) $2\pi : n \sin\frac{2\pi}{n}$.

435. The length of the common chord of two circles of radii 15 cm and 20 cm, whose centres are 25 cm apart, is (in cm)

(A) 24; (B) 25; (C) 15; (D) 20.

436. A circle of radius $\sqrt{3} - 1$ units with both coordinates of the centre negative, touches the straight lines $y - \sqrt{3}x = 0$ and $x - \sqrt{3}y = 0$. The equation of the circle is

(A) $x^2 + y^2 + 2(x + y) + (\sqrt{3} - 1)^2 = 0$;
(B) $x^2 + y^2 + 2(x + y) + (\sqrt{3} + 1)^2 = 0$;
(C) $x^2 + y^2 + 4(x + y) + (\sqrt{3} - 1)^2 = 0$;
(D) $x^2 + y^2 + 4(x + y) + (\sqrt{3} + 1)^2 = 0$.

437. Two circles $APQC$ and $PBDQ$ intersect each other at the points P and Q and APB and CQD are two parallel straight lines. Then only one of the following statements is *always* true. Which one is it?

(A) $ABDC$ is a cyclic quadrilateral. (B) AC is parallel to BD.
(C) $ABDC$ is a rectangle. (D) $\angle ACQ$ is a right angle.

438. The area of the triangle whose vertices are $(a, a), (a + 1, a + 1), (a + 2, a)$ is

(A) a^3; (B) $2a$; (C) 1; (D) $\sqrt{2}$.

439. In a trapezium, the lengths of the two parallel sides are 6 and 10 units. If one of the oblique sides has length 1 unit, then the length of the other oblique side must be

(A) greater than 3 units but less than 4 units;

(B) greater than 3 units but less than 5 units;
(C) less than or equal to 3 units;
(D) greater than 5 units but less than 6 units.

440. If in a triangle, the radius of the circumcircle is double the radius of the inscribed circle, then the triangle is

(A) equilateral; (B) isosceles; (C) right-angled;
(D) not necessarily any of the foregoing types.

441. If in a triangle ABC with a, b, c denoting sides opposite to angles A, B and C respectively, $a = 2b$ and $A = 3B$, then the triangle

(A) is isosceles;
(B) is right-angled but not isosceles;
(C) is right-angled and isosceles;
(D) need not necessarily be any of the above types.

442. Let the bisector of the angle at C of a triangle ABC intersect the side AB in a point D. Then the geometric mean of CA and CB

(A) is less than CD;
(B) is equal to CD;
(C) is greater than CD;
(D) does not always satisfy any one of the foregoing properties.

443. Suppose $ABCD$ is a cyclic quadrilateral within a circle of radius r. The bisector of the angle A cuts the circle at point P and the bisector of angle C cuts the circle at point Q. Then

(A) $AP = 2r$; (B) $PQ = 2r$; (C) $BQ = DP$; (D) $PQ = AP$.

444. In a triangle ABC, let C_1 be any point on the side AB other than A or B. Join CC_1. The line passing through A and parallel to CC_1 intersects the line BC extended at A_1. The line passing through B and parallel to CC_1 intersects the line AC extended at B_1. The lengths AA_1, BB_1, CC_1 are given to be p, q, r units respectively. Then

(A) $r = \frac{pq}{p+q}$; (B) $r = \frac{p+q}{4}$; (C) $r = \frac{\sqrt{pq}}{2}$;
(D) none of the foregoing statements is true.

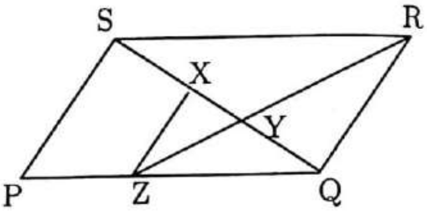
445. In a triangle ABC, D and E are points on AB and AC respectively such that $\angle BDC = \angle BEC$. Then

(A) $\angle BED = \angle BCD$; (B) $\angle CBE = \angle BED$;
(C) $\angle BED + \angle CDE = \angle BAC$; (D) $\angle BED + \angle BCD = \angle BAC$.

446. In the picture, $PQRS$ is a parallelogram. PS is parallel to ZX and $\frac{PZ}{ZQ}$ equals $\frac{2}{3}$. Then $\frac{XY}{SQ}$ equals

(A) $\frac{1}{4}$; (B) $\frac{9}{40}$;

(C) $\frac{1}{5}$; (D) $\frac{9}{25}$.



447. Let A, B, C be three points on a straight line, B lying between A and C. Consider all circles passing through B and C. The points of contact of the tangents from A to these circles lie on

(A) a straight line; (B) a circle; (C) a parabola;

(D) a curve of none of the foregoing types.

448. ABC is a triangle with $AB = 13; BC = 14$ and $CA = 15$. AD and BE are the altitudes from A and B to BC and AC respectively. H is the point of intersection of AD and BE. Then the ratio $\frac{HD}{HB}$ is

(A) $\frac{3}{5}$; (B) $\frac{12}{13}$; (C) $\frac{4}{5}$; (D) $\frac{5}{9}$.

449. ABC is a triangle such that $AB = AC$. Let D be the foot of the perpendicular from C to AB and E the foot of the perpendicular from B to AC. Then

(A) $BC^3 < BD^3 + BE^3$; (B) $BC^3 = BD^3 + BE^3$; (C) $BC^3 > BD^3 + BE^3$;

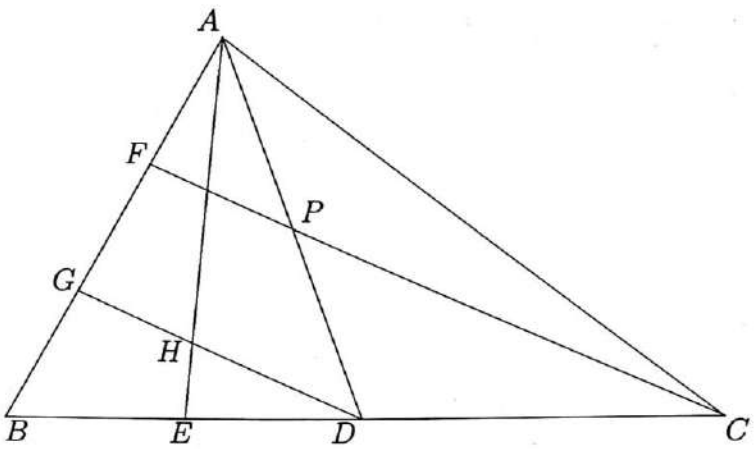
(D) none of the foregoing statements need always be true.

450. Through the centroid of an equilateral triangle, a line parallel to the base is drawn. On this line, an arbitrary point P is taken inside the triangle. Let h denote the distance of P from the base of the triangle. Let h_1 and h_2 be the distances of P from the other two sides of the triangle. Then

(A) $h = \frac{h_1+h_2}{2}$; (B) $h = \sqrt{h_1 h_2}$; (C) $h = \frac{2h_1h_2}{h_1+h_2}$;

(D) none of the foregoing conditions is necessarily true.

451. In the figure that follows, $BD = CD$, $BE = DE$, $AP = PD$ and $DG \parallel CF$.



Then $\frac{\text{area of } \Delta\, ADH}{\text{area of } \Delta\, ABC}$ is equal to

(A) $\frac{1}{6}$; (B) $\frac{1}{4}$; (C) $\frac{1}{5}$; (D) none of the foregoing quantities.

452. Let A be the fixed point (0,4) and B be a moving point $(2t, 0)$. Let M be the mid-point of AB and let the perpendicular bisector of AB meet the y-axis at R. The locus of the mid-point P of MR is

(A) $y + x^2 = 2$; (B) $x^2 + (y-2)^2 = \frac{1}{4}$; (C) $(y-2)^2 - x^2 = \frac{1}{4}$;
(D) none of the foregoing curves.

453. Let l_1 and l_2 be a pair of intersecting lines in the plane. Then the locus of the points P such that the distance of P from l_1 is twice the distance of P from l_2 is

(A) an ellipse; (B) a parabola;
(C) a hyperbola; (D) a pair of straight lines.

454. A triangle ABC has fixed base AB and the ratio of the other two unequal sides is a constant. The locus of the vertex C is

(A) A straight line parallel to AB.
(B) A straight line which is perpendicular to AB.
(C) A circle with AB as a diameter.
(D) A circle with centre on AB.

455. P is a variable point on a circle C and Q is a fixed point outside of C. R is a point on PQ dividing it in the ratio $p : q$, where $p > 0$ and $q > 0$ are fixed. Then the locus of R is

(A) a circle; (B) an ellipse;
(C) a circle if $p = q$ and an ellipse otherwise;
(D) none of the foregoing curves.

456. Let r be the length of the chord intercepted by the ellipse $9x^2 + 16y^2 = 144$ on the line $3x + 4y = 12$. Then

(A) $r = 5$; (B) $r > 5$; (C) $r = 3$; (D) $r = \sqrt{7}$.

457. The angles A, B and C of a triangle ABC are in arithmetic progression. $AB = 6$ and $BC = 7$. Then AC is

(A) 5; (B) 7; (C) 8; (D) none of the foregoing numbers.

458. ABC is a triangle. P, Q and R are respectively the mid-points of AB, BC and CA. The area of the triangle ABC is 20. Then the area of the triangle PQR is

(A) 4; (B) 5; (C) 6; (D) 8.

459. Let AC and CE be perpendicular line segments, each of length 18. Suppose B and D are the mid-points of AC and CE respectively. If F is the point of intersection of EB and AD, then the area of the triangle DEF is

(A) 18; (B) $18\sqrt{2}$; (C) 27; (D) $\frac{5}{2}\sqrt{85}$.

460. In a triangle ABC, the medians AM and CN to the sides BC and AB respectively, intersect at the point O. Let P be the mid-point of AC and let MP intersect CN at Q. If the area of the triangle OMQ is s square units, the area of ABC is

(A) $16s$; (B) $18s$; (C) $21s$; (D) $24s$.

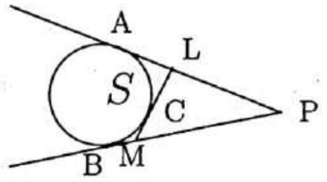
461. Let F be a point on the side AD of a square $ABCD$ of area 256. Suppose the perpendicular to the line FC at C meets the line segment AB extended at E. If the area of the triangle CEF is 200, then the length of BE is

(A) 12; (B) 14; (C) 15; (D) 20.

462. Consider the circle with center $C = (1, 2)$ which passes through the points $P = (1, 7)$ and $Q = (4, -2)$. If R is the point of intersection of the tangents to the circle drawn at P and Q, then the area of the quadrilateral $CPRQ$ is

(A) 50; (B) $50\sqrt{2}$; (C) 75; (D) 100.

463. PA and PB are tangents to a circle S touching S at points A and B. C is a point on S in between A and B as shown in the figure. LCM is a tangent to S intersecting PA and PB in points L and M, respectively. Then the perimeter of the triangle PLM depends on



(A) A, B, C and P;

(B) P, but not on C;

(C) P and C only;

(D) the radius of S only.

464. A and B are two points lying outside a plane Π, but on the same side of it. P and Q are, respectively, the feet of perpendiculars from A and B to Π. Let X be any point of Π. Then $(AX + XB)$ is minimum when X

(A) lies on PQ and $\angle AXP = \angle BXQ$;
(B) is the mid-point of PQ;
(C) is any point of Π with $\angle AXP = \angle BXQ$;
(D) is any point on the perpendicular bisector of PQ in Π.

465. The vertices of a triangle are the points (0,0), (4,4) and (0,8). The radius of the circumcircle of the triangle is

(A) $3\sqrt{2}$; (B) $2\sqrt{2}$; (C) 3; (D) 4.

466. The number of different angles θ satisfying the equation $\cos\theta + \cos 2\theta = -1$, and at the same time satisfying the condition $0 < \theta < 360°$ is

(A) 0; (B) 4; (C) 2; (D) 3.

467. ABC is a right-angled triangle with right angle at B. D is a point on AC such that $\angle ABD = 45°$. If $AC = 6$ cm and $AD = 2$ cm then AB is

(A) $\frac{6}{\sqrt{5}}$ cm; (B) $3\sqrt{2}$ cm; (C) $\frac{12}{\sqrt{5}}$ cm; (D) 2 cm.

468. In the triangle ABC, $AB = 6, BC = 5, CA = 4$. AP bisects the angle A and P lies on BC. Then BP equals

(A) 3; (B) 3.1; (C) 2.9; (D) 4.5.

469. In a triangle ABC, the internal bisector of the angle A meets BC at D. If $AB = 4, AC = 3$ and $\angle A = 60°$, then the length of AD is

(A) $2\sqrt{3}$; (B) $\frac{12\sqrt{3}}{7}$; (C) $\frac{15\sqrt{3}}{8}$; (D) none of these numbers.

470. ABC is a triangle with $BC = a, CA = b$ and $\angle BCA = 120°$. CD is the bisector of $\angle BCA$ meeting AB at D. Then the length of CD is

(A) $\frac{a+b}{4}$; (B) $\frac{ab}{a+b}$; (C) $\frac{a^2+b^2}{2(a+b)}$; (D) $\frac{a^2+ab+b^2}{3(a+b)}$.

471. The diagonal of the square $PQRS$ is $a + b$. The perimeter of a square with twice the area of $PQRS$ is

(A) $2(a + b)$; (B) $4(a + b)$; (C) $\sqrt{8}(a + b)$; (D) $8ab$.

472. A string of length 12 inches is bent first into a square $PQRS$ and then into a right-angled triangle PQT by keeping the side PQ of the square fixed. Then the area of $PQRS$ equals

(A) area of PQT; (B) 2(area of PQT);
(C) $\frac{3(\text{area of } PQT)}{2}$; (D) none of the foregoing numbers.

473. Instead of walking along two adjacent sides of a rectangular field, a boy took a short-cut along the diagonal and saved a distance equal to half the longer side. Then the ratio of the shorter side to the longer side is

(A) $\frac{1}{2}$; (B) $\frac{2}{3}$; (C) $\frac{1}{4}$; (D) $\frac{3}{4}$.

474. Consider a parallelogram ABCD with E as the midpoint of its diagonal BD. The point E is connected to a point F on DA such that DF = $\frac{1}{3}$DA. Then, the ratio of the area of the triangle DEF to the area of the quadrilateral ABEF is

(A) 1 : 2; (B) 1 : 3; (C) 1 : 5; (D) 1 : 4.

475. The external length, breadth and height of a closed box are 10 cm, 9 cm and 7 cm respectively. The total inner surface area of the box is 262 sq cm. If the walls of the box are of uniform thickness d cm, then d equals

(A) 1.5; (B) 2; (C) 2.5; (D) 1.

476. A hollow spherical ball whose inner radius is 4 cm is full of water. Half of the water is transferred to a conical cup and it completely fills the cup. If the height of the cup is 2 cm, then the radius of the base of the cone, in cm, is

(A) 4; (B) 8π; (C) 8; (D) 16.

477. $PQRS$ is a trapezium with PQ and RS parallel, $PQ = 6$ cm, $QR = 5$ cm, $RS = 3$ cm, $PS = 4$ cm. The area of $PQRS$

(A) is 27 cm^2; (B) 12 cm^2; (C) 18 cm^2;
(D) cannot be determined from the given information.

478. Suppose P, Q, R and S are the midpoints of the sides AB, BC, CD and DA, respectively, of a rectangle $ABCD$. If the area of the rectangle is Δ, then the area of the figure bounded by the straight lines AQ, BR, CS and DP is

(A) $\frac{\Delta}{4}$; (B) $\frac{\Delta}{5}$; (C) $\frac{\Delta}{8}$; (D) $\frac{\Delta}{2}$.

479. The ratio of the area of a triangle ABC to the area of the triangle whose sides are equal to the medians of the triangle ABC is

(A) $2:1$; (B) $3:1$; (C) $4:3$; (D) $3:2$.

480. Let C_1 and C_2 be the inscribed and circumscribed circles of a triangle with sides 3 cm, 4 cm and 5 cm. Then $\frac{\text{area of } C_1}{\text{area of } C_2}$ equals

(A) $\frac{16}{25}$; (B) $\frac{4}{25}$; (C) $\frac{9}{25}$; (D) $\frac{9}{16}$.

481. An isosceles triangle with base 6 cm and base angles 30° each is inscribed in a circle. A second circle touches the first circle and also touches the base of the triangle at its midpoint. If the second circle is situated outside the triangle, then its radius (in cm) is

(A) $\frac{3\sqrt{3}}{2}$; (B) $\frac{\sqrt{3}}{2}$; (C) $\sqrt{3}$; (D) $\frac{4}{\sqrt{3}}$.

482. In an isoceles triangle ABC, $\angle A = \angle C = \frac{\pi}{6}$ and the radius of its circumcircle is 4. The radius of its incircle is

(A) $4\sqrt{3}-6$; (B) $4\sqrt{3}+6$; (C) $2\sqrt{3}-2$; (D) $2\sqrt{3}+2$.

483. $PQRS$ is a quadrilateral in which PQ and SR are parallel (that is, $PQRS$ is a trapezium). Further, $PQ = 10, QR = 5, RS = 4, SP = 5$. Then the area of the quadrilateral is

(A) 25; (B) 28; (C) 20; (D) $10\sqrt{10}$.

484. The area of quadrilateral $ABCD$ with sides a, b, c, d is given by the formula $[(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\theta]^{\frac{1}{2}}$, where $2s$ is the perimeter and 2θ is the sum of opposite angles A and C. Then the area of the quadrilateral circumscribing a circle is given by

(A) $\tan\theta\sqrt{abcd}$; (B) $\cos\theta\sqrt{abcd}$; (C) $\sin\,\theta\sqrt{abcd}$;
(D) none of the foregoing formulæ.

485. Consider a unit square $ABCD$. Two equilateral triangles PAB and QCD are drawn so that AP, DQ intersect in R, and BP, CQ intersect in S. The area of the quadrilateral $PRQS$ is equal to

(A) $\frac{2-\sqrt{3}}{6}$ (B) $\frac{2-\sqrt{3}}{3}$ (C) $\frac{2+\sqrt{3}}{6\sqrt{3}}$ (D) $\frac{2-\sqrt{3}}{\sqrt{3}}$.

486. Through an arbitrary point lying inside a triangle, three straight lines parallel to its sides are drawn. These lines divide the triangle into six parts, three of which are triangles. If the areas of these triangles are S_1, S_2 and S_3, then the area of the given triangle equals

(A) $3(S_1 + S_2 + S_3)$; (B) $(\sqrt{S_1S_2} + \sqrt{S_2S_3} + \sqrt{S_3S_1})^2$;
(C) $(\sqrt{S_1} + \sqrt{S_2} + \sqrt{S_3})^2$; (D) none of the foregoing quantities.

487. The sides of a triangle are given by $\sqrt{b^2+c^2}$, $\sqrt{c^2+a^2}$ and $\sqrt{a^2+b^2}$, where a, b, c are positive. Then the area of the triangle equals

(A) $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$; (B) $\frac{1}{2}\sqrt{a^4+b^4+c^4}$;
(C) $\frac{\sqrt{3}}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$; (D) $\frac{\sqrt{3}}{2}(bc+ca+ab)$.

488. Two sides of a triangle are 4 and 5. Then, for the area of the triangle, which one of the following bounds is the sharpest?

(A) < 10. (B) ≤ 10. (C) ≤ 8. (D) > 5.

489. The area of a regular hexagon (that is, a six-sided polygon) inscribed in a circle of radius 1 is

(A) $\frac{3\sqrt{3}}{2}$; (B) 3; (C) 4; (D) $2\sqrt{3}$.

490. Chords AB and CD of a circle intersect at a point E at right angles to each other. If the segments AE, EB and ED are of lengths 2, 6 and 3 units respectively, then the diameter of the circle is

(A) $\sqrt{65}$; (B) 12; $\sqrt{52}$; (D) $\sqrt{63}$.

491. In a circle with centre O, OA and OB are two radii perpendicular to each other. Let AC be a chord and D the foot of the perpendicular drawn from B to AC. If the length of BD is 4 cm then the length of CD (in cm) is

(A) 4; (B) $2\sqrt{2}$; (C) $2\sqrt{3}$; (D) $3\sqrt{2}$.

492. ABC is a triangle and P is a point inside it such that $\angle BPC = \angle CPA = \angle APB$. Then P is

(A) the point of intersection of medians;
(B) the incentre;
(C) the circumcentre;
(D) none of the foregoing points.

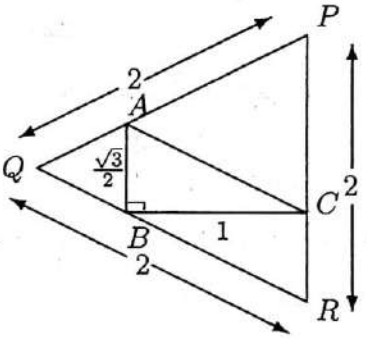
493. Suppose the circumcentre of a triangle ABC lies on BC. Then the orthocentre of the triangle is

(A) the point A; (B) the incentre of the triangle;
(C) the mid-point of the line segment joining the mid-points ofAB and AC;
(D) the centroid of the triangle.

494. ABC is a triangle inscribed in a circle. AD, AE are straight lines drawn from the vertex A to the base BC parallel to the tangents at B and C respectively. If $AB = 5$ cm, AC = 6 cm, and $CE = 9$ cm, then the length of BD (in cm) equals

(A) 7.5; (B) 10.8; (C) 7.0; (D) 6.25.

495. ABC is a triangle with $AB = \sqrt{3}/2$, $BC = 1$ and $\angle B = 90°$. PQR is an equilateral triangle with sides PQ, QR, RP passing through the points A, B, C respectively and each having length 2. Then the length of the segment BR is



(A) $\frac{2}{\sqrt{3}} \sin 75°$;
(B) $\frac{4}{2+\sqrt{3}}$;
(C) either 1 or $\frac{15}{13}$;
(D) $2 - \sin 75°$.

496. The equation $x^2y - 2xy + 2y = 0$ represents

(A) a straight line; (B) a circle; (C) a hyperbola;
(D) none of the foregoing curves.

497. The equation $r = 2a\cos\theta + 2b\sin\theta$, in polar coordinates, represents

(A) a circle passing through the origin;
(B) a circle with the origin lying outside it;
(C) a circle with radius $2\sqrt{a^2 + b^2}$;
(D) a circle with the centre at the origin.

498. The curve whose equation in polar coordinates is $r\sin^2\theta - \sin\theta - r = 0$, is

(A) an ellipse; (B) a parabola;
(C) a hyperbola; (D) none of the foregoing curves.

499. A point P on the line $3x + 5y = 15$ is equidistant from the coordinate axes. P can lie in

(A) quadrant I only; (B) quadrant I or quadrant II;
(C) quadrant I or quadrant III; (D) any quadrant.

500. The set of all points (x, y) in the plane satisfying the equation $5x^2y - xy + y = 0$ forms

(A) a straight line; (B) a parabola; (C) a circle;
(D) none of the foregoing curves.

501. The equation of the line through the intersection of the lines

$$2x + 3y + 4 = 0 \text{ and } 3x + 4y - 5 = 0$$

and perpendicular to $7x - 5y + 8 = 0$ is

(A) $5x + 7y - 1 = 0$; (B) $7x + 5y + 1 = 0$;
(C) $5x - 7y + 1 = 0$; (D) $7x - 5y - 1 = 0$.

502. Two equal sides of an isosceles triangle are given by the equations $y = 7x$ and $y = -x$ and its third side passes through $(1, -10)$. Then the equation of the third side is

(A) $3x + y + 7 = 0$ or $x - 3y - 31 = 0$;
(B) $x + 3y + 29 = 0$ or $-3x + y + 13 = 0$;
(C) $3x + y + 7 = 0$ or $x + 3y + 29 = 0$;
(D) $x - 3y - 31 = 0$ or $-3x + y + 13 = 0$.

503. The equations of two adjacent sides of a rhombus are given by $y = x$ and $y = 7x$. The diagonals of the rhombus intersect each other at the point (1,2). The area of the rhombus is

(A) $\frac{10}{3}$; (B) $\frac{20}{3}$; (C) $\frac{50}{3}$; (D) none of the foregoing quantities.

504. It is given that three distinct points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear. Then a necessary and sufficient condition for (x_2, y_2) to lie on the line segment joining (x_3, y_3) to (x_1, y_1) is

(A) either $x_1 + y_1 < x_2 + y_2 < x_3 + y_3$ or $x_3 + y_3 < x_2 + y_2 < x_1 + y_1$;
(B) either $x_1 - y_1 < x_2 - y_2 < x_3 - y_3$ or $x_3 - y_3 < x_2 - y_2 < x_1 - y_1$;
(C) either $0 < \frac{x_2 - x_3}{x_1 - x_3} < 1$ or $0 < \frac{y_2 - y_3}{y_1 - y_3} < 1$;
(D) none of the foregoing statements.

505. Let $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3), D(x_4, y_4)$ be four points such that x_1, x_2, x_3, x_4 and y_1, y_2, y_3, y_4 are both in A.P. If Δ denotes the area of the quadrilateral ABCD, then

(A) $\Delta = 0$; (B) $\Delta > 1$; (C) $\Delta < 1$;
(D) Δ depends on the coordinates of A, B, C and D.

506. The number of points (x, y) satisfying (i) $3y - 4x = 20$ and (ii) $x^2 + y^2 \leq 16$ is

(A) 0; (B) 1; (C) 2; (D) infinite.

507. The equation of the line parallel to the line $3x + 4y = 0$ and touching the circle $x^2 + y^2 = 9$ in the first quadrant is

(A) $3x + 4y = 9$; (B) $3x + 4y = 45$; (C) $3x + 4y = 15$;
(D) none of the foregoing equations.

508. The difference between the radii of the largest and the smallest circles, which have their centres on the circumference of the circle $x^2 + 2x + y^2 + 4y = 4$ and pass through the point (a, b) lying outside the given circle, is

(A) 6; (B) $\sqrt{(a+1)^2 + (b+2)^2}$; (C) 3; (D) $\sqrt{(a+1)^2 + (b+2)^2} - 3$.

509. The perimeter of the region bounded by $x^2 + y^2 \leq 100$ and $x^2 + y^2 - 10x - 10(2 - \sqrt{3})y \leq 0$ is
(A) $\frac{5\pi}{3}(5 + \sqrt{6} - \sqrt{2})$; (B) $\frac{5\pi}{3}(1 + \sqrt{6} - \sqrt{2})$; (C) $\frac{5\pi}{3}(1 + 2\sqrt{6} - 2\sqrt{2})$; (D) $\frac{5\pi}{3}(5 + 2\sqrt{6} - 2\sqrt{2})$.

510. The equation of the circle which has both coordinate axes as its tangents and which touches the circle $x^2 + y^2 = 6x + 6y - 9 - 4\sqrt{2}$ is

(A) $x^2 + y^2 = 2x + 2y + 1$; (B) $x^2 + y^2 = 2x - 2y + 1$;
(C) $x^2 + y^2 = 2x - 2y - 1$; (D) $x^2 + y^2 = 2x + 2y - 1$.

511. A circle and a square have the same perimeter. Then

(A) their areas are equal;
(B) the area of the circle is larger;
(C) the area of the square is larger;
(D) the area of the circle is π times the area of the square.

512. The equation $x^2 + y^2 - 2xy - 1 = 0$ represents

(A) two parallel straight lines; (B) two perpendicular straight lines;
(C) a circle; (D) a hyperbola.

513. The equation $x^3 - yx^2 + x - y = 0$ represents

(A) a straight line;
(B) a parabola and two straight lines;
(C) a hyperbola and two straight lines;
(D) a straight line and a circle.

514. The equation $x^3y + xy^3 + xy = 0$ represents

(A) a circle; (B) a circle and a pair of straight lines;
(C) a rectangular hyperbola; (D) a pair of straight lines.

515. A circle of radius r touches the parabola $x^2 + 4ay = 0$ $(a > 0)$ at the vertex of the parabola. The centre of the circle lies below the vertex and the circle lies entirely within the parabola. Then the largest possible value of r is

(A) a; (B) $2a$; (C) $4a$; (D) none of the foregoing expressions.

516. The equation $16x^4 - y^4 = 0$ represents

(A) a pair of straight lines; (B) one straight line;
(C) a point; (D) a hyperbola.

517. The equation of the straight line which passes through the point of intersection of the lines $x + 2y + 3 = 0$ and $3x + 4y + 7 = 0$ and is perpendicular to the straight line $y - x = 8$ is

(A) $6x + 6y - 8 = 0$; (B) $x + y + 2 = 0$;
(C) $4x + 8y + 12 = 0$; (D) $3x + 3y - 6 = 0$.

518. Two circles with equal radii are intersecting at the points $(0, 1)$ and $(0, -1)$. The tangent at $(0, 1)$ to one of the circles passes through the centre of the other circle. Then the centres of the two circles are at

(A) $(2, 0)$ and $(-2, 0)$; (B) $(0.75, 0)$ and $(-0.75, 0)$; (C) $(1, 0)$ and $(-1, 0)$;
(D) none of the foregoing pairs of points.

519. The number of distinct solutions (x, y) of the system of equations

$$x^2 = y^2$$

$$\text{and} \quad (x - a)^2 + y^2 = 1,$$

where a is any real number, can only be

(A) 0, 1, 2, 3, 4 or 5; (B) 0, 1 or 3; (C) 0, 1, 2 or 4; (D) 0, 2, 3 or 4.

520. The number of distinct points common to the curves $x^2+4y^2=1$ and $4x^2+y^2=4$ is

(A) 0; (B) 1; (C) 2; (D) 4.

521. The centres of the three circles

$$x^2+y^2-10x+9=0,\quad x^2+y^2-6x+2y+1=0,\quad x^2+y^2-9x-4y+2=0$$

(A) lie on the straight line $x-2y=5$; (B) lie on the straight line $y-2x=5$;
(C) lie on the straight line $2y-x=5$; (D) do not lie on a straight line.

522. In a parallelogram $ABCD$, A is the point (1,3), B is the point (5,6), C is the point (4,2). Then D is the point

(A) $(0,-1)$; (B) $(-1,0)$; (C) $(-1,1)$; (D) $(1,-1)$.

523. A square, whose side is 2 metres, has its corners cut away so as to form a regular octagon. Then the area of the octagon, in square metres, equals

(A) 2; (B) $\frac{8}{\sqrt{2}+1}$; (C) $4(3-2\sqrt{2})$;
(D) none of the foregoing numbers.

524. The equation of the line passing through the intersection of the lines $3x+4y=-5$, $4x+6y=6$ and perpendicular to $7x-5y+3=0$ is

(A) $5x+7y-2=0$; (B) $5x-7y+2=0$;
(C) $7x-5y+2=0$; (D) $5x+7y+2=0$.

525. The area of the triangle formed by the straight lines whose equations are $y=4x+2$, $2y=x+3$ and $x=0$, is

(A) $\frac{25}{7\sqrt{2}}$; (B) $\frac{\sqrt{2}}{28}$; (C) $\frac{1}{28}$; (D) $\frac{15}{7}$.

526. A circle is inscribed in an equilateral triangle and a square is inscribed in the circle. The ratio of the area of the triangle to the area of the square is

(A) $\sqrt{3}:\sqrt{2}$; (B) $3\sqrt{3}:2$; (C) $3:\sqrt{2}$; (D) $\sqrt{3}:1$.

527. If the area of the circumcircle of a regular polygon with n sides is A then the area of the circle inscribed in the polygon is

(A) $A\cos^2\frac{2\pi}{n}$; (B) $\frac{A}{2}\left(\cos\frac{2\pi}{n}+1\right)$; (C) $\frac{A}{2}\cos^2\frac{\pi}{n}$; (D) $A\left(\cos\frac{2\pi}{n}+1\right)$.

528. A rectangle $ABCD$ is inscribed in a circle. Let PQ be the diameter of the circle parallel to the side AB. If $\angle BPC = 30°$, then the ratio of the area of the rectangle to that of the circle is

(A) $\frac{\sqrt{3}}{\pi}$; (B) $\frac{\sqrt{3}}{2\pi}$; (C) $\frac{3}{\pi}$; (D) $\frac{\sqrt{3}}{9\pi}$.

529. Consider a circle passing through the points $(0, 1-a)$, $(a, 1)$ and $(0, 1+a)$. If a parallelogram with two adjacent sides having lengths a and b and an angle $150°$ between them has the same area as the circle, then b equals

(A) πa; (B) $2\pi a$; (C) $\frac{1}{2}\pi a$; (D) none of these numbers.

530. A square is inscribed in a *quarter-circle* in such a manner that two of its adjacent vertices lie on the two radii at an equal distance from the centre, while the other two vertices lie on the circular arc. If the square has sides of length x, then the radius of the circle is

(A) $\frac{16x}{\pi+4}$; (B) $\frac{2x}{\sqrt{\pi}}$; (C) $\frac{\sqrt{5}x}{\sqrt{2}}$; (D) $\sqrt{2}x$.

531. Let $Q = (x_1, y_1)$ be an exterior point and P a point on the circle centred at the origin and with radius r. Let θ be the angle which the line joining P to the centre makes with the positive direction of the x-axis. If the line PQ is tangent to the circle, then $x_1 \cos\theta + y_1 \sin\theta$ is equal to

(A) r; (B) r^2; (C) $\frac{1}{r}$; (D) $\frac{1}{r^2}$.

532. A straight line is drawn through the point (1,2) making an angle θ, $0 < \theta \leq \frac{\pi}{3}$, with the positive direction of the x-axis to intersect the line $x + y = 4$ at a point P so that the distance of P from the point (1,2) is $\frac{\sqrt{6}}{3}$. Then the value of θ is

(A) $\frac{\pi}{18}$; (B) $\frac{\pi}{12}$; (C) $\frac{\pi}{10}$; (D) $\frac{\pi}{3}$.

533. The area of intersection of two circular discs each of radius r and with the boundary of each disc passing through the centre of the other is

(A) $\frac{\pi r^2}{3}$; (B) $\frac{\pi r^2}{6}$; (C) $\frac{\pi r^2}{4}(2\pi - \frac{\sqrt{3}}{2})$; (D) $\frac{r^2}{6}(4\pi - 3\sqrt{3})$.

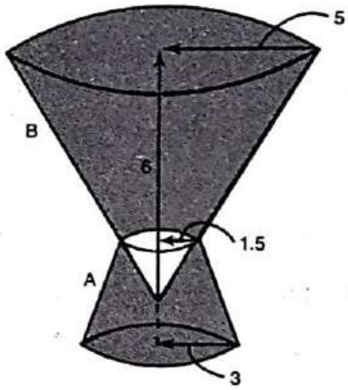
534. Three cylinders each of height 16 cm and radius of base 4 cm are placed on a plane so that each cylinder touches the other two. Then the volume of the region enclosed between the three cylinders is, in cm^3,

(A) $98(4\sqrt{3} - \pi)$; (B) $98(2\sqrt{3} - \pi)$; (C) $98(\sqrt{3} - \pi)$; (D) $128(2\sqrt{3} - \pi)$.

535. From a solid right circular cone made of iron with base of radius 2 cm and height 5 cm, a hemisphere of diameter 2 cm and centre coinciding with the centre of the base of the cone is scooped out. The resultant object is then dropped in a right circular cylinder whose inner diameter is 6 cm and inner height is 10 cm. Water is then poured into the cylinder to fill it up to the brim. The volume of the water required is

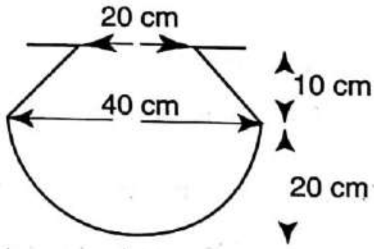
(A) 80π cm^3; (B) $\frac{250\pi}{3}$ cm^3; (C) $\frac{270\pi}{4}$ cm^3; (D) 84π cm^3.

536. A right-circular cone A with base radius 3 units and height 5 units is truncated in such a way that the radius of the circle at the top is 1.5 units and the top is parallel to the base. A second right-circular cone B with base radius 5 units and height 6 units is placed vertically inside the cone A as shown in the diagram. The total volume of the portion of the cone B that is outside cone A and the portion of the cone A excluding the portion of cone B that is inside A (that is, the total volume of the shaded portion in the diagram) is



(A) $\frac{1867}{40}\pi$; (B) $\frac{1913}{40}\pi$; (C) $\frac{2417}{40}\pi$; (D) $\frac{2153}{40}\pi$.

537. A cooking pot has a spherical bottom, while the upper part is a truncated cone. Its vertical cross-section is shown in the figure. If the volume of food increases by 15% during cooking, the maximum initial volume of food that can be cooked without spilling is, in cc,



(A) $14450\frac{\pi}{3}$; (B) $19550\frac{\pi}{3}$;
(C) $\frac{340000}{23}\frac{\pi}{3}$; (D) $20000\frac{\pi}{3}$.

538. A sealed cylindrical drum of radius r is 9% filled with paint. If the drum is tilted to rest on its side, the fraction of its *curved* surface area (not counting the flat sides) that will be under the paint is

(A) less than $\frac{1}{12}$; (B) between $\frac{1}{12}$ and $\frac{1}{6}$;
(C) between $\frac{1}{6}$ and $\frac{1}{4}$; (D) greater then $\frac{1}{4}$.

539. The number of tangents that can be drawn from the point (2,3) to the parabola $y^2 = 8x$ is

(A) 1; (B) 2; (C) 0; (D) 3.

540. A ray of light passing through the point $(1, 2)$ is reflected on the x-axis at a point P, and then passes through the point $(5, 3)$. Then the abscissa of the point P is

(A) $2\frac{1}{5}$; (B) $2\frac{2}{5}$; (C) $2\frac{3}{5}$; (D) $2\frac{4}{5}$.

541. If P, Q and R are three points with coordinates $(1, 4), (4, 2)$ and $(m, 2m - 1)$ respectively, then the value of m for which $PR + RQ$ is minimum is

(A) $\frac{17}{8}$; (B) $\frac{5}{2}$; (C) $\frac{7}{2}$; (D) $\frac{3}{2}$.

542. Let A be the point $(1, 2)$ and L be the line $x + y = 4$. Let M be the line passing through A such that the distance between A and the point of intersection of L and M is $\sqrt{\frac{2}{3}}$. Then the angle which M makes with L is

(A) 45°; (B) 60°; (C) 75°; (D) 30°.

543. The equation $x^2 + y^2 - 2x - 4y + 5 = 0$ represents

(A) a circle; (B) a pair of straight lines; (C) an ellipse; (D) a point.

544. The line $x = y$ is tangent at (0,0) to a circle of radius 1. The centre of the circle is

(A) (1,0); (B) either $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ or $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$;
(C) either $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ or $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$; (D) none of the foregoing points.

545. Let C be the circle $x^2 + y^2 + 4x + 6y + 9 = 0$. The point $(-1, -2)$ is

(A) inside C but not the centre of C; (B) outside C;
(C) on C; (D) the centre of C.

546. The equation of the circle circumscribing the triangle formed by the points (0,0), (1,0), (0,1) is

(A) $x^2 + y^2 + x + y = 0$; (B) $x^2 + y^2 + x - y + 2 = 0$;
(C) $x^2 + y^2 + x - y - 2 = 0$; (D) $x^2 + y^2 - x - y = 0$.

547. The equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy = 0$ at the origin is

(A) $fx + gy = 0$; (B) $gx + fy = 0$; (C) $x = 0$; (D) $y = 0$.

548. The equation of the circle circumscribing the triangle formed by the points (3,4), (1,4) and (3,2) is

(A) $x^2 - 4x + y^2 - 6y + 11 = 0$; (B) $x^2 + y^2 - 4x - 4y + 3 = 0$; ;
(C) $8x^2 + 8y^2 - 16x - 13y = 0$; (D) none of the foregoing equations.

549. The equation of the diameter of the circle $x^2 + y^2 + 2x - 4y = 4$ that is parallel to $3x + 5y = 4$ is

(A) $3x + 5y = 7$; (B) $3x - 5y = 7$; (C) $3x + 5y = -7$; (D) $3x - 5y = -7$.

550. Let C_1 and C_2 be the circles given by the equations $x^2 + y^2 - 4x - 5 = 0$ and $x^2 + y^2 + 8y + 7 = 0$. Then the circle having the common chord of C_1 and C_2 as its diameter has

(A) centre at $(-1, -1)$ and radius 2; (B) centre at $(1, -2)$ and radius $2\sqrt{3}$;
(C) centre at $(1, -2)$ and radius 2; (D) centre at $(3, -3)$ and radius 2.

551. The equation of a circle which passes through the origin, whose radius is a and for which $y = mx$ is a tangent is

(A) $\sqrt{1+m^2}(x^2 + y^2) + 2max + 2ay = 0$;
(B) $\sqrt{1+m^2}(x^2 + y^2) + 2ax - 2may = 0$;
(C) $\sqrt{1+m^2}(x^2 + y^2) - 2max + 2ay = 0$;
(D) $\sqrt{1+m^2}(x^2 + y^2) + 2ax + 2may = 0$.

552. The circles $x^2 + y^2 + 4x + 2y + 4 = 0$ and $x^2 + y^2 - 2x = 0$

(A) intersect at two points; (B) touch at one point;
(C) do not intersect; (D) satisfy none of the foregoing properties.

553. Let P be the point of intersection of the lines $ax + by - a = 0$ and $bx - ay + b = 0$. A circle with centre (1,0) passes through P. The tangent to this circle at P meets the x-axis at the point $(d, 0)$. Then the value of d is

(A) $\frac{2ab}{a^2+b^2}$; (B) 0; (C) -1; (D) none of the foregoing values.

554. The circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 8x - 6y + c = 0$ touch each other externally. That is, the circles are mutually tangential and they lie outside each other. Then the value of c is

(A) 9; (B) 8; (C) 6; (D) 4.

555. The circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ will touch if

(A) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$; (B) $a^2 + b^2 = c^2$;
(C) $a + b = c$; (D) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$.

556. Two circles are said to *cut each other orthogonally* if the tangents at a point of intersection are perpendicular to each other. The locus of the center of a circle that cuts the circle $x^2 + y^2 = 1$ orthogonally and touches the line $x = 2$ is

(A) a pair of straight lines; (B) an ellipse;
(C) a hyperbola; (D) a parabola.

557. The equation of the circle circumscribing the triangle formed by the lines $y = 0, y = x$ and $2x + 3y = 10$ is

(A) $x^2 + y^2 + 5x - y = 0$; (B) $x^2 + y^2 - 5x - y = 0$;
(C) $x^2 + y^2 - 5x + y = 0$; (D) $x^2 + y^2 - x + 5y = 0$.

558. Two gas companies X and Y, where X is situated at (40,0) and Y at (0,30) (unit = 1 km), offer to install equally priced gas furnaces in buyers' houses. Company X adds a charge of Rs. 40 per km of distance (measured along a straight line) between its location and the buyer's house, while company Y charges Rs. 60 per km of distance measured in the same way. Then the region where it is cheaper to have the furnace installed by company X is

(A) the inside of the circle $(x - 54)^2 + (y + 30)^2 = 3600$;
(B) the inside of the circle $(x - 24)^2 + (y - 12)^2 = 2500$;
(C) the outside of the circle $(x + 32)^2 + (y - 54)^2 = 3600$;
(D) the outside of the circle $(x + 24)^2 + (y - 12)^2 = 2500$.

559. Let C be the circle $x^2 + y^2 - 4x - 4y - 1 = 0$. The number of points common to C and the sides of the rectangle determined by the lines $x = 2, x = 5, y = -1$ and $y = 5$, equals

(A) 5; (B) 1; (C) 2; (D) 3.

560. A circle of radius a with both coordinates of its centre positive, touches the x-axis and also the line $3y = 4x$. Then its equation is

(A) $x^2 + y^2 - 2ax - 2ay + a^2 = 0$; (B) $x^2 + y^2 - 6ax - 4ay + 12a^2 = 0$;
(C) $x^2 + y^2 - 4ax - 2ay + 4a^2 = 0$; (D) none of the foregoing equations.

561. The equation of the circle with centre in the first quadrant and radius $\frac{1}{2}$ such that the line $15y = 8x$ and the X–axis are both tangents to the circle, is

(A) $x^2 + y^2 - 8x - y + 16 = 0$; (B) $x^2 + y^2 - 4x - y + 4 = 0$;
(C) $x^2 + y^2 - x - 4y + 4 = 0$; (D) $x^2 + y^2 - x - 8y + 16 = 0$.

562. The centre of the circle $x^2 + y^2 - 8x - 2fy - 11 = 0$ lies on a straight line which passes through the point $(0, -1)$ and makes an angle of 45° with the positive direction of the horizontal axis. The circle

(A) touches the vertical axis; (B) touches the horizontal axis;
(C) passes through the origin; (D) meets the axes at four points.

563. Let P and Q be any two points on the circles $x^2 + y^2 - 2x - 3 = 0$ and $x^2 + y^2 - 8x - 8y + 28 = 0$, respectively. If d is the distance between P and Q, then the set of all possible values of d is

(A) $0 \le d \le 9$; (B) $0 \le d \le 8$; (C) $1 \le d \le 8$; (D) $1 \le d \le 9$.

564. All points whose distance from the nearest point on the circle $(x-1)^2 + y^2 = 1$ is half the distance from the line $x = 5$ lie on

(A) an ellipse; (B) a pair of straight lines; (C) a parabola; (D) a circle.

565. If $P = (0,0), Q = (1,0)$ and $R = (\frac{1}{2}, \frac{\sqrt{3}}{2})$, then the centre of the circle for which the lines PQ, QR and RP are tangents, is

(A) $(\frac{1}{2}, \frac{1}{4})$; (B) $(\frac{1}{2}, \frac{\sqrt{3}}{4})$; (C) $(\frac{1}{2}, \frac{1}{2\sqrt{3}})$; (D) $(\frac{1}{2}, -\frac{1}{\sqrt{3}})$.

566. The equations of the pair of straight lines parallel to the x-axis and tangent to the curve $9x^2 + 4y^2 = 36$ are

(A) $y = -3, y = 6$; (B) $y = 3, y = -6$; (C) $y = \pm 6$; (D) $y = \pm 3$.

567. If the parabola $y = x^2 + bx + c$ is tangent to the straight line $x = y$ at the point (1,1), then

(A) $b = -1, c = +1$; (B) $b = +1, c = -1$;
(C) $b = -1, c$ arbitrary; (D) $b = 0, c = -1$.

568. The condition that the line $\frac{x}{a} + \frac{y}{b} = 1$ be a tangent to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ is

(A) $a^2 + b^2 = 2$; (B) $a^2 + b^2 = 1$; (C) $\frac{1}{a^2} + \frac{1}{b^2} = 1$; (D) $a^2 + b^2 = \frac{2}{3}$.

569. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(A) $x - 1 = 0$; (B) $2x + 1 = 0$; (C) $x + 1 = 0$; (D) $2x - 1 = 0$.

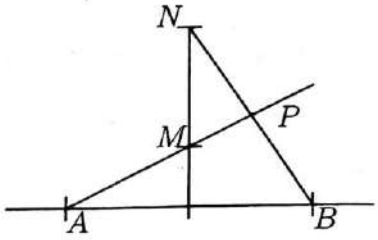
570. Let A be the point (0,0) and let B be the point (1,0). A point P moves so that the angle APB measures $\frac{\pi}{6}$. The locus of P is

(A) a parabola;
(B) arcs of two circles with centres $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$;

(C) arcs of two circles each of radius 1;

(D) a pair of straight lines.

571. Let $A = (-4, 0)$ and $B = (4, 0)$. Let M and N be points on the y-axis, with M below N, and $MN = 4$. Let P be the point of intersection of AM and BN. This is illustrated in the figure. Then the locus of P is



(A) $x^2 - 2xy = 16$;

(B) $x^2 + 2xy = 16$;

(C) $x^2 + 2xy + y^2 = 64$;

(D) $x^2 - 2xy + y^2 = 64$.

572. Consider a circle in the XY plane with diameter 1, passing through the origin O and through the point $A(1, 0)$. For any point B on the circle, let C be the point of intersection of the line OB with the vertical line through A. If M is the point on the line OBC such that OM and BC are of equal length, then the locus of the point M as B varies is given by the equation

(A) $y = \sqrt{x(x^2 + y^2)}$; (B) $y^2 = x$;

(C) $(x^2 + y^2)x - y^2 = 0$; (D) $y = x\sqrt{x^2 + y^2}$.

573. The locus of the foot of perpendicular from any focus upon any tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is

(A) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$; (B) $x^2 + y^2 = a^2 + b^2$;

(C) $x^2 + y^2 = a^2$; (D) none of the foregoing curves.

574. The area of the triangle formed by a tangent of slope m to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the two coordinate axes is

(A) $\frac{|m|}{2}(a^2 + b^2)$; (B) $\frac{1}{2|m|}(a^2 + b^2)$;

(C) $\frac{|m|}{2}(a^2m^2 + b^2)$; (D) $\frac{1}{2|m|}(a^2m^2 + b^2)$.

575. Consider the locus of a moving point $P = (x, y)$ in the plane which satisfies the law

$$2x^2 = r^2 + r^4, \text{ where } r^2 = x^2 + y^2.$$

Then only one of the following statements is true. Which one is it?

(A) For every positive real number d, there is a point (x, y) on the locus such that $r = d$.
(B) For every value d, $0 < d < 1$, there are exactly four points on the locus, each of which is at a distance d from the origin.
(C) The point P always lies in the first quadrant.
(D) The locus of P is an ellipse.

576. Let A be any variable point on the X–axis and B the point (2, 3). The perpendicular at A to the line AB meets the Y–axis at C. Then the locus of the mid-point of the segment AC as A moves is given by the equation

(A) $2x^2 - 2x + 3y = 0$; (B) $3x^2 - 3x + 2y = 0$;
(C) $3x^2 - 3x - 2y = 0$; (D) $2x - 2x^2 + 3y = 0$.

577. A straight line segment AB of length a moves with its ends on the axes. Then the locus of the point P such that $AP : BP = 2 : 1$ is

(A) $9(x^2 + y^2) = 4a^2$; (B) $9(x^2 + 4y^2) = 4a^2$;
(C) $9(y^2 + 4x^2) = 4a^2$; (D) $9x^2 + 4y^2 = a^2$.

578. Let P be a point moving on the straight line $\sqrt{3}x + y = 2$. Denote the origin by O. Suppose now that the line-segment OP is rotated, with O fixed, by an angle of 30° in anti-clockwise direction, to get OQ. The locus of Q is

(A) $\sqrt{3}x + 2y = 2$; (B) $2x + \sqrt{3}y = 2$;
(C) $\sqrt{3}x + 2y = 1$; (D) $x + \sqrt{3}y = 2$.

579. Consider an ellipse with centre at the origin. From any arbitrary point P on the ellipse, perpendiculars PA and PB are dropped on the axes of the ellipse. Then the locus of the point Q that divides AB in the fixed ratio $m : n$ is

(A) a circle; (B) an ellipse;
(C) a hyperbola; (D) none of the foregoing curves.

580. Let A and C be two distinct points in the plane and B a point on the line segment AC such that $AB = 2\,BC$. Then the locus of the point P lying in the plane and satisfying the condition $AP^2 + CP^2 = 2\,BP^2$ is

(A) a straight line parallel to the line AC;
(B) a straight line perpendicular to the line AC;
(C) a circle passing through A and C;
(D) none of the foregoing curves.

581. Let C be a circle and L a line on the same plane such that C and L do not intersect. Let P be a moving point such that the circle drawn with centre at P to touch L also touches C. Then the locus of P is

(A) a straight line parallel to L not intersecting C;
(B) a circle concentric with C;
(C) a parabola whose focus is centre of C and whose directrix is L;
(D) a parabola whose focus is the centre of C and whose directrix is a straight line parallel to L.

582. A right triangle with sides 3, 4 and 5 lies inside the circle $2x^2 + 2y^2 = 25$. The triangle is moved inside the circle in such a way that its hypotenuse always forms a chord of the circle. The locus of the vertex opposite to the hypotenuse is
(A) $2x^2 + 2y^2 = 1$ (B) $x^2 + y^2 = 1$ (C) $x^2 + y^2 = 2$ (D) $2x^2 + 2y^2 = 5$

583. Let P be the point $(-3, 0)$ and Q be a moving point $(0, 3t)$. Let PQ be trisected at R so that R is nearer to Q. RN is drawn perpendicular to PQ meeting the x-axis at N. The locus of the mid-point of RN is

(A) $(x+3)^2 - 3y = 0$; (B) $(y+3)^2 - 3x = 0$; (C) $x^2 - y = 1$; (D) $y^2 - x = 1$.

584. The maximum distance between two points of the unit cube is

(A) $\sqrt{2} + 1$; (B) $\sqrt{2}$; (C) $\sqrt{3}$; (D) $\sqrt{2} + \sqrt{3}$.

585. Each side of a cube is increased by 50%. Then the surface area of the cube is increased by

(A) 50%; (B) 100%; (C) 125%; (D) 150%.

586. A variable plane passes through a fixed point (a, b, c) and cuts the coordinate axes at P, Q, R. Then the coordinates (x, y, z) of the centre of the sphere passing through P, Q, R and the origin satisfy the equation

(A) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$; (B) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$;
(C) $ax + by + cz = 1$; (D) $ax + by + cz = a^2 + b^2 + c^2$.

587. Let $A = (0, 10)$ and $B = (30, 20)$ be two points in the plane and let $P = (x, 0)$ be a moving point on the x-axis. The value of x for which the sum of the distances of P from A and B is minimum equals

(A) 0; (B) 10; (C) 15; (D) 20.

588. The number of solutions to the pair of equations

$$\begin{aligned} \sin\left(\frac{x+y}{2}\right) &= 0 \\ |x| + |y| &= 1 \end{aligned}$$

is

(A) 2; (B) 3; (C) 4; (D) 1.

589. The equation $r^2 \cos\theta + 2ar \sin^2 \frac{\theta}{2} - a^2 = 0$ (a positive) represents

(A) a circle; (B) a circle and a straight line;
(C) two straight lines; (D) none of the foregoing curves.

590. The number of distinct solutions of $\sin 5\theta \cos 3\theta = \sin 9\theta \cos 7\theta$, in the interval $0 \leq \theta \leq \frac{\pi}{2}$ is

(A) 5; (B) 4; (C) 8; (D) 9.

591. The value of $\sin 15°$ is

(A) $\frac{\sqrt{6}-\sqrt{2}}{4}$; (B) $\frac{\sqrt{6}+\sqrt{2}}{4}$; (C) $\frac{\sqrt{5}+1}{2}$; (D) $\frac{\sqrt{5}-1}{2}$.

592. The value of $\sin 25° \sin 35° \sin 85°$ is equal to

(A) $\frac{\sqrt{3}}{4}$; (B) $\frac{1}{4}\sqrt{2-\sqrt{3}}$; (C) $\frac{5\sqrt{3}}{9}$; (D) $\frac{1}{4}\sqrt{\frac{1}{2}+\frac{\sqrt{3}}{4}}$.

593. The angle made by the complex number $\frac{1}{(\sqrt{3}+i)^{100}}$ with the positive real axis is

(A) 135°; (B) 120°; (C) 240°; (D) 180°.

594. The value of $\tan(\frac{\pi}{4}\sin^2 x)$, $-\infty < x < \infty$, lies between

(A) -1 and $+1$; (B) 0 and 1; (C) 0 and ∞; (D) $-\infty$ and $+\infty$.

595. If $\tan(\pi \cos\theta) = \cot(\pi \sin\theta)$, then the value of $\cos(\theta - \frac{\pi}{4})$ is

(A) $\pm\frac{1}{2\sqrt{2}}$; (B) $\pm\frac{1}{2}$; (C) $\pm\frac{1}{\sqrt{2}}$; (D) 0.

596. If $f(x) = \frac{1-x}{1+x}$, then $f(f(\cos x))$ equals

(A) x; (B) $\cos x$; (C) $\tan^2(\frac{x}{2})$; (D) none of the foregoing expressions.

597. If $\frac{\cos x}{\cos y} = \frac{a}{b}$, then $a \tan x + b \tan y$ equals

(A) $(a+b)\cot\frac{x+y}{2}$; (B) $(a+b)\tan\frac{x+y}{2}$;

(C) $(a+b)(\tan\frac{x}{2} + \tan\frac{y}{2})$; (D) $(a+b)(\cot\frac{x}{2} + \cot\frac{y}{2})$.

598. Let θ be an angle in the second quadrant (that is, $90^\circ \leq \theta < 180^\circ$) with $\tan\theta = -\frac{2}{3}$. Then the value of

$$\frac{\tan(90^\circ+\theta)+\cos(180^\circ+\theta)}{\sin(270^\circ-\theta)-\cot(-\theta)}$$

is

(A) $\frac{2+\sqrt{13}}{2-\sqrt{13}}$; (B) $\frac{2-\sqrt{13}}{2+\sqrt{13}}$; (C) $\frac{2+\sqrt{39}}{2-\sqrt{39}}$; (D) $2+3\sqrt{13}$.

599. Let P be a moving point such that if PA and PB are the two tangents drawn from P to the circle $x^2+y^2=1$ (A, B being the points of contact), then $\angle AOB = 60^\circ$, where O is the origin. Then the locus of P is

(A) a circle of radius $\frac{2}{\sqrt{3}}$; (B) a circle of radius 2;
(C) a circle of radius $\sqrt{3}$; (D) none of the foregoing curves.

600. A ring of 10 cm in diameter is suspended from a point 12 cm vertically above the centre by six equal strings. The strings are attached to the circumference of the ring at equal intervals, thus keeping the ring in a horizontal plane. The cosine of the angle between two adjacent strings is

(A) $\frac{2}{\sqrt{13}}$; (B) $\frac{313}{338}$; (C) $\frac{5}{\sqrt{26}}$; (D) $\frac{5\sqrt{651}}{338}$.

601. If, inside a big circle, exactly $n (n \geq 3)$ small circles, each of radius r, can be drawn in such a way that each small circle touches the big circle and also touches both its adjacent small circles (as shown in the picture), then the radius of the big circle is

(A) $r\,\text{cosec}\frac{\pi}{n}$;
(B) $r(1+\text{cosec}\frac{2\pi}{n})$;
(C) $r(1+\text{cosec}\frac{\pi}{2n})$;
(D) $r(1+\text{cosec}\frac{\pi}{n})$.

602. The range of values taken by $4\cos^3 A - 3\cos A$ is

(A) all negative values;
(B) all positive and negative values between $-\frac{4}{3}$ and $+\frac{4}{3}$;
(C) all positive and negative values between -1 and $+1$;
(D) all positive values.

603. If $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, then $\cos\theta - \sin\theta$ is

(A) always negative; (B) sometimes zero;
(C) always positive; (D) sometimes positive, sometimes negative.

604. For all angles A

$$\frac{\sin 2A \cos A}{(1+\cos 2A)(1+\cos A)}$$

equals

(A) $\sin \frac{A}{2}$; (B) $\cos \frac{A}{2}$; (C) $\tan \frac{A}{2}$; (D) $\sin A$.

605. If the angle θ with $0 < \theta < \frac{\pi}{2}$ is measured in radians, then $\cos\theta$ always lies between

(A) 0 and $1 - \frac{1}{2}\theta^2$; (B) $1 - \frac{1}{2}\theta^2 + \theta$ and 1;
(C) $1 - \frac{1}{3}\theta^2$ and 1; (D) $1 - \frac{1}{2}\theta^2$ and 1.

606. All possible values of x in $[0, 2\pi]$ satisfying the inequality $\sin 2x < \sin x$, are given by

(A) $\frac{\pi}{3} < x < \frac{5\pi}{3}$; (B) $\frac{\pi}{3} < x < \frac{2\pi}{3}$ and $\frac{4\pi}{3} < x < \frac{5\pi}{3}$;
(C) $\frac{\pi}{3} < x < \pi$ and $\frac{4\pi}{3} < x < 2\pi$; (D) $\frac{\pi}{3} < x < \pi$ and $\frac{5\pi}{3} < x < 2\pi$.

607. If $0 \leq \alpha \leq \pi/2$, then which of the following is true?

(A) $\sin(\cos\alpha) < \cos(\sin\alpha)$;
(B) $\sin(\cos\alpha) \leq \cos(\sin\alpha)$, and equality holds for some $\alpha \in [0, \pi/2]$;
(C) $\sin(\cos\alpha) > \cos(\sin\alpha)$;
(D) $\sin(\cos\alpha) \geq \cos(\sin\alpha)$, and equality holds for some α in $[0, \pi/2]$.

608. The value of $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$ is

(A) $\frac{3}{4}$; (B) $\frac{1}{\sqrt{2}}$; (C) $\frac{3}{2}$; (D) $\frac{\sqrt{3}}{2}$.

609. The expression

$$\tan\theta + 2\tan(2\theta) + 2^2\tan(2^2\theta) + \ldots + 2^{14}\tan(2^{14}\theta) + 2^{15}\cot(2^{15}\theta)$$

is equal to

(A) $2^{16}\tan(2^{16}\theta)$; (B) $\tan\theta$; (C) $\cot\theta$; (D) $2^{16}[\tan(2^{16}\theta) + \cot(2^{16}\theta)]$.

610. If α and β are two different solutions, lying between $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$, of the equation $2\tan\theta + \sec\theta = 2$, then $\tan\alpha + \tan\beta$ is

(A) 0; (B) 1; (C) $\frac{4}{3}$; (D) $\frac{8}{3}$.

611. Given that $\tan\theta = \frac{b}{a}$, the value of $a\cos 2\theta + b\sin 2\theta$ is

(A) $a^2(1-\frac{b^2}{a^2}) + 2b^2$; (B) $\frac{a^2+b^2}{a}$; (C) a; (D) $\frac{a^2+b^2}{a^2}$.

612. It $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then the value of $\cos(\theta - \frac{\pi}{4})$ is

(A) $\frac{1}{2}$; (B) $\pm\frac{1}{2\sqrt{2}}$; (C) $-\frac{1}{2\sqrt{2}}$; (D) $\frac{1}{2\sqrt{2}}$.

613. If $\tan x = \frac{2}{5}$, then $\sin 2x$ equals

(A) $\frac{20}{29}$; (B) $\pm\frac{10}{\sqrt{29}}$; (C) $-\frac{20}{29}$; (D) none of the foregoing numbers.

614. If $x = \tan 15°$, then

(A) $x^2 + 2\sqrt{3}x - 1 = 0$; (B) $x^2 + 2\sqrt{3}x + 1 = 0$; (C) $x = \frac{1}{2\sqrt{3}}$; (D) $x = \frac{2}{\sqrt{3}}$.

615. The value of $2\sin(\frac{\theta}{2})\cos(\frac{3\theta}{2}) + 4\sin\theta\sin^2(\frac{\theta}{2})$ equals

(A) $\sin(\frac{\theta}{2})$; (B) $\sin(\frac{\theta}{2})\cos\theta$; (C) $\sin\theta$; (D) $\cos\theta$.

616. If a and b are given positive numbers, then the values of c and θ with $0 \leq \theta \leq \pi$ for which $a\sin x + b\cos x = c\sin(x+\theta)$ is true for all x are given by

(A) $c = \sqrt{a^2+b^2}$ and $\tan\theta = \frac{a}{b}$; (B) $c = -\sqrt{a^2+b^2}$ and $\tan\theta = \frac{b}{a}$;
(C) $c = a^2 + b^2$ and $\tan\theta = \frac{b}{a}$; (D) $c = \sqrt{a^2+b^2}$ and $\tan\theta = \frac{b}{a}$.

617. The value of $\sin 330° + \tan 45° - 4\sin^2 120° + 2\cos^2 135° + \sec^2 180°$ is

(A) $\frac{1}{2}$; (B) $\frac{\sqrt{3}}{2}$; (C) $-\frac{\sqrt{3}}{2}$; (D) $-\frac{1}{2}$.

618. Given that $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, the value of $\tan\frac{5\pi}{8}$ is

(A) $-(\sqrt{2}+1)$; (B) $-\frac{1}{\sqrt{2}+1}$; (C) $1-\sqrt{2}$; (D) $\frac{1}{\sqrt{2}-1}$.

619. $\sin^6\frac{\pi}{49} + \cos^6\frac{\pi}{49} - 1 + 3\sin^2\frac{\pi}{49}\cos^2\frac{\pi}{49}$ equals

(A) 0; (B) $\tan^6\frac{\pi}{49}$; (C) $\frac{1}{2}$; (D) none of the foregoing numbers.

620. If $a\sin\theta = b\cos\theta$, then the value of $\sqrt{\frac{a-b}{a+b}} + \sqrt{\frac{a+b}{a-b}}$ equals

(A) $2\cos\theta$; (B) $\frac{2\cos\theta}{\sqrt{\cos 2\theta}}$; (C) $\frac{2\sin\theta}{\sqrt{\cos 2\theta}}$; (D) $\frac{2}{\sqrt{\cos 2\theta}}$.

621. The sides of a triangle are given to be $x^2 + x + 1, 2x + 1$ and $x^2 - 1$. Then the largest of the three angles of the triangle is

(A) 75°; (B) $\frac{x}{x+1}\pi$; (C) 120°; (D) 135°.

622. If A, B, C are the angles of a triangle, then the value of

$$1 - (\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2})$$

equals

(A) $2 \sin A \sin B \sin C$; (B) $2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$;
(C) $4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$; (D) $4 \sin A \sin B \sin C$.

623. In any triangle, if $\tan \frac{A}{2} = \frac{5}{6}$, $\tan \frac{B}{2} = \frac{20}{37}$ and $\tan \frac{C}{2} = \frac{2}{5}$, then
(A) $a + c = 2b$; (B) $a + b = 2c$; (C) $b + c = 2a$; (D) none of these holds.

624. Let $\cos(\alpha - \beta) = -1$. Then only one of the following statements is *always* true. Which one is it?

(A) α is not less than β.
(B) $\sin \alpha + \sin \beta = 0$ and $\cos \alpha + \cos \beta = 0$.
(C) Angles α and β are both positive.
(D) $\sin \alpha + \sin \beta = 0$ but $\cos \alpha + \cos \beta$ may not be zero.

625. If the trigonometric equation $1 + \sin^2 x\theta = \cos \theta$ has a nonzero solution in θ, then x must be

(A) an integer; (B) a rational number;
(C) an irrational number; (D) strictly between 0 and 1.

626. It is given that $\tan A$ and $\tan B$ are the two roots of the equation $x^2 - bx + c = 0$. The value of $\sin^2(A + B)$ is

(A) $\frac{b^2}{b^2+(1-c)^2}$; (B) $\frac{b^2}{b^2+c^2}$; (C) $\frac{b^2}{(b+c)^2}$; (D) $\frac{b^2}{c^2+(1-b)^2}$.

627. If $\cos x + \cos y + \cos z = 0$, $\sin x + \sin y + \sin z = 0$, then $\cos \frac{(x-y)}{2}$ is

(A) $\pm\frac{\sqrt{3}}{2}$; (B) $\pm\frac{1}{2}$; (C) $\pm\frac{1}{\sqrt{2}}$; (D) 0.

628. If x, y, z are in G.P. and $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are in A.P., then

(A) $x = y = z$ or $y = \pm 1$;
(B) $z = \frac{1}{x}$;
(C) $x = y = z$, but their common value is not necessarily 0;
(D) $x = y = z = 0$.

629. If α and β satisfy the equation $\sin\alpha + \sin\beta = \sqrt{3}(\cos\alpha - \cos\beta)$, then

(A) $\sin 3\alpha + \sin 3\beta = 1$; (B) $\sin 3\alpha + \sin 3\beta = 0$;
(C) $\sin 3\alpha - \sin 3\beta = 0$; (D) $\sin 3\alpha - \sin 3\beta = 1$.

630. If $\cos 2\theta = \sqrt{2}(\cos\theta - \sin\theta)$, then $\tan\theta$ is

(A) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$ or 1; (B) 1; (C) 1 or -1;
(D) none of the foregoing values.

631. The number of roots between 0 and π of the equation $2\sin^2 x + 1 = 3\sin x$ equals

(A) 2; (B) 4; (C) 1; (D) 3.

632. The equation in θ given by

$$\operatorname{cosec}^2\theta - \frac{2}{3}\sqrt{3}\operatorname{cosec}\theta\sec\theta - \sec^2\theta = 0$$

has solutions

(A) only in the first and third quadrants;
(B) only in the second and fourth quadrants;
(C) only in the third quadrant;
(D) in all the four quadrants.

633. If $\tan\theta + \cot\theta = 4$, then θ, for some integer n, is

(A) $\frac{n\pi}{2} + (-1)^n\frac{\pi}{12}$; (B) $n\pi + (-1)^n\frac{\pi}{12}$; (C) $n\pi + \frac{\pi}{12}$; (D) $n\pi - \frac{\pi}{12}$.

634. The equation $\sin x(\sin x + \cos x) = k$ has real solutions if and only if k is a real number such that

(A) $0 \le k \le \frac{1+\sqrt{2}}{2}$; (B) $2 - \sqrt{3} \le k \le 2 + \sqrt{3}$;
(C) $0 \le k \le 2 - \sqrt{3}$; (D) $\frac{1-\sqrt{2}}{2} \le k \le \frac{1+\sqrt{2}}{2}$.

635. The number of solutions of the equation $2\sin\theta + 3\cos\theta = 4$ for $0 \le \theta \le 2\pi$ is

(A) 0; (B) 1; (C) 2; (D) more than 2.

636. The number of values of x satisfying the equation

$$\sqrt{\sin x} - \frac{1}{\sqrt{\sin x}} = \cos x$$

is

(A) 1; (B) 2; (C) 3; (D) more than 3.

637. The number of times the function

$$f(x) = |\text{minimum}\{\sin x, \cos x\}|$$

takes the value 0.8 between $\frac{20}{3}\pi$ and $\frac{43}{6}\pi$ is

(A) 2; (B) more than 2; (C) 0; (D) 1.

638. The number of roots of the equation

$$2x = 3\pi(1 - \cos x),$$

where x is measured in radians, is

(A) 3; (B) 5; (C) 4; (D) 2.

639. Let $f(x) = \sin x - ax$ and $g(x) = \sin x - bx$, where $0 < a, b < 1$. Suppose that the number of real roots of $f(x) = 0$ is greater than that of $g(x) = 0$. Then

(A) $a < b$; (B) $a > b$; (C) $ab = \pi/6$;
(D) none of the foregoing relations hold.

640. The number of solutions θ with $0 < \theta < \frac{\pi}{2}$ of the equation

$$\sin 7\theta - \sin\theta = \sin 3\theta$$

is

(A) 1; (B) 2; (C) 3; (D) more than 3.

641. The number of solutions of the equation $\tan 5\theta = \cot 2\theta$ such that $0 \leq \theta \leq \pi/2$ is

(A) 1; (B) 2; (C) 3; (D) 4.

642. If $\sin^{-1}\frac{1}{\sqrt{5}}$ and $\cos^{-1}\frac{3}{\sqrt{10}}$ are angles in $\left[0, \frac{\pi}{2}\right]$, then their sum is equal to

(A) $\frac{\pi}{6}$; (B) $\frac{\pi}{4}$; (C) $\frac{\pi}{3}$; (D) $\sin^{-1}\frac{1}{\sqrt{50}}$.

643. If $\cot(\sin^{-1}\sqrt{\frac{13}{17}}) = \sin(\tan^{-1}\alpha)$, then α is

(A) $\frac{4}{17}$; (B) $\sqrt{\frac{17^2-13^2}{17\times 13}}$; (C) $\sqrt{\frac{17^2-13^2}{17^2+13^2}}$; (D) $\frac{2}{3}$.

644. The minimum value of $\sin 2\theta - \theta$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ is

(A) $-\frac{\sqrt{3}}{2} + \frac{\pi}{6}$; (B) $-\pi$; (C) $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$; (D) $-\frac{\pi}{2}$.

645. The number of solutions θ in the range $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and satisfying the equation

$$\sin^3\theta + \sin^2\theta + \sin\theta - \sin\theta\sin 2\theta - \sin 2\theta - 2\cos\theta = 0$$

is

(A) 0; (B) 1; (C) 2; (D) 3.

646. The number of roots of the equation

$$\cos^8\theta - \sin^8\theta = 1$$

in the interval $[0, 2\pi]$ is

(A) 4; (B) 8; (C) 3; (D) 6.

647. If $\sin 6\theta = \sin 4\theta - \sin 2\theta$, then θ must be, for some integer n, equal to

(A) $\frac{n\pi}{4}$; (B) $n\pi \pm \frac{\pi}{6}$; (C) $\frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{6}$; (D) $\frac{n\pi}{2}$.

648. Consider the solutions of the equation $\sqrt{2}\tan^2 x - \sqrt{10}\tan x + \sqrt{2} = 0$ in the range $0 \leq x \leq \frac{\pi}{2}$. Then only one of the following statements is true. Which one is it?

(A) No solutions for x exist in the given range.
(B) Two solutions x_1 and x_2 exist with $x_1 + x_2 = \frac{\pi}{4}$.
(C) Two solutions x_1 and x_2 exist with $x_1 - x_2 = \frac{\pi}{4}$.
(D) Two solutions x_1 and x_2 exist with $x_1 + x_2 = \frac{\pi}{2}$.

649. The set of all values of θ which satisfy the equation $\cos 2\theta = \sin\theta + \cos\theta$ is

(A) $\theta = 0$;
(B) $\theta = n\pi + \frac{\pi}{2}$, where n is any integer;
(C) $\theta = 2n\pi$ or $\theta = 2n\pi - \frac{\pi}{2}$ or $\theta = n\pi - \frac{\pi}{4}$, where n is any integer;
(D) $\theta = 2n\pi$ or $\theta = n\pi + \frac{\pi}{4}$, where n is any integer.

650. The equation $2x = (2n+1)\pi(1 - \cos x)$, where n is a positive integer, has

(A) infinitely many real roots; (B) exactly $2n+1$ real roots;
(C) exactly one real root; (D) exactly $2n+3$ real roots.

651. The number of roots of the equation

$$\sin 2x + 2\sin x - \cos x - 1 = 0$$

in the range $0 \leq x \leq 2\pi$ is

(A) 1; (B) 2; (C) 3; (D) 4.

652. If $2\sec 2\alpha = \tan\beta + \cot\beta$, then one possible value of $\alpha+\beta$ is

(A) $\frac{\pi}{2}$; (B) $\frac{\pi}{4}$; (C) $\frac{\pi}{3}$; (D) 0.

653. The equation

$$[3\sin^4\theta - 2\cos^6\theta + y - 2\sin^6\theta + 3\cos^4\theta]^2 = 9,$$

is true

(A) for any value of θ and $y = 2$ or -4.
(B) only for $\theta = \frac{\pi}{4}$ or π and $y = -2$ or 4.
(C) only for $\theta = \frac{\pi}{2}$ or π and $y = 2$ or -4.
(D) only for $\theta = 0$ or $\frac{\pi}{2}$ and $y = 2$ or -2.

654. If the shadow of a tower standing on the level plane is found to be 60 feet (ft) longer when the sun's altitude is 30° than when it is 45°, then the height of the tower is, in ft,

(A) $30(1+\frac{\sqrt{3}}{2})$; (B) 45; (C) $30(1+\sqrt{3})$; (D) 30.

655. Two poles, AB of length 2 metres and CD of length 20 metres are erected vertically with bases at B and D. The two poles are at a distance not less than twenty metres. It is observed that $\tan\angle ACB = \frac{2}{77}$. The distance between the two poles, in metres, is

(A) 72; (B) 68; (C) 24; (D) 24·27.

656. The elevation of the top of a tower from a point A is 45°. From A, a man walks 10 meters up a path sloping at an angle of 30°. After this the slope becomes steeper and after walking up another 10 meters the man reaches the top. Then the distance of A from the foot of the tower is

(A) $5(\sqrt{3}+1)$ meters; (B) 5 meters;
(C) $10\sqrt{2}$ meters; (D) $5\sqrt{2}$ meters.

657. A man standing x metres to the north of a tower finds the angle of elevation of its top to be 30°. He then starts walking towards the tower. After walking a distance of $\frac{1}{2}x$ metres, he turns east and walks $\frac{1}{2}x$ metres. Then again he turns south and walks $\frac{1}{2}x$ metres. The angle of elevation of the top of the tower from his new position is

(A) 30°; (B) $\tan^{-1}\sqrt{\frac{2}{3}}$; (C) $\tan^{-1}\frac{2}{\sqrt{3}}$; (D) none of the foregoing quantities.

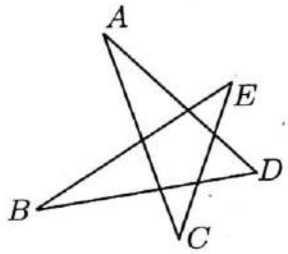
658. The elevation of the summit of a mountain is found to be 45°. After ascending one km towards the summit, up a slope of 30° inclination, the elevation is found to be 60°. Then the height of the mountain is, in km,

(A) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$; (B) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$; (C) $\frac{1}{\sqrt{3}-1}$; (D) $\frac{1}{\sqrt{3}+1}$.

659. The distance at which a vertical pillar, of height 33 feet, subtends an angle of $12''$ (that is, 12 seconds) is, approximately in yards (1 yard = 3 feet),

(A) $\frac{11000000}{6\pi}$; (B) $\frac{864000}{11\pi}$; (C) $\frac{594000}{\pi}$; (D) $\frac{864000}{\pi}$.

660. If the points A, B, C, D and E in the figure lie on a circle, then $\frac{AD}{BE}$



(A) equals $\frac{\sin(A+D)}{\sin(B+E)}$;

(B) equals $\frac{\sin B}{\sin D}$;

(C) equals $\frac{\sin(B+C)}{\sin(C+D)}$;

(D) cannot be found unless the radius of the circle is given.

661. A man stands at a point A on the bank AB of a straight river and observes that the line joining A to a post C on the opposite bank makes with AB an angle of 30°. He then goes 200 metres along the bank to B, and finds that BC makes an angle of 60° with the bank. If b is the breadth of the river, then

(A) $50\sqrt{3}$ is the only possible value of b;

(B) $100\sqrt{3}$ is the only possible value of b;

(C) $50\sqrt{3}$ and $100\sqrt{3}$ are the only possible values of b;

(D) none of the foregoing statements is correct.

662. A straight pole A subtends a right angle at a point B of another pole at a distance of 30 metres from A, the top of A being 60° above the horizontal line joining the point B to the pole A. The length of the pole A is, in metres,

(A) $20\sqrt{3}$; (B) $40\sqrt{3}$; (C) $60\sqrt{3}$; (D) $\frac{40}{\sqrt{3}}$.

663. The angle of elevation of a bird from a point h metres above a lake is α and the angle of depression of its image in the lake from the same point is β. The height of the bird above the lake is, in metres,

(A) $\frac{h\sin(\beta-\alpha)}{\sin\beta\cos\alpha}$; (B) $\frac{h\sin(\beta+\alpha)}{\sin\alpha\cos\beta}$; (C) $\frac{h\sin(\beta-\alpha)}{\sin(\alpha+\beta)}$; (D) $\frac{h\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$.

664. Two persons who are 500 metres apart, observe the direction and the angular elevation of a balloon at the same instant. One finds the elevation to be 60° and the direction South-West, while the other finds the elevation to be 45° and the direction West. Then the height of the balloon is, in metres,

(A) $500\sqrt{(\frac{12+3\sqrt{6}}{10})}$; (B) $500\sqrt{(\frac{12-3\sqrt{6}}{10})}$; (C) $250\sqrt{3}$;
(D) none of the foregoing numbers.

665. Standing far from a hill, an observer records its elevation. The elevation increases by 15° as he walks $1+\sqrt{3}$ miles towards the hill, and by a further 15° as he walks another mile in the same direction. Then, the height of the hill is
(A) $\frac{\sqrt{3}+1}{2}$ miles; (B) $\frac{\sqrt{3}-1}{\sqrt{2}-1}$ miles; (C) $\frac{\sqrt{3}-1}{2}$ miles; (D) none of these.

666. A man finds that at a point due south of a tower the angle of elevation of the tower is 60°. He then walks due west $10\sqrt{6}$ metres on a horizontal plane and finds that the angle of elevation of the tower at that point is 30°. Then the original distance of the man from the tower is, in metres

(A) $5\sqrt{3}$; (B) $15\sqrt{3}$; (C) 15; (D) 180.

667. A man stands a metres due east of a tower and finds the angle of elevation of the top of the tower to be θ. He then walks x metres north west and finds the angle of elevation to be θ again. Then the value of x is

(A) a; (B) $\sqrt{2}a$; (C) $\frac{a}{\sqrt{2}}$; (D) none of the foregoing expressions.

668. The angle of elevation of the top of a hill from a point A is α. After walking a distance d towards the top, up a slope inclined to the horizon at an angle θ, which is less than α, the angle of elevation is β. The height h of the hill equals

(A) $\frac{d\sin\alpha\sin\theta}{\sin(\beta-\alpha)}$; (B) $\frac{d\sin(\beta-\alpha)\sin\theta}{\sin\alpha\sin\beta}$; (C) $\frac{d\sin(\alpha-\theta)\sin(\beta-\alpha)}{\sin(\alpha-\theta)}$; (D) $\frac{d\sin\alpha\sin(\beta-\theta)}{\sin(\beta-\alpha)}$.

669. A person observes the angle of elevation of a peak from a point A on the ground to be α. He goes up an incline of inclination β, where $\beta < \alpha$, to the horizontal level towards the top of the peak and observes that the angle of elevation of the peak now is γ. If B is the second place of observation and AB is y metres, the height of the peak above the ground is

(A) $y\sin\beta + y\sin(\alpha-\beta)\text{cosec}(\gamma-\alpha)\sin\gamma$;
(B) $y\sin\beta + y\sin(\beta-\alpha)\sec(\gamma-\alpha)\sin\gamma$;
(C) $y\sin\beta + y\sin(\alpha-\beta)\sec(\alpha-\gamma)\sin\gamma$;
(D) $y\sin\beta + y\sin(\alpha-\beta)\text{cosec}(\alpha-\gamma)\sin\gamma$.

670. Standing on one side of a 10 meter wide straight road, a man finds that the angle of elevation of a statue located on the same side of the road is α. After crossing the road by the shortest possible distance, the angle reduces to β. The height of the statue is

(A) $\frac{10\tan\alpha\tan\beta}{\sqrt{\tan^2\alpha-\tan^2\beta}}$; (B) $\frac{10\sqrt{\tan^2\alpha-\tan^2\beta}}{\tan\alpha\tan\beta}$;

(C) $10\sqrt{\tan^2\alpha-\tan^2\beta}$; (D) $\frac{10}{\sqrt{\tan^2\alpha-\tan^2\beta}}$.

671. The complete set of solutions of the equation

$$\sin^{-1}x = 2\tan^{-1}x$$

is

(A) ±1; (B) 0; (C) $\pm1, 0$; (D) $\pm\frac{1}{2}, \pm1, 0$.

672. For a regular octagon (a polygon with 8 equal sides) inscribed in a circle of radius 1, the product of the distances from a fixed vertex to the other seven vertices is

(A) 4; (B) 8; (C) 12; (D) 16.

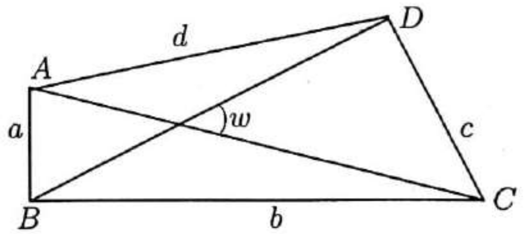
673. In the quadrilateral in the figure, the lengths of AC and BD are x and y respectively. Then the value of $2xy\cos w$ equals

(A) $b^2+d^2-a^2-c^2$;

(B) $b^2+a^2-c^2-d^2$;

(C) $a^2+c^2-b^2-d^2$;

(D) $a^2+d^2-b^2-c^2$.



674. In a triangle ABC with sides $a=5, b=3$ and $c=7$, the value of $3\cos C+7\cos B$ is

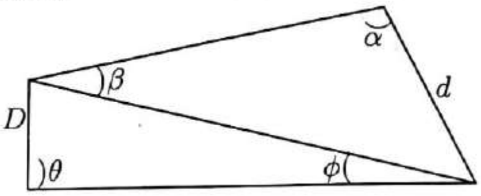
(A) 3; (B) 7; (C) 10; (D) 5.

675. If in a triangle ABC, the bisector of the angle $\angle A$ meets the side BC at the point D, then the length AD equals

(A) $\frac{2bc\cos\frac{A}{2}}{b+c}$; (B) $\frac{bc\cos\frac{A}{2}}{b+c}$; (C) $\frac{bc\cos A}{b+c}$; (D) $\frac{2bc\sin\frac{A}{2}}{b+c}$.

676. In an arbitrary quadrilateral with sides and angles as marked in the figure, the value of d is equal to

(A) $\frac{D\sin\theta\sin\alpha}{\sin\phi\sin\beta}$;
(B) $\frac{D\sin\phi\sin\beta}{\sin\theta\sin\alpha}$;
(C) $\frac{D\sin\theta\sin\beta}{\sin\phi\sin\alpha}$;
(D) $\frac{D\sin\theta\sin\phi}{\sin\alpha\sin\beta}$.



677. Suppose the internal bisectors of the angles of a quadrilateral form another quadrilateral. Then the sum of the cosines of the angles of the second quadrilateral

(A) is a constant independent of the first quadrilateral;
(B) always equals the sum of the sines of the angles of the first quadrilateral;
(C) always equals the sum of the cosines of the angles of the first quadrilateral;
(D) depends on the angles as well as the sides of the first quadrilateral.

678. Consider the following two statements:
P: all cyclic quadrilaterals $ABCD$ satisfy $\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}\tan\frac{D}{2} = 1$,
Q: all trapeziums $ABCD$ satisfy $\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}\tan\frac{D}{2} = 1$.
Then

(A) both P and Q are true;
(B) P is true but Q is not true;
(C) P is not true and Q is true;
(D) neither P nor Q is true.

679. Let a, b, c denote the three sides of a triangle and A, B, C the corresponding opposite angles. Only one of the expressions below has the same value for all triangles. Which one is it?

(A) $\sin A + \sin B + \sin C$.
(B) $\tan A\tan B + \tan B\tan C + \tan C\tan A$.
(C) $\frac{a+b+c}{\sin A+\sin B+\sin C}$.
(D) $\cot A\cot B + \cot B\cot C + \cot C\cot A$.

680. In a triangle ΔABC, $2\sin C\cos B = \sin A$ holds. Then one of the following statements is correct. Which one is it?

(A) The triangle must be equilateral.
(B) The triangle must be isosceles but not necessarily equilateral.
(C) C must be an obtuse angle.
(D) None of the foregoing statements is necessarily true.

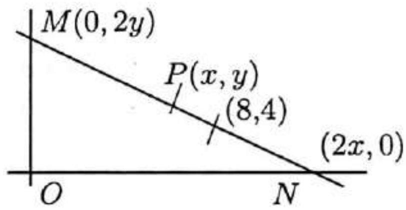
681. If A, B, C are the angles of a triangle and $\sin^2 A + \sin^2 B = \sin^2 C$, then C equals

(A) 30°; (B) 90°; (C) 45°; (D) none of the foregoing angles.

682. The value of $\dfrac{\cos 37° + \sin 37°}{\cos 37° - \sin 37°}$ equals

(A) $\tan 8°$; (B) $\cot 8°$; (C) $\sec 8°$; (D) cosec 8°.

683. A straight line passes through the fixed point (8,4) and cuts the y-axis at M and the x-axis at N as in figure. Then the locus of the middle point P of MN is



(A) $xy - 4x - 2y + 8 = 0$;

(B) $xy - 2x - 4y = 0$;

(C) $xy + 2x + 4y = 64$;

(D) $xy + 4x + 2y = 72$.

684. In a triangle ABC, a, b and c denote the sides opposite to angles A, B and C respectively. If $\sin A = 2 \sin C \cos B$, then

(A) $b = c$; (B) $c = a$; (C) $a = b$;
(D) none of the foregoing statements is true.

685. The lengths of the sides CB and CA of a triangle ABC are given by a and b, and the angle C is $\frac{2\pi}{3}$. The line CD bisects the angle C and meets AB at D. Then the length of CD is

(A) $\frac{1}{a+b}$; (B) $\frac{a^2+b^2}{a+b}$; (C) $\frac{ab}{2(a+b)}$; (D) $\frac{ab}{a+b}$.

686. Suppose in a triangle ABC, $b \cos B = c \cos C$. Then the triangle

(A) is right-angled; (B) is isosceles; (C) is equilateral;
(D) need not necessarily be any of the above types.

687. Let $V_0 = 2, V_1 = 3$ and for any natural number $k \geq 1$, let $V_{k+1} = 3V_k - 2V_{k-1}$. Then for any $n \geq 0$, V_n equals

(A) $\frac{1}{2}(n^2 + n + 4)$; (B) $\frac{1}{6}(n^3 + 5n + 12)$; (C) $2^n + 1$;
(D) none of the foregoing expressions.

688. If $a_n = \frac{1000^n}{n!}$, for $n = 1, 2, 3, \ldots$, then the sequence $\{a_n\}$

(A) does not have a maximum;
(B) attains maximum at exactly one value of n;
(C) attains maximum at exactly two values of n;
(D) attains maximum for infinitely many values of n.

689. Let f be a function of a real variable such that it satisfies

$$f(r+s) = f(r) + f(s), \quad \text{for all } r, s.$$

Let m and n be integers. Then $f(\frac{m}{n})$ equals

(A) $\frac{m}{n}$; (B) $\frac{f(m)}{f(n)}$; (C) $\frac{m}{n} f(1)$;
(D) none of the foregoing expressions, in general.

690. Let $f(x)$ be a real-valued function defined for all real numbers x such that $|f(x) - f(y)| \le \frac{1}{2}|x - y|$ for all x, y. Then the number of points of intersection of the graph of $y = f(x)$ and the line $y = x$ is

(A) 0; (B) 1; (C) 2; (D) none of the foregoing numbers.

691. The limit of $\frac{1}{n^4} \sum_{k=1}^{n} k(k+2)(k+4)$ as $n \to \infty$

(A) exists and equals $\frac{1}{4}$; (B) exists and equals 0;
(C) exists and equals $\frac{1}{8}$; (D) does not exist.

692. The limit of the sequence $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots$ is

(A) 1; (B) 2; (C) $2\sqrt{2}$; (D) ∞.

693. Let

$$P_n = \frac{2^3 - 1}{2^3 + 1} \cdot \frac{3^3 - 1}{3^3 + 1} \cdots \frac{n^3 - 1}{n^3 + 1}; \qquad n = 2, 3, \ldots$$

$\lim_{n \to \infty} P_n$ is
(A) $\frac{3}{4}$; (B) $\frac{7}{11}$; (C) $\frac{2}{3}$; (D) $\frac{1}{2}$.

694. Let $a_1 = 1$ and $a_n = n(a_{n-1} + 1)$ for $n = 2, 3, \ldots$. Define

$$P_n = \left(1 + \frac{1}{a_1}\right)\left(1 + \frac{1}{a_2}\right) \cdots \left(1 + \frac{1}{a_n}\right).$$

Then $\lim_{n \to \infty} P_n$ is
(A) $1 + e$; (B) e; (C) 1; (D) ∞.

695. Let x be a real number. Let $a_0 = x$, $a_1 = \sin x$ and, in general, $a_n = \sin a_{n-1}$. Then the sequence $\{a_n\}$

(A) oscillates between -1 and $+1$, unless x is a multiple of π;

(B) converges to 0 whatever be x;

(C) converges to 0 if and only if x is a multiple of π;

(D) sometimes converges and sometimes oscillates depending on x.

696. If k is an integer such that

$$\lim_{n\to\infty}\left[\left(\cos\frac{k\pi}{4}\right)^n - \left(\cos\frac{k\pi}{6}\right)^n\right] = 0,$$

then

(A) k is divisible neither by 4 nor by 6;

(B) k must be divisible by 12, but not necessarily by 24;

(C) k must be divisible by 24;

(D) either k is divisible by 24 or k is divisible neither by 4 nor by 6.

697. The limit of $\sqrt{x}(\sqrt{x+4} - \sqrt{x})$ as $x \to \infty$

(A) does not exist;

(B) exists and equals 0;

(C) exists and equals $\frac{1}{2}$;

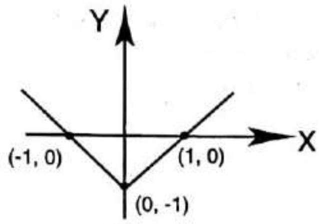
(D) exists and equals 2.

698. Four graphs marked G_1, G_2, G_3 and G_4 are given in the figure which are graphs of the four functions

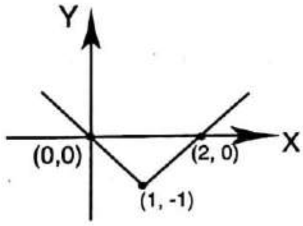
$$f_1(x) = |x-1| - 1, \quad f_2(x) = ||x-1| - 1|,$$

$$f_3(x) = |x| - 1, \quad f_4(x) = 1 - |x|,$$

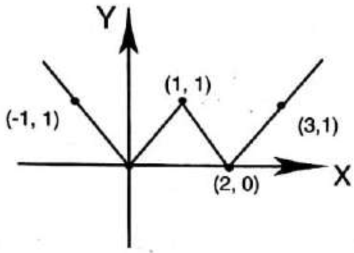
not necessarily in the correct order.



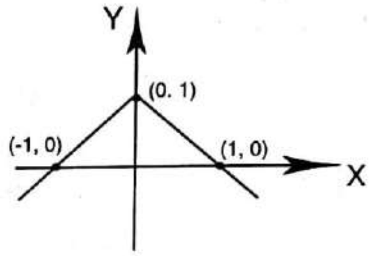
G_1



G_2



G_3

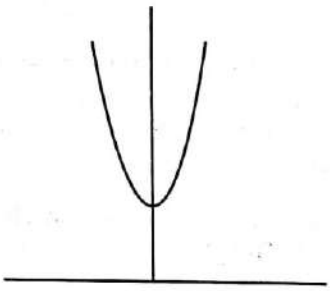


G_4

The correct order is

(A) G_2, G_1, G_3, G_4; (B) G_3, G_4, G_1, G_2;
(C) G_2, G_3, G_1, G_4; (D) G_4, G_3, G_1, G_2.

699. The adjoining figure is the graph of



(A) $y = 2e^x$;

(B) $y = 2e^{-x}$;

(C) $y = e^x + e^{-x}$;

(D) $y = e^x - e^{-x} + 2$.

700. Suppose that the three distinct real numbers a, b, c are in G.P. and $a+b+c = xb$. Then

(A) $-3 < x < 1$; (B) $x > 1$ or $x < -3$;
(C) $x < -1$ or $x > 3$; (D) $-1 < x < 3$.

701. The maximum value attained by the function $y = 10 - |x - 10|$ in the range $-9 \leq x \leq 9$ is

(A) 10; (B) 9; (C) $+\infty$; (D) 1.

702. Let $f(x)$ be a real-valued function of a real variable. Then the function is said to be 'one-to-one' if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$. The function is said to be 'onto' if it takes all real values. Suppose now $f(x) = x^3 - 3x^2 + 6x - 5$. Then

(A) f is one-to-one and onto;
(B) f is one-to-one but not onto;
(C) f is onto but not one-to-one;
(D) f is neither one-to-one nor onto.

703. Let f be a function from a set X to X such that $f(f(x)) = x$ for all $x \in X$. Then

(A) f is one-to-one but need not be onto;
(B) f is onto but need not be one-to-one;
(C) f is both one-to-one and onto;
(D) none of the foregoing statements is necessarily true.

Directions for Items 704 to 706:

A real-valued function $f(x)$ of a real variable x is said to be periodic if there is a strictly positive real number p such that $f(x+p) = f(x)$ for every x. The smallest p satisfying the above property is called the period of f.

704. Only one of the following functions is *not* periodic. Which one is it?

(A) $e^{\sin x}$.
(B) $\frac{1}{10+\sin x+\cos x}$.
(C) $\log_e(\cos x)$.
(D) $\sin(e^x)$.

705. Suppose f is periodic with period greater than h. Then

(A) for all $h' > h$ and for all $x, f(x+h') = f(x)$;
(B) for all $x, f(x+h) \neq f(x)$;
(C) for some $x, f(x+h) \neq f(x)$;
(D) none of the foregoing statements is true.

706. Suppose f is a function with period a and g is a function with period b. Then the function $h(x) = f(g(x))$

(A) may not have any period;
(B) has period a;
(C) has period b;
(D) has period ab.

707. A function f is said to be odd if $f(-x) = -f(x)$ for all x. Which of the following is *not* odd?

(A) A function f such that $f(x+y) = f(x) + f(y)$ for all x, y;
(B) $f(x) = \frac{xe^{x/2}}{1+e^x}$;

(C) $f(x) = x - [x]$, where $[x]$ denotes the greatest integer which is less than or equal to x;
(D) $f(x) = x^2 \sin x + x^3 \cos x$.

708. If n stands for the number of negative roots and p for the number of positive roots of the equation $e^x = x$, then

(A) $n = 1, p = 0$; (B) $n = 0, p = 1$; (C) $n = 0, p > 1$; (D) $n = 0, p = 0$.

709. In the interval $(-2\pi, 0)$, the function $f(x) = \sin(\frac{1}{x^3})$

(A) never changes sign;
(B) changes sign only once;
(C) changes sign more than once, but a finite number of times;
(D) changes sign infinite number of times.

710. If $f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + \ldots + a_n \cos nx$, where $a_0, a_1, \ldots, a_n$ are nonzero real numbers and $a_n > |a_0| + |a_1| + \ldots + |a_{n-1}|$, then the number of roots of $f(x) = 0$ in $0 \leq x \leq 2\pi$, is

(A) at most n; (B) more than n but less than $2n$;
(C) at least $2n$; (D) zero.

711. The number of roots of the equation $x^2 + \sin^2 x = 1$ in the closed interval $[0, \frac{\pi}{2}]$ is

(A) 0; (B) 1; (C) 2; (D) 3.

712. The number of roots of the equation $x \sin x = 1$ in the interval $0 < x \leq 2\pi$ is

(A) 0; (B) 1; (C) 2; (D) 4.

713. The number of points in the rectangle

$$\{(x, y) \mid -10 \leq x \leq 10 \text{ and } -3 \leq y \leq 3\}$$

which lie on the curve $y^2 = x + \sin x$ and at which the tangent to the curve is parallel to the x-axis, is

(A) 0; (B) 2; (C) 4; (D) 8.

714. The set of all real numbers x satisfying the inequality $x^3(x+1)(x-2) \geq 0$ can be written

(A) as $2 \leq x < \infty$; (B) as $0 \leq x < \infty$;
(C) as $-1 \leq x < \infty$; (D) in none of the foregoing forms.

715. A set S is said to *have a minimum if there is an element a in S such that $a \leq y$ for all y in S*. Similarly, S is said to *have a maximum if there is an element b in S such that $b \geq y$ for all y in S*. If $S = \left\{y : y = \frac{2x+3}{x+2},\ x \geq 0\right\}$, which one of the following statements is correct?

(A) S has both a maximum and a minimum.
(B) S has neither a maximum nor a minimum.
(C) S has a maximum but no minimum.
(D) S has a minimum but no maximum.

716. $\lim\limits_{x\to\infty} \dfrac{20 + 2\sqrt{x} + 3\sqrt[3]{x}}{2 + \sqrt{4x-3} + \sqrt[3]{8x-4}}$ is

(A) 10; (B) $\frac{3}{2}$; (C) 1; (D) 0.

717. $\lim\limits_{x\to\infty} [x\sqrt{x^2+a^2} - \sqrt{x^4+a^4}]$ is

(A) ∞; (B) $\frac{a^2}{2}$; (C) a^2; (D) 0.

718. The limit of $x^3\{\sqrt{x^2+\sqrt{x^4+1}} - x\sqrt{2}\}$ as $x \to \infty$

(A) exists and equals $\dfrac{1}{2\sqrt{2}}$;
(B) exists and equals $\dfrac{1}{4\sqrt{2}}$;
(C) does not exist;
(D) exists and equals $\dfrac{3}{4\sqrt{2}}$.

719. If $f(x) = \sqrt{\frac{x-\cos^2 x}{x+\sin x}}$, then the limit of $f(x)$ as $x \to \infty$ is

(A) 0; (B) 1; (C) ∞; (D) none of 0, 1 or ∞.

720. Consider the function $f(x) = \tan^{-1}(2\tan\frac{x}{2})$, where $-\frac{\pi}{2} \leq f(x) \leq \frac{\pi}{2}$.
($\lim\limits_{x\to\pi-0}$ means limit from the left at π and $\lim\limits_{x\to\pi+0}$ means limit from the right.)
Then

(A) $\lim\limits_{x\to\pi-0} f(x) = \dfrac{\pi}{2}$, $\lim\limits_{x\to\pi+0} f(x) = -\dfrac{\pi}{2}$;
(B) $\lim\limits_{x\to\pi-0} f(x) = -\dfrac{\pi}{2}$, $\lim\limits_{x\to\pi+0} f(x) = \dfrac{\pi}{2}$;
(C) $\lim\limits_{x\to\pi} f(x) = \dfrac{\pi}{2}$;
(D) $\lim\limits_{x\to\pi} f(x) = -\dfrac{\pi}{2}$.

721. The value of $\lim_{x\to a} \frac{x\sin a - a\sin x}{x-a}$ is

(A) non-existent; (B) $\sin a + a\cos a$; (C) $a\sin a - \cos a$; (D) $\sin a - a\cos a$.

722. The limit

$$\lim_{x\longrightarrow 0} \frac{\cos x - \sec x}{x^2(x+1)}$$

(A) is 0; (B) is 1; (C) is -1; (D) does not exist.

723. The limit

$$\lim_{x\to 0+} \frac{\tan x - x}{x - \sin x}$$

equals

(A) -1; (B) 0; (C) 1; (D) 2.

724. $\lim_{x\to 0} \frac{(1+x)^{\frac{1}{2}} - 1}{(1+x)^{\frac{1}{3}} - 1}$ is

(A) 1; (B) 0; (C) $\frac{3}{2}$; (D) ∞.

725. A right circular cylindrical container closed on both sides is to contain a fixed volume of motor oil. Suppose its base has diameter d and its height is h. The overall surface area of the container is minimum when

(A) $h = \frac{4}{3}\pi d$; (B) $h = 2d$; (C) $h = d$;
(D) conditions other than the foregoing are satisfied.

726. $\lim_{x\to\infty} (\log x - x)$

(A) equals $+\infty$; (B) equals e; (C) equals $-\infty$; (D) does not exist.

727. $\lim_{x\to 0} x\tan\frac{1}{x}$

(A) equals 0; (B) equals 1; (C) equals ∞; (D) does not exist.

728. The limit

$$\lim_{h\to 0} \int_{-1}^{1} \frac{h}{h^2+x^2}dx$$

(A) equals 0; (B) equals π; (C) equals $-\pi$; (D) does not exist.

729. If the area of an expanding circular region increases at a constant rate (with respect to time), then the rate of increase of the perimeter with respect to time

(A) varies inversely as the radius;
(B) varies directly as the radius;
(C) varies directly as the square of the radius;
(D) remains constant.

730. Let $y = \tan^{-1}(\frac{\sqrt{1+x^2}-1}{x})$. Then $\frac{dy}{dx}$ equals

(A) $\frac{1}{2(1+x^2)}$; (B) $\frac{2}{1+x^2}$; (C) $-\frac{1}{2}\cdot\frac{1}{1+x^2}$; (D) $-\frac{2}{1+x^2}$.

731. If θ is an acute angle then the largest value of $3\sin\theta + 4\cos\theta$ is

(A) 4; (B) $3(1+\frac{\sqrt{3}}{2})$; (C) $5\sqrt{2}$; (D) 5.

732. Let $f(x) = (x-1)e^x + 1$. Then

(A) $f(x) \geq 0$ for all $x \geq 0$ and $f(x) < 0$ for all $x < 0$;
(B) $f(x) \geq 0$ for all $x \geq 1$ and $f(x) < 0$ for all $x < 1$;
(C) $f(x) \geq 0$ for all x;
(D) none of the foregoing statements is true.

733. A ladder AB, 25 feet (ft) (1 ft=12 inches (in)) long leans against a vertical wall. The lower end A, which is at a distance of 7 ft from the bottom of the wall, is being moved away along the ground from the wall at the rate of 2 ft/sec. Then the upper end B will start moving towards the bottom of the wall at the rate of (in in/sec)

(A) 10; (B) 17; (C) 7; (D) 5.

734. Let

$$f(x) = \begin{cases} \left|\, |x-1| - 1 \,\right|, & \text{if } x < 1 \\ [x], & \text{if } x \geq 1, \end{cases}$$

where, for any real number x, $[x]$ denotes the largest integer $\leq x$ and $|\, y \,|$ denotes the absolute value of y. Then, the set of discontinuity-points of the function f consists precisely of

(A) all integers ≥ 0; (B) all integers ≥ 1;
(C) all integers > 1; (D) the integer 1.

735. Let f and g be two functions defined on an interval I such that $f(x) \geq 0$ and $g(x) \leq 0$ for all $x \in I$, and f is strictly decreasing on I while g is strictly increasing on I. Then

(A) the product function fg is strictly increasing on I;
(B) the product function fg is strictly decreasing on I;
(C) the product function fg is increasing but not necessarily strictly increasing on I;
(D) nothing can be said about the monotonicity of the product function fg.

736. Given that f is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real x, it follows that

(A) $f(x)$ is an increasing function;
(B) $f(x)$ is a decreasing function;
(C) $|f(x)|$ is an increasing function;
(D) $|f(x)|$ is a decreasing function.

737. Let x and y be positive numbers. Which of the following always implies $x^y \geq y^x$?

(A) $x \leq e \leq y$; (B) $y \leq e \leq x$;
(C) $x \leq y \leq e$ or $e \leq y \leq x$; (D) $y \leq x \leq e$ or $e \leq x \leq y$.

738. Let f be the function $f(x) = \cos x - 1 + \frac{x^2}{2}$. Then

(A) $f(x)$ is an increasing function on the real line;
(B) $f(x)$ is a decreasing function on the real line;
(C) $f(x)$ is an increasing function in the interval $-\infty < x \leq 0$ and decreasing in the interval $0 \leq x < \infty$;
(D) $f(x)$ is a decreasing function in the interval $-\infty < x \leq 0$ and increasing in the interval $0 \leq x < \infty$.

739. Consider the function $f(n)$ defined for all positive integers as follows:

$$f(n) = \begin{cases} n+1 & \text{if } n \text{ is odd,} \\ \frac{n}{2} & \text{if } n \text{ is even.} \end{cases}$$

Let $f^{(k)}$ denote f applied k times; e.g., $f^{(1)}(n) = f(n), f^{(2)}(n) = f(f(n))$ and so on. Then

(A) there exists one integer k_0 such that for all $n \geq 2$, $f^{(k_0)}(n) = 1$;
(B) for each $n \geq 2$, there exists an integer k (depending on n) such that $f^{(k)}(n) = 1$;
(C) for each $n \geq 2$, there exists an integer k (depending on n) such that $f^{(k)}(n)$ is a multiple of 4;
(D) for each n, $f^{(k)}(n)$ is a decreasing function of k.

740. Let $p_n(x), n = 0, 1, \ldots$ be polynomials defined by $p_0(x) = 1, p_1(x) = x$ and $p_n(x) = xp_{n-1}(x) - p_{n-2}(x)$ for $n \geq 2$. Then $p_{10}(0)$ equals

(A) 0; (B) 10; (C) 1; (D) −1.

741. Consider the function $f(x) = x(x-1)(x+1)$ from **R** to **R**, where **R** is the set of all real numbers. Then,

(A) f is one-one and onto; (B) f is neither one-one nor onto;
(C) f is one-one but not onto; (D) f is not one-one but onto.

742. For all integers $n \geq 2$, define $f_n(x) = (x+1)^{1/n} - x^{1/n}$, where $x > 0$. Then, as a function of x

(A) f_n is increasing for all n;
(B) f_n is decreasing for all n;
(C) f_n is increasing for n odd and f_n is decreasing for n even;
(D) f_n is decreasing for n odd and f_n is increasing for n even.

743. Let

$$g(x) = \int_{-10}^{x} tf'(t)\,dt \quad \text{for } x \geq -10,$$

where f is an increasing function. Then

(A) $g(x)$ is an increasing function of x;
(B) $g(x)$ is a decreasing function of x;
(C) $g(x)$ is increasing for $x > 0$ and decreasing for $-10 < x < 0$;
(D) none of the foregoing conclusions is necessarily true.

744. Let $f(x) = \begin{cases} x^3 - x + 3 & \text{for } 0 < x \leq 1, \\ 2x + 1 & \text{for } 1 < x \leq 2, \\ x^2 + 1 & \text{for } 2 < x < 3. \end{cases}$

Then

(A) $f(x)$ is differentiable at $x = 1$ and at $x = 2$;
(B) $f(x)$ is differentiable at $x = 1$ but not at $x = 2$;
(C) $f(x)$ is differentiable at $x = 2$ but not at $x = 1$;
(D) $f(x)$ is differentiable neither at $x = 1$ nor at $x = 2$.

745. If the function

$$f(x) = \begin{cases} \dfrac{x^2 - 2x + A}{\sin x} & \text{when } x \neq 0, \\ B & \text{when } x = 0, \end{cases}$$

is continuous at $x = 0$, then

(A) $A = 0, B = 0$;
(B) $A = 0, B = -2$;
(C) $A = 1, B = 1$;
(D) $A = 1, B = 0$.

746. The function

$$f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2} & \text{if } x < 0, \\ a & \text{if } x = 0, \\ \dfrac{2\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & \text{if } x > 0, \end{cases}$$

is continuous at $x = 0$ for

(A) $a = 8$; (B) $a = 4$; (C) $a = 16$; (D) no value of a.

747. Consider the function

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ x^2 & \text{if } x \text{ is irrational} \end{cases}$$

Then only one of the following statements is true. Which one is it?

(A) f is differentiable at $x = 0$ but not continuous at any other point.
(B) f is not continuous anywhere.
(C) f is continuous but not differentiable at $x = 0$.
(D) None of the foregoing statements is true.

748. Let $f(x) = x \sin \frac{1}{x}$, if $x \neq 0$, and let $f(x) = 0$, if $x = 0$. Then f is

(A) not continuous at 0;
(B) continuous but not differentiable at 0;
(C) differentiable at 0 and $f'(0) = 1$;
(D) differentiable at 0 and $f'(0) = 0$.

749. Let $f(x)$ be the function defined on the interval $(0, 1)$ by

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational,} \\ 1 - x & \text{otherwise.} \end{cases}$$

Then f is continuous

(A) at no point in $(0, 1)$;
(B) at exactly one point in $(0, 1)$;
(C) at more than one, but finitely many points in $(0, 1)$;
(D) at infinitely many points in $(0, 1)$.

750. The function $f(x) = [x] + \sqrt{x - [x]}$, where $[x]$ denotes the largest integer smaller than or equal to x, is

(A) continuous at every real number x;
(B) continuous at every real number x except at negative integer values;
(C) continuous at every real number x except at integer values;
(D) continuous at every real number x except at $x = 0$.

751. For any positive real number x and any positive integer n, we can uniquely write

$$x = mn + r,$$

where m is an integer (positive, negative or zero) and $0 \leq r < n$. With this notation we define

$$x \bmod n = r.$$

For example, $13.2 \bmod 3 = 1.2$.

The number of discontinuity points of the function

$$f(x) = (x \bmod 2)^2 + (x \bmod 4)$$

in the interval $0 < x < 9$ is

(A) 0; (B) 2; (C) 4; (D) 6.

752. Let $f(x)$ and $g(x)$ be defined as follows:

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0. \end{cases}$$

$$g(x) = \begin{cases} x^2, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0. \end{cases}$$

Then

(A) f and g are both differentiable at $x = 0$;
(B) f is differentiable at $x = 0$ but g is not;
(C) g is differentiable at $x = 0$ but f is not;
(D) neither f nor g is differentiable at $x = 0$.

753. The number of points at which the function

$$f(x) = \begin{cases} \min\{|x|, x^2\} & \text{if } -\infty < x < 1 \\ \min\{2x - 1, x^2\} & \text{otherwise} \end{cases}$$

is not differentiable is

(A) 0; (B) 1; (C) 2; (D) more than 2.

754. The function $f(x)$ is defined as

$$f(x) = \begin{cases} \frac{1}{|x|}, & \text{for } |x| > 2 \\ a + bx^2 & \text{for } |x| \leq 2, \end{cases}$$

where a and b are known constants. Then, only one of the following statements is true. Which one is it?

(A) $f(x)$ is differentiable at $x = -2$ if and only if $a = \frac{3}{4}$ and $b = -\frac{1}{16}$.
(B) $f(x)$ is differentiable at $x = -2$, whatever be the values of a and b.
(C) $f(x)$ is differentiable at $x = -2$, if $b = -\frac{1}{16}$, whatever be the value of a.
(D) $f(x)$ is differentiable at $x = -2$, if $b = \frac{1}{16}$ whatever be the value of a.

755. The function

$$f(x) = \begin{cases} \frac{\sin x^2}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(A) is continuous, but not differentiable at $x = 0$;
(B) is differentiable at $x = 0$, but the derivative is not continuous at $x = 0$;
(C) is differentiable at $x = 0$, and the derivative is continuous at $x = 0$;
(D) is not continuous at $x = 0$.

756. Let $f(x) = x[x]$ where $[x]$ denotes the greatest integer smaller than or equal to x. When x is not an integer, what is $f'(x)$?

(A) $2x$. (B) $[x]$. (C) $2[x]$. (D) It does not exist.

757. If $f(x) = (\sin x)(\sin 2x)\ldots(\sin nx)$, then $f'(x)$ is

(A) $\sum_{k=1}^{n}(k \cos kx) f(x)$;
(B) $(\cos x)(2\cos 2x)(3\cos 3x)\ldots(n\cos nx)$;
(C) $\sum_{k=1}^{n}(k\cos kx)(\sin kx)$;
(D) $\sum_{k=1}^{n}(k\cot kx) f(x)$.

758. Let $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$ where a_0, a_1, a_2 and a_3 are constants. Then only one of the following statements is correct. Which one is it?

(A) $f(x)$ is differentiable at $x = 0$ whatever be a_0, a_1, a_2, a_3.

(B) $f(x)$ is not differentiable at $x = 0$ whatever be a_0, a_1, a_2, a_3.
(C) If $f(x)$ is differentiable at $x = 0$, then $a_1 = 0$.
(D) If $f(x)$ is differentiable at $x = 0$, then $a_1 = 0$ and $a_3 = 0$.

759. Consider the function $f(x) = |\sin x| + |\cos x|$ defined for x in the interval $(0, 2\pi)$. Then

(A) $f(x)$ is differentiable everywhere;

(B) $f(x)$ is not differentiable at $x = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ and differentiable everywhere else;

(C) $f(x)$ is not differentiable at $x = \frac{\pi}{2}$, π and $\frac{3\pi}{2}$ and differentiable everywhere else;

(D) none of the foregoing statements is true.

760. A curve in the XY plane is given by the the parametric equations $x = t^2 + t + 1$, $y = t^2 - t + 1$, where the parameter t varies over all nonnegative real numbers. The number of straight lines passing through the point (1, 1) which are tangent to the curve, is

(A) 2; (B) 0; (C) 1; (D) 3.

761. If $f(x) = (\frac{a+x}{b+x})^{a+b+2x}$, then $f'(0)$ equals

(A) $\frac{b^2-a^2}{b^2}(\frac{a}{b})^{a+b-1}$; (B) $(2\log\frac{a}{b} + \frac{b^2-a^2}{ab})(\frac{a}{b})^{a+b}$;
(C) $2\log(\frac{a}{b}) + \frac{b^2-a^2}{ab}$; (D) none of the foregoing expressions.

762. If $y = 2\sin^{-1}\sqrt{(1-x)} + \sin^{-1}(2\sqrt{x(1-x)})$ for $0 < x < \frac{1}{2}$ then $\frac{dy}{dx}$ equals

(A) $\frac{2}{\sqrt{x(1-x)}}$; (B) $\sqrt{\frac{(1-x)}{x}}$; (C) $\frac{-1}{\sqrt{x(1-x)}}$; (D) 0.

763. If $y = \sin^{-1}(3x - 4x^3)$, then $\frac{dy}{dx}$ equals

(A) $3x$; (B) 3; (C) $\frac{3}{\sqrt{1-x^2}}$;
(D) none of the foregoing expressions.

764. If $y = 3^{\frac{\sin ax}{\cos bx}}$, then $\frac{dy}{dx}$ is

(A) $3^{\frac{a\cos ax\cos bx + b\sin ax\sin bx}{\cos^2 bx}}$;
(B) $3^{\frac{\sin ax}{\cos bx}}\ \frac{a\cos ax\cos bx + b\sin ax\sin bx}{\cos^2 bx}\log 3$;

(C) $3^{\frac{\sin ax}{\cos bx}}\ \frac{a\cos ax\cos bx - b\sin ax\sin bx}{\cos^2 bx}\log 3$;

(D)$3^{\frac{\sin ax}{\cos bx}}\ \log 3$.

765. If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, then the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ equals

(A) $-\frac{1}{a}$; (B) $-\frac{1}{4a}$; (C) $-a$; (D) none of the foregoing numbers.

766. Let $F(x) = e^x$, $G(x) = e^{-x}$ and $H(x) = G(F(x))$, where x is a real number. Then $\frac{dH}{dx}$ at $x = 0$ is

(A) 1; (B) -1; (C) $-\frac{1}{e}$; (D) $-e$.

767. Let $f(x) = |\sin^3 x|$ and $g(x) = \sin^3 x$, both being defined for x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then

(A) $f'(x) = g'(x)$ for all x;
(B) $f'(x) = -g'(x)$ for all x;
(C) $f'(x) = |g'(x)|$ for all x;
(D) $g'(x) = |f'(x)|$ for all x.

768. Consider the functional equation $f(x - y) = f(x)/f(y)$. If $f'(0) = p$ and $f'(5) = q$, then $f'(-5)$ is

(A) $\frac{p^2}{q}$; (B) $\frac{q}{p}$; (C) $\frac{p}{q}$; (D) q.

769. Let f be a polynomial. Then the second derivative of $f(e^x)$ is

(A) $f''(e^x)\cdot e^x + f'(e^x)$; (B) $f''(e^x)\cdot e^{2x} + f'(x)\cdot e^x$;
(C) $f''(e^x)$; (D) $f''(e^x)\cdot e^{2x} + f'(e^x)\cdot e^x$.

770. If $A(t)$ is the area of the region enclosed by the curve $y = e^{-|x|}$ and the portion of the x-axis between $-t$ and $+t$, then $\lim_{t\to\infty} A(t)$

(A) is 1; (B) is ∞; (C) is 2; (D) does not exist.

771. $\lim_{x\to 0} \frac{(e^x - 1)\tan^2 x}{x^3}$

(A) does not exist; (B) exists and equals 0;
(C) exists and equals $\frac{2}{3}$; (D) exists and equals 1.

772. If $f(x) = \sin x$, $g(x) = x^2$ and $h(x) = \log_e x$, and if $F(x) = h(g(f(x)))$, then $\frac{d^2F}{dx^2}$ equals

(A) $-2\operatorname{cosec}^2 x$; (B) $2\operatorname{cosec}^3 x$;
(C) $2\cot(x^2) - 4x^2\operatorname{cosec}^2(x^2)$; (D) $2x\cot(x^2)$.

773. A lamp is placed on the ground 100 feet (ft) away from a wall. A man six ft tall is walking at a speed of 10 ft/sec from the lamp to the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of his shadow is (in ft/sec)

(A) 2.4; (B) 3; (C) 12; (D) 3.6.

774. A water tank has the shape of a right-circular cone with its vertex down. The radius of the top is 15 ft and the height is 10 ft. Water is poured into the tank at a constant rate of C cubic feet per second. Water leaks out from the bottom at a constant rate of one cubic foot per second. The value of C for which the water level will be rising at the rate of four ft per second at the time point when the water is two ft deep, is given by

(A) $C = 1 + 36\pi$; (B) $C = 1 + 9\pi$; (C) $C = 1 + 4\pi$; (D) $C = 1 + 18\pi$.

775. Let $f(x) = a|\sin x| + be^{|x|} + c|x|^3$. If $f(x)$ is differentiable at $x = 0$ then

(A) $a = b = c = 0$;
(B) $a = b = 0$ and c can be any real number;
(C) $b = c = 0$ and a can be any real number;
(D) $c = a = 0$ and b can be any real number.

776. A necessary and sufficient condition for the function $f(x)$ defined by

$$f(x) = \begin{cases} x^2 + 2x & \text{if } x \le 0 \\ ax + b & \text{if } x > 0 \end{cases}$$

to be differentiable at the point $x = 0$ is that

(A) $a = 0$ and $b = 0$; (B) $a = 0$ while b can be arbitrary;
(C) $a = 2$ while b can be arbitrary; (D) $a = 2$ and $b = 0$.

777. If $f(x) = \log_{x^2}(e^x)$ defined for $x > 1$, then the derivative $f'(x)$ of $f(x)$ is

(A) $\frac{\log x - 1}{2(\log x)^2}$; (B) $\frac{\log x - 1}{(\log x)^2}$; (C) $\frac{\log x + 1}{2(\log x)^2}$; (D) $\frac{\log x + 1}{(\log x)^2}$.

778. For $x > 0$, if $g(x) = x^{\log x}$ and $f(x) = e^{g(x)}$, then $f'(x)$ equals

(A) $(2x^{(\log x - 1)}\log x)f(x)$; (B) $(x^{(2\log x - 1)}\log x)f(x)$;
(C) $(1 + x)e^x$; (D) none of the foregoing expressions.

779. Suppose f and g are functions having second derivatives f'' and g'' everywhere. If $f(x)g(x) = 1$ for all x and f' and g' are never zero, then $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)}$ equals

(A) $-\frac{2f'(x)}{f(x)}$; (B) 0; (C) $-\frac{f'(x)}{f(x)}$; (D) $\frac{2f'(x)}{f(x)}$.

780. If $f(x) = a_1 \cdot e^{|x|} + a_2 \cdot |x|^5$, where a_1, a_2 are constants, is differentiable at $x = 0$, then

(A) $a_1 = a_2$; (B) $a_1 = a_2 = 0$; (C) $a_1 = 0$; (D) $a_2 = 0$.

781. If $y = (\cos^{-1} x)^2$, then the value of $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$ is

(A) -1; (B) -2; (C) 1; (D) 2.

782. The n^{th} derivative of the function $f(x) = \frac{1}{1-x^2}$ at the point $x = 0$, where n is even, is

(A) $n\binom{n}{2}$; (B) 0; (C) $n!$; (D) none of the foregoing quantities.

783. Let $f(x) = \dfrac{x^n(1-x)^n}{n!}$. Then for any integer $k \geq 0$, the k-th derivatives $f^{(k)}(0)$ and $f^{(k)}(1)$

(A) are both 0; (B) are both rational numbers but not necessarily integers;
(C) are both integers; (D) do not satisfy any of the foregoing properties.

784. Let $f_1(x) = e^x$, $f_2(x) = e^{f_1(x)}$, $f_3(x) = e^{f_2(x)} \ldots$ and, in general, $f_{n+1}(x) = e^{f_n(x)}$ for any $n \geq 1$. Then for any fixed n, the value of $\frac{d}{dx}f_n(x)$ equals

(A) $f_n(x)$; (B) $f_n(x)f_{n-1}(x)$;
(C) $f_n(x)f_{n-1}(x)\ldots f_2(x)f_1(x)$; (D) $f_n(x)f_{n-1}(x)\ldots f_1(x)e^x$.

785. The maximum value of $5\sin\theta + 12\cos\theta$ is

(A) 5; (B) 12; (C) 13; (D) 17.

786. Let A and B be the points $(1, 0)$ and $(3, 0)$ respectively. Let P be a variable point on the y-axis. Then the maximum value of the angle $\angle APB$ is

(A) $22\frac{1}{2}^\circ$; (B) 30°; (C) 45°; (D) none of the foregoing quantities.

787. The least value of the expression $\dfrac{1+x^2}{1+x}$, for values of $x \geq 0$, is

(A) $\sqrt{2}$; (B) 1; (C) $2\sqrt{2} - 2$; (D) none of the foregoing numbers.

788. The minimum value of $3x + 4y$, subject to the condition $x^2y^3 = 6$ and x and y are positive, is

(A) 10; (B) 14; (C) 7; (D) 13.

789. A window is in the form of a rectangle with a semicircular bend on the top. If the perimeter of the window is 10 metres, the radius, in metres, of the semicircular bend that maximizes the amount of light admitted is

(A) $\frac{20}{4+\pi}$; (B) $\frac{10}{4+\pi}$; (C) $10 - 2\pi$;
(D) none of the foregoing numbers.

790. $ABCD$ is a fixed rectangle with $AB = 2\,\text{cm}$ and $BC = 4\,\text{cm}$. $PQRS$ is a rectangle such that A, B, C and D lie on PQ, QR, RS and SP respectively. Then the maximum possible area of $PQRS$ is

(A) $16\,\text{cm}^2$; (B) $18\,\text{cm}^2$; (C) $20\,\text{cm}^2$; (D) $22\,\text{cm}^2$.

791. The curve $y = \frac{2x}{1+x^2}$ has

(A) exactly three points of inflection separated by a point of maximum and a point of minimum;
(B) exactly two points of inflection with a point of maximum lying between them;
(C) exactly two points of inflection with a point of minimum lying between them;
(D) exactly three points of inflection separated by two points of maximum.

792. As x varies over all real numbers, the range of the function $f(x) = \frac{x^2-3x+4}{x^2+3x+4}$ is

(A) $\left[\frac{1}{7}, 7\right]$; (B) $\left[-\frac{1}{7}, 7\right]$; (C) $[-7, 7]$; (D) $\left(-\infty, \frac{1}{7}\right) \cup (7, \infty)$.

793. The minimum value of $f(x) = x^8 + x^6 - x^4 - 2x^3 - x^2 - 2x + 9$ is

(A) 5; (B) 1; (C) 0; (D) 9.

794. The number of minima of the polynomial $10x^6 - 24x^5 + 15x^4 + 40x^2 + 108$ is

(A) 0; (B) 1; (C) 2; (D) 3.

795. The number of local maxima of the function $f(x) = x + \sin x$ is

(A) 1; (B) 2; (C) infinite; (D) 0.

796. The maximum value of $\log_{10}(4x^3 - 12x^2 + 11x - 3)$ in the interval [2,3] is

(A) $\log_{10} 3$; (B) $1 + \log_{10} 5$; (C) $-\frac{3}{2}\log_{10} 3$; (D) none of these.

797. The maximum value of the function

$$f(x) = \frac{(1+x)^{0.3}}{1+x^{0.3}}$$

in the interval $0 \leq x \leq 1$ is

(A) 1; (B) $2^{0.7}$; (C) $2^{-0.7}$; (D) none of these.

798. The number of local maxima of the function $f(x) = x - \sin x$ is

(A) infinitely many; (B) two; (C) one; (D) zero.

799. From a square tin sheet of side 12 feet (ft) a box with its top open is made by cutting away equal squares at the four corners and then bending the tin sheet so as to form the sides of the box. The side of the removed square for which the box has the maximum possible volume is, in ft,

(A) 3; (B) 1; (C) 2; (D) none of the foregoing numbers.

800. A rectangular box of volume 48 cu ft is to be constructed, so that its length is twice its width. The material to be used for the top and the four sides is three times costlier per sq ft than that used for the bottom. Then, the box that minimises the cost has height equal to (in ft)

(A) $\frac{8}{27}$; (B) $\frac{8\sqrt[3]{4}}{3}$; (C) $\frac{4}{27}$; (D) $\frac{8}{3}$.

801. A truck is to be driven 300 km on a highway at a constant speed of x kmph. Speed rules of the highway require that $30 \leq x \leq 60$. The fuel costs Rs. 10 per litre and is consumed at the rate of $2 + \frac{x^2}{600}$ litres per hour. The wages of the driver are Rs. 200 per hour. The most economical speed to drive the truck, in kmph, is

(A) 30; (B) 60; (C) $30\sqrt{3.3}$; (D) $20\sqrt{33}$.

802. Let P be a point in the first quadrant lying on the ellipse $\frac{x^2}{8} + \frac{y^2}{18} = 1$. Let AB be the tangent at P to the ellipse meeting the x-axis at A and the y-axis at B. If O is the origin, the minimum possible area of triangle OAB is

(A) 4π; (B) 9π; (C) 9; (D) 12.

803. Consider the parabola $y^2 = 4x$. Let P and Q be the points $(4, -4)$ and $(9,6)$ of the parabola. Let R be a moving point on the arc of the parabola between P and Q. Then the area of the triangle RPQ is largest when

(A) $\angle PRQ = 90°$; (B) $R = (4, 4)$; (C) $R = (\frac{1}{4}, 1)$;
(D) condition other than the foregoing conditions is satisfied.

804. Out of a circular sheet of paper of radius a, a sector with central angle θ is cut out and folded into the shape of a conical funnel. The volume of this funnel is maximum when θ equals

(A) $\frac{2\pi}{\sqrt{2}}$; (B) $2\pi\sqrt{\frac{2}{3}}$; (C) $\frac{\pi}{2}$; (D) π.

805. Let $f(x) = 5 - 4(\sqrt[3]{x-2})^2$. Then at $x = 2$, the function $f(x)$

(A) attains a minimum value;
(B) attains a maximum value;
(C) attains neither a minimum value nor a maximum value;
(D) is undefined.

806. A given right circular cone has a volume p, and the largest right circular cylinder that can be inscribed in the given cone has a volume q. Then the ratio $p : q$ equals

(A) 9:4; (B) 8:3; (C) 7:2; (D) none of the foregoing ratios.

807. If $[x]$ stands for the largest integer not exceeding x, then the integral $\int_{-1}^{2}[x]dx$ is

(A) 3; (B) 0; (C) 1; (D) 2.

808. For any real number x, let $[x]$ denote the greatest integer m such that $m \leq x$. Then

$$\int_{-2}^{2}[x^2 - 1]dx$$

equals

(A) $2(3 - \sqrt{3} - \sqrt{2})$; (B) $2(5 - \sqrt{3} - \sqrt{2})$;
(C) $2(1 - \sqrt{3} - \sqrt{2})$; (D) none of these.

809. Let $f(x)$ be a continuous function such that its first two derivatives $f'(x), f''(x)$ are continuous. The tangents to the graph of $f(x)$ at the points with abscissa $x = a$ and $x = b$ make with the X-axis angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ respectively. Then the value of the integral $\int_a^b f'(x)f''(x)\,dx$ equals

(A) $1 - \sqrt{3}$; (B) 0; (C) 1; (D) -1.

810. The integral $\int_0^{100} e^{x-[x]}dx$ is

(A) $\frac{e^{100}-1}{100}$; (B) $\frac{e^{100}-1}{e-1}$; (C) $100(e-1)$; (D) $\frac{e-1}{100}$.

811. If $S = \int_0^1 \frac{e^t}{t+1}dt$ then $\int_{a-1}^{a} \frac{e^{-t}}{t-a-1}dt$ is

(A) Se^a; (B) Se^{-a}; (C) $-Se^{-a}$; (D) $-Se^a$.

812. If the value of the integral $\int_1^2 e^{x^2}dx$ is α, then the value of $\int_e^{e^4} \sqrt{\log x}dx$ is

(A) $e^4 - e - \alpha$; (B) $2e^4 - e - \alpha$; (C) $2(e^4 - e) - \alpha$;
(D) none of the foregoing quantities.

813. The value of the integral $\int_0^{\pi} |1 + 2\cos x|dx$ is

(A) $\frac{\pi}{3} + \sqrt{3}$; (B) $\frac{\pi}{3} + 2\sqrt{3}$; (C) $\frac{\pi}{3} + 4\sqrt{3}$; (D) $\frac{2\pi}{3} + 4\sqrt{3}$.

814. The value of the integral $\int_0^{u} \sqrt{1 + \sin\frac{x}{2}}\, dx$, where $0 \le u \le \pi$, is

(A) $4 + 4(\sin\frac{u}{4} - \cos\frac{u}{4})$; (B) $4 + 4(\cos\frac{u}{4} - \sin\frac{u}{4})$;
(C) $4 + \frac{1}{4}(\cos\frac{u}{4} - \sin\frac{u}{4})$; (D) $4 + \frac{1}{4}(\sin\frac{u}{4} - \cos\frac{u}{4})$.

815. The definite integral $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + (\tan x)^{101}}$ equals

(A) π; (B) $\frac{\pi}{2}$; (C) 0; (D) $\frac{\pi}{4}$.

816. If $f(x)$ is a nonnegative continuous function such that $f(x) + f(\frac{1}{2} + x) = 1$ for all $x, 0 \le x \le \frac{1}{2}$, then $\int_0^1 f(x)dx$ is equal to

(A) $\frac{1}{2}$; (B) $\frac{1}{4}$; (C) 1; (D) 2.

817. The value of the integral

$$\int_0^{\frac{\pi}{4}} \log_e(1 + \tan\theta)d\theta$$

is

(A) $\frac{\pi}{8}$; (B) $\frac{\pi}{8}\log_e 2$; (C) 1; (D) $2\log_e 2 - 1$.

818. Define the real-valued function f on the set of real numbers by

$$f(x) = \int_0^1 \frac{x^2 + t^2}{2 - t}\, dt.$$

Consider the curve $y = f(x)$. It represents

(A) a straight line; (B) a parabola;
(C) a hyperbola; (D) an ellipse.

819. $\lim_{n\to\infty} \frac{1}{n} \sum_{r=0}^{n-1} \cos\left(\frac{r\pi}{2n}\right)$

(A) is 1; (B) is 0; (C) is $\frac{2}{\pi}$; (D) does not exist.

820. $\lim_{n\to\infty} \frac{\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n-1}}{n\sqrt{n}}$ is equal to

(A) $\frac{1}{2}$; (B) $\frac{1}{3}$; (C) $\frac{2}{3}$; (D) 0.

821. The value of

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \left[\sqrt{4i/n}\right],$$

where $[x]$ is the largest integer smaller than or equal to x, is

(A) 3; (B) $\frac{3}{4}$; (C) $\frac{4}{3}$; (D) none of the foregoing numbers.

822. Let $\alpha = \lim_{n\to\infty} \frac{1^2+2^2+\cdots+n^2}{n^3}$,
and $\beta = \lim_{n\to\infty} \frac{(1^3-1^2)+(2^3-2^2)+\ldots+(n^3-n^2)}{n^4}$.
Then

(A) $\alpha = \beta$; (B) $\alpha < \beta$; (C) $4\alpha - 3\beta = 0$; (D) $3\alpha - 4\beta = 0$.

823. The value of the integral

$$\int_{-4}^{4} |x - 3|\, dx$$

is

(A) 13; (B) 8; (C) 25; (D) 24.

824. The value of $\int_{-2}^{2} |x(x-1)|dx$ is

(A) $\frac{11}{3}$; (B) $\frac{13}{3}$; (C) $\frac{16}{3}$; (D) $\frac{17}{3}$.

825. $\displaystyle\int_{-1}^{3/2} |x \sin \pi x| dx$ is equal to

(A) $\frac{3\pi+1}{\pi^2}$; (B) $\frac{\pi+1}{\pi^2}$; (C) $\frac{1}{\pi^2}$; (D) $\frac{3\pi-1}{\pi^2}$.

826. The set of values of a for which the integral $\displaystyle\int_0^2 (|x-a|-|x-1|)\, dx$ is nonnegative, is

(A) all numbers $a \geq 1$; (B) all real numbers;
(C) all numbers a with $0 \leq a \leq 2$; (D) all numbers $a \leq 1$.

827. The maximum value of $\int_{a-1}^{a+1} e^{-(x-1)^2}\, dx$, where a is a real number, is attained at

(A) $a = 0$; (B) $a = 1$; (C) $a = -1$; (D) $a = 2$.

828. Let

$$f(x) = \begin{cases} \int_0^x \{5 + |1-y|\} dy & \text{if } x > 2 \\ 5x + 1 & \text{if } x \leq 2. \end{cases}$$

Then

(A) $f(x)$ is continuous but not differentiable at $x = 2$;
(B) $f(x)$ is not continuous at $x = 2$;
(C) $f(x)$ is differentiable everywhere;
(D) the right derivative of $f(x)$ at $x = 2$ does not exist.

829. Consider the function $f(x) = \displaystyle\int_0^x [t]\, dt$ where $x > 0$ and $[t]$ denotes the largest integer less than or equal to t. Then

(A) $f(x)$ is not defined for $x = 1, 2, 3, \ldots$;
(B) $f(x)$ is defined for all $x > 0$ but is not continuous at $x = 1, 2, 3, \ldots$;
(C) $f(x)$ is continuous at all $x > 0$ but is not differentiable at $x = 1, 2, 3, \ldots$;
(D) $f(x)$ is differentiable at all $x > 0$.

830. Let

$$f(x) = \begin{cases} 2 & \text{if } 0 \le x \le 1, \\ 3 & \text{if } 1 < x \le 2. \end{cases}$$

Define $g(x) = \int_0^x f(t)dt$, for $0 \le x \le 2$. Then

(A) g is not differentiable at $x = 1$; (B) $g'(1) = 2$;
(C) $g'(1) = 3$; (D) none of the above holds.

831. Let $[x]$ denote the greatest integer which is less than or equal to x. Then the value of the integral

$$\int_0^{\pi/4} [3\tan^2 x]dx$$

is

(A) $\pi/3 - \tan^{-1}(\sqrt{\frac{2}{3}})$; (B) $\pi/4 - \tan^{-1}(\sqrt{\frac{2}{3}})$;
(C) $3 - [\frac{3\pi}{4}]$; (D) $[3 - \frac{3\pi}{4}]$.

832. Consider continuous functions f on the interval $[0,1]$ which satisfy the following two conditions:
(i) $f(x) \le \sqrt{5}$ for all $x \in [0,1]$; and
(ii) $f(x) \le \frac{2}{x}$ for all $x \in [\frac{1}{2}, 1]$.

Then, the smallest real number α such that the inequality $\int_0^1 f(x)dx \le \alpha$ holds for *any such* f is

(A) $\sqrt{5}$; (B) $\frac{\sqrt{5}}{2} + 2\log 2$; (C) $2 + 2\log\frac{\sqrt{5}}{2}$; (D) $2 + \log\frac{\sqrt{5}}{2}$.

833. Let

$$f(x) = \int_0^x e^{-t^2}dt, \quad \text{for all } x > 0.$$

Then for all $x > 0$,

(A) $xe^{-x^2} < f(x)$; (B) $x < f(x)$; (C) $1 < f(x)$;
(D) none of the foregoing statements is necessarily true.

834. Let $f(x) = \int_0^x \cos\left(\frac{t^2+2t+1}{5}\right)dt$, where $0 \le x \le 2$. Then

(A) $f(x)$ increases monotonically as x increases from 0 to 2;
(B) $f(x)$ decreases monotonically as x increases from 0 to 2;

(C) $f(x)$ has a maximum at $x = \alpha$ such that $2\alpha^2 + 4\alpha = 5\pi - 2$;
(D) $f(x)$ has a minimum at $x = \alpha$ such that $2\alpha^2 + 4\alpha = 5\pi - 2$.

835. The maximum value of the integral

$$\int_{a-1}^{a+1} \frac{1}{1+x^8}\, dx$$

is attained

(A) exactly at two values of a;
(B) only at one value of a which is positive;
(C) only at one value of a which is negative;
(D) only at $a = 0$.

836. The value of the integral $\int \cos\log x\, dx$ is

(A) $x[\cos\log x + \sin\log x]$;
(B) $\frac{x}{2}[\cos\log x - \sin\log x]$;
(C) $\frac{x}{2}[\sin\log x - \cos\log x]$;
(D) $\frac{x}{2}[\cos\log x + \sin\log x]$.

837. If $u_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$ for $n \geq 2$, then $u_n + u_{n-2}$ equals

(A) $\frac{1}{n-1}$; (B) $\frac{1}{n}$; (C) $\frac{1}{n+1}$; (D) $\frac{1}{n} + \frac{1}{n-2}$.

838. $\int_0^1 \tan^{-1} x dx$ is equal to

(A) $\frac{\pi}{4} - \log_e \sqrt{2}$; (B) $\frac{\pi}{4} + \log_e \sqrt{2}$; (C) $\frac{\pi}{4}$; (D) $\log_e \sqrt{2}$.

839. $\int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$ equals

(A) $\frac{\pi}{4}$; (B) $\frac{\pi}{2}$; (C) $3\frac{\pi}{4}$; (D) $\frac{\pi}{3}$.

840. The indefinite integral $\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$ equals

(A) $\frac{2}{3}\sin^{-1}(\frac{x}{a})^{\frac{3}{2}} + C$, where C is a constant;
(B) $\cos^{-1}(\frac{x}{a})^{\frac{3}{2}} + C$, where C is a constant;
(C) $\frac{2}{3}\cos^{-1}(\frac{x}{a})^{\frac{3}{2}} + C$, where C is a constant;
(D) none of the foregoing functions.

841. The value of the integral $\int_0^{\log 5} \frac{e^x\sqrt{e^x - 1}}{e^x + 3} dx$ is

(A) 4π; (B) 4; (C) $\frac{\pi}{2}$; (D) $4 - \pi$.

842. The area of the region $\{(x, y) : x^2 \leq y \leq |x|\}$ is

(A) $\frac{1}{3}$; (B) $\frac{1}{6}$; (C) $\frac{1}{2}$; (D) 1.

843. The area bounded by the curves $y = \sqrt{x}$ and $y = x^2$ is

(A) $\frac{1}{3}$; (B) 1; (C) $\frac{2}{3}$;
(D) none of the foregoing numbers.

844. The area bounded by the curve $y = \log_e x$, the x-axis and the straight line $x = e$ equals

(A) e; (B) 1; (C) $1 - \frac{1}{e}$;
(D) none of the foregoing numbers.

845. The area of the region in the first quadrant bounded by

$$y = \sin x \qquad \text{and} \qquad \frac{2y-1}{\sqrt{3}-1} = \frac{6x-\pi}{\pi}$$

equals
(A) $\frac{\sqrt{3}-1}{2} - \frac{\pi}{24}(\sqrt{3}+1)$; (B) $\frac{\sqrt{3}+1}{2} - \frac{\pi}{24}(\sqrt{3}-1)$;
(C) $\frac{\sqrt{3}-1}{2}(1 - \frac{\pi}{12})$; (D) none of the above quantities.

846. The area of the region bounded by the straight lines $x = \frac{1}{2}$ and $x = 2$, and the curves given by the equations $y = \log_e x$ and $y = 2^x$ is

(A) $\frac{1}{\log_e 2}(4 + \sqrt{2}) - \frac{5}{2}\log_e 2 + \frac{3}{2}$;
(B) $\frac{1}{\log_e 2}(4 - \sqrt{2}) - \frac{5}{2}\log_e 2$;
(C) $\frac{1}{\log_e 2}(4 - \sqrt{2}) - \frac{5}{2}\log_e 2 + \frac{3}{2}$;
(D) is not equal to any of the foregoing expressions.

847. The area of the bounded region enclosed between the curves $y^3 = x^2$ and $y = 2 - x^2$ is

(A) $2\frac{4}{15}$; (B) $1\frac{1}{15}$; (C) $2\frac{2}{15}$; (D) $2\frac{14}{15}$.

848. The area of the region enclosed between the curve $y = \frac{1}{2}x^2$ and the straight line $y = 2$ equals (in sq. units)

(A) $\frac{4}{3}$; (B) $\frac{8}{3}$; (C) $\frac{16}{3}$; (D) $\frac{32}{3}$.

849. The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-\frac{x^2}{2}} \sin x dx$ is

(A) $\frac{\pi}{2} - 1$; (B) $\frac{\pi}{2}$; (C) $\sqrt{2\pi}$; (D) none of the foregoing numbers.

850. The area of the region of the plane bounded by $\max(|x|, |y|) \leq 1$ and $xy \leq \frac{1}{2}$ is

(A) $\frac{1}{2} + \log 2$; (B) $3 + \log 2$; (C) $7\frac{3}{4}$;
(D) none of the foregoing numbers.

851. The largest area of a rectangle which has one side on the x-axis and two vertices on the curve $y = e^{-x^2}$ is

(A) $\frac{1}{\sqrt{2}}e^{-\frac{1}{2}}$; (B) $\frac{1}{2}e^{-2}$; (C) $\sqrt{2}e^{-\frac{1}{2}}$; (D) $\sqrt{2}e^{-2}$.

852. Approximate values of the integral $I(x) = \int_0^x (\cos t)e^{-\frac{t^2}{10}} dt$ are given in the following table.

x	$\pi/2$	π	$3\pi/2$	2π
$I(x)$	0.95	0.44	0.18	0.22

Which of the following numbers best approximates the value of the integral $\int_0^{5\pi/4} (\cos t)e^{-\frac{t^2}{10}} dt$?

(A) 0.16; (B) 0.23; (C) 0.32; (D) 0.40.

853. The maximum of the areas of the isosceles triangles lying between the curve $y = e^{-x}$ and the x-axis, with base on the positive x-axis, is

(A) $1/e$; (B) 1; (C) $1/2$; (D) e.

854. The area bounded by the straight lines $x = -1$ and $x = 1$ and the graphs of $f(x)$ and $g(x)$, where $f(x) = x^3$ and

$$g(x) = \begin{cases} x^5 & \text{if } -1 \leq x \leq 0, \\ x & \text{if } \ 0 \leq x \leq 1, \end{cases}$$

is

(A) $1/3$; (B) $1/8$; (C) $1/2$; (D) $1/4$.

855. A right circular cone is cut from a solid sphere of radius a, the vertex and the circumference of the base being on the surface of the sphere. The height of the cone when its volume is maximum is

(A) $\frac{4a}{3}$; (B) $\frac{3a}{2}$; (C) a; (D) $\frac{6a}{5}$.

856. For any choice of *five* distinct points in the unit square (that is, a square with side 1 unit), we can assert that there is a number c such that there are at least two points whose distance is less than or equal to c. The smallest value c for which such an assertion can be made is

(A) $\frac{1}{\sqrt{2}}$; (B) $\frac{2}{3}$; (C) $\frac{1}{2}$; (D) none of the foregoing numbers.

857. The largest volume of a cube that can be enclosed in a sphere of diameter 2 cm, is, in cm^3,

(A) 1; (B) $2\sqrt{2}$; (C) π; (D) $\frac{8}{3\sqrt{3}}$.

858. A lane runs perpendicular to a road 64 feet wide. If it is just possible to carry a pole 125 feet long from the road into the lane, keeping it horizontal, then the minimum width of the lane must be (in feet)

(A) $(\frac{125}{\sqrt{2}} - 64)$; (B) 61; (C) 27; (D) 36.

B.Stat. (Hons.) Admission Test: 2007

Multiple-Choice Test
Time: 2 hours

1. Let x be an irrational number. If a, b, c and d are rational numbers such that $\frac{ax+b}{cx+d}$ is a rational number, which of the following must be true?

(A) $ad = bc$ (B) $ac = bd$ (C) $ab = cd$ (D) $a = d = 0$.

2. Let $z = x+iy$ be a complex number which satisfies the equation $(z+\overline{z})z = 2+4i$. Then

(A) $y = \pm 2$; (B) $x = \pm 2$; (C) $x = \pm 3$; (D) $y = \pm 1$.

3. Suppose a, b and n are positive integers, all greater than one. If $a^n + b^n$ is prime, what can you say about n?

(A) The integer n must be 2;
(B) The integer n need not be 2, but must be a power of 2;
(C) The integer n need not be a power of 2, but must be even;
(D) None of the above is necessarily true.

4. For how many real values of p do the equations $x^2+px+1 = 0$ and $x^2+x+p = 0$ have *exactly one* common root?

(A) 0; (B) 1; (C) 2; (D) 3.

5. The limit

$$\lim_{n\to\infty}\left(1+\frac{1}{n^2}\right)^{\frac{8n^3}{\pi}\sin\left(\frac{\pi}{2n}\right)}$$

is

(A) ∞; (B) 1; (C) e^4; (D) 4.

6. Water falls from a tap of circular cross section at the rate of 2 metres/sec and fills up a hemispherical bowl of inner diameter 0.9 metres. If the inner diameter of the tap is 0.01 metres, then the time needed to fill the bowl is

(A) 40.5 minutes; (B) 81 minutes;
(C) 60.75 minutes; (D) 20.25 minutes.

7. Let AB be a fixed line segment. Let P be a moving point such that $\angle APB$ is equal to a constant *acute* angle. Then, which of the following curves does the point P move along?

(A) a circle; (B) an ellipse;

(C) the boundary of the common region of 2 identical intersecting circles with centres outside the common region;
(D) the boundary of the union of 2 identical intersecting circles with centres outside the common region.

8. A circular pit of radius r metres and depth 2 metres is dug and the removed soil is piled up as a cone with the bottom of the pit as its base. What proportion of the volume of the cone is above the ground level?

(A) $\frac{2}{3}$; (B) $\frac{8}{27}$; (C) $\frac{4r^2}{9}$; (D) $\frac{r^3}{27}$.

9. The algebraic sum of the perpendicular distances from $A\ (x_1, y_1)$, $B\ (x_2, y_2)$, $C\ (x_3, y_3)$, to a line is zero. Then the line must pass through the

(A) orthocentre of $\triangle ABC$;
(B) centroid of $\triangle ABC$;
(C) incentre of $\triangle ABC$;
(D) circumcentre of $\triangle ABC$.

10. The value of the integral

$$\int_{\pi/2}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}}\, dx$$

equals

(A) 1; (B) π; (C) e; (D) none of these.

11. If the function $f(x) = \frac{(x-1)(x-2)}{x-a}$, for $x \neq a$, takes all values in $(-\infty, \infty)$, then we must have

(A) $a \leq 1$; (B) $a \geq 2$; (C) $a \leq 1$ or $a \geq 2$; (D) $1 \leq a \leq 2$.

12. In how many ways can you choose three distinct numbers from the set $\{1, 2, 3, \ldots, 19, 20\}$ such that their product is divisible by 4?

(A) 795; (B) 810; (C) 855; (D) 1665.

13. Consider the function $f(x) = e^{2x} - x^2$. Then

(A) $f(x) = 0$ for some $x < 0$ but $f(x) \neq 0$ for every $x > 0$;
(B) $f(x) = 0$ for some $x > 0$ but $f(x) \neq 0$ for every $x < 0$;
(C) $f(x_1) = 0$ for some $x_1 < 0$ and $f(x_2) = 0$ for some $x_2 > 0$;
(D) $f(x) \neq 0$ for every x.

14. For $k \geq 1$, the value of

$$\binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \cdots + \binom{n+k}{k}$$

equals

(A) $\binom{n+k+1}{n+k}$; (B) $(n+k+1)\binom{n+k}{n+1}$;

(C) $\binom{n+k+1}{n+1}$; (D) $\binom{n+k+1}{n}$.

15. In a triangle ABC, angle A is twice the angle B. Then which of the following has to be true?

(A) $a^2 = b(b+c)$; (B) $b^2 = a(a+c)$;

(C) $c^2 = a(a+b)$; (D) $ab = c(a+c)$.

16. The point (x, y) on the line $x + y = 10$ for which $\min\{4-x, 5-y\}$ is the largest is

(A) $(\frac{9}{2}, \frac{11}{2})$; (B) $(5, 5)$; (C) $(\frac{11}{2}, \frac{9}{2})$; (D) none of these.

17. The value of

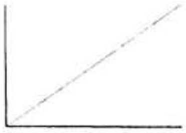
$$\sin^{-1} \cot \left[\sin^{-1} \left\{ \frac{1}{2} \left(1 - \sqrt{\frac{5}{6}} \right) \right\} + \cos^{-1} \sqrt{\frac{2}{3}} + \sec^{-1} \sqrt{\frac{8}{3}} \right]$$

is

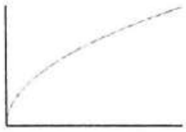
(A) 0; (B) $\pi/6$; (C) $\pi/4$; (D) $\pi/2$.

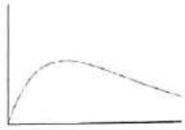
18. Which of the following graphs represents the function

$$f(x) = \int_0^{\sqrt{x}} e^{-u^2/x} du, \quad \text{for} \quad x > 0 \quad \text{and} \quad f(0) = 0?$$

(A) 

(B) 

(C) 

(D) 

19. Consider a triangle ABC with the sides a, b, c in A.P. Then the largest possible value of the angle B is

(A) $60°$; (B) $67\frac{1}{2}°$; (C) $75°$; (D) $82\frac{1}{2}°$.

20. If $a_n = \left(1 + \frac{1}{n^2}\right)\left(1 + \frac{2^2}{n^2}\right)^2\left(1 + \frac{3^2}{n^2}\right)^3 \cdots \left(1 + \frac{n^2}{n^2}\right)^n$, then

$$\lim_{n\to\infty} a_n^{-1/n^2}$$

is

(A) 0; (B) 1; (C) e; (D) $\sqrt{e}/2$.

21. If $f(x) = e^x \sin x$, then $\frac{d^{10}}{dx^{10}} f(x) \Big|_{x=0}$ equals

(A) 1; (B) -1; (C) 10; (D) 32.

22. Consider a circle with centre O. Two chords AB and CD extended intersect at a point P outside the circle. If $\angle AOC = 43°$ and $\angle BPD = 18°$, then the value of $\angle BOD$ is

(A) $36°$; (B) $29°$; (C) $7°$; (D) $25°$.

23. Consider a triangle ABC. The median AD meets the side BC at the point D. A point E on AD is such that $AE : DE = 1 : 3$. The straight line BE extended meets the side AC at a point F. Then $AF : FC$ equals

(A) 1:6; (B) 1:7; (C) 1:4; (D) 1:3.

24. A person standing at a point A finds the angle of elevation of a nearby tower to be $60°$. From A, the person walks a distance of 100 feet to a point B and then walks again to another point C such that $\angle ABC = 120°$. If the angles of elevation of the tower at both B and C are also $60°$ each, then the height of the tower is

(A) 50 feet; (B) $50\sqrt{3}$ feet; (C) $100\sqrt{3}$ feet; (D) 100 feet.

25. A box contains 10 red cards numbered $1, \ldots, 10$ and 10 black cards numbered $1, \ldots, 10$. In how many ways can we choose 10 out of the 20 cards so that there are exactly 3 *matches*, where a *match* means a red card and a black card with the same number?

(A) $\binom{10}{3}\binom{7}{4}2^4$; (B) $\binom{10}{3}\binom{7}{4}$; (C) $\binom{10}{3}2^7$; (D) $\binom{10}{3}\binom{14}{4}$.

26. Let P be a point on the ellipse $x^2 + 4y^2 = 4$ which does not lie on the axes. If the normal at the point P intersects the major and minor axes at C and D respectively, then the ratio $PC : PD$ equals

(A) 2; (B) 1/2; (C) 4; (D) 1/4.

27. Let α denote the absolute value of the difference between the lengths of the two segments of a focal chord of a parabola. Let β denote the length of a chord passing through the vertex and parallel to that focal chord. Then which of the following is always true?

(A) $\alpha^2 = 2\beta$; (B) $\alpha = 2\beta$; (C) $\alpha = \beta$; (D) $\beta^2 = 2\alpha$.

28. The directrix of the parabola traced out by the centre of a moving circle, which touches both the straight line $y = -x$ and the circle $(x-3)^2 + (y-4)^2 = 9$, is

(A) $y = -x + 3$; (B) $y = -x - 3$; (C) $y = -x + 3\sqrt{2}$; (D) $y = x - 3\sqrt{2}$.

29. For a real number x, let $[x]$ denote the largest integer smaller than or equal to x and (x) denote the smallest integer larger than or equal to x. Let $f(x) = \min(x - [x], (x) - x)$ for $0 \leq x \leq 12$. The volume of the solid obtained by rotating the curve $y = f(x)$ about the X-axis is

(A) π; (B) 4π; (C) $\pi/2$; (D) $\pi/4$.

30. For a real number x, let $[x]$ denote the largest integer smaller than or equal to x. The value of $\int_{-100}^{100} [t^3]\, dt$ is

(A) 0; (B) 100; (C) -100; (D) -100^3.

B.Math. (Hons.) Admission Test: 2007

Multiple-Choice Test
Time: 2 hours

1. The number of ways of going up 7 steps if we take one or two steps at a time is

(A) 19; (B) 20; (C) 21; (D) 22.

2. Consider the surface defined by $x^2 + 2y^2 - 5z^2 = 0$. If we cut the surface by the plane given by the equation $x = z$, then we obtain a

(A) hyperbola; (B) circle; (C) parabola; (D) pair of straight lines.

3. Let a, b be real numbers. The number of real solutions of the system of equations $x + y = a$ and $xy = b$ is

(A) at most 1; (B) at most 2; (C) at least 1; (D) at least 2.

4. If a fair coin is tossed 100 times, then the probability of getting at least one head is

(A) $\frac{100}{2^{100}}$; (B) $\frac{99}{100}$; (C) $1 - \frac{1}{100!}$; (D) $1 - \frac{1}{2^{100}}$.

5. Let $f(x)$ be a degree five polynomial with real coefficients. Then the number of real roots of f must be

(A) 1; (B) 2 or 4; (C) 1 or 3 or 5;
(D) none of the above.

6. The number of ways in which 3 girls and 2 boys can sit on a bench so that no two girls are adjacent is

(A) 6; (B) 12; (C) 32; (D) 120.

7. Let $R_n = 2 + \sqrt{2 + \sqrt{2 + \ldots + \sqrt{2}}}$ (n square root signs). Then $\lim_{n\to\infty} R_n$ equals

(A) 4; (B) 8; (C) 16; (D) e^2.

8. Let a_n be the sequence whose nth term is the sum of the digits of the natural number $9n$. For example, $a_1 = 9$, $a_{11} = 18$ etc. The minimum m such that $a_m = 81$ is

(A) 110111112; (B) 119111113; (C) 111111111;
(D) none of the above.

9. $\lim_{n\to\infty} [2\log(3n) - \log(n^2+1)]$

(A) is 0; (B) is $2\log 3$; (C) is $4\log 6$; (D) does not exist.

10. Let $S = \{x \in \mathbb{R} \mid 1 \leq |x| \leq 100\}$ be a subset of the real line. Let M be a non-empty subset of S such that for all x, y in M, their product xy is also in M. Then M can have

(A) only one element; (B) at most 2 elements;
(C) more than 2 but only finitely many elements;
(D) infinitely many elements.

11. An astronaut lands on a planet and meets a native of the planet. She asks the native "How many days do you have in your year?" He answers "It is the sum of the squares of three consecutive natural numbers but it is also the sum of the squares of the next two numbers". The answer to the astronaut's question is

(A) 365; (B) 1095; (C) 30000; (D) 10^{10}.

12. Let $a_1 = 2$ and for all natural number n, define $a_{n+1} = a_n(a_n + 1)$. Then, as $n \to \infty$, the number of prime factors of a_n

(A) goes to infinity; (B) goes to a finite limit;
(C) oscillates boundedly; (D) oscillates unboundedly.

13. Suppose that the equation $ax^2 + bx + c = 0$ has a rational solution. If a, b, c are integers then

(A) at least one of a, b, c is even; (B) all of a, b, c are even;
(C) at most one of a, b, c is odd; (D) all of a, b, c are odd.

14. Let $S = \{1, 2, 3, 4\}$. The number of functions $f : S \to S$ such that $f(i) \leq 2i$ for all $i \in S$ is

(A) 32; (B) 64; (C) 128; (D) 256.

15. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^2 - \frac{x^2}{1+x^2}$. Then

(A) f is one-one but not onto; (B) f is onto but not one-one;
(C) f is both one-one and onto; (D) f is neither one-one nor onto.

16. Define the sequence $\{a_n\}$ by $a_1 = 1, a_2 = \frac{e}{2}, a_3 = \frac{e^2}{4}, a_4 = \frac{e^3}{8}, \ldots$. Then $\lim_{n\to\infty} a_n$ is

(A) 0; (B) 1; (C) e^e; (D) infinite.

17. Let C be the circle of radius 1 around 0 in the complex plane and z_0 be a fixed point on C. Then the number of *ordered pairs* (z_1, z_2) of points on C such that $z_0 + z_1 + z_2 = 0$ is

(A) 0; (B) 1; (C) 2; (D) ∞.

18. The number of real solutions of $e^x + x^2 = \sin x$ is

(A) 0; (B) 1; (C) 2; (D) ∞.

19. The set of complex numbers z such that $|z+1| \leq |z-1|$ is the half plane

(A) of complex numbers that lie above the real axis;
(B) of complex numbers that lie below the real axis;
(C) of complex numbers that lie left of the imaginary axis;
(D) of complex numbers that lie right of the imaginary axis.

20. $\lim_{x \to 0} \cos(\sin x)$ is

(A) -1; (B) 0; (C) 1/2; (D) 1.

21. The number of rational roots of the polynomial $x^3 - 3x - 1$ is

(A) 0; (B) 1; (C) 2; (D) 3.

22. $\sum_{n=1}^{\infty} \frac{n^2}{n!}$ equals

(A) e; (B) $2e$; (C) e^2; (D) ∞.

23. Let $n > 1$ be a natural number and let $A = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$. Then

(A) $A^n = Id$; (B) $A^{n^2+1} = Id$; (C) $A^{n^4+1} = Id$;
(D) none of these numbers.

24. The number of ways of breaking a stick of length $n > 1$ into n pieces of unit length (at each step break one of the pieces with length > 1 into two pieces of integer lengths) is

(A) $(n-1)!$; (B) $n! - 1$; (C) 2^{n-2}; (D) $2^{n-1} - 1$.

25. Let ABC be a right angled triangle in the plane with area s. Then the maximum area of a rectangle inside ABC is

(A) $s/4$; (B) $s/3$; (C) $s/2$; (D) s.

B.Stat. (Hons.) Admission Test: 2008

Multiple-Choice Test
Time: 2 hours

1. Let C be the circle $x^2 + y^2 + 4x + 6y + 9 = 0$. The point $(-1, -2)$ is

(A) inside C but not the centre of C; (B) outside C;
(C) on C; (D) the centre of C.

2. The number of distinct real roots of the equation

$$\left(x + \frac{1}{x}\right)^2 - 5\left(x + \frac{1}{x}\right) + 6 = 0$$

is

(A) 1; (B) 2; (C) 3; (D) 4.

3. The set of complex numbers z satisfying the equation

$$(3 + 7i)z + (10 - 2i)\bar{z} + 100 = 0$$

represents, in the Argand plane,

(A) a straight line; (B) a pair of intersecting straight lines;
(C) a pair of distinct parallel straight lines; (D) a point.

4. Let X be the set $\{1, 2, 3, \ldots, 10\}$ and P the subset $\{1, 2, 3, 4, 5\}$. The number of subsets Q of X such that $P \cap Q = \{3\}$ is

(A) 1; (B) 2^4; (C) 2^5; (D) 2^9.

5. The number of triplets (a, b, c) of integers such that $a < b < c$ and a, b, c are sides of a triangle with perimeter 21 is

(A) 7; (B) 8; (C) 11; (D) 12.

6. Suppose a, b and c are three numbers in G.P. If the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then $\frac{d}{a}$, $\frac{e}{b}$ and $\frac{f}{c}$ are in

(A) A.P.; (B) G.P.; (C) H.P.; (D) none of the above.

7. The number of solutions of the equation $\sin^{-1} x = 2\tan^{-1} x$ is

(A) 1; (B) 2; (C) 3; (D) 5.

8. Suppose $x^2 + px + q = 0$ has two real roots α and β with $|\alpha| \neq |\beta|$. If α^4 and β^4 are the roots of $x^2 + rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 + r = 0$ has

(A) one positive and one negative root; (B) two distinct positive roots;
(C) two distinct negative roots; (D) no real roots.

9. Suppose $ABCD$ is a quadrilateral such that $\angle BAC = 50°$, $\angle CAD = 60°$, $\angle CBD = 30°$ and $\angle BDC = 25°$. If E is the point of intersection of AC and BD, then the value of $\angle AEB$ is

(A) 75°; (B) 85°; (C) 95°; (D) 110°.

10. Let $\mathbb{R}$ be the set of all real numbers. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 - 3x^2 + 6x - 5$ is

(A) one-to-one, but not onto; (B) one-to-one and onto;
(C) onto, but not one-to-one; (D) neither one-to-one nor onto.

11. The angles of a convex pentagon are in A.P. Then, the minimum possible value of the smallest angle is

(A) 30°; (B) 36°; (C) 45°; (D) 54°.

12. The number of points (b, c) lying on the circle $x^2 + (y-3)^2 = 8$, such that the quadratic equation $t^2 + bt + c = 0$ has real roots, is

(A) infinite; (B) 2; (C) 4; (D) 0.

13. Let L be the point $(t, 2)$ and M be a point on the y-axis such that LM has slope $-t$. Then the locus of the midpoint of LM, as t varies over all real values, is

(A) $y = 2 + 2x^2$; (B) $y = 1 + x^2$; (C) $y = 2 - 2x^2$; (D) $y = 1 - x^2$.

14. Suppose $x, y \in (0, \pi/2)$ and $x \neq y$. Which of the following statements is true?

(A) $2\sin(x+y) < \sin 2x + \sin 2y$ for all x, y;
(B) $2\sin(x+y) > \sin 2x + \sin 2y$ for all x, y;
(C) There exist x, y such that $2\sin(x+y) = \sin 2x + \sin 2y$;
(D) None of the above.

15. A triangle ABC has a fixed base BC. If $AB : AC = 1 : 2$, then the locus of the vertex A is

(A) a circle whose centre is the midpoint of BC;
(B) a circle whose centre is on the line BC but not the midpoint of BC;
(C) a straight line; (D) none of the above.

16. Suppose $e^{-x}\sin x - e^{-x}\cos x + \cos x = 0$ for some $x > 0$. Then

(A) $\sin x > 0$; (B) $\sin x \cos x > 0$; (C) $\cos x > 0$; (D) $\sin x \cos x < 0$.

17. Let $f(x) = x^6 - 3x^2 - 10$. The set of all values taken by $f(x)$ as x varies over the interval $[-2, 2]$ is

(A) $[-12, -10]$; (B) $[-10, 42]$; (C) $[-12, 42]$; (D) $[-10, 12]$.

18. N is a 50 digit number. All the digits except the 26th from the right are 1. If N is divisible by 13, then the unknown digit is

(A) 1; (B) 3; (C) 7; (D) 9.

19. If $f(x) = x^{n-1}\log x$, then the n-th derivative of f equals

(A) $\frac{(n-1)!}{x}$; (B) $\frac{n}{x}$; (C) $(-1)^{n-1}\frac{(n-1)!}{x}$; (D) $\frac{1}{x}$.

20. Suppose $a < b$. The maximum value of the integral

$$\int_a^b \left(\frac{3}{4} - x - x^2\right) dx$$

over all possible values of a and b is

(A) $\frac{3}{4}$; (B) $\frac{4}{3}$; (C) $\frac{3}{2}$; (D) $\frac{2}{3}$.

21. For any $n \geq 5$, the value of $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2^n - 1}$ lies between

(A) 0 and $\frac{n}{2}$; (B) $\frac{n}{2}$ and n;
(C) n and $2n$; (D) none of the above.

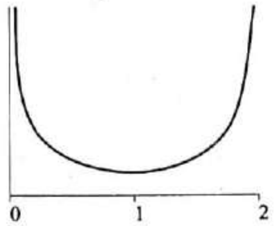
22. Let ω denote a cube root of unity which is not equal to 1. Then the number of distinct elements in the set

$$\{(1 + \omega + \omega^2 + \cdots + \omega^n)^m \ : \ m, n = 1, 2, 3, \cdots\}$$

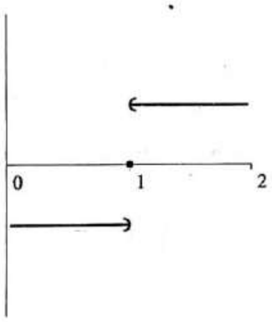
is

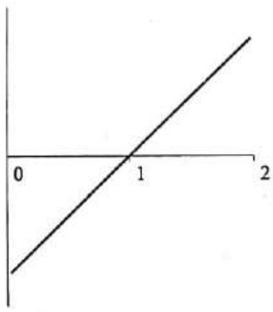
(A) 4; (B) 5; (C) 7; (D) infinite.

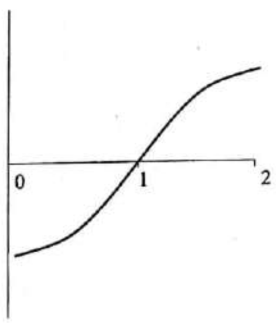
23. The graph of a function f defined on $(0,2)$ with the property $\lim_{x\to 0+} f(x) = \lim_{x\to 2-} f(x) = \infty$ is as follows :

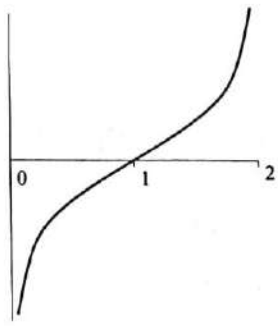


The graph of the derivative of the above function looks like :

(A)


(B)


(C)


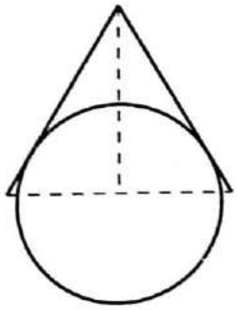
(D)


24. The value of the integral

$$\int_2^3 \frac{dx}{\log_e x}$$

(A) is less than 2;
(B) is equal to 2;
(C) lies in the interval $(2,3)$;
(D) is greater than 3.

25. A hollow right circular cone rests on a sphere as shown in the figure. The height of the cone is 4 metres and the radius of the base is 1 metre. The volume of the sphere is the same as that of the cone. Then, the distance between the centre of the sphere and the vertex of the cone is



(A) 4 metres; (B) $\sqrt{17}$ metres; (C) $\sqrt{15}$ metres; (D) 5 metres.

26. For each positive integer n, define a function f_n on $[0,1]$ as follows:

$$f_n(x) = \begin{cases} 0 & \text{if} \quad x = 0 \\ \sin\frac{\pi}{2n} & \text{if} \quad 0 < x \le \frac{1}{n} \\ \sin\frac{2\pi}{2n} & \text{if} \quad \frac{1}{n} < x \le \frac{2}{n} \\ \sin\frac{3\pi}{2n} & \text{if} \quad \frac{2}{n} < x \le \frac{3}{n} \\ \vdots & \vdots \quad \vdots \\ \sin\frac{n\pi}{2n} & \text{if} \quad \frac{n-1}{n} < x \le 1. \end{cases}$$

Then, the value of $\lim_{n\to\infty} \int_0^1 f_n(x)\,dx$ is

(A) π; (B) 1; (C) $\frac{1}{\pi}$; (D) $\frac{2}{\pi}$.

27. The limit

$$\lim_{n\to\infty} \left(1 + \frac{1}{n^2 + \cos n}\right)^{n^2+n}$$

(A) does not exist; (B) equals 1; (C) equals e; (D) equals e^2.

28. Let K be the set of all points (x, y) such that $|x| + |y| \leq 1$. Given a point A in the plane, let F_A be the point in K which is closest to A. Then the points A for which $F_A = (1, 0)$ are

(A) all points $A = (x, y)$ with $x \geq 1$;
(B) all points $A = (x, y)$ with $x \geq y + 1$ and $x \geq 1 - y$;
(C) all points $A = (x, y)$ with $x \geq 1$ and $y = 0$;
(D) all points $A = (x, y)$ with $x \geq 0$ and $y = 0$.

29. In a win-or-lose game, the winner gets 2 points whereas the loser gets 0. *Six* players A, B, C, D, E and F play each other in a preliminary round from which the top *three* players move to the final round. After each player has played *four* games, A has 6 points, B has 8 points and C has 4 points. It is also known that E won against F. In the next set of games D, E and F win their games against A, B and C respectively. If A, B and D move to the final round, the final scores of E and F are, respectively,

(A) 4 and 2; (B) 2 and 4; (C) 2 and 2; (D) 4 and 4.

30. The number of ways in which one can select six distinct integers from the set $\{1, 2, 3, \cdots, 49\}$, such that no two consecutive integers are selected, is

(A) $\binom{49}{6} - 5\binom{48}{5}$; (B) $\binom{43}{6}$; (C) $\binom{25}{6}$; (D) $\binom{44}{6}$.

B.Math. (Hons.) Admission Test: 2008

Multiple-Choice Test
Time: 2 hours

1. Let a, b and c be fixed positive real numbers. Let $u_n = \frac{n^2 a}{b+n^2 c}$ for $n \geq 1$. Then as n increases,

(A) u_n increases; (B) u_n decreases;
(C) u_n increases first and then decreases;
(D) none of the above is necessarily true.

2. The number of polynomials of the form $x^3 + ax^2 + bx + c$ which are divisible by $x^2 + 1$ and where a, b and c belong to $\{1, 2, \ldots, 10\}$, is

(A) 1; (B) 10; (C) 11; (D) 100.

3. How many integers n are there such that $1 \leq n \leq 1000$ and the highest common factor of n and 36 is 1?

(A) 333; (B) 667; (C) 166; (D) 361.

4. The value of Σij, where the summation is over all i and j such that $1 \leq i, j \leq 10$, is

(A) 1320; (B) 2640; (C) 3025;
(D) none of the above.

5. Let $d_1, d_2, \ldots, d_k$ be all the factors of a positive integer n including 1 and n. Suppose $d_1 + d_2 + \cdots + d_k = 72$. Then the value of

$$\frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_k}$$

(A) is $\frac{k^2}{72}$; (B) is $\frac{72}{k}$; (C) is $\frac{72}{n}$; (D) cannot be computed.

6. The inequality $\sqrt{x+6} \geq x$ is satisfied for real x if and only if

(A) $-3 \leq x \leq 3$; (B) $-2 \leq x \leq 3$;
(C) $-6 \leq x \leq 3$; (D) $0 \leq x \leq 6$.

7. In the Cartesian plane the equation $x^3y + xy^3 + xy = 0$ represents

(A) a circle; (B) a circle and a pair of straight lines;
(C) a rectangular hyperbola ; (D) a pair of straight lines.

8. P is a variable point on a circle C and Q is a fixed point on the outside of C. R is a point in PQ dividing it in the ratio $p : q$, where $p > 0$ and $q > 0$ are fixed. Then the locus of R is

(A) a circle; (B) an ellipse;
(C) a circle if $p = q$ and an ellipse otherwise;
(D) none of the above curves.

9. ABC is a right-angled triangle with right angle at B. D is a point on AC such that $\angle ABD = 45°$. If $AC = 6$cm and $AD = 2$cm then AB is

(A) $\frac{6}{\sqrt{5}}$ cm; (B) $3\sqrt{2}$ cm; (C) $\frac{12}{\sqrt{5}}$ cm; (D) 2 cm

10. The maximum value of the integral

$$\int_{a-1}^{a+1} \frac{1}{1+x^8} dx$$

is attained

(A) exactly at two values of a ;
(B) only at one value of a which is positive;
(C) only at one value of a which is negative; (D) only at $a = 0$.

11. Let f be a function from a set X to X such that $f(f(x)) = x$ for all $x \in X$. Then

(A) f is one-to-one but need not be onto;
(B) f is onto but need not be one-to-one;
(C) f is both one-to-one and onto;
(D) none of the above is necessarily true.

12. The value of the sum

$$\cos\frac{2\pi}{1000} + \cos\frac{4\pi}{1000} + \cdots + \cos\frac{1998\pi}{1000}$$

equals

(A) -1; (B) 0; (C) 1; (D) an irrational number.

13. A box contains 100 balls of different colours: 28 red, 17 blue, 21 green, 10 white, 12 yellow and 12 black. The smallest number n such that any n balls drawn from the box will contain at least 15 balls of the same colour is

(A) 73; (B) 77; (C) 81; (D) 85.

14. The sum

$$(1 \cdot 1!) + (2 \cdot 2!) + (3 \cdot 3!) + \cdots + (50 \cdot 50!)$$

equals

(A) 51!; (B) 2.5!; (C) 51!-1; (D) 51!+1.

15. The remainder $R(x)$ obtained by dividing the polynomial x^{100} by the polynomial $x^2 - 3x + 2$ is

(A) $2^{100} - 1$; (B) $(2^{100} - 1)x - 2(2^{99} - 1)$;
(C) $2^{100}x - 3 \cdot 2^{100}$; (D) $(2^{100} - 1)x + 2(2^{99} - 1)$.

16. If three prime numbers, all greater than 3, are in A.P., then their common difference

(A) must be divisible by 2 but not necessarily by 3;
(B) must be divisible by 3 but not necessarily by 2;
(C) must be divisible by both 2 and 3;
(D) must not be divisible by any of 2 and 3.

17. Let P denote the set of all positive integers and

$$S = \{(x, y) \in P \times P : x^2 - y^2 = 666\}.$$

Then S

(A) is an empty set; (B) contains exactly one element;
(C) contains exactly two elements; (D) contains more than two elements.

18. For any real number x, let $[x]$ denote the greatest integer m such that $m \leq x$. The number of points in the open interval $(-2, 2)$ where $f(x) = [x^2 - 1]$ is not continuous equals

(A) 5; (B) 6; (C) 7; (D) ∞.

19. The equation $\log_3 x - \log_x 3 = 2$ has

(A) no real solution; (B) exactly one real solution;
(C) exactly two real solutions; (D) infinitely many real solutions.

20. Let l_1 and l_2 be a pair of intersecting lines in the plane. Then the locus of the points P such that the distance of P from l_1 is twice the distance of P from l_2 is

(A) an ellipse; (B) a parabola;
(C) a hyperbola; (D) a pair of straight lines.

21. If $c\int_0^1 xf(2x)dx = \int_0^2 tf(t)dt$, where f is a positive continuous function; then the value of c is

(A) $\frac{1}{2}$; (B) 4; (C) 2; (D) 1.

22. The equations $x^3 + 2x^2 + 2x + 1 = 0$ and $x^{200} + x^{130} + 1 = 0$ have

(A) exactly one common root ; (B) no common root;
(C) exactly three common roots; (D) exactly two common roots.

23. The set of complex numbers z such that $z(1-z)$ is a real number forms

(A) a line and circle; (B) a pair of lines;
(C) a line and a parabola; (D) a line and a hyperbola.

24. The numbers $12n + 1$ and $30n + 2$ are relatively prime for

(A) any positive integer n; (B) infinitely many, but not all, integers n;
(C) for finitely many integers n; (D) no positive integer n.

25. Let $f, g : \mathbb{R} \to \mathbb{R}$ be two differentiable functions. If $f(a) = 2,\ f'(a) = 1,\ g(a) = -1,\ g'(a) = 2$, then the limit

$$\lim_{x\to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$$

is

(A) 2; (B) 3; (C) 4; (D) 5.

26. Let $f : (-1, 1) \to (-1, 1)$ be continuous, $f(x) = f(x^2)$ for every x and $f(0) = \frac{1}{2}$. Then $f(\frac{1}{4})$ is

(A) $\frac{1}{2}$; (B) $\sqrt{\frac{3}{2}}$; (C) $\frac{3}{\sqrt{2}}$; (D) $\frac{\sqrt{2}}{3}$.

27. The number of ways in which a team of 6 members containing at least 2 left-handers can be formed from 7 right-handers and 4 left-handers is:

(A) 210; (B) 371; (C) $\binom{11}{6}$; (D) $\binom{11}{2}$.

28. The sum of the coefficients of the polynomial $(x-1)^2(x-2)^4(x-3)^6$ is

(A) 6; (B) 0; (C) 28; (D) 18.

29. Let $f : \{1, 2, 3\} \to \{1, 2, 3\}$ be a function. Then the number of functions $g : \{1, 2, 3\} \to \{1, 2, 3\}$ such that $f(x) = g(x)$ for at least one $x \in \{1, 2, 3\}$ is

(A) 11; (B) 19; (C) 23; (D) 27.

30. The polynomial $p(x) = x^4 - 4x^2 + 1$ has

(A) no roots in the interval $[0, 3]$;
(B) exactly one root in the interval $[0, 3]$;
(C) exactly two roots in the interval $[0, 3]$;
(D) more than two roots in the interval $[0, 3]$.

B.Stat. (Hons.) Admission Test: 2009

Multiple-Choice Test

Time: 2 hours

Group A

Each of the following questions has exactly one correct option and you have to identify it.

1. If k times the sum of the first n natural numbers is equal to the sum of the squares of the first n natural numbers, then $\cos^{-1}\left(\frac{2n-3k}{2}\right)$ is

(A) $\frac{5\pi}{6}$; (B) $\frac{2\pi}{3}$; (C) $\frac{\pi}{3}$; (D) $\frac{\pi}{6}$.

2. Two circles touch each other at P. The two common tangents to the circles, none of which pass through P, meet at E. They touch the larger circle at C and D. The larger circle has radius 3 units and CE has length 4 units. Then the radius of the smaller circle is

(A) 1; (B) $\frac{5}{7}$; (C) $\frac{3}{4}$; (D) $\frac{1}{2}$.

3. Suppose $ABCDEFGHIJ$ is a ten-digit number, where the digits are all distinct. Moreover, $A > B > C$ satisfy $A+B+C = 9$, $D > E > F$ are consecutive even digits and $G > H > I > J$ are consecutive odd digits. Then A is

(A) 8; (B) 7; (C) 6; (D) 5.

4. Let ABC be a right angled triangle with $AB > BC > CA$. Construct three equilateral triangles BCP, CQA and ARB, so that A and P are on opposite sides of BC; B and Q are on opposite sides of CA; C and R are on opposite sides of AB. Then

(A) $CR > AP > BQ$; (B) $CR < AP < BQ$;
(C) $CR = AP = BQ$; (D) $CR^2 = AP^2 + BQ^2$.

5. The value of $(1+\tan 1^\circ)(1+\tan 2^\circ)\cdots(1+\tan 44^\circ)$ is

(A) 2; (B) a multiple of 22;
(C) not an integer; (D) a multiple of 4.

6. Let $y = x/(1+x)$, where

$$x = \omega^{2009^{2009^{\cdots\text{upto 2009 times}}}}$$

and ω is a complex cube root of 1. Then y is

(A) ω; (B) $-\omega$; (C) ω^2; (D) $-\omega^2$.

7. The number of solutions of θ in the interval $[0, 2\pi]$ satisfying

$$\left(\log_{\sqrt{3}} \tan\theta\right) \sqrt{\log_{\tan\theta} 3 + \log_{\sqrt{3}} 3\sqrt{3}} = -1$$

is

(A) 0; (B) 2; (C) 4; (D) 6.

8. A building with ten storeys, each storey of height 3 metres, stands on one side of a wide street. From a point on the other side of the street directly opposite to the building, it is observed that the three uppermost storeys together subtend an angle equal to that subtended by the two lowest storeys. The width of the street is

(A) $6\sqrt{35}$ metres; (B) $6\sqrt{70}$ metres;
(C) 6 metres; (D) $6\sqrt{3}$ metres.

9. A collection of black and white balls are to be arranged on a straight line, such that each ball has at least one neighbour of different colour. If there are 100 black balls, then the maximum number of white balls that allows such an arrangement is

(A) 100; (B) 101; (C) 202; (D) 200.

10. Let $f(x)$ be a real-valued function satisfying $af(x) + bf(-x) = px^2 + qx + r$, where a and b are distinct real numbers and p, q and r are non-zero real numbers. Then $f(x) = 0$ will have real solution when

(A) $(\frac{a+b}{a-b})^2 \leq \frac{q^2}{4pr}$; (B) $(\frac{a+b}{a-b})^2 \leq \frac{4pr}{q^2}$;
(C) $(\frac{a+b}{a-b})^2 \geq \frac{q^2}{4pr}$; (D) $(\frac{a+b}{a-b})^2 \geq \frac{4pr}{q^2}$.

11. A circle is inscribed in a square of side x, then a square is inscribed in that circle, a circle is inscribed in the latter square, and so on. If S_n is the sum of the areas of the first n circles so inscribed, then, $\lim_{n\to\infty} S_n$ is

(A) $\frac{\pi x^2}{4}$; (B) $\frac{\pi x^2}{3}$; (C) $\frac{\pi x^2}{2}$; (D) πx^2.

12. Let $1, 4, \ldots$ and $9, 14, \ldots$ be two arithmetic progressions. Then the number of distinct integers in the collection of first 500 terms of each of the progressions is

(A) 833; (B) 835; (C) 837; (D) 901.

13. Consider all the 8-letter words that can be formed by arranging the letters in BACHELOR in all possible ways. Any two such words are called *equivalent* if those two words maintain the same relative order of the letters A, E and O. For example, BACOHELR and CABLROEH are equivalent. How many words are there which are equivalent to BACHELOR?

(A) $\binom{8}{3} \times 3!$; (B) $\binom{8}{3} \times 5!$; (C) $2 \times \binom{8}{3}^2$; (D) $5! \times 3! \times 2!$.

14. The limit

$$\lim_{n\to\infty}\left(\frac{1}{6}+\frac{1}{24}+\frac{1}{60}+\frac{1}{120}+\cdots+\frac{1}{n^3-n}\right)$$

equals

(A) 1; (B) $\frac{1}{2}$; (C) $\frac{1}{4}$; (D) $\frac{1}{8}$.

15. Let a and b be real numbers satisfying $a^2+b^2 \neq 0$. Then the set of real numbers c, such that the equations $al + bm = c$ and $l^2 + m^2 = 1$ have real solutions for l and m is

(A) $[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$; (B) $[-|a+b|, |a+b|]$;
(C) $[0, a^2+b^2]$; (D) $(-\infty, \infty)$.

16. Let f be an onto and differentiable function defined on $[0,1]$ to $[0,T]$, such that $f(0) = 0$. Which of the following statements is necessarily true?

(A) $f'(x)$ is greater than or equal to T *for all* x;
(B) $f'(x)$ is smaller than T *for all* x;
(C) $f'(x)$ is greater than or equal to T *for some* x;
(D) $f'(x)$ is smaller than T *for some* x.

17. The area of the region bounded by $|x| + |y| + |x+y| \leq 2$ is

(A) 2; (B) 3; (C) 4; (D) 6.

18. Let f and g be two positive valued functions defined on $[-1,1]$, such that $f(-x) = 1/f(x)$ and g is an even function with $\int_{-1}^{1} g(x)dx = 1$. Then $I = \int_{-1}^{1} f(x)g(x)dx$ satisfies

(A) $I \geq 1$; (B) $I \leq 1$; (C) $\frac{1}{3} < I < 3$; (D) $I = 1$.

19. How many possible values of (a,b,c,d), with a,b,c,d real, are there such that $abc = d$, $bcd = a$, $cda = b$ and $dab = c$?

(A) 1; (B) 6; (C) 9; (D) 17.

20. What is the maximum possible value of a positive integer n, such that for any choice of seven distinct elements from $\{1, 2, \ldots, n\}$, there will exist two numbers x and y satisfying $1 < x/y \leq 2$?

(A) 2×7; (B) $2^7 - 2$; (C) $7^2 - 2$; (D) $7^7 - 2$.

Group B

Each of the following questions has either one or two correct options and you have to identify all the correct options.

21. Which of the following are roots of the equation $x^7 + 27x = 0$?

(A) $-\sqrt{3}i$; (B) $\frac{\sqrt{3}}{2}(-1 + \sqrt{3}i)$;
(C) $-\frac{\sqrt{3}}{2}(1 + i)$; (D) $\frac{\sqrt{3}}{2}(\sqrt{3} - i)$.

22. The equation $|x^2 - x - 6| = x + 2$ has

(A) two positive roots; (B) two real roots;
(C) three real roots; (D) none of the above.

23. If $0 < x < \pi/2$, then

(A) $\cos(\cos x) > \sin x$; (B) $\sin(\sin x) > \sin x$;
(C) $\sin(\cos x) > \cos x$; (D) $\cos(\sin x) > \sin x$.

24. Suppose $ABCD$ is a quadrilateral such that the coordinates of A, B and C are $(1, 3)$, $(-2, 6)$ and $(5, -8)$ respectively. For which choices of coordinates of D will $ABCD$ be a trapezium?

(A) $(3, -6)$; (B) $(6, -9)$; (C) $(0, 5)$; (D) $(3, -1)$.

25. Let x and y be two real numbers such that $2\log(x - 2y) = \log x + \log y$ holds. Which of the following are possible values of x/y?

(A) 4; (B) 3; (C) 2; (D) 1.

26. Let f be a differentiable function satisfying $f'(x) = f'(-x)$ for all x. Then

(A) f is an odd function;
(B) $f(x) + f(-x) = 2f(0)$ for all x;
(C) $\frac{1}{2}f(x) + \frac{1}{2}f(y) = f(\frac{1}{2}(x + y))$ for all x, y;
(D) If $f(1) = f(2)$, then $f(-1) = f(-2)$.

27. Consider the function

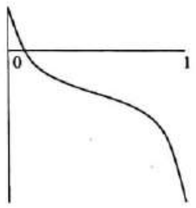
$$f(x) = \begin{cases} \frac{\max\{x, \frac{1}{x}\}}{\min\{x, \frac{1}{x}\}}, & \text{when } x \neq 0, \\ 1, & \text{when } x = 0. \end{cases}$$

Then

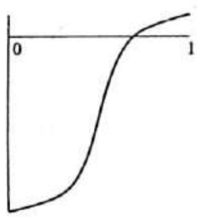
(A) $\lim_{x \to 0+} f(x) = 0$;
(B) $\lim_{x \to 0-} f(x) = 0$;
(C) $f(x)$ is continuous for all $x \neq 0$;
(B) $f(x)$ is differentiable for all $x \neq 0$.

28. Which of the following graphs represent functions whose derivatives have a maximum in the interval $(0, 1)$?

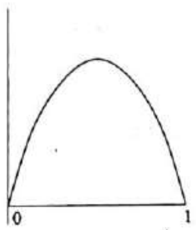
(A)



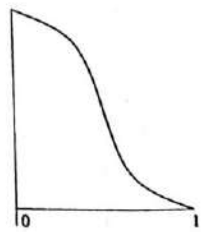
(B)



(C)



(D)



29. A collection of geometric figures is said to satisfy *Helly property* if the following condition holds:

> for any choice of three figures A, B, C from the collection satisfying $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$ and $C \cap A \neq \emptyset$, one must have $A \cap B \cap C \neq \emptyset$.

Which of the following collections satisfy Helly property?

(A) A set of circles;
(B) A set of hexagons;
(C) A set of squares with sides parallel to the axes;
(D) A set of horizontal line segments.

30. Consider an array of m rows and n columns obtained by arranging the first mn integers in some order. Let b_i be the maximum of the numbers in the i-th row and c_j be the minimum of the numbers in the j-th column. If

$$b = \min_{1 \le i \le m} b_i \quad \text{and} \quad c = \max_{1 \le j \le n} c_j,$$

then which of the following statements are necessarily true?

(A) $m \le c$; (B) $n \ge b$; (C) $c \ge b$; (D) $c \le b$.

B.Math. (Hons.) Admission Test: 2009

Multiple-Choice Test
Time: 2 hours

1. The domain of definition of $f(x) = -\log(x^2 - 2x - 3)$ is:

(A) $(0, \infty)$ (B) $(-\infty, -1)$ (C) $(-\infty, -1) \cup (3, \infty)$ (D) $(-\infty, -3) \cup (1, \infty)$.

2. ABC is a right-angled triangle with the right angle at B. If $\overline{AB} = 7$ and $\overline{BC} = 24$, then the length of the perpendicular from B to AC is

(A) 12.2; (B) 6.72; (C) 7.2; (D) 3.36.

3. If the points z_1 and z_2 are on the circles $|z| = 2$ and $|z| = 3$, respectively and the angle included between these vectors is $60°$, then $\frac{|z_1+z_2|}{z_1-z_2|}$ equals

(A) $\sqrt{\frac{19}{7}}$; (B) $\sqrt{19}$; (C) $\sqrt{7}$; (D) $\sqrt{133}$.

4. Let a, b, c and d be positive integers such that $\log_a(b) = 3/2$ and $\log_c(d) = 5/4$. If $a - c = 9$, then $b - d$ equals

(A) 55; (B) 23; (C) 89; (D) 93.

5. $1 - x - e^{-x} > 0$ for:

(A) all $x \in \mathbf{R}$; (B) no $x \in \mathbf{R}$; (C) $x > 0$; (D) $x < 0$.

6. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + bx + c$, where $ac \neq 0$, then the equation $P(x)Q(x) = 0$ has:

(A) only real roots; (B) no real root;
(C) at least two real roots; (D) exactly two real roots.

7. $\lim\limits_{x\to\infty} \left|\sqrt{x^2+x} - x\right|$ is equal to

(A) $\frac{1}{2}$; (B) 0; (C) ∞; (D) 2.

8. $\lim\limits_{n\to\infty} \frac{\pi}{2^n} \sum\limits_{j=1}^{2^n} \sin\left(\frac{j\pi}{2^n}\right)$ is equal to

(A) 0; (B) π; (C) 2; (D) 1.

9. Let $f : \mathbf{R} \to \mathbf{R}$ be given by $f(x) = x(x-1)(x+1)$. Then,

(A) f is 1-1 and onto; (B) f is neither 1-1 nor onto;
(C) f is 1-1 but not onto; (D) f is onto but not 1-1.

10. The last digit of 22^{22} is:

(A) 2; (B) 4; (C) 6; (D) 0.

11. The average of scores of 10 students in a test is 25. The lowest score is 20. Then, the highest score is at most

(A) 100; (B) 30; (C) 70; (D) 75.

12. The coefficient of t^3 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is

(A) 10; (B) 12; (C) 8; (D) 9.

13. Let $p_n(x)$, $n \geq 0$ be polynomials defined by $p_0(x) = 1$, $p_1(x) = x$ and $p_n(x) = xp_{n-1}(x) - p_{n-2}(x)$ for $n \geq 2$. Then $p_{10}(0)$ equals

(A) 0; (B) 10; (C) 1; (D) -1.

14. Suppose A, B are matrices satisfying $AB + BA = 0$. Then A^5B^2 is equal to

(A) 0; (B) B^2A^5; (C) $-B^2A^5$; (D) AB.

15. The number of terms in the expansion of $(x+y+z+w)^{2009}$ is

(A) $\binom{2009}{4}$; (B) $\binom{2013}{4}$; (C) $\binom{2012}{3}$; (D) $(2010)^4$.

16. If a, b and c are positive real numbers satisfying $ab + bc + ca = 12$, then the maximum value of abc is

(A) 8; (B) 9; (C) 6; (D) 12.

17. If at least 90 per cent students in a class are good in sports, and at least 80 per cent are good in music and at least 70 per cent are good in studies, then the percentage of students who are good in all three is at least

(A) 25; (B) 40; (C) 20; (D) 50.

18. If $\cot\left(\sin^{-1}\sqrt{13/17}\right) = \sin\left(\tan^{-1}\theta\right)$, then θ is

(A) $\frac{2}{\sqrt{17}}$; (B) $\sqrt{\frac{13}{17}}$; (C) $\sqrt{\frac{2}{\sqrt{13}}}$; (D) $\frac{2}{3}$.

19. Let $f(t) = \frac{t+1}{t-1}$. Then $f(f(2010))$ equals

(A) $\frac{2011}{2009}$; (B) 2010; (C) $\frac{2010}{2009}$; (D) none of the above.

20. If each side of a cube is increased by 60%, then the surface area of the cube increases by

(A) 156%; (B) 160%; (C) 120%; (D) 240%.

21. If $a > 2$, then

(A) $\log_e(a) + \log_a(10) < 0$; (B) $\log_e(a) + \log_a(10) > 0$; (C) $e^a < 1$;
(D) none of the above is true.

22. The number of complex numbers w such that $|w| = 1$ and imaginary part of w^4 is 0, is

(A) 4; (B) 2; (C) 8; (D) infinite.

23. Let $f(x) = c \cdot \sin(x)$ for all $x \in \mathbf{R}$. Suppose $f(x) = \sum_{k=1}^{\infty} \frac{f(x+k\pi)}{2^k}$ for all $x \in \mathbf{R}$. Then

(A) $c = 1$; (B) $c = 0$; (C) $c < 0$; (D) $c = -1$.

24. The number of points at which the function $f(x) = \max(1+x, 1-x)$ if $x < 0$ and
$f(x) = \min(1+x, 1+x^2)$ if $x \geq 0$ is not differentiable, is

(A) 1; (B) 0; (C) 2; (D) none of the above.

25. The greatest value of the function $f(x) = \sin^2(x)\cos(x)$ is

(A) $\frac{2}{3\sqrt{3}}$; (B) $\sqrt{\frac{2}{3}}$; (C) $\frac{2}{9}$; (D) $\frac{\sqrt{2}}{3\sqrt{3}}$.

26. Let $g(t) = \int_{-10}^{t} (x^2+1)^{10} dx$ for all $t \geq -10$. Then

(A) g is not differentiable;
(B) g is constant;
(C) g is increasing in $(-10, \infty)$;
(D) g is decreasing in $(-10, \infty)$.

27. Let $p(x)$ be a continuous function which is positive for all x and $\int_2^3 p(x)dx = c\int_0^2 p\left(\frac{x+4}{2}\right) dx$. Then

(A) $c = 4$; (B) $c = 1/2$; (C) $c = 1/4$; (D) $c = 2$.

28. Let $f : [0,1] \to (1,\infty)$ be a continuous function. Let $g(x) = 1/x$ for $x > 0$. Then, the equation $f(x) = g(x)$ has

(A) no solution;
(B) all points in (0,1] as solutions;
(C) at least one solution;
(D) none of the above.

29. Let $0 \leq \theta, \phi < 2\pi$ be two angles. Then, the equation $\sin\theta + \sin\phi = \cos\theta + \cos\phi$

(A) determines θ uniquely in terms of ϕ;
(B) gives two values of θ for each value of ϕ;
(C) gives more than two values of θ for each value of ϕ;
(D) none of the above.

30. Ten players are to play in a tennis tournament. The number of pairings for the first round is

(A) $\frac{10!}{2^5 5!}$; (B) 2^{10}; (C) $\binom{10}{2}$; (D) ${}^{10}P_2$.

B.Stat. (Hons.) Admission Test: 2010

Multiple-Choice Test
Time: 2 hours

1. There are 8 balls numbered 1,2,...,8 and 8 boxes numbered 1,2,...,8. The number of ways one can put these balls in the boxes so that each box gets one ball and exactly 4 balls go in their corresponding numbered boxes is

(A) $3 \times \binom{8}{4}$; (B) $6 \times \binom{8}{4}$; (C) $9 \times \binom{8}{4}$; (D) $12 \times \binom{8}{4}$.

2. Let α and β be two positive real numbers. For every integer $n > 0$, define

$$a_n = \int_{\beta}^{n} \frac{\alpha}{u(u^{\alpha} + 2 + u^{-\alpha})} du.$$

Then $\lim_{n \to \infty} a_n$ is equal to

(A) $\frac{1}{1+\beta^{\alpha}}$; (B) $\frac{\beta^{\alpha}}{1+\beta^{-\alpha}}$;
(C) $\frac{\beta^{\alpha}}{1+\beta^{\alpha}}$; (D) $\frac{\beta^{-\alpha}}{1+\beta^{\alpha}}$.

3. Let $f : \mathbb{R} \to \mathbb{R}^2$ be a function given by $f(x) = (x^m, x^n)$, where $x \in \mathbb{R}$ and m, n are fixed positive integers. Suppose that f is one-one. Then

(A) both m and n must be odd;
(B) at least one of m and n must be odd;
(C) exactly one of m and n must be odd;
(D) neither m nor n can be odd.

4. $\lim_{x \to 2} \frac{e^{x^2} - e^{2x}}{(x-2)e^{2x}}$ equals

(A) 0; (B) 1; (C) 2; (D) 3.

5. A circle is inscribed in a triangle with sides 8, 15 and 17 centimetres. The radius of the circle in centimetres is

(A) 3; (B) 22/7; (C) 4; (D) none of the above.

6. Let α, β and γ be the angles of an acute angled triangle. Then the quantity $\tan\alpha \tan\beta \tan\gamma$

(A) can have any real value; (B) is $\leq 3\sqrt{3}$;
(C) is $\geq 3\sqrt{3}$; (D) none of the above.

7. Let $f(x) = |x| \sin x + |x - \pi| \cos x$ for $x \in \mathbb{R}$. Then

(A) f is differentiable at $x = 0$ and $x = \pi$;
(B) f is not differentiable at $x = 0$ and $x = \pi$;
(C) f is differentiable at $x = 0$ but not differentiable at $x = \pi$;
(D) f is not differentiable at $x = 0$ but differentiable at $x = \pi$.

8. Consider a rectangular cardboard box of height 3, breadth 4 and length 10 units. There is a lizard in one corner A of the box and an insect in the corner B which is farthest from A. The length of the shortest path between the lizard and the insect along the surface of the box is

(A) $\sqrt{5^2 + 10^2}$ units; (B) $\sqrt{7^2 + 10^2}$ units;
(C) $4 + \sqrt{3^2 + 10^2}$ units; (D) $3 + \sqrt{10^2 + 4^2}$ units.

9. *Recall that, for any non-zero complex number w which does not lie on the negative real axis,* $\arg w$ *denotes the unique real number θ in $(-\pi, \pi)$ such that*

$$w = |w|\,(\cos\theta + i \sin\theta).$$

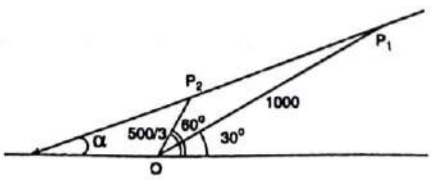
Let z be any complex number such that its real and imaginary parts are both non-zero. Further, suppose that z satisfies the relations

$$\arg z > \arg(z + 1) \quad \text{and} \quad \arg z > \arg(z + i).$$

Then $\cos(\arg z)$ can take

(A) any value in the set $(-1/2, 0) \cup (0, 1/2)$ but none from outside;
(B) any value in the interval $(-1, 0)$ but none from outside;
(C) any value in the interval $(0, 1)$ but none from outside;
(D) any value in the set $(-1, 0) \cup (0, 1)$ but none from outside.

10. An aeroplane P is moving in the air along a straight line path which passes through the points P_1 and P_2, and makes an angle α with the ground. Let O be the position of an observer as shown in the figure below. When the plane is at the position P_1 its angle of elevation is 30° and when it is at P_2 its angle of elevation is 60° from the position of the observer. Moreover, the distances of the observer from the points P_1 and P_2 respectively are 1000 metres and 500/3 metres.



Then α is equal to

(A) $\tan^{-1}(\frac{2-\sqrt{3}}{2\sqrt{3}-1})$; (B) $\tan^{-1}(\frac{2\sqrt{3}-3}{4-2\sqrt{3}})$;
(C) $\tan^{-1}(\frac{2\sqrt{3}-2}{5-\sqrt{3}})$; (D) $\tan^{-1}(\frac{6-\sqrt{3}}{6\sqrt{3}-1})$.

11. The sum of all even positive divisors of 1000 is (A) 2170; (B) 2184; (C) 2325; (D) 2340.

12. The equation $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ has two real roots α and β. If $a > 0$, then the area under the curve $f(x) = x^2 + \frac{b}{a}x + \frac{c}{a}$ between α and β is

(A) $\frac{b^2-4ac}{2a}$; (B) $\frac{(b^2-4ac)^{3/2}}{6a^3}$;
(C) $-\frac{(b^2-4ac)^{3/2}}{6a^3}$; (D) $-\frac{b^2-4ac}{2a}$.

13. The minimum value of $x_1^2 + x_2^2 + x_3^2 + x_4^2$ subject to $x_1 + x_2 + x_3 + x_4 = a$ and $x_1 - x_2 + x_3 - x_4 = b$ is

(A) $\frac{a^2+b^2}{4}$; (B) $\frac{a^2+b^2}{2}$;
(C) $\frac{(a+b)^2}{4}$; (D) $\frac{(a+b)^2}{2}$.

14. The value of

$$\lim_{n\to\infty} \frac{\sum_{r=0}^{n} \binom{2n}{2r} 3^r}{\sum_{r=0}^{n-1} \binom{2n}{2r+1} 3^r}$$

is

(A) 0; (B) 1; (C) $\sqrt{3}$; (D) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$.

15. For any real number x, let $\tan^{-1}(x)$ denote the unique real number θ in $(-\pi/2, \pi/2)$ such that $\tan\theta = x$. Then

$$\lim_{n\to\infty} \sum_{m=1}^{n} \tan^{-1} \frac{1}{1+m+m^2}$$

(A) is equal to $\pi/2$; (B) is equal to $\pi/4$;
(C) does not exist; (D) none of the above.

16. Let n be an integer. The number of primes which divide both $n^2 - 1$ and $(n+1)^2 - 1$ is

(A) at most one; (B) exactly one;
(C) exactly two; (D) none of the above.

17. The value of

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{6n}{9n^2-r^2}$$

is

(A) 0; (B) $\log\frac{3}{2}$; (C) $\log\frac{2}{3}$; (D) $\log 2$.

18. A person X standing at a point P on a flat plane starts walking. At each step, he walks exactly 1 foot in one of the directions North, South, East or West. Suppose that after 6 steps X comes back to the original position P. Then the number of distinct paths that X can take is

(A) 196; (B) 256; (C) 344; (D) 400.

19. Consider the branch of the rectangular hyperbola $xy = 1$ in the first quadrant. Let P be a fixed point on this curve. The locus of the mid-point of the line segment joining P and an arbitrary point Q on the curve is part of

(A) a hyperbola; (B) a parabola; (C) an ellipse; (D) none of the above.

20. The digit at the unit place of $(1! - 2! + 3! - \cdots + 25!)^{(1!-2!+3!-\cdots+25!)}$ is

(A) 0; (B) 1; (C) 5; (D) 9.

21. Let $A_1, A_2, \ldots, A_n$ be the vertices of a regular polygon and A_1A_2, A_2A_3, $\ldots$, $A_{n-1}A_n$, A_nA_1 be its n sides. If

$$\frac{1}{A_1A_2}-\frac{1}{A_1A_4}=\frac{1}{A_1A_3},$$

then the value of n is

(A) 5; (B) 6; (C) 7; (D) 8.

22. Suppose that α and β are two distinct numbers in the interval $(0,\pi)$. If

$$\sin\alpha+\sin\beta=\sqrt{3}(\cos\alpha-\cos\beta)$$

then the value of $\sin 3\alpha + \sin 3\beta$ is

(A) 0; (B) $2\sin\frac{3(\alpha+\beta)}{2}$; (C) $2\cos\frac{3(\alpha-\beta)}{2}$; (D) $\cos\frac{3(\alpha-\beta)}{2}$.

23. Consider the function $h(x) = x^2 - 2x + 2 + \frac{4}{x^2-2x+2}$, $x \in \mathbb{R}$. Then $h(x) - 5 = 0$ has

(A) no solution; (B) only one solution;
(C) exactly two solutions; (D) exactly three solutions.

24. Consider the quadratic equation $x^2 + bx + c = 0$. The number of pairs (b, c) for which the equation has solutions of the form $\cos\alpha$ and $\sin\alpha$ for some α is

(A) 0; (B) 1; (C) 2; (D) infinite.

25. Let $\theta_1 = \frac{2\pi}{3}$, $\theta_2 = \frac{4\pi}{7}$, $\theta_3 = \frac{7\pi}{12}$. Then

(A) $(\sin\theta_1)^{\sin\theta_1} < (\sin\theta_2)^{\sin\theta_2} < (\sin\theta_3)^{\sin\theta_3}$;
(B) $(\sin\theta_2)^{\sin\theta_2} < (\sin\theta_1)^{\sin\theta_1} < (\sin\theta_3)^{\sin\theta_3}$;
(C) $(\sin\theta_3)^{\sin\theta_3} < (\sin\theta_1)^{\sin\theta_1} < (\sin\theta_2)^{\sin\theta_2}$;
(D) $(\sin\theta_1)^{\sin\theta_1} < (\sin\theta_3)^{\sin\theta_3} < (\sin\theta_2)^{\sin\theta_2}$.

26. Consider the following two curves on the interval $(0, 1)$:

$$C_1 : y = 1 - x^4 \quad \text{and} \quad C_2 : y = \sqrt{1 - x^2}.$$

Then on $(0, 1)$

(A) C_1 lies above C_2; (B) C_2 lies above C_1;
(C) C_1 and C_2 intersect at exactly one point;
(D) none of the above.

27. Let f be a real valued function on $\mathbb{R}$ such that f is twice differentiable. Suppose that f' vanishes only at 0 and f'' is everywhere negative. Define a function h by $h(x) = (x - a)^2 - f(x)$, where $a > 0$. Then

(A) h has a local minima in $(0, a)$;
(B) h has a local maxima in $(0, a)$;
(C) h is monotonically increasing in $(0, a)$;
(D) h is monotonically decreasing in $(0, a)$.

28. Consider the triangle with vertices $(1, 2)$, $(-5, -1)$ and $(3, -2)$. Let Δ denote the region enclosed by the above triangle. Consider the function $f : \Delta \to \mathbb{R}$ defined by $f(x, y) = |10x - 3y|$. Then the range of f is the interval

(A) $[0, 36]$; (B) $[0, 47]$; (C) $[4, 47]$; (D) $[36, 47]$.

29. For every positive integer n, let $\langle n \rangle$ denote the integer closest to $\sqrt{n}$. Let $A_k = \{n > 0 : \langle n \rangle = k\}$. The number of elements in A_{49} is

(A) 97; (B) 98; (C) 99; (D) 100.

30. Consider a square $ABCD$ inscribed in a circle of radius 1. Let A' and C' be two points on the (smaller) arcs AD and CD respectively, such that $A'ABCC'$ is a pentagon in which $AA' = CC'$. If P denotes the area of the pentagon $A'ABCC'$ then

(A) P can not be equal to 2;
(B) P lies in the interval $(1, 2]$;
(C) P is greater than or equal to 2;
(D) none of the above.

B.Math. (Hons.) Admission Test: 2010

Multiple-Choice Test
Time: 2 hours

1. The product of the first 100 positive integers ends with

(A) 21 zeroes; (B) 22 zeroes; (C) 23 zeroes; (D) 24 zeroes.

2. Given four 1-gm stones, four 5-gm stones, four 25-gm stones and four 125-gm stones each, it is possible to weigh material of any integral weight up to

(A) 600 gms; (B) 625 gms; (C) 624 gms; (D) 524 gms.

3. The function $f(x) = |x| + \sin x + \cos^3 x$ is

(A) continuous but not differentiable at $x = 0$;
(B) differentiable at $x = 0$;
(C) a bounded function which is not continuous at $x = 0$;
(D) a bounded function which is continuous at $x = 0$.

4. The sum of the first n terms of an arithmetic progression whose first term is a (not necessarily positive) integer and the common difference is 2, is known to be 153. If $n > 1$, then the number of possible values of n is

(A) 2; (B) 3; (C) 4; (D) 5.

5. Consider a cubical box of 1 m side which has one corner placed at $(0,0,0)$ and the opposite corner placed at $(1,1,1)$. The least distance that an ant crawling from the point $(0,0,0)$ to the point $(1,1,1)$ must travel is

(A) $\sqrt{6}$ m; (B) $\sqrt{5}$ m; (C) $2\sqrt{3}$ m; (D) $1+\sqrt{3}$ m.

6. Let $x_1 < -1$ and $x_{n+1} = \frac{x_n}{1+x_n}$ for all $n \geq 1$. Then

(A) $\{x_n\} \to -1$ as $n \to \infty$; (B) $\{x_n\} \to 1$ as $n \to \infty$;
(C) $\{x_n\} \to 0$ as $n \to \infty$ (D) $\{x_n\}$ diverges.

7. The number of perfect cubes among the first 4000 positive integers is

(A) 16; (B) 15; (C) 14; (D) 13.

8. The roots of the equation $x^4 + x^2 = 1$ are

(A) all real and positive; (B) never real;
(C) 2 positive and 2 negative; (D) 1 positive, 1 negative and 2 non-real.

9. If $(\frac{x+a}{x-a})^x \to 9$ as $x \to \infty$, then $a =$

(A) 3^e; (B) $\log 3$; (C) $\log 9$; (D) 3.

10. The number of multiples of 4 among all 10 digit numbers is

(A) 25×10^8; (B) 25×10^7; (C) 225×10^7; (D) 234×10^7.

11. The larger diagonal of a parallelogram of area 8 must have length

(A) at least 4; (B) equal to 8; (C) at most 4; (D) equal to $\sqrt{8}$.

12. Let $\{a_n\}$ be a sequence of positive real numbers such that $\lim_{n\to\infty} 2^n(a_n + a_{n+1})$ is finite. Then

(A) $\{a_n\}$ converges to 1;
(B) $\{a_n\}$ converges to 0;
(C) $\{a_n\}$ converges to 1/2;
(D) $\{a_n\}$ converges to $1/\sqrt{2}$.

13. Consider the following two statements about a positive integer n and choose the correct option below.
(I) n is a perfect square.
(II) The number of positive integer divisors of n is odd.

(A) I and II are equivalent; (B) I implies II but not conversely;
(C) II implies I but not conversely; (D) Neither statement implies the other.

14. For triangles ABC and PQR, it is given that AB=PQ,BC=QR and the angle ACB equals the angle QRP. Then, the triangles ABC and PQR

(A) are congruent; (B) cannot be congruent;
(C) need not be congruent but must be similar;
(D) need not be similar but, if they are, then they must be congruent.

15. A particle starts at the origin and travels along the positive x-axis. For the first one second, its speed is 1 m/sec. Thereafter, its speed at any time t is at the most (9/10)-ths of its speed at time $t - 1$. Then

(A) the particle reaches any point $x > 0$ at some finite time;
(B) the particle must reach $x = 10$;
(C) the particle may or may not reach $x = 9$ but it will never reach $x = 10$;
(D) nothing of the above nature can be predicted without knowing the exact speed.

16. Let $f(x) = \min\ (e^x, e^{-x})$ for any real number x. Then

(A) f has no maximum;
(B) f attains its maximum at a point where $f'(x) = 0$;
(C) f attains its maximum at a point where it is not differentiable;
(D) $M := \max\ (f(x) : x \text{ real }) < \infty$ but there is no number x_0 such that $f(x_0) = M$.

17. If θ is an acute angle, the maximal value of $3\sin\theta + 4\cos\theta$ is

(A) 4; (B) 5; (C) $5\sqrt{2}$; (D) $3(1+\sqrt{3}/2)$.

18. I sold 2 books for Rs.30 each. My profit on one was 25 per cent and the loss on the other was 25 per cent. Then, on the whole, I

(A) lost Rs. 5; (B) lost Rs. 4; (C) gained Rs. 4; (D) neither gained nor lost.

19. Suppose x is an irrational number and, a, b, c, d are non-zero rational numbers. If $\frac{ax+b}{cx+d}$ is rational, then we must have

(A) $a = c = 0$; (B) $a = c, b = d$; (C) $ad = bc$; (D) $a + d = b + c$.

20. If a, b, c are real numbers so that $x^3 + ax^2 + bx + c = (x^2+1)g(x)$ for some polynomial g, then

(A) $b = 1, a = c$; (B) $b = 0 = c$; (C) $a = 0$; (D) none of the above.

21. The average of scores of 12 students in a test is 74. The highest score is 79. Then, the minimum possible lowest score must be

(A) 25; (B) 12; (C) 19; (D) 28.

22. If $x > y$ are positive integers such that $3x + 11y$ leaves a remainder 2 when divided by 7 and $9x + 5y$ leaves a remainder 3 when divided by 7, then the remainder when $x - y$ is divided by 7, equals

(A) 3; (B) 4; (C) 5; (D) 6.

23. The set of all real numbers which satisfy $\frac{x^2-2x+3}{\sqrt{x^2-2x+2}} \geq 2$ is

(A) the set of all integers;
(B) the set of all rational numbers;
(C) the set of all positive real numbers;
(D) the set of all real numbers.

24. Let ABC be a triangle such that the three medians divide it into six parts of equal area. Then, the triangle

(A) cannot exist; (B) can be any triangle; (C) must be equilateral;
(D) need not be equilateral but must be isosceles.

25. From a bag containing 10 distinct objects, the number of ways one can select an odd number of objects is

(A) 2^{10}; (B) 2^9; (C) 10!; (D) 5.

26. Consider the two statements: (I) between any two rational numbers, there is an irrational number;
(II) between any two irrational numbers, there is a rational number.
Then

(A) both (I) and (II) are true;
(B) (I) is true but (II) is not;
(C) (I) is false but (II) is not;
(D) both (I) and (II) are false.

27. Let $f(x) = ax^2 + bx + c$ where a, b, c are real numbers. Suppose $f(x) \neq x$ for any real number x. Then the number of solutions of $f(f(x)) = x$ in real numbers x is

(A) 4; (B) 2; (C) 0; (D) cannot be determined.

28. Let $s = \sum_{n=1}^{\infty} ne^{-n}$. Then
(a) $s \leq 1$ (b) $1 < s < \infty$ (c) s is infinite (d) $s = 0$

29. Let $f(x) = ax^3 + bx^2 + cx + d$ be a polynomial of degree 3 where a, b, c, d are real. Then

(A) $f(x) \to \infty$ as $x \to \infty$;
(B) f is 1-1 as well as onto;
(C) the graph of $f(x)$ meets the x-axis in one or three points;
(D) f must be onto but need not be 1-1.

30. Let $a_1, \cdots, a_n$ be arbitrary integers and suppose $b_1, \cdots, b_n$ is a permutation of the a_i's. Then the value $|a_1 - b_1| + |a_2 - b_2| + \cdots + |a_n - b_n|$

(A) is less than or equal to n; (B) can be an arbitrary positive integer;
(C) can be any even nonnegative integer; (D) must be zero.

B.Stat. (Hons.) Admission Test: 2011

Multiple-Choice Test

Time: 2 hours

Group A

Each of the following questions has exactly one correct option and you have to identify it.

1. The limit

$$\lim_{x\to 0} \frac{1-\cos(\sin^2 \alpha x)}{x}$$

(A) equals 1; (B) equals α; (C) equals 0; (D) does not exist.

2. The set of all x for which the function $f(x) = \log_{\frac{1}{2}}(x^2 - 2x - 3)$ is defined and monotone increasing is

(A) $(-\infty, 1)$; (B) $(-\infty, -1)$; (C) $(1, \infty)$; (D) $(3, \infty)$.

3. Let a line with slope of $60°$ be drawn through the focus F of the parabola $y^2 = 8(x+2)$. If the two points of intersection of the line with the parabola are A and B and the perpendicular bisector of the chord AB intersects the x-axis at the point P, then the length of the segment PF is

(A) $\frac{16}{3}$; (B) $\frac{8}{3}$; (C) $\frac{16\sqrt{3}}{3}$; (D) $8\sqrt{3}$.

4. Suppose z is a complex number with $|z| < 1$. Let $w = \frac{1+z}{1-z}$. Which of the following is always true?

[Re(w) is the real part of w and Im(w) is the imaginary part of w.]

(A) $\text{Re}(w) > 0$; (B) $\text{Im}(w) \geq 0$; (C) $|w| \leq 1$; (D) $|w| \geq 1$.

5. Among all the factors of $4^6 6^7 21^8$, the number of factors which are perfect squares is

(A) 240; (B) 360; (C) 400; (D) 640.

6. Let A be the set $\{1, 2, ..., 20\}$. Fix two disjoint subsets S_1 and S_2 of A, each with exactly three elements. How many 3-element subsets of A are there, which have exactly one element common with S_1 and at least one element common with S_2?

(A) 51; (B) 102; (C) 135; (D) 153.

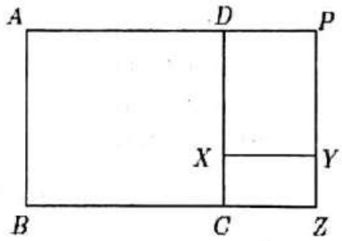
7. In how many ways can 3 couples sit around a round table such that men and women alternate and none of the couples sit together?

(A) 1; (B) 2; (C) $\frac{5!}{3}$; (D) none of these.

8. The equation $x^3 + y^3 = xy(1 + xy)$ represents

(A) two parabolas intersecting at two points;
(B) two parabolas touching at one point;
(C) two non-intersecting hyperbolas;
(D) one parabola passing through the origin.

9. Consider the diagram below where $ABZP$ is a rectangle and $ABCD$ and $CXYZ$ are squares whose areas add up to 1.



The maximum possible area of the rectangle $ABZP$ is

(A) $1 + \frac{1}{\sqrt{2}}$; (B) $2 - \sqrt{2}$; (C) $1 + \sqrt{2}$; (D) $\frac{1+\sqrt{2}}{2}$.

10. Let A be the set $\{1, 2, ..., 6\}$. How many functions f from A to A are there such that the range of f has exactly 5 elements?

(A) 240; (B) 720; (C) 1800; (D) 10800.

11. Let C_1, C_2 and C_3 be three circles lying in the same quadrant, each touching both the axes. Suppose also that C_1 touches C_2 and C_2 touches C_3. If the area of the smallest circle is 1 unit, then the area of the largest circle is

(A) $\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^4$; (B) $(1 + \sqrt{2})^2$; (C) $(2 + \sqrt{2})^2$; (D) 2^4.

12. Let $[x]$ denote the largest integer less than or equal to x. Then $\int_0^{n^{\frac{1}{k}}} [x^k + n]\, dx$ equals

(A) $n^2 + \sum_{i=1}^{n} i^{\frac{1}{k}}$; (B) $2n^{\frac{1+k}{k}} - \sum_{i=1}^{n} i^{\frac{1}{k}}$; (C) $2n^{\frac{1+k}{k}} - \sum_{i=1}^{n-1} i^{\frac{1}{k}}$; (D) None of these.

13. Consider the function

$$f(x) = \begin{cases} x(x-1)e^{2x}, & \text{if } x \leq 0, \\ x(1-x)e^{-2x}, & \text{if } x > 0. \end{cases}$$

Then $f(x)$ attains its maximum value at

(A) $1 - \frac{1}{\sqrt{2}}$; (B) $1 + \frac{1}{\sqrt{2}}$; (C) $-\frac{1}{\sqrt{2}}$; (D) $\frac{1}{\sqrt{2}}$.

14. Consider the function $f(x) = \frac{x^n(1-x)^n}{n!}$, where $n \geq 1$ is a fixed integer. Let $f^{(k)}$ denote the k-th derivative of f. Which of the following is true for all $k \geq 1$?

(A) $f^{(k)}(0)$ and $f^{(k)}(1)$ are integers;
(B) $f^{(k)}(0)$ is an integer, but not $f^{(k)}(1)$;
(C) $f^{(k)}(1)$ is an integer, but not $f^{(k)}(0)$;
(D) Neither $f^{(k)}(1)$ nor $f^{(k)}(0)$ is an integer.

15. The number of solutions of the equation $\sin(\cos\theta) = \theta, \ -1 \leq \theta \leq 1$, is

(A) 0; (B) 1; (C) 2; (D) 3.

16. Suppose $ABCD$ is a parallelogram and P, Q are points on the sides BC and CD respectively, such that $PB = \alpha BC$ and $DQ = \beta DC$. If the area of the triangles ΔABP, ΔADQ and ΔPCQ are 15, 15 and 4 respectively, then the area of ΔAPQ is

(A) 14; (B) 15; (C) 16; (D) 18.

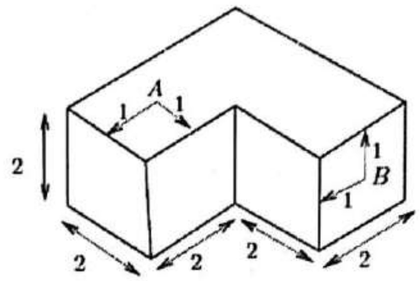
17. Consider an equilateral triangle ABC with side 2.1 cm. You want to place a number of smaller equilateral triangles, each with side 1 cm., over the triangle ABC, so that the triangle ABC is fully covered. What is the minimum number of smaller triangles that you need?

(A) 4; (B) 5; (C) 6; (D) 7.

18. A regular tetrahedron has all its vertices on a sphere of radius R. Then the length of each edge of the tetrahedron is

(A) $\frac{\sqrt{2}}{\sqrt{3}}R$; (B) $\frac{\sqrt{3}}{2}R$; (C) $\frac{4}{3}R$; (D) $\frac{2\sqrt{2}}{\sqrt{3}}R$.

19. Consider the L-shaped brick in the diagram below.



If an ant starts from A, find the minimum distance it has to travel along the surface to reach B.

(A) $\sqrt{5}$; (B) $2\sqrt{5}$; (C) $\frac{3\sqrt{5}}{2}$; (D) $3\sqrt{5}$.

20. Let $f(x) = (\tan x)^{\frac{3}{2}} - 3\tan x + \sqrt{\tan x}$. Consider the three integrals

$$I_1 = \int_0^1 f(x)dx, \quad I_2 = \int_{0.3}^{1.3} f(x)dx, \quad I_3 = \int_{0.5}^{1.5} f(x)dx.$$

Then,

(A) $I_1 > I_2 > I_3$; (B) $I_2 > I_1 > I_3$; (C) $I_3 > I_1 > I_2$; (D) $I_1 > I_3 > I_2$.

Group B

Each of the following questions has either one or two correct options and you have to identify all the correct options.

21. Let $a < b < c$ be three real numbers and w denote a complex cube root of unity. If $(a + bw + cw^2)^3 + (a + bw^2 + cw)^3 = 0$, then which of the following must be true?

(A) $a + b + c = 0$; (B) $abc = 0$; (C) $ab + bc + ca = 0$; (D) $b = \frac{c+a}{2}$.

22. Suppose f is continuously differentiable up to 3rd order and satisfies

$$\int_0^1 \{6f(x) + x^3 f'''(x)\}\, dx = f''(1).$$

Which of the following must be true?

(A) $f(1) = 0$; (B) $f'(1) = 2f(1)$; (C) $f'(1) = f(1)$; (D) $f'(1) = 0$.

23. Let $f(x) = ax^2 + bx + c$ for some real numbers a, b and c. If $f(-5) \geq 10$, $f(-3) < 6$ and $f(2) \geq 7$, then which of the following cannot be true?

(A) $f(3) = 6$; (B) $f(3) \geq 16$; (C) $f(4) = 5$; (D) $f(4) \geq 6.2$.

24. Consider the sequence x_n, $n \geq 1$, defined as:

$$x_n = \left(1 + \frac{2}{n^a}\right)^{-n^b} n^c$$

where a, b and c are real numbers. Which of the following are true?

(A) if $b < a$, $x_n \to 0$ as $n \to \infty$;
(B) if $a < b$, $x_n \to 0$ as $n \to \infty$;
(C) if $a = b$ and $c > 0$, $x_n \to \infty$ as $n \to \infty$;
(D) if $a = b$ and $c < 0$, $x_n \to \infty$ as $n \to \infty$.

25. The value of $n^{\frac{1}{n}} - 1$

(A) tends to 0 as $n \to \infty$;
(B) is greater than $\frac{\log n}{n}$ for all $n \geq 3$;
(C) is greater than $\log n$ for all $n \geq 3$;
(D) is greater than $\frac{1}{\sqrt{n}}$ for all $n \geq 3$.

26. If the complex numbers $1 + i$ and $5 - 3i$ represent two diagonally opposite vertices of a square, which of the following complex numbers can represent another vertex of the square?

(A) $5 + 2i$; (B) $3 + 2\sqrt{2} - i$; (C) $1 - 3i$; (D) $4 + 2\sqrt{2} + 2\sqrt{2}i$.

27. Suppose x and y are two positive numbers satisfying the equation $x^y = y^x$. Which of the following are true?

(A) For all $x > 1$, there always exist a $y > x$ such that the above equation holds;
(B) For all $x > e$, there is always a $y > x$ such that the above equation holds;
(C) For all $1 < x < e$, there is always a $y > x$ such that the above equation holds;
(D) If $x < 1$, then y must be equal to x.

28. Consider 6 points on the plane no three of which are collinear. An edge is a straight line joining one point to another. Two points are called connected if one can go from one point to another through edges. Suppose you are only told how many edges are there in total, but not where they are. Which of the following are true?

(A) If you are told that there are 7 edges, you cannot be sure that all pairs of points are connected;
(B) If you are told that there are 9 edges, you can always ensure that all pairs of points are connected;
(C) If you are told that there are 12 edges, you cannot be sure that all pairs of points are connected;
(D) If you are told that there are 13 edges, you can always ensure that all pairs of points are connected.

29. Suppose $f : \mathbb{R} \to \mathbb{R}$ is differentiable everywhere. Which of the following conditions imply that $|f(x)|$ is also differentiable?

(A) $f(x) = 0$ whenever $f'(x) = 0$;
(B) $f'(x) = 0$ whenever $f(x) = 0$;
(C) $f'(x)$ never takes the value 0;
(D) $f(x)$ never takes the value 0.

30. Let the coordinates of the centre of a circle be $\left(-\frac{7}{10}, 2\sqrt{2}\right)$. Then the number of points (x, y) on the circle such that both x and y are rational

(A) cannot be 3 or more;
(B) at least 1, but at most 2;
(C) at least 2, but infinitely many;
(D) infinitely many.

B.Math. (Hons.) Admission Test: 2011

Multiple-Choice Test
Time: 2 hours

1. The equation of the circle of smallest radius which passes through the points $(-1,0)$ and $(0,-1)$ is:

(A) $x^2+y^2+2xy=0$;
(B) $x^2+y^2+x+y=0$;
(C) $x^2+y^2-x-y=0$;
(D) $x^2+y^2+x+y+1/4=0$.

2. The function $f(x)=x^2e^{-|x|}$ defined on the entire real line is

(A) not continuous at exactly one point;
(B) continuous everywhere but not differentiable at exactly one point;
(C) differentiable everywhere;
(D) differentiable everywhere.

3. Let c_1 and c_2 be positive real numbers. Consider the function

$$f(x)=\begin{cases} c_1x, & 0\le x<\frac{1}{3};\\ c_2(1-x), & \frac{1}{3}\le x\le 1.\end{cases}$$

If f is continuous and $\int_0^1 f(x)dx=1$, the value of c_2 is

(A) 2; (B) 1; (C) 3; (D) $\frac{1}{2}$.

4. Mr. Gala purchased 10 plots of land in the year 2007, all plots costing the same amount. He made a profit of 25 percent on each of the 6 plots which he sold in 2008. He had a loss of 25 percent on each of the remaining plots when he sold them in 2009. If he ended with a total profit of Rs. 2 crores in this project, his total purchase price was

(A) 8 crores; (B) 40 crores; (C) 10 crores; (D) 20 crores.

5. Let $f(x)=x\sin(1/x)$ for $x>0$. Then

(A) f is unbounded;
(B) f is bounded, but $\lim_{x\to\infty} f(x)$ does not exist;
(C) $\lim_{x\to\infty} f(x)=1$;
(D) $\lim_{x\to\infty} f(x)=0$.

6. Let

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Then

(A) f is discontinuous at $x = 0$; (B) f is differentiable, and f' is continuous; (C) f is not differentiable at $x = 0$; (D) f is differentiable at every x, but f' is discontinuous at $x = 0$.

7. Let $P(x) = a_0 + a_1x + \cdots + a_nx^n$ be a polynomial of degree n with real coefficients a_i. Suppose that there is a constant $C > 0$ and an integer $k \geq 0$ such that $|P(x)| < Cx^k$ for all $x > 0$. Then

(A) n must be equal to k; (B) the given information is not sufficient to say anything about n;
(C) $n \geq k$; (D) $n \leq k$.

8. Let f be a strictly increasing function on $\mathbb{R}$, that is, $f(x) < f(y)$ whenever $x < y$. Then

(A) f is a continuous function; (B) f is a bounded function;
(C) f is an unbounded function; (D) the given information is not sufficient to say anything about continuity or boundedness of f.

9. The minimum value of $x^2 + y^2$ subject to $x + y = 1$ is

(A) 0; (B) 1/2; (C) 1/4; (D) 1.

10. The number 2532645918 is divisible by

(A) 3 but not 11; (B) 11 but not 3; (C) both 3 and 11; (D) neither 3 nor 11.

11. Let $p > 3$ be a prime number. Which of the following is always false?

(A) $p + 2$ is a prime number;
(B) $p + 4$ is a prime number;
(C) both $p + 2$ and $p + 4$ are prime numbers;
(D) neither $p + 2$ nor $p + 4$ are prime numbers.

12. By a diagonal of a convex polygon, we mean a line segment between any two non-consecutive vertices. The number of diagonals of a convex polygon of 8 sides is:

(A) 15; (B) 20; (C) 28; (D) 35.

13. The coefficients of three consecutive terms in the expansion of $(1 + t)^n$ are $120, 210, 252$. Then, n must be

(A) 10; (B) 12; (C) 14; (D) 16.

14. If $2\sec(2\alpha) = \tan(\beta) + \cot(\beta)$, then $\alpha + \beta$ can have the value

(A) $\pi/2$; (B) $\pi/3$; (C) $\pi/4$; (D) 0.

15. Consider the unit circle $x^2 + y^2 = 1$. The locus of a point P such that the tangents PA, PB at the points A, B respectively of the circle are so that $\angle AOB = 60^o$, where O is the origin, is

(A) a circle of radius $\frac{2}{\sqrt{3}}$ with centre O;
(B) a circle of radius $\sqrt{3}$ with centre O;
(C) a circle of radius 2 with centre O;
(D) a pair of straight lines.

16. Let x, y be integers. Consider the two statements: (I) $10x + y$ is divisible by 7, and, (II) $x + 5y$ is divisible by 7. Then

(A) (I) implies (II) but not conversely;
(B) (II) implies (I) but not conversely;
(C) the two statements are equivalent;
(D) neither statement implies the other.

17. The number of solutions of the equation $6m + 15n = 8$ in integers m and n are

(A) zero;
(B) one;
(C) more than one but finitely many;
(D) infinitely many.

18. Let A, B be real numbers both greater than 0. The graph of the function $f(x) = Bx^5 + 2Ax + Asin(2x)$ passes through the two points $P = (-1, 2)$ and $Q = (0, 1)$ for

(A) finitely many values of A and infinitely many values of B;
(B) infinitely many values of A and infinitely many values of B;
(C) no values of A or B;
(D) none of the above.

19. A triangle in the plane has area 1. Then its perimeter (=sum of the lengths of its three sides) p must satisfy

(A) $p < 1$; (B) $p < 2$; (C) $p > 2$; (D) $p = 2$.

20. A sequence $\{a_n\}$ is defined by $a_1 = 1$ and the inductive formula $a_{n+1} = \sqrt{1 + a_n^\delta}$ where δ is a real number greater than 0. If this sequence converges to a finite limit then δ must be

(A) > 0; (B) > 2; (C) < 2; (D) $= 2$.

21. Let a, b, c be three nonzero real numbers. If $f(x) = ax^2 + bx + c$ has equal roots, then a, b, c are in

(A) arithmetic progression;
(B) geometric progression;
(C) harmonic progression;
(D) none of the above.

22. Let a, b, c be real numbers such that $3b > a^2$. Then the function $g : \mathbb{R} \to \mathbb{R}$ given by $g(x) = x^3 + ax^2 + bx + c$ is

(A) one-one and onto;
(B) onto but not one-one;
(C) one-one but not onto;
(D) neither one-one nor onto.

23. Express the polynomial $f(x) = (2 + x)^n$ as $f(x) = c_0 + c_1x + c_2x^2 + \cdots + c_nx^n$, where n is a positive integer. If $\sum_{j=0}^{n} c_j = 81$, then the largest coefficient c_j of f is

(A) 64; (B) 16; (C) 24; (D) 32.

24. Let l, m, n be any three positive integers such that $l^2 + m^2 = n^2$. Then,

(A) 3 always divides mn;
(B) 3 always divides lm;
(C) 3 always divides ln;
(D) 3 does not divide lmn.

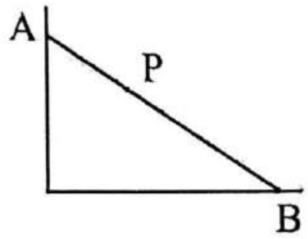
25. Let $a_1 = 10$, $a_2 = 20$ and define $a_{n+1} = a_{n-1} - \frac{4}{a_n}$ for $n > 1$. The smallest k for which $a_k = 0$

(A) does not exist;
(B) is 200;
(C) is 50;
(D) is 52.

B.Stat. (Hons.) & B.Math. (Hons.) Admission Test: 2012

Multiple-Choice Test
Time: 2 hours

1. A rod AB of length 3 rests on a wall as follows:



P is a point on AB such that $AP : PB = 1 : 2$. If the rod slides along the wall, then the locus of P lies on

(A) $2x + y + xy = 2$

(B) $4x^2 + y^2 = 4$

(C) $4x^2 + xy + y^2 = 4$

(D) $x^2 + y^2 - x - 2y = 0$.

2. Consider the equation $x^2 + y^2 = 2007$. How many solutions (x, y) exist such that x and y are positive integers?

(A) None

(B) Exactly two

(C) More than two but finitely many

(D) Infinitely many.

3. Consider the functions $f_1(x) = x$, $f_2(x) = 2 + \log_e x$, $x > 0$ (where e is the base of natural logarithm). The graphs of the functions intersect

(A) once in $(0, 1)$ and never in $(1, \infty)$

(B) once in $(0, 1)$ and once in (e^2, ∞)

(C) once in $(0, 1)$ and once in (e, e^2)

(D) more than twice in $(0, \infty)$.

4. Consider the sequence

$$u_n = \sum_{r=1}^{n} \frac{r}{2^r}, n \geq 1.$$

Then the limit of u_n as $n \to \infty$ is

(A) 1 (B) 2 (C) e (D) 1/2.

5. Suppose that z is any complex number which is not equal to any of $\{3, 3\omega, 3\omega^2\}$ where ω is a complex cube root of unity. Then

$$\frac{1}{z-3} + \frac{1}{z-3\omega} + \frac{1}{z-3\omega^2}$$

equals

(A) $\frac{3z^2+3z}{(z-3)^3}$ (B) $\frac{3z^2+3\omega z}{z^3-27}$ (C) $\frac{3z^2}{z^3-3z^2+9z-27}$ (D) $\frac{3z^2}{z^3-27}$.

6. Consider all functions $f : \{1,2,3,4\} \to \{1,2,3,4\}$ which are one-one, onto and satisfy the following property:

$$\text{if } f(k) \text{ is odd then } f(k+1) \text{ is even, } k = 1, 2, 3.$$

The number of such functions is

(A) 4 (B) 8 (C) 12 (D) 16.

7. A function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} e^{-\frac{1}{x}}, & x > 0 \\ 0 & x \leq 0. \end{cases}$$

Then

(A) f is not continuous

(B) f is differentiable but f' is not continuous

(C) f is continuous but $f'(0)$ does not exist

(D) f is differentiable and f' is continuous.

8. The last digit of $9! + 3^{9966}$ is

(A) 3 (B) 9 (C) 7 (D) 1.

9. Consider the function

$$f(x) = \frac{2x^2 + 3x + 1}{2x - 1}, \; 2 \le x \le 3.$$

Then

(A) maximum of f is attained inside the interval $(2, 3)$

(B) minimum of f is $28/5$

(C) maximum of f is $28/5$

(D) f is a decreasing function in $(2, 3)$.

10. A particle P moves in the plane in such a way that the angle between the two tangents drawn from P to the curve $y^2 = 4ax$ is always $90°$. The locus of P is

(A) a parabola (B) a circle (C) an ellipse (D) a straight line.

11. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = |x^2 - 1|, \; x \in \mathbb{R}.$$

Then

(A) f has a local minima at $x = \pm 1$ but no local maximum

(B) f has a local maximum at $x = 0$ but no local minima

(C) f has a local minima at $x = \pm 1$ and a local maximum at $x = 0$

(D) none of the above is true.

12. The number of triples (a, b, c) of positive integers satisfying

$$2^a - 5^b 7^c = 1$$

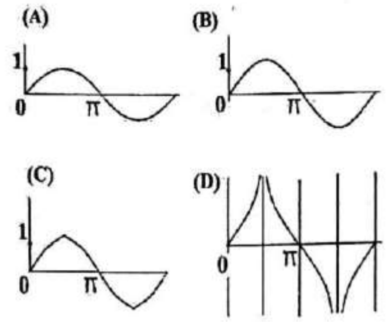
is

(A) infinite (B) 2 (C) 1 (D) 0.

13. Let a be a fixed real number greater than -1. The locus of $z \in \mathbb{C}$ satisfying $|z - ia| = Im(z) + 1$ is

(A) parabola (B) ellipse (C) hyperbola (D) not a conic.

14. Which of the following is closest to the graph of $\tan(\sin x), x > 0$?



15. Consider the function $f : \mathbb{R} \setminus \{1\} \to \mathbb{R} \setminus \{2\}$ given by

$$f(x) = \frac{2x}{x-1}.$$

Then

(A) f is one-one but not onto

(B) f is onto but not one-one

(C) f is neither one-one nor onto

(D) f is both one-one and onto.

16. Consider a real valued continuous function f satisfying $f(x+1) = f(x)$ for all $x \in \mathbb{R}$. Let

$$g(t) = \int_0^t f(x)\, dx, \quad t \in \mathbb{R}.$$

Define $h(t) = \lim_{n\to\infty} \frac{g(t+n)}{n}$, provided the limit exists. Then

(A) $h(t)$ is defined only for $t = 0$

(B) $h(t)$ is defined only when t is an integer

(C) $h(t)$ is defined for all $t \in \mathbb{R}$ and is independent of t

(D) none of the above is true.

17. Consider the sequence $a_1 = 24^{1/3}$, $a_{n+1} = (a_n + 24)^{1/3}, n \geq 1$. Then the integer part of a_{100} equals

(A) 2 (B) 10 (C) 100 (D) 24.

18. Let $x, y \in (-2, 2)$ and $xy = -1$. Then the minimum value of

$$\frac{4}{4-x^2} + \frac{9}{9-y^2}$$

is

(A) 8/5 (B) 12/5 (C) 12/7 (D) 15/7.

19. What is the limit of

$$\left(1 + \frac{1}{n^2+n}\right)^{n^2+\sqrt{n}}$$

as $n \to \infty$?

(A) e (B) 1 (C) 0 (D) ∞.

20. Consider the function $f(x) = x^4 + x^2 + x - 1, x \in (-\infty, \infty)$. The function

(A) is zero at $x = -1$, but is increasing near $x = -1$

(B) has a zero in $(-\infty, -1)$

(C) has two zeros in $(-1, 0)$

(D) has exactly one local minimum in $(-1, 0)$.

21. Consider a sequence of 10 A's and 8 B's placed in a row. By a run we mean one or more letters of the same type placed side by side. Here is an arrangement of 10 A's and 8 B's which contains 4 runs of A and 4 runs of B:

$$AAABBABBBAABAAAABB$$

In how many ways can 10 A's and 8 B's be arranged in a row so that there are 4 runs of A and 4 runs of B?

(A) $2\binom{9}{3}\binom{7}{3}$ (B) $\binom{9}{3}\binom{7}{3}$ (C) $\binom{10}{4}\binom{8}{4}$ (D) $\binom{10}{5}\binom{8}{5}$.

22. Suppose $n \geq 2$ is a fixed positive integer and

$$f(x) = x^n|x|, x \in \mathbb{R}.$$

Then

(A) f is differentiable everywhere only when n is even

(B) f is differentiable everywhere except at 0 if n is odd

(C) f is differentiable everywhere

(D) none of the above is true.

23. The line $2x + 3y - k = 0$ with $k > 0$ cuts the x axis and y axis at points A and B respectively. Then the equation of the circle having AB as diameter is

(A) $x^2 + y^2 - \frac{k}{2}x - \frac{k}{3}y = k^2$

(B) $x^2 + y^2 - \frac{k}{3}x - \frac{k}{2}y = k^2$

(C) $x^2 + y^2 - \frac{k}{2}x - \frac{k}{3}y = 0$

(D) $x^2 + y^2 - \frac{k}{3}x - \frac{k}{2}y = 0$.

24. Let $\alpha > 0$ and consider the sequence

$$x_n = \frac{(\alpha+1)^n + (\alpha-1)^n}{(2\alpha)^n}, n = 1, 2, \ldots.$$

Then $\lim_{n\to\infty} x_n$ is

(A) 0 for any $\alpha > 0$

(B) 1 for any $\alpha > 0$

(C) 0 or 1 depending on what $\alpha > 0$ is

(D) 0, 1 or ∞ depending on what $\alpha > 0$ is.

25. If $0 < \theta < \pi/2$ then

(A) $\theta < \sin\theta$

(B) $\cos(\sin\theta) < \cos\theta$

(C) $\sin(\cos\theta) < \cos(\sin\theta)$

(D) $\cos\theta < \sin(\cos\theta)$.

26. Consider a cardboard box in the shape of a prism as shown below. The length of the prism is 5. The two triangular faces ABC and $A'B'C'$ are congruent and isosceles with side lengths 2,2,3. The shortest distance between B and A' along the surface of the prism is

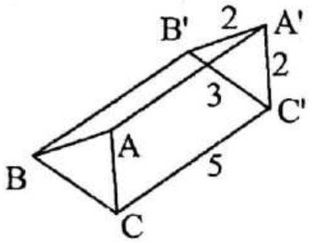
(A) $\sqrt{29}$ (B) $\sqrt{28}$ (C) $\sqrt{29-\sqrt{5}}$ (D) $\sqrt{29-\sqrt{3}}$

27. Assume the following inequalities for positive integer k:

$$\frac{1}{2\sqrt{k+1}} < \sqrt{k+1} - \sqrt{k} < \frac{1}{2\sqrt{k}}.$$

The integer part of

$$\sum_{k=2}^{9999} \frac{1}{\sqrt{k}}$$



equals

(A) 198 (B) 197 (C) 196 (D) 195.

28. Consider the sets defined by the inequalities

$$A = \{(x, y) \in \mathbb{R}^2 : x^4 + y^2 \leq 1\},\ B = \{(x, y) \in \mathbb{R}^2 : x^6 + y^4 \leq 1\}.$$

Then

(A) $B \subseteq A$

(B) $A \subseteq B$

(C) each of the sets $A - B, B - A$ and $A \cap B$ is non-empty

(D) none of the above is true.

29. The number

$$\left(\frac{2^{10}}{11}\right)^{11}$$

is

(A) strictly larger than $\binom{10}{1}^2\binom{10}{2}^2\binom{10}{3}^2\binom{10}{4}^2\binom{10}{5}$

(B) strictly larger than $\binom{10}{1}^2\binom{10}{2}^2\binom{10}{3}^2\binom{10}{4}^2$ but strictly smaller than $\binom{10}{1}^2\binom{10}{2}^2\binom{10}{3}^2\binom{10}{4}^2\binom{10}{5}$

(C) less than or equal to $\binom{10}{1}^2\binom{10}{2}^2\binom{10}{3}^2\binom{10}{4}^2$

(D) equal to $\binom{10}{1}^2\binom{10}{2}^2\binom{10}{3}^2\binom{10}{4}^2\binom{10}{5}$.

30. If the roots of the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ are in geometric progression then

(A) $b^2 = ac$ (B) $a^2 = b$ (C) $a^2b^2 = c^2$ (D) $c^2 = a^2d$.

B.Stat. (Hons.) & B.Math. (Hons.) Admission Test: 2013

Multiple-Choice Test

Time: 2 hours

1. Let $i = \sqrt{-1}$ and $S = \{i + i^2 + \cdots + i^n : n \geq 1\}$. The number of distinct *real* numbers in the set S is

(A) 1 (B) 2 (C) 3 (D) infinite.

2. From a square of unit length, pieces from the corners are removed to form a regular octagon. Then, the value of the area removed is

(A) $1/2$ (B) $1/\sqrt{2}$ (C) $\sqrt{2} - 1$ (D) $(\sqrt{2} - 1)^2$.

3. We define the *dual* of a line $y = mx + c$ to be the point $(m, -c)$. Consider a set of n non-vertical lines, $n > 3$, passing through the point $(1, 1)$. Then the duals of these lines will always

(A) be the same (B) lie on a circle (C) lie on a line

(D) form the vertices of a polygon with positive area.

4. Suppose α, β and γ are three real numbers satisfying $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$. Then the value of $\cos(\alpha - \beta)$ is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$.

5. The value of $\lim_{x \to \infty} (3^x + 7^x)^{\frac{1}{x}}$ is

(A) 7 (B) 10 (C) e^7 (D) ∞.

6. The distance between the two foci of the rectangular hyperbola defined by $xy = 2$ is

(A) 2 (B) $2\sqrt{2}$ (C) 4 (D) $4\sqrt{2}$.

7. Suppose f is a differentiable and increasing function on $[0, 1]$ such that $f(0) < 0 < f(1)$. Let $F(t) = \int_0^t f(x)dx$. Then

(A) F is an increasing function on $[0, 1]$

(B) F is a decreasing function on $[0, 1]$

(C) F has a unique maximum in the open interval $(0, 1)$

(D) F has a unique minimum in the open interval $(0, 1)$.

8. In an isosceles triangle ΔABC, the angle $\angle ABC = 120^o$. Then the ratio of two sides $AC : AB$ is

(A) 2:1 (B) 3: 1 (C) $\sqrt{2} : 1$ (D) $\sqrt{3} : 1$.

9. Let x, y, z be positive real numbers. If the equation

$$x^2 + y^2 + z^2 = (xy + yz + zx)\sin\theta$$

has a solution for θ, then x, y and z must satisfy

(A) $x = y = z$ (B) $x^2 + y^2 + z^2 \leq 1$

(C) $xy + yz + zx = 1$ (D) $0 < x,\ y,\ z \leq 1$.

10. Suppose $\sin\theta = \frac{4}{5}$ and $\sec\alpha = \frac{7}{4}$ where $0 \leq \theta \leq \frac{\pi}{2}$ and $-\frac{\pi}{2} \leq \alpha \leq 0$. Then $\sin(\theta + \alpha)$ is

(A) $\dfrac{3\sqrt{33}}{35}$ (B) $-\dfrac{3\sqrt{33}}{35}$ (C) $\dfrac{16 + 3\sqrt{33}}{35}$ (D) $\dfrac{16 - 3\sqrt{33}}{35}$.

11. Let $i = \sqrt{-1}$ and $z_1, z_2, \ldots$ be a sequence of complex numbers defined by $z_1 = i$ and $z_{n+1} = z_n^2 + i$ for $n \geq 1$. Then $|z_{2013} - z_1|$ is

(A) 0 (B) 1 (C) 2 (D) $\sqrt{5}$.

12. The last digit of the number $2^{100} + 5^{100} + 8^{100}$ is

(A) 1 (B) 3 (C) 5 (D) 7.

13. The maximum value of $|x - 1|$ subject to the condition $|x^2 - 4| \leq 5$ is

(A) 2 (B) 3 (C) 4 (D) 5.

14. Which of the following is correct?

(A) $ex \leq e^x$ for all x. (B) $ex < e^x$ for $x < 1$ and $ex \geq e^x$ for $x \geq 1$.

(C) $ex \geq e^x$ for all x. (D) $ex < e^x$ for $x > 1$ and $ex \geq e^x$ for $x \leq 1$.

15. The area of a regular polygon of 12 sides that can be inscribed in the circle $x^2 + y^2 - 6x + 5 = 0$ is

(A) 6 units (B) 9 units (C) 12 units (D) 15 units.

16. Let $f(x) = \sqrt{\log_2 x - 1} + \frac{1}{2}\log_{\frac{1}{2}} x^3 + 2$. The set of all real values of x for which the function $f(x)$ is defined and $f(x) < 0$ is

(A) $x > 2$ (B) $x > 3$ (C) $x > e$ (D) $x > 4$.

17. Let a be the largest integer *strictly* smaller than $\frac{7}{8}b$ where b is also an integer. Consider the following inequalities:

(1) $\frac{7}{8}b - a \leq 1$ (2) $\frac{7}{8}b - a \geq \frac{1}{8}$

and find which of the following is correct.

(A) Only (1) is correct. (B) Only (2) is correct.

(C) Both (1) and (2) are correct. (D) None of them is correct.

18. The value of $\lim_{x \to -\infty} \sum_{k=1}^{1000} \frac{x^k}{k!}$ is

(A) $-\infty$ (B) ∞ (C) 0 (D) e^{-1}.

19. For integers m and n, let $f_{m,n}$ denote the function from the set of integers to itself, defined by

$$f_{m,n}(x) = mx + n.$$

Let $\mathcal{F}$ be the set of all such functions,

$$\mathcal{F} = \{f_{m,n} : m, n \text{ integers }\}.$$

Call an element $f \in \mathcal{F}$ *invertible* if there exists an element $g \in \mathcal{F}$ such that $g(f(x)) = f(g(x)) = x$ for all integers x. Then which of the following is true?

(A) Every element of $\mathcal{F}$ is invertible.

(B) $\mathcal{F}$ has infinitely many invertible and infinitely many non-invertible elements.

(C) $\mathcal{F}$ has finitely many invertible elements.

(D) No element of $\mathcal{F}$ is invertible.

20. Consider six players P_1, P_2, P_3, P_4, P_5 and P_6. A team consists of two players. (Thus, there are 15 distinct teams.) Two teams play a match exactly once if there is no common player. For example, team $\{P_1, P_2\}$ can not play with $\{P_2, P_3\}$ but will play with $\{P_4, P_5\}$. Then the total number of possible matches is

(A) 36 (B) 40 (C) 45 (D) 54.

21. The minimum value of $f(\theta) = 9\cos^2\theta + 16\sec^2\theta$ is

(A) 25 (B) 24 (C) 20 (D) 16.

22. The number of 0's at the end of the integer

$$100! - 101! + \cdots - 109! + 110!$$

is

(A) 24 (B) 25 (C) 26 (D) 27.

23. We denote the largest integer less than or equal to z by $[z]$. Consider the identity

$$(1+x)(10+x)(10^2+x)\cdots(10^{10}+x) = 10^a + 10^b x + a_2x^2 + \cdots + a_{11}x^{11}.$$

Then

(A) $[a] > [b]$ (B) $[a] = [b]$ and $a > b$

(C) $[a] < [b]$ (D) $[a] = [b]$ and $a < b$.

24. The number of four tuples (a, b, c, d) of *positive integers* satisfying all three equations

$$\begin{aligned} a^3 &= b^2 \\ c^3 &= d^2 \\ c - a &= 64 \end{aligned}$$

is

(A) 0 (B) 1 (C) 2 (D) 4.

25. The number of real roots of $e^x = x^2$ is

(A) 0 (B) 1 (C) 2 (D) 3.

26. Suppose $\alpha_1, \alpha_2, \alpha_3$ and α_4 are the roots of the equation $x^4 + x^2 + 1 = 0$. Then the value of $\alpha_1^4 + \alpha_2^4 + \alpha_3^4 + \alpha_4^4$ is

(A) -2 (B) 0 (C) 2 (D) 4.

27. *Among the four time instances* given in the options below, when is the angle between the minute hand and the hour hand the smallest?

(A) 5:25 p.m. (B) 5:26 p.m. (C) 5:29 p.m. (D) 5:30 p.m.

28. Suppose all roots of the polynomial $P(x) = a_{10}x^{10} + a_9x^9 + \cdots + a_1x + a_0$ are real and smaller than 1. Then, for any such polynomial, the function

$$f(x) = a_{10}\frac{e^{10x}}{10} + a_9\frac{e^{9x}}{9} + \cdots + a_1e^x + a_0x, \ x > 0$$

(A) is increasing (B) is either increasing or decreasing

(C) is decreasing (D) is neither increasing nor decreasing.

29. Consider a quadrilateral $ABCD$ in the XY-plane with all of its angles less than 180^o. Let P be an arbitrary point in the plane and consider the six triangles each of which is formed by the point P and two of the points A, B, C, D. Then the total area of these six triangles is minimum when the point P is

(A) outside the quadrilateral

(B) one of the vertices of the quadrilateral

(C) intersection of the diagonals of the quadrilateral

(D) none of the points given in (A), (B) or (C).

30. The graph of the equation $x^3 + 3x^2y + 3xy^2 + y^3 - x^2 + y^2 = 0$ comprises

(A) one point

(B) union of a line and a parabola

(C) one line

(D) union of a line and a hyperbola.

B.Stat. (Hons.) & B.Math. (Hons.) Admission Test: 2014

Multiple-Choice Test
Time: 2 hours

1. The system of inequalities

$$a - b^2 \geq \frac{1}{4},\ b - c^2 \geq \frac{1}{4},\ c - d^2 \geq \frac{1}{4},\ d - a^2 \geq \frac{1}{4} \quad \text{has}$$

(A) no solutions
(B) exactly one solution
(C) exactly two solutions
(D) infinitely many solutions.

2. Let $\log_{12} 18 = a$. Then $\log_{24} 16$ is equal to

(A) $\frac{8-4a}{5-a}$ (B) $\frac{1}{3+a}$ (C) $\frac{4a-1}{2+3a}$ (D) $\frac{8-4a}{5+a}$.

3. The number of solutions of the equation $\tan x + \sec x = 2\cos x$, where $0 \leq x \leq \pi$, is

(A) 0 (B) 1 (C) 2 (D) 3.

4. Using only the digits 2, 3 and 9, how many six digit numbers can be formed which are divisible by 6?

(A) 41 (B) 80 (C) 81 (D) 161

5. What is the value of the following integral?

$$\int_{\frac{1}{2014}}^{2014} \frac{\tan^{-1} x}{x}\, dx$$

(A) $\frac{\pi}{4} \log 2014$ (B) $\frac{\pi}{2} \log 2014$ (C) $\pi \log 2014$ (D) $\frac{1}{2} \log 2014$

6. A light ray travelling along the line $y = 1$, is reflected by a mirror placed along the line $x = 2y$. The reflected ray travels along the line

(A) $4x - 3y = 5$ (B) $3x - 4y = 2$ (C) $x - y = 1$ (D) $2x - 3y = 1$.

7. For a real number x, let $[x]$ denote the greatest integer less than or equal to x. Then the number of real solutions of $\left|2x - [x]\right| = 4$ is

(A) 1 (B) 2 (C) 3 (D) 4.

8. What is the ratio of the areas of the regular pentagons inscribed inside and circumscribed around a given circle?

(A) $\cos 36°$ (B) $\cos^2 36°$ (C) $\cos^2 54°$ (D) $\cos^2 72°$

9. Let z_1, z_2 be nonzero complex numbers satisfying $|z_1 + z_2| = |z_1 - z_2|$. The circumcentre of the triangle with the points z_1, z_2, and the origin as its vertices is given by

(A) $\frac{1}{2}(z_1 - z_2)$ (B) $\frac{1}{3}(z_1 + z_2)$ (C) $\frac{1}{2}(z_1 + z_2)$ (D) $\frac{1}{3}(z_1 - z_2)$.

10. In how many ways can 20 identical chocolates be distributed among 8 students so that each student gets at least one chocolate and exactly two students get at least two chocolates each?

(A) 308 (B) 364 (C) 616 (D) $\binom{8}{2}\binom{17}{7}$

11. Two vertices of a square lie on a circle of radius r, and the other two vertices lie on a tangent to this circle. Then, each side of the square is

(A) $\frac{3r}{2}$ (B) $\frac{4r}{3}$ (C) $\frac{6r}{5}$ (D) $\frac{8r}{5}$.

12. Let P be the set of all numbers obtained by multiplying five distinct integers between 1 and 100. What is the largest integer n such that 2^n divides at least one element of P?

(A) 8 (B) 20 (C) 24 (D) 25

13. Consider the function $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are real numbers with $a > 0$. If f is strictly increasing, then the function $g(x) = f'(x) - f''(x) + f'''(x)$ is

(A) zero for some $x \in \mathbb{R}$ (B) positive for all $x \in \mathbb{R}$
(C) negative for all $x \in \mathbb{R}$ (D) strictly increasing.

14. Let A be the set of all points (h, k) such that the area of the triangle formed by (h, k), $(5, 6)$ and $(3, 2)$ is 12 square units. What is the least possible length of a line segment joining $(0, 0)$ to a point in A?

(A) $\frac{4}{\sqrt{5}}$ (B) $\frac{8}{\sqrt{5}}$ (C) $\frac{12}{\sqrt{5}}$ (D) $\frac{16}{\sqrt{5}}$

15. Let $P = \{abc : a, b, c \text{ positive integers}, a^2 + b^2 = c^2, \text{ and } 3 \text{ divides } c\}$. What is the largest integer n such that 3^n divides every element of P?

(A) 1 (B) 2 (C) 3 (D) 4

16. Let $A_0 = \emptyset$ (the empty set). For each $i = 1, 2, 3, \ldots$, define the set $A_i = A_{i-1} \cup \{A_{i-1}\}$. The set A_3 is

(A) $\emptyset$ (B) $\{\emptyset\}$ (C) $\{\emptyset, \{\emptyset\}\}$ (D) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

17. Let $f(x) = \dfrac{1}{x-2}$ · The graphs of the functions f and f^{-1} intersect at

(A) $(1+\sqrt{2}, 1+\sqrt{2})$ and $(1-\sqrt{2}, 1-\sqrt{2})$

(B) $(1+\sqrt{2}, 1+\sqrt{2})$ and $(\sqrt{2}, -1-\frac{1}{\sqrt{2}})$

(C) $(1-\sqrt{2}, 1-\sqrt{2})$ and $(-\sqrt{2}, -1+\frac{1}{\sqrt{2}})$

(D) $(\sqrt{2}, -1-\frac{1}{\sqrt{2}})$ and $(-\sqrt{2}, -1+\frac{1}{\sqrt{2}})$

18. Let N be a number such that whenever you take N consecutive positive integers, at least one of them is coprime to 374. What is the smallest possible value of N?

(A) 4 (B) 5 (C) 6 (D) 7

19. Let $A_1, A_2, \ldots, A_{18}$ be the vertices of a regular polygon with 18 sides. How many of the triangles $\triangle A_iA_jA_k$, $1 \le i < j < k \le 18$, are isosceles but not equilateral?

(A) 63 (B) 70 (C) 126 (D) 144

20. The limit $\lim\limits_{x \to 0} \dfrac{\sin^{\alpha} x}{x}$ exists *only when*

(A) $\alpha \ge 1$ (B) $\alpha = 1$

(C) $|\alpha| \le 1$ (D) α is a positive integer.

21. Consider the region $R = \{(x, y) : x^2 + y^2 \le 100, \sin(x+y) > 0\}$. What is the area of R?

(A) 25π (B) 50π (C) 50 (D) $100\pi - 50$

22. Consider a cyclic trapezium whose circumcentre is on one of the sides. If the ratio of the two parallel sides is 1 : 4, what is the ratio of the sum of the two oblique sides to the longer parallel side?

(A) $\sqrt{3} : \sqrt{2}$ (B) $3 : 2$ (C) $\sqrt{2} : 1$ (D) $\sqrt{5} : \sqrt{3}$

23. Consider the function $f(x) = \left\{\log_e\left(\dfrac{4+\sqrt{2x}}{x}\right)\right\}^2$ for $x > 0$. Then,

(A) f decreases upto some point and increases after that

(B) f increases upto some point and decreases after that

(C) f increases initially, then decreases and then again increases

(D) f decreases initially, then increases and then again decreases.

24. What is the number of ordered triplets (a, b, c), where a, b, c are positive integers (not necessarily distinct), such that $abc = 1000$?

(A) 64 (B) 100 (C) 200 (D) 560

25. Let $f : (0, \infty) \to (0, \infty)$ be a function differentiable at 3, and satisfying $f(3) = 3f'(3) > 0$. Then the limit

$$\lim_{x\to\infty} \left(\frac{f\left(3 + \frac{3}{x}\right)}{f(3)} \right)^x$$

(A) exists and is equal to 3
(B) exists and is equal to e
(C) exists and is always equal to $f(3)$
(D) need not always exist.

26. Let z be a non-zero complex number such that $\left| z - \frac{1}{z} \right| = 2$. What is the maximum value of $|z|$?

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) $1 + \sqrt{2}$.

27. The minimum value of

$$\left| \sin x + \cos x + \tan x + \operatorname{cosec} x + \sec x + \cot x \right| \quad \text{is}$$

(A) 0 (B) $2\sqrt{2} - 1$ (C) $2\sqrt{2} + 1$ (D) 6

28. For any function $f : X \to Y$ and any subset A of Y, define

$$f^{-1}(A) = \{x \in X : f(x) \in A\}.$$

Let A^c denote the complement of A in Y. For subsets A_1, A_2 of Y, consider the following statements:

(i) $f^{-1}(A_1^c \cap A_2^c) = (f^{-1}(A_1))^c \cup (f^{-1}(A_2))^c$
(ii) If $f^{-1}(A_1) = f^{-1}(A_2)$ then $A_1 = A_2$.

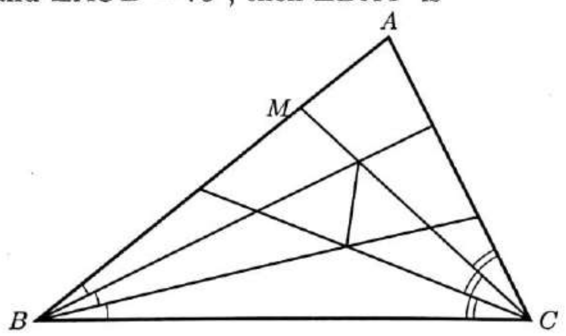
Then,

(A) both (i) and (ii) are always true
(B) (i) is always true, but (ii) may not always be true
(C) (ii) is always true, but (i) may not always be true
(D) neither (i) nor (ii) is always true.

29. Let f be a function such that $f''(x)$ exists, and $f''(x) > 0$ for all $x \in [a, b]$. For any point $c \in [a, b]$, let $A(c)$ denote the area of the region bounded by $y = f(x)$, the tangent to the graph of f at $x = c$ and the lines $x = a$ and $x = b$. Then

(A) $A(c)$ attains its minimum at $c = \frac{1}{2}(a+b)$ for any such f
(B) $A(c)$ attains its maximum at $c = \frac{1}{2}(a+b)$ for any such f
(C) $A(c)$ attains its minimum at both $c = a$ and $c = b$ for any such f
(D) the points c where $A(c)$ attains its minimum depend on f.

30. In $\triangle ABC$, the lines BP, BQ trisect $\angle ABC$ and the lines CM, CN trisect $\angle ACB$. Let BP and CM intersect at X and BQ and CN intersect at Y. If $\angle ABC = 45°$ and $\angle ACB = 75°$, then $\angle BXY$ is



(A) 45°   (B) $47\frac{1}{2}°$ (C) 50°   (D) 55°

B.Stat. (Hons.) & B.Math. (Hons.) Admission Test: 2015

Multiple-Choice Test
Time: 2 hours

1. Let $\mathbb{C}$ denote the set of complex numbers and $S = \{z \in \mathbb{C} \mid \overline{z} = z^2\}$, where $\overline{z}$ denotes the complex conjugate of z. Then S has:

   (A) two elements (B) three elements
   (C) four elements (D) six elements.

2. The number of one-to-one functions from a set with 3 elements to a set with 6 elements is

   (A) 20 (B) 120 (C) 216 (D) 720.

3. Two sides of a triangle are of length 2 cm and 3 cm. Then, the maximum possible area (in cm^2) of the triangle is:

   (A) 2 (B) 3 (C) 4 (D) 6.

4. The number of factors of $2^{15} \times 3^{10} \times 5^6$ which are either perfect squares or perfect cubes (or both) is:

   (A) 252 (B) 256 (C) 260 (D) 264.

5. The minimum value of the function $f(x) = x^2 + 4x + \frac{4}{x} + \frac{1}{x^2}$ where $x > 0$, is

   (A) 9.5 (B) 10 (C) 15 (D) 20.

6. The minimum area of the triangle formed by any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the coordinate axes is

   (A) ab (B) $\frac{a^2+b^2}{2}$
   (C) $\frac{(a+b)^2}{4}$ (D) $\frac{a^2+ab+b^2}{3}$.

7. The angle between the hyperbolas $xy = 1$ and $x^2 - y^2 = 1$ (at their point of intersection) is

   (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$.

8. The population of a city doubles in 50 years. In how many years will it triple, under the assumption that the rate of increase is proportional to the number of inhabitants?

(A) 75 years
(B) 100 years
(C) 50 $\log_2(3)$ years
(D) 50 $\log_e(\frac{3}{2})$ years.

9. We define a set $\{f_1, f_2, \dots, f_n\}$ of polynomials to be a linearly dependent set if there exist real numbers $c_1, c_2, \dots, c_n$, not all zero, such that $c_1 f_1(x) + \dots + c_n f_n(x) = 0$ for all real numbers x. Which of the following is a linearly dependent set?

(A) $\{x, x^2, x^3\}$
(B) $\{x^2 - x, 2x, x^2 + 3x\}$
(C) $\{x, 2x^3, 5x^2\}$
(D) $\{x^2 - 1, 2x + 5, x^2 + 1\}$.

10. A set of numbers S is said to be multiplicatively closed if $ab \in S$ whenever both $a \in S$ and $b \in S$. Let $i = \sqrt{-1}$ and ω be a non-real cube root of unity. Let

$$S_1 = \{a + b\,i \mid a, b \text{ are integers}\} \text{ and} S_2 = \{a + b\,\omega \mid a, b \text{ are integers}\}.$$

Which one of the following statements is true?

(A) Both S_1 and S_2 are multiplicatively closed.
(B) S_1 is multiplicatively closed but S_2 is not.
(C) S_2 is multiplicatively closed but S_1 is not.
(D) Neither S_1 nor S_2 is multiplicatively closed.

11. When the product of four consecutive odd positive integers is divided by 5, the set of remainder(s) is

(A) $\{0\}$ (B) $\{0, 4\}$ (C) $\{0, 2, 4\}$ (D) $\{0, 2, 3, 4\}$.

12. Consider the equation $x^2 + y^2 = 2015$ where $x \geq 0$ and $y \geq 0$. How many solutions (x, y) exist such that both x and y are non-negative integers?

(A) None
(B) Exactly one
(C) Exactly two
(D) Greater than two.

13. Let P be a point on the circle $x^2 + y^2 - 9 = 0$ above the x-axis, and Q be a point on the circle $x^2 + y^2 - 20x + 96 = 0$ below the x-axis such that the line joining P and Q is tangent to both these circles. Then the length of PQ is

(A) $5\sqrt{2}$ units (B) $5\sqrt{3}$ units (C) $5\sqrt{6}$ units (D) $6\sqrt{5}$ units.

14. Let $S = \{(x, y) | \; x, y$ are positive integers$\}$ viewed as a subset of the plane. For every point P in S, let d_P denote the sum of the distances from P to the point $(8, 0)$ and the point $(0, 12)$. The number of points P in S such that d_P is the least among all elements in the set S, is

(A) 0 (B) 3 (C) 8 (D) 1.

15. Let A, B and C be the angles of a triangle. Suppose that $\tan A$ and $\tan B$ are the roots of the equation $x^2 - 8x + 5 = 0$. Then $\cos^2 C - 8\cos C \sin C + 5\sin^2 C$ equals

(A) -1 (B) 0 (C) 1 (D) 2.

16. Let $A = \{a_1, a_2, \ldots, a_{10}\}$ and $B = \{1, 2\}$. The number of functions $f : A \to B$ for which the sum $f(a_1) + \cdots + f(a_{10})$ is an even number, is

(A) 128 (B) 256 (C) 512 (D) 768.

17. Define $\operatorname{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$

Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = (x+1)\operatorname{sgn}(x^2 - 1)$ where $\mathbb{R}$ is the set of real numbers. Then the number of discontinuities of f is:

(A) 0 (B) 1 (C) 2 (D) 3.

18. Suppose X is a subset of real numbers and $f : X \to X$ is a bijection (that is, one-to-one and onto) satisfying $f(x) > x$ for all $x \in X$. Then X cannot be:

(A) the set of integers (B) the set of positive integers
(C) the set of positive real numbers (D) the set of real numbers

19. The set of real numbers x satisfying the inequality

$$\frac{4x^2}{\left(1 - \sqrt{1+2x}\right)^2} < 2x + 9$$

is:

(A) $[-\frac{1}{2}, 0) \cup (0, \frac{45}{8})$ (B) $[-\frac{1}{2}, 0) \cup (\frac{45}{8}, \infty)$
(C) $[-\frac{1}{2}, 0) \cup (0, \infty)$ (D) $(0, \frac{45}{8}) \cup (\frac{45}{8}, \infty)$.

20. Let $ABCDEFGHIJ$ be a 10-digit number, where all the digits are distinct. Further, $A > B > C$, $A + B + C = 9$, $D > E > F > G$ are consecutive odd numbers and $H > I > J$ are consecutive even numbers. Then A is

(A) 8 (B) 7 (C) 6 (D) 5.

21. Let $A = \{(a, b, c) : a, b, c \text{ are prime numbers}, a < b < c, a + b + c = 30\}$. The number of elements in A is

(A) 0 (B) 1 (C) 2 (D) 3.

22. Let $f(x) = \begin{cases} \dfrac{|\sin x|}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

Then $\int_{-1}^{1} f(x)dx$ is equal to

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{8}$ (C) $-\frac{\pi}{4}$ (D) 0.

23. Let $f : (0,2) \cup (4,6) \to \mathbb{R}$ be a differentiable function. Suppose also that $f'(x) = 1$ for all $x \in (0,2) \cup (4,6)$. Which of the following is ALWAYS true?
(A) f is increasing
(B) f is one-to-one
(C) $f(x) = x$ for all $x \in (0,2) \cup (4,6)$
(D) $f(5.5) - f(4.5) = f(1.5) - f(0.5)$

24. Consider 50 evenly placed points on a circle with centre at the origin and radius R such that the arc length between any two consecutive points is the same. The complex numbers represented by these points form
(A) an arithmetic progression with common difference $\left(\cos(\frac{2\pi}{50}) + i\sin(\frac{2\pi}{50})\right)$
(B) an arithmetic progression with common difference $\left(R\cos(\frac{2\pi}{50}) + iR\sin(\frac{2\pi}{50})\right)$
(C) a geometric progression with common ratio $\left(\cos(\frac{2\pi}{50}) + i\sin(\frac{2\pi}{50})\right)$
(D) a geometric progression with common ratio $\left(R\cos(\frac{2\pi}{50}) + iR\sin(\frac{2\pi}{50})\right)$

25. Given two complex numbers z, w with unit modulus (i.e., $|z| = |w| = 1$), which of the following statements will ALWAYS be correct?
(A) $|z+w| < \sqrt{2}$ and $|z-w| < \sqrt{2}$
(B) $|z+w| \leq \sqrt{2}$ and $|z-w| \geq \sqrt{2}$
(C) $|z+w| \geq \sqrt{2}$ or $|z-w| \geq \sqrt{2}$
(D) $|z+w| < \sqrt{2}$ or $|z-w| < \sqrt{2}$

26. The number of points in the region $\{(x,y) : x^2 + y^2 \leq 4\}$ satisfying $\tan^4 x + \cot^4 x + 1 = 3\sin^2 y$ is

(A) 1 (B) 2 (C) 3 (D) 4.

27. If all the roots of the equation $x^4 - 8x^3 + ax^2 + bx + 16 = 0$ are positive, then $a + b$
(A) must be -8
(B) can be any number strictly between -16 and -8
(C) must be -16
(D) can be any number strictly between -8 and 0

28. Let O denote the origin and A, B denote respectively the points $(-10, 0)$ and $(7, 0)$ on the x-axis. For how many points P on the y-axis will the lengths of all the line segments PA, PO and PB be positive integers?

(A) 0 (B) 2 (C) 4 (D) infinite.

29. Let $G(x) = \int_{-x^3}^{x^3} f(t)dt$, where x is any real number and f is a continuous function such that $f(t) > 1$ for all real t. Then,
(A) $G'(0) = 0$ and G has a local maximum or minimum at $x = 0$.
(B) For any real number c, the equation $G(x) = c$ has a unique solution.
(C) There exists a real number c such that $G(x) = c$ has no solution.
(D) There exists a real number c such that $G(x) = c$ has more than one solution.

30. There are $2n + 1$ real numbers having the property that the sum of any n of them is less than the sum of the remaining $n + 1$. Then,
(A) all the numbers must be positive
(B) all the numbers must be negative
(C) all the numbers must be equal
(D) such a system of real numbers cannot exist.

B.Stat. (Hons.) & B.Math. (Hons.) Admission Test: 2016

Multiple-Choice Test
Time: 2 hours

1. The largest integer n for which $n+5$ divides n^5+5 is

(A) 3115 (B) 3120 (C) 3125 (D) 3130.

2. Let p, q be primes and a, b be integers. If pa is divided by q, then the remainder is 1. If qb is divided by p, then also the remainder is 1. The remainder when $pa+qb$ is divided by pq is

(A) 1 (B) 0 (C) -1 (D) 2.

3. The polynomial x^7+x^2+1 is divisible by

(A) $x^5-x^4+x^2-x+1$ (B) x^5+x^4+1
(C) $x^5+x^4+x^2+x+1$ (D) $x^5-x^4+x^2+x+1$.

4. Let $\alpha > 0$. If the equation $p(x) = x^3 - 9x^2 + 26x - \alpha$ has three positive real roots, then α must satisfy

(A) $\alpha \leq 27$ (B) $\alpha > 81$ (C) $27 < \alpha \leq 54$ (D) $54 < \alpha \leq 81$.

5. The largest integer which is less than or equal to $(2+\sqrt{3})^4$ is

(A) 192 (B) 193 (C) 194 (D) 195.

6. Consider a circle of unit radius and a chord of that circle that has unit length. The area of the largest triangle that can be inscribed in the circle with that chord as its base is

(A) $\frac{1}{2}+\frac{\sqrt{2}}{4}$ (B) $\frac{1}{2}+\frac{\sqrt{2}}{2}$ (C) $\frac{1}{2}+\frac{\sqrt{3}}{4}$ (D) $\frac{1}{2}+\frac{\sqrt{3}}{2}$.

7. Let $z_1 = 3+4i$. If z_2 is a complex number such that $|z_2| = 2$, then the greatest and the least values of $|z_1 - z_2|$ are respectively

(A) 7 and 3 (B) 5 and 1
(C) 9 and 5 (D) $4+\sqrt{7}$ and $\sqrt{7}$.

8. Consider two distinct arithmetic progressions (AP) each of which has a positive first term and a positive common difference. Let S_n and T_n be the sums of the first n terms of these AP. Then $\lim_{n\to\infty} \frac{S_n}{T_n}$ equals

(A) ∞ or 0 depending on which AP has larger first term
(B) ∞ or 0 depending on which AP has larger common difference
(C) the ratio of the first terms of the AP
(D) the ratio of the common differences of the AP.

9. Let $f(x) = \max\{\cos x, x^2\}$, $0 < x < \frac{\pi}{2}$. If x_0 is the solution of the equation

$$\cos x = x^2 \text{ in } (0, \frac{\pi}{2}),$$

then
(A) f is continuous only at x_0
(B) f is not continuous at x_0
(C) f is continuous everywhere and differentiable only at x_0
(D) f is differentiable everywhere except at x_0.

10. The set of all real numbers in $(-2, 2)$ satisfying

$$2^{|x|} - |2^{x-1} - 1| = 2^{x-1} + 1$$

is
(A) $\{-1, 1\}$
(B) $\{-1\} \cup [1, 2)$
(C) $(-2, -1] \cup [1, 2)$
(D) $(-2, -1] \cup \{1\}$.

11. Let $S(k)$ denote the set of all one-to-one and onto functions from $\{1, 2, 3, \ldots, k\}$ to itself. Let p, q be positive integers. Let $S(p, q)$ be the set of all τ in $S(p + q)$ such that $\tau(1) < \tau(2) < \cdots < \tau(p)$ and $\tau(p + 1) < \tau(p + 2) < \cdots < \tau(p + q)$. The number of elements in the set $S(13, 29)$ is

(A) 377 (B) $(42)!$ (C) $\binom{42}{13}$ (D) $\frac{42!}{29!}$.

12. Suppose that both the roots of the equation $x^2 + ax + 2016 = 0$ are positive even integers. The number of possible values of a is

(A) 6 (B) 12 (C) 18 (D) 24.

13. Let $b \neq 0$ be a fixed real number. Consider the family of parabolas given by the equations

$$y^2 = 4ax + b, \quad \text{where } a \in \mathbb{R}.$$

The locus of the points on the parabolas at which the tangents to the parabolas make 45° angle with the x-axis is
(A) a straight line
(B) a pair of straight lines
(C) a parabola
(D) a hyperbola.

14. Consider the curve represented by the equation

$$ax^2 + 2bxy + cy^2 + d = 0$$

in the plane, where $a > 0$, $c > 0$ and $ac > b^2$. Suppose that the normals to the curve drawn at 5 distinct points on the curve all pass through the origin. Then

(A) $a = c$ and $b > 0$
(B) $a \neq c$ and $b = 0$
(C) $a \neq c$ and $b < 0$
(D) None of the above.

15. Let P be a 12-sided regular polygon and T be an equilateral triangle with its incircle having radius 1. If the area of P is the same as the area of T, then the length of the side of P is

(A) $\sqrt{\sqrt{3}\cot 15^\circ}$
(B) $\sqrt{\sqrt{3}\tan 15^\circ}$
(C) $\sqrt{3\sqrt{2}\tan 15^\circ}$
(D) $\sqrt{3\sqrt{2}\cot 15^\circ}$.

16. Let ABC be a right-angled triangle with $\angle ABC = 90^\circ$. Let P be the midpoint of BC and Q be a point on AB. Suppose that the length of BC is $2x$, $\angle ACQ = \alpha$, and $\angle APQ = \beta$. Then the length of AQ is

(A) $\dfrac{3x}{2\cot\alpha - \cot\beta}$
(B) $\dfrac{2x}{3\cot\alpha - 2\cot\beta}$
(C) $\dfrac{3x}{\cot\alpha - 2\cot\beta}$
(D) $\dfrac{2x}{2\cot\alpha - 3\cot\beta}$.

17. Let $[x]$ denote the greatest integer less than or equal to x. The value of the integral

$$\int_1^n [x]^{x-[x]}\,dx$$

is equal to

(A) $1 + \dfrac{2^3}{\log_e 2} - \dfrac{2^2}{\log_e 2} + \dfrac{3^4}{\log_e 3} - \dfrac{3^3}{\log_e 3} + \cdots + \dfrac{(n-1)^n}{\log_e(n-1)} - \dfrac{(n-1)^{n-1}}{\log_e(n-1)}$

(B) $1 + \dfrac{1}{\log_e 2} + \dfrac{2}{\log_e 3} + \cdots + \dfrac{n-2}{\log_e(n-1)}$

(C) $\dfrac{1}{2} + \dfrac{2^2}{3} + \cdots + \dfrac{n^{n+1}}{n+1}$

(D) $\dfrac{2^3-1}{3} + \dfrac{3^4-2^3}{4} + \cdots + \dfrac{n^{n+1}-(n-1)^n}{n+1}$.

18. Let $\alpha > 0, \beta \geq 0$ and $f : \mathbb{R} \to \mathbb{R}$ be continuous at 0 with $f(0) = \beta$. If $g(x) = |x|^\alpha f(x)$ is differentiable at 0, then

(A) $\alpha = 1$ and $\beta = 1$
(B) $0 < \alpha < 1$ and $\beta = 0$
(C) $\alpha \geq 1$ and $\beta = 0$
(D) $\alpha > 0$ and $\beta > 0$.

19. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable and strictly decreasing function such that $f(0) = 1$ and $f(1) = 0$. For $x \in \mathbb{R}$, let

$$F(x) = \int_0^x (t-2)f(t)\, dt.$$

Then

(A) F is strictly increasing in $[0, 3]$
(B) F has a unique minimum in $(0, 3)$ but has no maximum in $(0, 3)$
(C) F has a unique maximum in $(0, 3)$ but has no minimum in $(0, 3)$
(D) F has a unique maximum and a unique minimum in $(0, 3)$.

20. Let $f : \mathbb{R} \to \mathbb{R}$ be a nonzero function such that $\lim_{x\to\infty} \dfrac{f(xy)}{x^3}$ exists for all $y > 0$. Let $g(y) = \lim_{x\to\infty} \dfrac{f(xy)}{x^3}$. If $g(1) = 1$, then for all $y > 0$

(A) $g(y) = 1$ (B) $g(y) = y$
(C) $g(y) = y^2$ (D) $g(y) = y^3$.

21. Let $D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$. Then the maximum number of points in D such that the distance between any pair of points is at least 1 will be

(A) 5 (B) 6 (C) 7 (D) 8.

22. The number of 3-digit numbers abc such that we can construct an isosceles triangle with sides a, b and c is

(A) 153 (B) 163 (C) 165 (D) 183.

23. The function

$$f(x) = x^{1/2} - 3x^{1/3} + 2x^{1/4}, \quad x \geq 0$$

(A) has more than two zeros
(B) is always nonnegative
(C) is negative for $0 < x < 1$
(D) is one-to-one and onto.

24. Let $X = \{a + \sqrt{-5}\, b : a, b \in \mathbb{Z}\}$. An element $x \in X$ is called special if there exists $y \in X$ such that $xy = 1$. The number of special elements in X is

(A) 2 (B) 4 (C) 6 (D) 8.

25. For a set X, let $P(X)$ denote the set of all subsets of X. Consider the following statements.

(I) $P(A) \cap P(B) = P(A \cap B)$.
(II) $P(A) \cup P(B) = P(A \cup B)$.
(III) $P(A) = P(B) \Longrightarrow A = B$.
(IV) $P(\emptyset) = \emptyset$.

Then

(A) All the statements are true
(B) (I), (II), (III) are true and (IV) is false
(C) (I), (III) are true and (II), (IV) are false
(D) (II), (III), (IV) are true and (I) is false.

26. Let a, b, c be real numbers such that $a + b + c < 0$. Suppose that the equation $ax^2 + bx + c = 0$ has imaginary roots. Then

(A) $a < 0$ and $c < 0$
(B) $a < 0$ and $c > 0$
(C) $a > 0$ and $c < 0$
(D) $a > 0$ and $c > 0$.

27. For $\alpha \in (0, \frac{3}{2})$, define $x_n = (n+1)^\alpha - n^\alpha$. Then $\lim_{n\to\infty} x_n$ is

(A) 1 for all α
(B) 1 or 0 depending on the value of α
(C) 1 or ∞ depending on the value of α
(D) 1, 0, or ∞ depending on the value of α.

28. Let f be a continuous strictly increasing function from $[0, \infty)$ onto $[0, \infty)$ and $g = f^{-1}$ (that is, $f(x) = y$ if and only if $g(y) = x$). Let $a, b > 0$ and $a \neq b$. Then

$$\int_0^a f(x)\, dx + \int_0^b g(y)\, dy$$

is

(A) greater than or equal to ab
(B) less than ab
(C) always equal to ab
(D) always equal to $\dfrac{af(a) + bg(b)}{2}$.

29. The sum of the series $\sum_{n=1}^{\infty} n^2 e^{-n}$ is

(A) $\dfrac{e^2}{(e-1)^3}$
(B) $\dfrac{e^2 + e}{(e-1)^3}$
(C) $\dfrac{3}{2}$
(D) ∞.

30. Let $f : [0, 1] \to [-1, 1]$ be a non-zero function such that

$$f(2x) = 3f(x), \quad x \in [0, \tfrac{1}{2}].$$

Then $\lim_{x \to 0+} f(x)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 0.

KEY TO MULTIPLE-CHOICE QUESTIONS

1. C	2. D	3. A	4. B
5. C	6. B	7. B	8. C
9. A	10. A	11. A	12. B
13. A	14. C	15. B	16. A
17. B	18. C	19. B	20. B
21. C	22. A	23. C	24. C
25. B	26. C	27. D	28. C
29. B	30. D	31. D	32. D
33. A	34. C	35. D	36. B
37. C	38. B	39. A	40. D
41. D	42. D	43. D	44. D
45. C	46. C	47. C	48. D
49. A	50. C	51. A	52. D
53. B	54. A	55. A	56. D
57. B	58. B	59. B	60. D
61. A	62. D	63. B	64. D
65. C	66. A	67. C	68. B
69. C	70. B	71. D	72. D
73. A	74. A	75. D	76. B
77. D	78. C	79. A	80. C
81. A	82. C	83. C	84. D
85. C	86. B	87. D	88. A
89. D	90. B	91. B	92. A
93. C	94. C	95. C	96. D
97. A	98. B	99. B	100. C

101. A	102. B	103. B	104. B
105. A	106. C	107. B	108. A
109. A	110. A	111. B	112. A
113. C	114. D	115. D	116. B
117. A	118. A	119. C	120. C
121. D	122. D	123. D	124. B
125. B	126. B	127. B	128. C
129. B	130. D	131. D	132. C
133. A	134. B	135. A	136. D
137. B	138. D	139. B	140. D
141. A	142. D	143. A	144. D
145. A	146. D	147. B	148. B
149. C	150. B	151. A	152. C
153. C	154. A	155. B	156. C
157. A	158. A	159. D	160. A
161. B	162. C	163. A	164. A
165. C	166. B	167. D	168. C
169. A	170. B	171. D	172. C
173. B	174. B	175. A	176. A
177. C	178. B	179. D	180. A
181. A	182. D	183. C	184. B
185. C	186. B	187. A	188. D
189. B	190. A	191. B	192. A
193. B	194. B	195. A	196. D
197. A	198. C	199. A	200. D

201. A	202. A	203. D	204. D
205. B	206. C	207. C	208. B
209. B	210. C	211. C	212. C
213. B	214. A	215. D	216. A
217. B	218. D	219. C	220. B
221. A	222. D	223. B	224. B
225. B	226. C	227. D	228. C
229. A	230. B	231. B	232. D
233. A	234. B	235. B	236. D
237. C	238. C	239. D	240. B
241. C	242. B	243. B	244. A
245. C	246. A	247. B	248. D
249. A	250. B	251. A	252. D
253. B	254. D	255. B	256. B
257. D	258. A	259. B	260. C
261. A	262. B	263. A	264. B
265. D	266. A	267. D	268. A
269. A	270. B	271. B	272. B
273. B	274. A	275. B	276. B
277. C	278. C	279. C	280. D
281. C	282. B	283. C	284. B
285. D	286. D	287. D	288. B
289. B	290. C	291. B	292. D
293. C	294. D	295. A	296. B
297. A	298. A	299. D	300. D

301. A	302. B	303. B	304. D
305. C	306. B	307. B	308. A
309. B	310. D	311. C	312. A
313. B	314. A	315. D	316. C
317. B	318. D	319. A	320. D
321. C	322. C	323. A	324. B
325. D	326. D	327. B	328. D
329. C	330. C	331. D	332. B
333. B	334. A	335. D	336. D
337. C	338. B	339. B	340. A
341. B	342. B	343. D	344. C
345. D	346. A	347. B	348. B
349. A	350. D	351. C	352. B
353. A	354. A	355. C	356. B
357. B	358. C	359. C	360. B
361. B	362. A	363. B	364. D
365. D	366. D	367. C	368. B
369. C	370. C	371. C	372. D
373. A	374. C	375. D	376. C
377. D	378. D	379. A	380. B
381. D	382. A	383. C	384. A
385. D	386. C	387. B	388. C
389. B	390. B	391. D	392. D
393. D	394. D	395. B	396. A
397. D	398. A	399. C	400. C

401. A	402. A	403. D	404. D
405. B	406. A	407. D	408. D
409. C	410. A	411. A	412. D
413. B	414. A	415. D	416. C
417. C	418. C	419. C	420. A
421. B	422. C	423. D	424. D
425. D	426. C	427. B	428. C
429. B	430. B	431. D	432. B
433. C	434. A	435. A	436. D
437. B	438. C	439. B	440. A
441. B	442. C	443. B	444. A
445. A	446. B	447. B	448. A
449. C	450. A	451. C	452. A
453. D	454. D	455. A	456. A
457. D	458. B	459. C	460. D
461. A	462. C	463. B	464. A
465. D	466. B	467. A	468. A
469. B	470. B	471. B	472. C
473. D	474. C	475. D	476. C
477. C	478. B	479. C	480. B
481. A	482. A	483. B	484. C
485. D	486. C	487. A	488. B
489. A	490. A	491. A	492. D
493. A	494. D	495. C	496. A
497. A	498. B	499. B	500. A

501. A	502. A	503. A	504. C
505. A	506. B	507. C	508. A
509. C	510. D	511. B	512. A
513. A	514. D	515. B	516. A
517. B	518. C	519. D	520. C
521. D	522. A	523. B	524. D
525. C	526. B	527. B	528. A
529. B	530. C	531. A	532. B
533. D	534. D	535. D	536. C
537. D	538. B	539. C	540. C
541. A	542. B	543. D	544. C
545. A	546. D	547. B	548. A
549. A	550. C	551. C	552. C
553. C	554. A	555. A	556. D
557. C	558. C	559. D	560. C
561. B	562. D	563. D	564. A
565. C	566. D	567. A	568. B
569. C	570. C	571. B	572. C
573. C	574. D	575. B	576. A
577. B	578. D	579. B	580. B
581. D	582. A	583. D	584. C
585. C	586. A	587. B	588. A
589. B	590. D	591. A	592. D
593. C	594. B	595. A	596. B
597. B	598. A	599. A	600. B

601. D	602. C	603. C	604. C
605. D	606. D	607. A	608. C
609. C	610. D	611. C	612. B
613. A	614. A	615. C	616. D
617. D	618. A	619. A	620. B
621. C	622. B	623. A	624. B
625. B	626. A	627. B	628. A
629. B	630. B	631. D	632. D
633. A	634. D	635. A	636. D
637. C	638. B	639. A	640. C
641. D	642. B	643. D	644. A
645. B	646. C	647. C	648. D
649. C	650. D	651. C	652. B
653. A	654. C	655. A	656. A
657. C	658. C	659. C	660. C
661. B	662. B	663. D	664. A
665. C	666. A	667. B	668. D
669. A	670. A	671. C	672. B
673. A	674. D	675. A	676. C
677. A	678. A	679. D	680. B
681. B	682. B	683. B	684. A
685. D	686. D	687. C	688. C
689. C	690. B	691. A	692. B
693. C	694. B	695. B	696. D
697. D	698. C	699. C	700. C

701. B	702. A	703. C	704. D
705. C	706. C	707. C	708. D
709. D	710. C	711. B	712. C
713. B	714. D	715. D	716. C
717. B	718. B	719. B	720. A
721. D	722. C	723. D	724. C
725. C	726. C	727. D	728. D
729. A	730. A	731. D	732. C
733. C	734. C	735. A	736. D
737. D	738. D	739. B	740. D
741. D	742. B	743. C	744. B
745. B	746. D	747. A	748. B
749. B	750. A	751. C	752. C
753. B	754. A	755. C	756. B
757. D	758. C	759. C	760. C
761. B	762. D	763. C	764. B
765. A	766. C	767. D	768. C
769. D	770. C	771. D	772. A
773. A	774. A	775. B	776. D
777. A	778. A	779. D	780. C
781. D	782. C	783. C	784. C
785. C	786. B	787. C	788. A
789. B	790. B	791. A	792. A
793. A	794. B	795. D	796. D
797. A	798. D	799. C	800. D

801. B	802. D	803. C	804. B
805. B	806. A	807. B	808. A
809. D	810. C	811. C	812. B
813. B	814. A	815. D	816. A
817. B	818. B	819. C	820. C
821. A	822. D	823. C	824. D
825. A	826. B	827. B	828. A
829. C	830. A	831. A	832. C
833. A	834. C	835. D	836. D
837. A	838. A	839. A	840. A
841. D	842. A	843. A	844. B
845. A	846. C	847. C	848. C
849. D	850. B	851. C	852. B
853. A	854. A	855. A	856. A
857. D	858. C		

KEY TO 2007 B.STAT. ADMISSION TEST: MULTIPLE-CHOICE

1. A	**2. A**	**3. B**	**4. B**
5. C	**6. D**	**7. D**	**8. B**
9. B	**10. B**	**11. D**	**12. A**
13. A	**14. C**	**15. A**	**16. A**
17. A	**18. C**	**19. A**	**20. D**
21. D	**22. C**	**23. A**	**24. C**
25. A	**26. C**	**27. C**	**28. B**
29. A	**30. C**		

KEY TO 2007 B.MATH. ADMISSION TEST: MULTIPLE-CHOICE

1. C	**2. D**	**3. B**	**4. D**
5. C	**6. B**	**7. A**	**8. C**
9. B	**10. B**	**11. A**	**12. A**
13. A	**14. C**	**15. D**	**16. D**
17. C	**18. A**	**19. C**	**20. D**
21. A	**22. B**	**23. D**	**24. A**
25. C			

KEY TO 2008 B.STAT. ADMISSION TEST: MULTIPLE-CHOICE

1. A	**2. C**	**3. D**	**4. C**
5. A	**6. A**	**7. C**	**8. A**
9. C	**10. B**	**11. B**	**12. B**
13. A	**14. B**	**15. B**	**16. D**
17. C	**18. D**	**19. A**	**20. D**
21. B	**22. C**	**23. D**	**24. A**
25. B	**26. D**	**27. C**	**28. B**
29. A	**30. D**		

KEY TO 2008 B.MATH. ADMISSION TEST: MULTIPLE-CHOICE

1. A	2. B	3. A	4. C
5. C	6. C	7. D	8. A
9. A	10. D	11. C	12. A
13. B	14. C	15. B	16. C
17. A	18. B	19. C	20. D
21. B	22. D	23. B	24. A
25. D	26. A	27. B	28. B
29. B	30. C		

KEY TO 2009 B.STAT. ADMISSION TEST: MULTIPLE-CHOICE

1. B	2. C	3. A	4. C
5. D	6. B	7. B	8. A
9. D	10. D	11. C	12. D
13. B	14. C	15. A	16. C
17. B	18. A	19. C	20. B
21. A, D	22. A, C	23. A	24. B, D
25. A	26. B, D	27. B, C	28. A, B
29. C, D	30. D		

KEY TO 2009 B.MATH. ADMISSION TEST: MULTIPLE-CHOICE

1. C	2. B	3. A	4. D
5. B	6. C	7. A	8. C
9. D	10. B	11. C	12. A
13. D	14. B	15. C	16. A
17. B	18. D	19. B	20. A
21. B	22. C	23. B	24. C
25. A	26. C	27. B	28. C
29. B	30. A		

KEY TO 2010 B.STAT. ADMISSION TEST: MULTIPLE-CHOICE

1. C	2. A	3. B	4. C
5. A	6. C	7. C	8. B
9. B	10. D	11. B	12. C
13. A	14. C	15. B	16. A
17. D	18. D	19. A	20. B
21. C	22. A	23. D	24. D
25. D	26. C	27. A	28. B
29. B	30. D		

KEY TO 2010 B.MATH. ADMISSION TEST: MULTIPLE-CHOICE

1. D	2. C	3. A	4. D
5. B	6. C	7. B	8. D
9. B	10. C	11. A	12. B
13. A	14. D	15. C	16. C
17. B	18. B	19. C	20. A
21. C	22. D	23. D	24. B
25. B	26. A	27. C	28. B
29. D	30. C		

KEY TO 2011 B.STAT. ADMISSION TEST: MULTIPLE-CHOICE

1. C	2. B	3. A	4. A
5. C	6. C	7. B	8. A
9. D	10. D	11. A	12. B
13. C	14. A	15. B	16. C
17. C	18. D	19. C	20. D
21. D	22. B	23. A, C	24. B, C
25. A, B	26. C	27. C, D	28. A, D
29. B, D	30. A		

KEY TO 2011 B.MATH. ADMISSION TEST: MULTIPLE-CHOICE

1. B	2. C	3. C	4. B
5. C	6. D	7. D	8. D
9. B	10. C	11. C	12. B
13. A	14. C	15. A	16. C
17. A	18. C	19. C	20. C
21. B	22. A	23. D	24. B
25. D			

KEY TO 2012 B.STAT–B.MATH. ADMISSION TEST: MULTIPLE-CHOICE

1. B	2. A	3. C	4. B
5. D	6. C	7. D	8. B
9. C	10. D	11. C	12. D
13. A	14. B	15. D	16. C
17. A	18. B	19. A	20. D
21. A	22. C	23. C	24. D
25. C	26. A	27. B	28. B
29. A	30. D		

KEY TO 2013 B.STAT–B.MATH. ADMISSION TEST: MULTIPLE-CHOICE

1. B	2. D	3. C	4. A
5. A	6. D	7. D	8. D
9. A	10. D	11. C	12. D
13. C	14. A	15. C	16. D
17. C	18. B	19. B	20. C
21. A	22. B	23. D	24. C
25. B	26. A	27. A	28. B
29. C	30. B		

KEY TO 2014 B.STAT–B.MATH. ADMISSION TEST: MULTIPLE-CHOICE

1. B	2. A	3. C	4. C
5. B	6. A	7. D	8. B
9. C	10. A	11. D	12. C
13. B	14. B	15. D	16. D
17. A	18. C	19. C	20. A
21. B	22. A	23. B	24. B
25. B	26. D	27. B	28. D
29. A	30. C		

KEY TO 2015 B.STAT–B.MATH. ADMISSION TEST: MULTIPLE-CHOICE

1. C	2. B	3. B	4. A
5. B	6. A	7. A	8. C
9. B	10. A	11. B	12. A
13. A	14. B	15. C	16. C
17. B	18. B	19. A	20. A
21. C	22. D	23. D	24. C
25. C	26. D	27. A	28. B
29. B	30. A		

KEY TO 2016 B.STAT–B.MATH. ADMISSION TEST: MULTIPLE-CHOICE

1. A	2. A	3. A	4. A
5. B	6. C	7. A	8. D
9. D	10. B	11. C	12. B
13. D	14. D	15. A	16. A
17. B	18. C	19. D	20. D
21. C	22. C	23. B	24. A
25. C	26. A	27. D	28. A
29. B	30. D		

Short-Answer Type Questions

1. A vessel contains x gallons of wine and another contains y gallons of water. From each vessel z gallons are taken out and transferred to the other. From the resulting mixture in each vessel, z gallons are again taken out and transferred to the other. If after the second transfer, the quantity of wine in each vessel remains the same as it was after the first transfer, then show that $z(x+y) = xy$.

2. Find the number of positive integers less than or equal to 6300 which are not divisible by 3, 5 and 7.

3. A troop 5 metres long starts marching. A soldier at the end of the file steps out and starts marching forward at a higher speed. On reaching the head of the column, he immediately turns around and marches back at the same speed. As soon as he reaches the end of the file, the troop stops marching, and it is found that the troop has moved by exactly 5 metres. What distance has the soldier travelled?

4. The following table gives the urban population in India and percentages of total population in rural and urban centres for the decades during 1901-81.

URBAN AND RURAL POPULATION OF INDIA: 1901-1981

Year	*Urban Population in million*	*Percentage of Total Population*	
		Rural	*Urban*
1901	25.8	89.0	11.0
1911	25.9	89.6	10.4
1921	28.0	88.7	11.3
1931	33.5	87.8	12.2
1941	44.1	85.9	14.1
1951	62.4	82.4	17.6
1961	78.9	81.7	18.3
1971	108.9	79.8	20.2
1981	162.2	76.3	23.7

Verify the following statements against the given data and classify each statement into one of the following categories: (A) True ; (B) False; (C) Does not necessarily follow from the given information.
[Note: Do not make any additional assumptions. Use only the information given.]

(i) The percent increase in urban population during 1901-81 is about 2.5 to 3 times the percent increase in total population in that period.

(ii) The density of population in urban centres has increased by 160% during 1951-81.

(iii) The largest rate of increase in urban population in a decade during 1901-1981 occurred in 1971-81.

(iv) The smallest rate of increase in urban population in a decade during 1931-1981 occurred in 1931-41.

(v) The relative degree of urbanization (i.e., change in the percentage of urban population) was highest in 1941-51 and 1971-81.

5. In a study of the disparities in the levels of living in rural areas among different states in India, estimates were obtained for the per-capita household consumption expenditure on all items per month, (denoted by PCE) in the years 1963-64 and 1973-74. They are presented in the table below.

(i) Which state has a PCE closest to the all-India PCE (a) in 1963-64? (b) in 1973-74?

(ii) Suppose the overall disparity among states is measured by the ratio of the largest to the smallest of the state PCEs in a year. Has disparity among states increased or decreased between 1963-64 and 1973-74?

(iii) By considering the ranks of the states according to PCE, separately for each year, find out which state has improved its rank most and which state has declined most between the two years.

PER CAPITA MONTHLY HOUSEHOLD CONSUMPTION EXPENDITURE IN RURAL AREAS BY STATES, 1963-64 AND 1973-74

State	PCE in 1963-64	PCE in 1973-74
Andhra Pradesh	20.91	50.69
Assam	26.28	52.01
Bihar	21.24	56.31
Gujarat	22.69	54.54
Himachal Pradesh	25.75	71.85
Jammu & Kashmir	27.99	54.14
Karnataka	20.35	52.29
Kerala	20.45	55.32
Madhya Pradesh	23.21	50.84
Maharashtra	21.75	52.91
Manipur	22.20	52.88
Orissa	19.47	42.61
Punjab & Haryana	29.11	75.09
Rajasthan	23.27	63.98
Tamil Nadu	23.52	47.68
Tripura	23.66	50.15
Uttar Pradesh	21.37	51.50
West Bengal	23.83	47.47
INDIA	22.38	55.90

6. The following table gives the distribution of the workers in a community by age, sex and type of work.

FREQUENCY DISTRIBUTION OF WORKERS BY AGE, SEX AND TYPE OF WORK

Age in Years (last birthday)	Type of Work			
	Manual		Non-manual	
	Male	Female	Male	Female
5-10	8	15	0	0
11-15	10	20	0	0
16-20	22	20	25	5
21-35	35	45	65	25
36-50	25	35	45	15
51-65	20	15	15	5
above 65	10	5	0	0

A similar table that was produced for the same community ten years ago is as follows:

FREQUENCY DISTRIBUTION OF WORKERS BY AGE AND TYPE OF WORK

Age in years (last birthday)	Type of Work	
	Manual	Non-manual
5-10	15	0
11-15	35	0
16-20	40	40
21-35	50	80
36-50	70	65
51-65	20	15
above 65	10	0

The break-up of workers by sex is not given in the old table. Assume that the male-female ratios in both the '35 and below' and 'above 35' age-groups have remained the same over the last ten years for manual as well as non-manual workers. In which of these two age-groups has the number of female manual workers increased more?

7. The following table gives the natural logarithm of the bodyweights (in Kg) for five individuals going through a weight-loss programme for five successive weeks.

Person \ Week	0	1	2	3	4	5
A	4.24	4.00	3.75	3.50	3.40	3.40
B	4.32	4.15	3.90	3.95	6.30	3.40
C	4.24	4.10	3.80	3.50	3.45	3.60
D	4.38	4.40	4.25	4.30	4.10	3.95
E	4.38	4.30	4.35	4.20	3.80	3.85

(i) For each week identify the individual for whom there are equal number of individuals with higher and lower bodyweights, respectively. Call this individual the *midperson* for that week.

(ii) Plot the logarithm of the bodyweight of the midperson for each week against the week (in plain paper).

(iii) What is your prediction for the logarithm of the bodyweight of the midperson for the sixth week? Give reasons.

(iv) Comment on any unusual observations that you find in the table.

(v) Write a brief report (in about five sentences) on how effective the weight-loss programme has been on different individuals.

8. In a club of 80 members, 10 members play none of the games Tennis, Badminton and Cricket. 30 members play exactly one of these three games and 30 members play exactly two of these games. 45 members play at least one of the games among Tennis and Badminton, whereas 18 members play both Tennis and Badminton. Determine the number of Cricket playing members.

9. Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$, $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)$, where $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n$ are real numbers. We write $\boldsymbol{x} > \boldsymbol{y}$, if for some k, $1 \leq k \leq (n-1)$, $x_1 = y_1, x_2 = y_2, \ldots, x_k = y_k$, but $x_{k+1} > y_{k+1}$. Show that for $\boldsymbol{u} = (u_1, ..., u_n)$, $\boldsymbol{v} = (v_1, ..., v_n)$, $\boldsymbol{w} = (w_1, ..., w_n)$ and $\boldsymbol{z} = (z_1, ..., z_n)$, if $\boldsymbol{u} > \boldsymbol{v}$ and $\boldsymbol{w} > \boldsymbol{z}$, then $(\boldsymbol{u} + \boldsymbol{w}) > (\boldsymbol{v} + \boldsymbol{z})$.

10. We say that a sequence $\{a_n\}$ has property P, if there exists a positive integer m such that $a_n \leq 1$ for every $n \geq m$. For each of the following sequences, determine whether it has the property P or not. [Do not use any result on limits.]

(i)

$$a_n = \begin{cases} 0.9 + \frac{200}{n} & \text{if } n \text{ is even} \\ \frac{1}{n} & \text{if } n \text{ is odd} \end{cases}$$

(ii)

$$a_n = \begin{cases} 1 + \frac{\cos \frac{n\pi}{2}}{n} & \text{if } n \text{ is even} \\ \frac{1}{n} & \text{if } n \text{ is odd.} \end{cases}$$

11. Let x and n be positive integers such that $1 + x + x^2 + \ldots + x^{n-1}$ is a prime number. Then show that n is a prime number.

12. Let

$$x_n = \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \ \cdots \ \cdot \frac{2n-1}{2n}.$$

Then show that

$$x_n \leq \frac{1}{\sqrt{3n+1}}, \quad \text{for all } n = 1, 2, 3, \ldots .$$

13. **(i)** In the identity

$$\frac{n!}{x(x+1)(x+2)\ldots(x+n)} = \sum_{k=0}^{n} \frac{A_k}{x+k},$$

prove that

$$A_k = (-1)^k \binom{n}{k}.$$

(ii) Deduce that:

$$\binom{n}{0}\frac{1}{1\cdot 2} - \binom{n}{1}\frac{1}{2\cdot 3} + \binom{n}{2}\frac{1}{3\cdot 4} - \ldots + (-1)^n \binom{n}{n}\frac{1}{(n+1)(n+2)} = \frac{1}{n+2}.$$

14. Show that

$$\frac{3}{1.2.4} + \frac{4}{2.3.5} + \frac{5}{3.4.6} + \ldots + \frac{n+2}{n(n+1)(n+3)}$$
$$= \frac{1}{6}\left[\frac{29}{6} - \frac{4}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}\right].$$

15. How many natural numbers less than 10^8 are there, whose sum of digits equals 7?

16. Suppose k, n are integers ≥ 1. Show that $(k \cdot n)!$ is divisible by $(k!)^n$.

17. If the coefficients of a quadratic equation $ax^2 + bx + c = 0, (a \neq 0)$, are all odd integers, show that the roots cannot be rational.

18. Let $D = a^2 + b^2 + c^2$, where a and b are successive positive integers and $c = ab$. Prove that $\sqrt{D}$ is an odd positive integer.

19. Prove that

$${}^nC_0 + {}^nC_3 + {}^nC_6 + \cdots + {}^nC_{3k} \leq \frac{1}{3}(2^n + 2),$$

where n is a positive integer and k is the largest integer for which $3k \leq n$.

20. Let $u_n = (3 + \sqrt{5})^n + (3 - \sqrt{5})^n$ for $n = 1, 2, \ldots$.

(i) Show that for each n, u_n is an integer.

(ii) Show that $u_{n+1} = 6u_n - 4u_{n-1}$ for all $n \geq 2$.

(iii) Use (ii) above, to show that u_n is divisible by 2^n.

21. For a natural number n, let $a_n = n^2 + 20$. If d_n denotes the greatest common divisor of a_n and a_{n+1}, then show that d_n divides 81.

22. Let $n \geq 2$ be an integer. Let m be the largest integer which is less than or equal to n, and which is a power of 2. Put l_n = the least common multiple of $1, 2, \ldots, n$. Show that l_n/m is odd, and that for every integer $k \leq n, k \neq m, l_n/k$ is even. Hence prove that

$$1 + \frac{1}{2} + \cdots + \frac{1}{n}$$

is not an integer.

23. By considering the expression $(1 + \sqrt{2})^n + (1 - \sqrt{2})^n$, where n is a positive integer, show that the integers $[(1 + \sqrt{2})^n]$ are alternatively even and odd as n takes values $1, 2, \ldots$. Here for any real number x, $[x]$ denotes the greatest integer less than or equal to x.

24. If n is a positive integer greater than 1 such that $3n + 1$ is perfect square, then show that $n + 1$ is the sum of three perfect squares.

25. Show that for every positive integer n, $\sqrt{n}$ is either an integer or an irrational number.

26. Show that $2^{2n} - 3n - 1$ is divisible by 9 for all $n \geq 1$.

27. Suppose that the roots of $x^2 + px + q = 0$ are rational numbers and p, q are integers. Then show that the roots are integers.

28. Let $f(x)$ and $g(x)$ be two quadratic polynomials all of whose coefficients are rational numbers. Suppose $f(x)$ and $g(x)$ have a common irrational root. Show that $g(x) = rf(x)$ for some rational number r.

29. Show that for every positive integer n, 7 divides $3^{2n+1} + 2^{n+2}$.

30. Show that if n is *any* odd integer greater than 1, then $n^5 - n$ is divisible by 80.

31. If k is an odd positive integer, prove that for any integer $n \geq 1$, $1^k + 2^k + \cdots + n^k$ is divisible by $\frac{n(n+1)}{2}$.

32. Show that the number $11\ldots1$ with 3^n digits is divisible by 3^n.

33. Let k be a fixed odd positive integer. Find the minimum value of x^2+y^2, where x, y are nonnegative integers and $x+y=k$.

34. Let $f(x,y)=x^2+y^2$. Consider the region, including the boundary, enclosed by $y=\frac{x}{2}$, $y=-\frac{x}{2}$ and $x=y^2+1$. Find the maximum value of $f(x,y)$ in this region.

35. (a) Prove that, for any odd integer n, n^4 when divided by 16 always leaves remainder 1.

(b) Hence or otherwise show that we cannot find integers $n_1, n_2, \ldots, n_8$ such that
$$n_1^4+n_2^4+\ldots+n_8^4=1993.$$

36. Let $a_1, a_2, \cdots, a_n$ be n numbers such that each a_i is either 1 or -1. If
$$a_1a_2a_3a_4+a_2a_3a_4a_5+\cdots+a_na_1a_2a_3=0,$$
then prove that 4 divides n.

37. Suppose p is a prime number such that $(p-1)/4$ and $(p+1)/2$ are also primes. Show that $p=13$.

38. Show that if a prime number p is divided by 30, then the remainder is either a prime or is 1.

39. Two integers m and n are called *relatively prime* if the greatest common divisor of m and n is 1. Prove that among any five consecutive positive integers there is one integer which is relatively prime to the other four integers. (*Hint*: For any two positive integers $m<n$, any common divisor has to be less than or equal to $n-m$).

40. (i) If k and l are positive integers such that k divides l, show that for every positive integer m, $1+(k+m)l$ and $1+ml$ are relatively prime.

(ii) Consider the smallest number in each of the $\binom{n}{r}$ subsets (of size r) of $S=\{1,2,\ldots,n\}$. Show that the arithmetic mean of the numbers so obtained is $\frac{n+1}{r+1}$.

41. Find the number of rational numbers m/n, where m, n are relatively prime positive integers satisfying $m<n$ and $mn=25!$.

42. Let $f(x)$ be a polynomial with integer coefficients. Suppose that there exist distinct integers a_1, a_2, a_3, a_4, such that $f(a_1)=f(a_2)=f(a_3)=f(a_4)=3$. Show that there does not exist any integer b with $f(b)=14$.

43. Show that the equation

$$x^3 + 7x - 14(n^2 + 1) = 0$$

has no integral root for any integer n.

44. Show that if $n > 2$, then $(n!)^2 > n^n$.

45. Let $J = \{0, 1, 2, 3, 4\}$. For x, y in J define $x \oplus y$ to be the remainder of the usual sum of x and y after division by 5 and $x \odot y$ to be the remainder of the usual product of x and y after division by 5. For example, $4 \oplus 3 = 2$ while $4 \odot 2 = 3$. Find x and y in J, satisfying the following equations simultaneously:

$$(3 \odot x) \oplus (2 \odot y) = 2, \quad (2 \odot x) \oplus (4 \odot y) = 1.$$

46. A *function* f from a set A into a set B is a rule which assigns to each element x in A, a unique (one and only one) element (denoted by $f(x)$) in B. A function f from A into B is called an *onto* function, if for each element y in B there is some element x in A, such that $f(x) = y$. Now suppose that $A = \{1, 2, \ldots, n\}$ and $B = \{1, 2, 3\}$. Determine the total number of onto functions from A into B.

47. For a finite set A, let $|A|$ denote the number of elements in the set A.

(a) Let F be the set of all functions

$$f : \{1, 2, \ldots, n\} \longrightarrow \{1, 2, \ldots, k\} \quad (n \geq 3,\ k \geq 2)$$

satisfying

$$f(i) \neq f(i+1) \quad \text{for every } i,\ 1 \leq i \leq n-1.$$

Show that $|F| = k(k-1)^{n-1}$.

(b) Let $c(n, k)$ denote the number of functions in F satisfying $f(n) \neq f(1)$. For $n \geq 4$, show that

$$c(n, k) = k(k-1)^{n-1} - c(n-1, k).$$

(c) Using (b) prove that for $n \geq 3$,

$$c(n, k) = (k-1)^n + (-1)^n(k-1).$$

48. Find the number of ways in which 5 different gifts can be presented to 3 children so that each child receives at least one gift.

49. x red balls, y black balls and z white balls are to be arranged in a row. Suppose that any two balls of the same colour are indistinguishable. Given that $x + y + z = 30$, show that the number of possible arrangements is the largest for $x = y = z = 10$.

50. All the permutations of the letters a, b, c, d, e are written down and arranged in alphabetical order as in a dictionary. Thus the arrangement *abcde* is in the first position and *abced* is in the second position. What is the position of the arrangement *debac*?

51. **(a)** Given m identical symbols, say H's, show that the number of ways in which you can distribute them in k boxes marked $1, 2, \ldots\ldots, k$, so that no box goes empty is $\binom{m-1}{k-1}$.

(b) In an arrangement of m H's and n T's, an uninterrupted sequence of one kind of symbol is called a *run*. (For example, the arrangement $HHHTHH$ $TTTH$ of 6 H's and 4 T's opens with an H-run of length 3, followed successively by a T-run of length 1, an H-run of length 2, a T-run of length 3 and, finally, an H-run of length 1.)
Find the number of arrangements of m H's and n T's in which there are *exactly* k H-runs. [You may use (a) above.]

52. **(i)** Find the number of all possible ordered k-tuples of non-negative integers $(n_1, n_2, \ldots, n_k)$ such that $\sum_{i=1}^{k} n_i = 100$.

(ii) Show that the number of all possible ordered 4-tuples of non-negative integers (n_1, n_2, n_3, n_4) such that $\sum_{i=1}^{4} n_i \leq 100$ is $\binom{104}{4}$.

53. Show that the number of ways one can choose a set of distinct positive integers, each smaller than or equal to 50, such that their sum is odd, is 2^{49}.

54. Let $S = \{1, 2, \ldots, n\}$. Find the number of unordered pairs $\{A, B\}$ of subsets of S such that A and B are disjoint, where A or B or both may be empty.

55. A *partition* of a set S is formed by disjoint, nonempty subsets of S whose union is S. For example, $\{\{1, 3, 5\}, \{2\}, \{4, 6\}\}$ is a partition of the set $T = \{1, 2, 3, 4, 5, 6\}$ consisting of subsets $\{1, 3, 5\}$, $\{2\}$ and $\{4, 6\}$. However, $\{\{1, 2, 3, 5\}, \{3, 4, 6\}\}$ is not a partition of T.

If there are k nonempty subsets in a partition, then it is called a partition into k classes. Let S_k^n stand for the number of different partitions of a set with n elements into k classes.

(i) Find S_2^n.

(ii) Show that $S_k^{n+1} = S_{k-1}^n + kS_k^n$.

56. Show that the number of ways in which four distinct integers can be chosen from $1, 2, \ldots, n, (n \geq 7)$ such that no two are consecutive is equal to $\binom{n-3}{4}$.

57. How many 6-letter words can be formed using the letters A, B and C so that each letter appears at least once in the word?

58. In a certain game, 30 balls of k different colours are kept inside a sealed box. You are told only the value of k, but not the number of balls of each colour. Based on this, you have to guess whether it is possible to split the balls into 10 groups of 3 each, such that in each group the three balls are of different colours. Your answer is to be a simple YES or NO. You win or lose a point according as your guess is correct or not. For what values of k, you can say NO and be sure of winning? For what values of k, you can say YES and be sure of winning? Justify your solution.

59. Consider the set of points

$$S = \{(x, y) : x, y \text{ are non-negative integers } \leq n\}.$$

Find the number of squares that can be formed with vertices belonging to S and sides parallel to the axes.

60. Consider the set S of all integers between and including 1000 and 99999. Call two integers x and y in S to be in the same equivalence class if the digits appearing in x and y are the same. For example, if $x = 1010$, $y = 1000$ and $z = 1201$, then x and y are in the same equivalence class, but y and z are not. Find the number of distinct equivalence classes that can be formed out of S.

61. Solve

$$6x^2 - 25x + 12 + \frac{25}{x} + \frac{6}{x^2} = 0.$$

62. Consider the system of equations $x + y = 2$, $ax + y = b$. Find conditions on a and b under which

(i) the system has exactly one solution;

(ii) the system has no solution;

(iii) the system has more than one solution.

63. If any one pair among the straight lines

$$ax + by = a + b,\ bx - (a + b)y = -a,\ (a + b)x - ay = b$$

intersect, then show that the three straight lines are concurrent.

64. If $f(x)$ is a real-valued function of a real variable x, such that $2f(x) + 3f(-x) = 15 - 4x$ for all x, find the function $f(x)$.

65. Show that for all real x, the expression $ax^2 + bx + c$ (where a, b, c are real constants with $a > 0$), has the minimum value $\frac{(4ac-b^2)}{4a}$. Also find the value of x for which this minimum value is attained.

66. If c is a real number with $0 < c < 1$, then show that the values taken by the function $y = \frac{x^2+2x+c}{x^2+4x+3c}$, as x varies over real numbers, range over all real numbers.

67. Describe the set of all real numbers x which satisfy $2\log_{2x+3} x < 1$.

68. (i) Determine m so that the equation

$$x^4 - (3m+2)x^2 + m^2 = 0$$

has four real roots in arithmetic progression.

(ii) Let a and b be two real numbers. If the roots of the equation

$$x^2 - ax - b = 0$$

have absolute value less than one, show that each of the following conditions holds:
(i) $|b| < 1$, (ii) $a + b < 1$ and (iii) $b - a < 1$.

69. Suppose that the three equations $ax^2 - 2bx + c = 0$, $bx^2 - 2cx + a = 0$ and $cx^2 - 2ax + b = 0$ all have only positive roots. Show that $a = b = c$.

70. Suppose that all roots of the polynomial equation

$$x^4 - 4x^3 + ax^2 + bx + 1 = 0$$

are positive real numbers. Show that all the roots of the polynomial are equal.

71. Consider the following simultaneous equations in x and y:

$$\begin{aligned} x + y + axy &= a \\ x - 2y - xy^2 &= 0 \end{aligned}$$

where a is a real constant. Show that these equations admit real solutions in x and y.

72. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose roots are $\alpha - \frac{1}{\beta\gamma}$, $\beta - \frac{1}{\alpha\gamma}$ and $\gamma - \frac{1}{\alpha\beta}$,

73. Consider the equation $x^3 + Gx + H = 0$, where G and H are complex numbers. Suppose that this equation has a pair of complex conjugate roots. Show that both G and H are real.

74. The sum of squares of the digits of a three-digit positive number is 146, while the sum of the two digits in the unit's and the ten's place is 4 times the digit in the hundred's place. Further, when the number is written in the reverse order, it is increased by 297. Find the number.

75. Show that there is at least one real value of x for which $\sqrt[3]{x} + \sqrt{x} = 1$.

76. Find the set of all values of m such that $y = \frac{x^2-x}{1-mx}$ can take all real values.

77. For $x > 0$, show that $\frac{x^n-1}{x-1} \geq nx^{\frac{n-1}{2}}$, where n is a positive integer.

78. For real numbers x, y and z, show that

$$|x| + |y| + |z| \leq |x + y - z| + |y + z - x| + |z + x - y|.$$

79. Let $\theta_1, \theta_2, \ldots, \theta_{10}$ be any values in the closed interval $[0, \pi]$. Show that

$$F = (1 + \sin^2\theta_1)(1 + \cos^2\theta_1)(1 + \sin^2\theta_2)(1 + \cos^2\theta_2) \ldots\ldots$$

$$(1 + \sin^2\theta_{10})(1 + \cos^2\theta_{10}) \leq (\frac{9}{4})^{10}.$$

What is the maximum value attainable by F and at what values of $\theta_1, \theta_2, \ldots, \theta_{10}$, is the maximum value attained?

80. If a, b, c are positive numbers, then show that

$$\frac{b^2 + c^2}{b + c} + \frac{c^2 + a^2}{c + a} + \frac{a^2 + b^2}{a + b} \geq a + b + c.$$

81. Find all possible real numbers a, b, c, d, e which satisfy the following set of equations:

$$\begin{aligned} 3a &= (b + c + d)^3, \\ 3b &= (c + d + e)^3, \\ 3c &= (d + e + a)^3, \\ 3d &= (e + a + b)^3, \\ 3e &= (a + b + c)^3. \end{aligned}$$

82. Let a, b, c, d be positive real numbers such that $abcd = 1$. Show that

$$(1+a)(1+b)(1+c)(1+d) \geq 16.$$

83. If a and b are positive real numbers such that $a + b = 1$, prove that

$$\left(a+\frac{1}{a}\right)^2 + \left(b+\frac{1}{b}\right)^2 \geq \frac{25}{2}.$$

84. Show that there is exactly one value of x which satisfies the equation

$$2\cos^2(x^3+x) = 2^x + 2^{-x}.$$

85. **(i)** Prove, from first principles, that

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta,$$

for every positive integer n.

(ii) Prove that

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta,$$

for every negative integer n.

86. Sketch the set $A \cap B$ in the Argand plane, where $A = \{z : |\frac{z+1}{z-1}| \leq 1\}$ and $B = \{z : |z| - \operatorname{Re} z \leq 1\}$.

87. Let $P(z) = az^2 + bz + c$, where a, b, c are complex numbers.

(a) If $P(z)$ is real for all real numbers z, show that a, b, c are real numbers.

(b) In addition to (a) above, assume that $P(z)$ is not real whenever z is not real. Show that $a = 0$.

88. A pair of complex numbers z_1, z_2 is said to have *property P* if for *every* complex number z, we can find *real numbers* r and s such that $z = rz_1 + sz_2$. Show that a pair z_1, z_2 has property P *if and only if* the points z_1, z_2 and 0 on the complex plane are *not* collinear.

89. Let a be a non-zero complex number such that $|a| \neq 1$. Let P be the point a in the complex plane, and let Q be the point $1/\overline{a}$. Let C_1 be the circle $\{z : |z| = 1\}$ and let C_2 be any circle passing through P and Q. Show that C_1 and C_2 intersect orthogonally. [*Two circles are said to* ***intersect orthogonally*** *if the tangents at a point of intersection are perpendicular to each other.*]

90. Draw the region of points (x, y) in the plane, which satisfy $|y| \leq |x| \leq 1$.

91. Show that a necessary and sufficient condition for the line $ax + by + c = 0$, where a, b, c are nonzero real numbers, to pass through the first quadrant is either $ac < 0$ or $bc < 0$.

92. Let a and b be real numbers such that the equations $2x+3y = 4$ and $ax-by = 7$ have exactly one solution. Then, show that the equations $12x - 8y = 9$ and $bx + ay = 0$ also have exactly one solution.

93. Let ABC be *any* triangle, right-angled at A, with D *any* point on the side AB. The line DE is drawn parallel to BC to meet the side AC at the point E. F is the foot of the perpendicular drawn from E to BC. If $AD = x_1, DB = x_2, BF = x_3, EF = x_4$ and $AE = x_5$, then show that

$$\frac{x_1}{x_5} + \frac{x_2}{x_5} = \frac{x_1x_3 + x_4x_5}{x_3x_5 - x_1x_4}.$$

94. Consider the circle of radius 1 with its centre at the point (0,1). From this initial position, the circle is rolled along the positive x-axis without slipping. Find the locus of the point P on the circumference of the circle which is on the origin at the initial position of the circle.

95. Let the circles

$$x^2 + y^2 - 2cy - a^2 = 0 \quad \text{and} \quad x^2 + y^2 - 2bx + a^2 = 0,$$

with centres at A and B intersect at P and Q. Show that the points A, B, P, Q and $O = (0, 0)$ lie on a circle.

96. Two intersecting circles are said to be *orthogonal* to each other, if the tangents to the two circles at any point of intersection, are perpendicular to each other. Show that every circle through the points $(2, 0)$ and $(-2, 0)$ is orthogonal to the circle $x^2 + y^2 - 5x + 4 = 0$.

97. Consider the circle C whose equation is

$$(x - 2)^2 + (y - 8)^2 = 1$$

and the parabola P with the equation

$$y^2 = 4x.$$

Find the minimum value of the length of the segment AB as A moves on the circle C and B moves on the parabola P.

98. Let $f(x, y) = xy$, where $x \geq 0$ and $y \geq 0$. Prove that the function f satisfies the following property:

$$f(\lambda x + (1 - \lambda)x', \lambda y + (1 - \lambda)y') > \min\{f(x, y), f(x', y')\}$$

for all $(x, y) \neq (x', y')$ and for all $\lambda \in (0, 1)$.

99. If a circle intersects the hyperbola $y = 1/x$ at four distinct points $(x_i, y_i), i = 1, 2, 3, 4$, then prove that $x_1x_2 = y_3y_4$.

100. Consider the parabola $y^2 = 4x$. Let $P = (a, b)$ be any point inside the parabola, i.e., $b^2 < 4a$, and let F be the focus of the parabola. Find the point Q on the parabola such that $FQ + QP$ is minimum. Also, show that the normal to the parabola at Q bisects the angle FQP.

101. Let E be the ellipse with centre at origin O whose major and minor axes are of length $2a$ and $2b$ respectively. Let θ be the acute angle at which E is cut by a circle with centre at the origin (i.e., θ is the acute angle of intersection of their tangents at a point of intersection). Prove that the maximum possible value of θ is $tan^{-1}(\frac{a^2-b^2}{2ab})$.

102. Suppose that AB is an arc of a circle with a given radius and centre subtending an angle θ ($0 < \theta < \pi$ is fixed) at the centre. Consider an arbitrary point P on this arc and the product $l(AP) \cdot l(PB)$, where $l(AP)$ and $l(PB)$ denote the lengths of the straight lines AP and PB, respectively. Determine possible location(s) of P for which this product will be maximized. Justify your answer.

103. Let P be the fixed point (3,4) and Q the point $(x, \sqrt{25 - x^2})$. If $M(x)$ is the slope of the line PQ, find $\lim_{x \to 3} M(x)$.

104. Let A and B be two fixed points 3 cm apart.

(a) Let P be any point not collinear with A and B, such that $PA = 2PB$. The tangent at P to the circle passing through the points P, A and B meets the extended line AB at the point K. Find the lengths of the segments KB and KP.

(b) Hence or otherwise, prove that the locus of all points P in the plane such that $PA = 2PB$ is a circle.

105. Tangents are drawn to a given circle from a point on a given straight line, which does not meet the given circle. Prove that the locus of the mid-point of the chord joining the two points of contact of the tangents with the circle is a circle.

106. The circles C_1, C_2 and C_3 with radii 1, 2 and 3, respectively, touch each other externally. The centres of C_1 and C_2 lie on the x-axis, while C_3 touches them from the top. Find the ordinate of the centre of the circle that lies in the region enclosed by the circles C_1, C_2 and C_3 and touches all of them.

107. If a, b and c are the lengths of the sides of a triangle ABC and if p_1, p_2 and p_3 are the lengths of the perpendiculars drawn from the circumcentre onto the sides BC, CA and AB respectively, then show that

$$\frac{a}{p_1} + \frac{b}{p_2} + \frac{c}{p_3} = \frac{abc}{4p_1p_2p_3}.$$

108. Inside an equilateral triangle ABC, an arbitrary point P is taken from which the perpendiculars PD, PE and PF are dropped onto the sides BC, CA and AB, respectively. Show that the ratio $\dfrac{PD+PE+PF}{BD+CE+AF}$ does not depend upon the choice of the point P and find its value.

109. Let P be an interior point of the triangle $\Delta\, ABC$. Assume that AP, BP and CP meet the opposite sides BC, CA and AB ar D, E and F, respectively. Show that

$$\frac{AF}{FB} + \frac{AE}{EC} = \frac{AP}{PD}.$$

110. Let $ABCD$ be a cyclic quadrilateral with lengths of sides $AB = p$, $BC = q$, $CD = r$ and $DA = s$. Show that

$$\frac{AC}{BD} = \frac{ps+qr}{pq+rs}.$$

111. AB is a chord of a circle C.

(a) Find a point P on the circumference of C such that $PA.PB$ is the maximum.

(b) Find a point P on the circumference of C which maximises $PA + PB$.

112. A rectangle $OACB$ with the two axes as two sides, the origin O as a vertex is drawn in which the length OA is four times the width OB. A circle is drawn passing through the points B and C and touching OA at its mid-point, thus dividing the rectangle into three parts. Find the ratio of the areas of these three parts.

113. Find the vertices of the two right angled triangles each having area 18 and such that the point $(2, 4)$ lies on the hypotenuse and the other two sides are formed by the x and y axes.

114. Let PQ be a line segment of a fixed length L with its two ends P and Q sliding along the X–axis and Y–axis respectively. Complete the rectangle $OPRQ$ where O is the origin. Show that the locus of the foot of the perpendicular drawn from R on PQ is given by

$$x^{2/3} + y^{2/3} = L^{2/3}.$$

115. If $\frac{\sin^4 x}{a} + \frac{\cos^4 x}{b} = \frac{1}{a+b}$, then show that $\frac{\sin^6 x}{a^2} + \frac{\cos^6 x}{b^2} = \frac{1}{(a+b)^2}$.

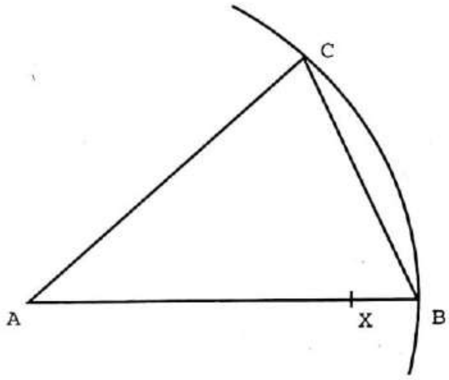
116. If A, B, C are the angles of a triangle, then show that $\sin A + \sin B - \cos C \leq \frac{3}{2}$.

117. Let $[x]$ denote the largest integer (positive, negative or zero) less than or equal to x. Let $y = f(x) = [x] + \sqrt{x - [x]}$ be defined for all real numbers x.

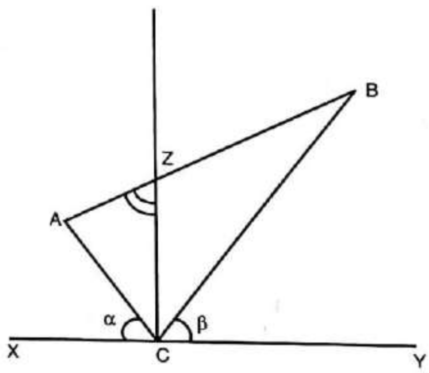
(i) Sketch on plain paper, the graph of the function $f(x)$ in the range $-5 \leq x \leq 5$.

(ii) Show that, given any real number y_0, there is a real number x_0, such that $y_0 = f(x_0)$.

118. Let X be a point on a straight line segment AB such that $AB \cdot BX = AX^2$. Let C be a point on the circle with centre at A and radius AB such that $BC = AX$. (See figure.) Show that the angle $BAC = 36°$.



119. In the adjoining figure CZ is perpendicular to XY and the ratio of the lengths AZ to ZB is 1:2. The angle ACX is α and the angle BCY is β. Find an expression for the angle AZC in terms of α and β.



120. **(i)** If $A + B + C = n\pi$ where n is a positive integer, show that

$$\sin 2A + \sin 2B + \sin 2C = (-1)^{n-1} 4 \sin A \sin B \sin C.$$

(ii) Let triangles ABC and DEF be inscribed in the same circle. If the triangles are of equal perimeter, then prove that

$$\sin A + \sin B + \sin C = \sin D + \sin E + \sin F.$$

(iii) State and prove the converse of (ii) above.

121. Let $\{x_n\}$ be a sequence such that $x_1 = 2, x_2 = 1$ and

$$2x_n - 3x_{n-1} + x_{n-2} = 0$$

for $n > 2$. Find an expression for x_n.

122. Sketch on plain paper, the graph of the function $y = \sin(x^2)$, in the range $0 \leq x \leq \sqrt{4\pi}$.

123. Let $[x]$ denote the largest integer less than or equal to x. For example, $[4\frac{1}{2}] = 4$; $[4] = 4$. Draw a rough sketch of the graphs of the following functions on plain paper:

(i) $f(x) = [x]$;

(ii) $g(x) = x - [x]$;

(iii) $h(x) = \frac{1}{[x]}$.

124. Sketch, on plain paper, the graph of $y = \frac{x^2+1}{x^2-1}$.

125. Let $f : \mathbb{N} \to \mathbb{N}$ be the function defined by $f(0) = 0, f(1) = 1$, and $f(n) = f(n-1) + f(n-2)$ for $n \geq 2$, where $\mathbb{N}$ is the set of all non-negative integers. Prove the following results:

(i) $f(n) < f(n+1)$ for all $n \geq 2$.

(ii) There exist precisely four non-negative integers n for which $f(f(n)) = f(n)$.

(iii) $f(5n)$ is divisible by 5, for all n.

126. Sketch, on plain paper, the regions represented on the plane by the following:

(i) $|y| = \sin x$;

(ii) $|x| - |y| \geq 1$.

127. Find all (x, y) such that

$$\sin x + \sin y = \sin(x + y) \quad \text{and} \quad |x| + |y| = 1.$$

128. Draw the graph (on plain paper) of

$$f(x) = \min\{|x| - 1, |x - 1| - 1, |x - 2| - 1\}.$$

129. Using calculus, sketch the graph of the following function on a plain paper:

$$f(x) = \frac{5 - 3x^2}{1 - x^2}.$$

130. **(a)** Study the derivatives of the function

$$f(x) = \frac{x + 1}{(x - 1)(x - 7)}$$

to make conclusions about the behaviour of the function as x ranges over all possible values for which the above formula for $f(x)$ is meaningful.

(b) Use the information obtained in (a) to draw a rough sketch of the graph of $f(x)$ on plain paper.

131. Sketch the curve $y = 4x^3 - 3x + a$ on plain paper and show that the equation (in x)

$$4x^3 - 3x + a = 0,$$

(where the real constant a is such that $0 < |a| < 1$) has three distinct real roots all of which have their absolute values smaller than 1.

132. For the following function f study its derivatives and use them to sketch its graph on plain paper.

$$f(x) = \frac{x - 1}{x + 1} + \frac{x + 1}{x - 1} \quad \text{for} \quad x \neq -1, 1.$$

133. Use the derivatives and the left and right limits at points of discontinuities, if any, of the function

$$f(x) = \frac{x}{x + 2} + \frac{x + 2}{x}$$

to make conclusions about the behaviour of the function as x ranges over all possible values. Using this, draw a rough sketch of the graph of the function $f(x)$ on plain paper.

134. Using Calculus, sketch on plain paper the graph of the function

$$f(x) = x^2 + x + \frac{1}{x} + \frac{1}{x^2} \quad \text{for} \quad x \neq 0.$$

Show that the function f defined as above for *positive real* numbers attains a unique minimum. What is the minimum value of the function? What is the value of x at which the minimum is attained?

135. Suppose $f(x)$ is a continuous function such that $f(x) = \int_0^x f(t)dt$. Prove that $f(x)$ is identically equal to zero.

136. Consider the function

$$f(x) = \begin{cases} \sin\left(\frac{\pi}{4}(x - [x])\right) & \text{if } [x] \text{ is odd, } x \geq 0 \\ \cos\left(\frac{\pi}{4}(1 - x + [x])\right) & \text{if } [x] \text{ is even, } x \geq 0 \end{cases}$$

where $[x]$ denotes the largest integer smaller than or equal to x.

(i) Sketch the graph of the function f on plain paper.

(ii) Determine the points of discontinuities of f and the points where f is not differentiable.

137. For a real number x, let $[x]$ denote the largest integer less than or equal to x and $< x >$ denote $x - [x]$. Find all the solutions of the equation

$$13[x] + 25 < x >= 271.$$

138. For any positive integer n, let $\langle n \rangle$ denote the integer nearest to $\sqrt{n}$.

(a) Given a positive integer k, describe all positive integers n such that $\langle n \rangle = k$.

(b) Show that

$$\sum_{n=1}^{\infty} \frac{2^{\langle n \rangle} + 2^{-\langle n \rangle}}{2^n} = 3.$$

139. Find all *positive* integers x such that $[x/5] - [x/7] = 1$, where, for any real number t, $[t]$ is the greatest integer less than or equal to t.

140. Consider the function

$$f(x) = \lim_{n \to \infty} \frac{\log_e(2 + x) - x^{2n} \sin x}{1 + x^{2n}}, \quad x > 0.$$

(i) Is $f(x)$ continuous at $x = 1$? Justify your answer.

(ii) Show that $f(x)$ does not vanish anywhere in the interval $0 \leq x \leq \frac{\pi}{2}$, and indicate the points where $f(x)$ changes its sign.

141. Consider the function $f(t) = e^{-\frac{1}{t}}, t > 0$. Let for each positive integer n, P_n be the polynomial such that $\frac{d^n}{dt^n} f(t) = P_n(\frac{1}{t})e^{-\frac{1}{t}}$ for all $t > 0$. Show that

$$P_{n+1}(x) = x^2(P_n(x) - \frac{d}{dx}P_n(x)).$$

142. Study the derivative of the function

$$f(x) = x^3 - 3x^2 + 4,$$

and roughly sketch the graph of $f(x)$, on plain paper.

143. Study the derivative of the function

$$f(x) = \log_e x - (x - 1), \text{for } x > 0,$$

and roughly sketch the graph of $f(x)$, on plain paper.

144. Suppose f is a real-valued differentiable function defined on $[1, \infty)$ with $f(1) = 1$. Suppose, moreover, that f satisfies

$$f'(x) = \frac{1}{x^2 + f^2(x)}.$$

Show that $f(x) \leq 1 + \pi/4$ for every $x \geq 1$.

145. Let $f(x)$ be a real valued function of a variable x such that $f'(x)$ takes both positive and negative values and $f''(x) > 0$ for all x. Show that there is a real number p such that $f(x)$ is an increasing function of x for all $x \geq p$.

146. Suppose f is a function such that $f(x) > 0$ and $f'(x)$ is continuous at every real number x. If $f'(t) \geq \sqrt{f(t)}$ for all t, then show that

$$\sqrt{f(x)} \geq \sqrt{f(1)} + \frac{1}{2}(x - 1).$$

for all $x \geq 1$.

147. A function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is called *periodic* if for some constant $a > 0$, $f(x + a) = f(x)$ for every real number x. Show that the function

$$f(x) = \cos\ x + \cos(\frac{\sqrt{3}}{2}x)$$

is not periodic.

148. Show that there is no real constant $c > 0$ such that $\cos\sqrt{x+c} = \cos\sqrt{x}$ for all real numbers $x \geq 0$.

149. Consider the real-valued function f, defined over $(-\infty, \infty)$ by $f(x) = x^4 + bx^3 + cx^2 + dx + e$, where b, c, d, e are real numbers and $3b^2 < 8c$. Show that the function f has a unique minimum.

150. Find the maximum among $1, 2^{1/2}, 3^{1/3}, 4^{1/4}, \cdots$.

151. Let x be a positive number. A sequence $\{x_n\}$ of real numbers is defined as follows:

$$x_1 = \frac{1}{2}(x + \frac{5}{x}), \quad x_2 = \frac{1}{2}(x_1 + \frac{5}{x_1}), \ldots, \text{and in general,}$$

$$x_{n+1} = \frac{1}{2}(x_n + \frac{5}{x_n}) \ \text{ for all } \ n \geq 1.$$

(**a**) Show that, for all $n \geq 1$, $\frac{x_n - \sqrt{5}}{x_n + \sqrt{5}} = (\frac{x-\sqrt{5}}{x+\sqrt{5}})^{2^n}$.

(**b**) Hence find $\lim\limits_{n\to\infty} x_n$.

152. Let a_0 and b_0 be any two positive integers. Define a_n, b_n for $n \geq 1$ using the relations $a_n = a_{n-1} + 2b_{n-1}$, $b_n = a_{n-1} + b_{n-1}$ and let $c_n = \frac{a_n}{b_n}$, for $n = 0, 1, 2, \ldots$.

(**a**) Write $(\sqrt{2} - c_{n+1})$ in terms of $(\sqrt{2} - c_n)$.

(**b**) Show that $|\sqrt{2} - c_{n+1}| < \frac{1}{1+\sqrt{2}}|\sqrt{2} - c_n|$.

(**c**) Show that $\lim\limits_{n\to\infty} c_n = \sqrt{2}$.

153. Suppose $x_1 = \tan^{-1} 2 > x_2 > x_3 > \cdots$ are positive real numbers satisfying

$$\sin(x_{n+1} - x_n) + 2^{-(n+1)} \sin x_n \sin x_{n+1} = 0 \qquad \text{for } n \geq 1.$$

Find $\cot x_n$. Also, show that $\lim\limits_{n\to\infty} x_n = \frac{\pi}{4}$.

154. Find the maximum and minimum values of the function $f(x) = x^2 - x\sin x$, in the closed interval $[0, \frac{\pi}{2}]$.

155. Evaluate

$$\lim_{n\to\infty}\{\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n}\}.$$

156. A man walking towards a building, on which a flagstaff is fixed vertically, observes the angle subtended by the flagstaff to be the greatest when he is at a distance d from the building. If θ is the observed greatest angle, show that the length of the flagstaff is $2d\tan\theta$.

157. Evaluate

$$\lim_{n\to\infty}\{(1+\frac{1}{2n})(1+\frac{3}{2n})(1+\frac{5}{2n})\dots(1+\frac{2n-1}{2n})\}^{\frac{1}{2n}}.$$

158. Find the value of

$$\int_2^{11}\frac{dx}{1-x}.$$

159. If $A=\int_0^{\pi}\frac{\cos x}{(x+2)^2}dx$, then show that

$$\int_0^{\pi/2}\frac{\sin x\cos x}{(x+1)}dx=\frac{1}{2}(\frac{1}{2}+\frac{1}{\pi+2}-A).$$

160. Prove by induction or otherwise that

$$\int_0^{\pi/2}\frac{\sin(2n+1)x}{\sin x}dx=\frac{\pi}{2}$$

for every integer $n\geq 0$.

161. Show that

$$\int_0^{\pi}\left|\frac{\sin nx}{x}\right|dx\geq\frac{2}{\pi}\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right).$$

162. Show that

$$2(\sqrt{251}-1)<\sum_{k=1}^{250}\frac{1}{\sqrt{k}}<2(\sqrt{250}).$$

163. Using the identity $\log x=\int_1^x\frac{dt}{t}$, $x>0$, or otherwise, prove that

$$\frac{1}{n+1}\leq\log(1+\frac{1}{n})\leq\frac{1}{n}$$

for all integers $n\geq 1$.

164. Show that the area of the bounded region enclosed between the curves $y^3 = x^2$ and $y = 2 - x^2$, is $2\frac{2}{15}$.

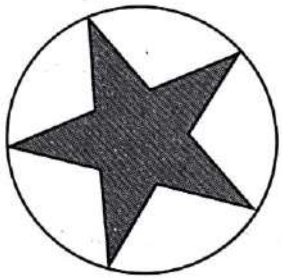
165. Find the area of the region in the xy-plane, bounded by the graphs of $y = x^2, x + y = 2$ and $y = -\sqrt{x}$.

166. A cow is grazing with a rope around her neck and the other end of the rope is tied to a pole. The length of the rope is 10 metres. There are two boundary walls perpendicular to each other, one at a distance of 5 metres to the east of the pole and another at a distance of $5\sqrt{2}$ metres to the north of the pole. Find the area the cow can graze on.

167. Show that the larger of the two areas into which the circle $x^2 + y^2 = 64$ is divided by the curve $y^2 = 12x$ is $\frac{16}{3}(8\pi - \sqrt{3})$.

168. Out of a circular sheet of paper of radius a, a sector with central angle θ is cut out and folded into the shape of a conical funnel. Show that the volume of the funnel is maximum when θ equals $2\pi\sqrt{\frac{2}{3}}$.

169. A *regular* five-pointed star is inscribed in a circle of radius r. (See the figure.) Show that the area of the region inside the star is $\frac{10r^2 \tan(\pi/10)}{3-\tan^2(\pi/10)}$.



170. Let $\{C_n\}$ be an infinite sequence of circles lying in the positive quadrant of the XY–plane, with strictly decreasing radii and satisfying the following conditions. Each C_n touches both the X–axis and the Y–axis. Further, for all $n \geq 1$, the circle C_{n+1} touches the circle C_n externally. If C_1 has radius 10 cm, then show that the sum of the areas of all these circles is $\frac{25\pi}{3\sqrt{2}-4}$ sq cm.

171. Let ABC be an isosceles triangle with $AB = BC = 1$ cm and $\angle A = 30°$. Find the volume of the solid obtained by revolving the triangle about the line AB.

172. Suppose there are k teams playing a round robin tournament; that is, each team plays against all the other teams and no game ends in a draw. Suppose the i^{th} team loses l_i games and wins w_i games. Show that

$$\sum_{i=1}^{k} l_i^2 = \sum_{i=1}^{k} w_i^2.$$

173. Let $P_1, P_2, \ldots, P_n$ be polynomials in x, each having all integer coefficients, such that $P_1 = P_1^2 + P_2^2 + \ldots + P_n^2$. Assume that P_1 is not the zero polynomial. Show that $P_1 = 1$ and $P_2 = P_3 = \ldots = P_n = 0$.

174. Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c and d are integers. The sums of the pairs of roots of $P(x)$ are given by 1, 2, 5, 6, 9 and 10. Find $P(\frac{1}{2})$.

175. Let $P(x) = x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots\cdots + a_1x + a_0$ be a polynomial with integer coefficients, such that, $P(0)$ and $P(1)$ are odd integers. Show that:

(a) $P(x)$ does not have any even integer roots.

(b) $P(x)$ does not have any odd integer roots.

176. Suppose that $P(x)$ is a polynomial of degree n such that

$$P(k) = \frac{k}{k+1} \quad \text{for} \quad k = 0, 1, \ldots, n.$$

Find the value of $P(n+1)$.

177. There are 1000 doors $D_1, D_2, \ldots, D_{1000}$ and 1000 persons $P_1, P_2, \ldots, P_{1000}$. Initially all the doors were closed. Person P_1 goes and opens all the doors. Then person P_2 closes doors $D_2, D_4, \ldots, D_{1000}$ and leaves the odd-numbered doors open. Next, P_3 changes the state of every third door, that is, $D_3, D_6, \ldots, D_{999}$. (For instance, P_3 closes the open door D_3 and opens the closed door D_6, and so on.) Similarly, P_m changes the state of the doors $D_m, D_{2m}, D_{3m}, \ldots, D_{nm}, \ldots$ while leaving the other doors untouched. Finally, P_{1000} opens D_{1000} if it were closed and closes it if it were open. At the end, how many doors remain open?

178. Let l, b be positive integers. Divide the $l \times b$ rectangle into lb unit squares in the usual manner. Consider one of the two diagonals of this rectangle. How many of these unit squares contain a segment of positive length of this diagonal?

179. Let $X = \{0, 1, 2, 3, \ldots, 99\}$. For a, b in X, we define $a * b$ to be the remainder obtained by dividing the product ab by 100. For example, $9 * 18 = 62$ and $7 * 5 = 35$. Let x be an element in X. An element y in X is called the inverse of x if $x * y = 1$. Find which of the elements $1, 2, 3, 4, 5, 6, 7$ have inverses and write down their inverses.

180. Each pair in a group of 20 persons is classified by the existence of kinship relation and friendship relation between them. The following table of data is obtained from such a classification

KINSHIP AND FRIENDSHIP RELATION AMONG 20 PERSONS

Friendship → Kinship ↓	Yes	No
Yes	27	31
No	3	129

Determine (with justifications) whether each of the following statements is supported by the above data:

(i) Most of the friends are kin.

(ii) Most of the kin are friends.

181. Suppose that one moves along the points (m, n) in the plane where m and n are integers in such a way that each move is a diagonal step, that is, consists of one unit to the right or left followed by one unit either up or down.

(a) Which points (p, q) can be reached from the origin?

(b) What is the minimum number of moves needed to reach such a point (p, q)?

182. In a competition, six teams A, B, C, D, E, F play each other in the preliminary round—called *round robin* tournament. Each game ends either in a win or a loss. The winner is awarded two points while the loser is awarded zero points. After the round robin tournament, the three teams with the highest scores move to the final round. Based on the following information, find the score of each team at the end of the round robin tournament.

(i) In the game between E and F, team E won.

(ii) After each team had played four games, team A had 6 points, team B had 8 points and team C had 4 points. The remaining matches yet to be played were

  (i) between A and D;

  (ii) between B and E; and

  (iii) between C and F.

(iii) The teams D, E and F had won their games against A, B and C respectively.

(iv) Teams A, B and D had moved to the final round of the tournament.

183. Let $N = \{1, 2, ..., n\}$ be a set of elements called voters. Let $\mathcal{C} = \{S : S \subseteq N\}$ be the set of all subsets of N. Members of $\mathcal{C}$ are called coalitions. Let f be a function from $\mathcal{C}$ to $\{0, 1\}$. A coalition $S \subseteq N$ is said to be *winning* if $f(S) = 1$; it is said to be a *losing* coalition if $f(S) = 0$. Such a function f is called a voting game if the following conditions hold.

(a) N is a winning coalition.
(b) The empty set ϕ is a losing coalition.
(c) If S is a winning coalition and $S \subseteq S'$, then S' also is winning.
(d) If both S and S' are winning coalitions, then $S \bigcap S' \neq \phi$, i.e., S and S' have a common voter.

Show that the maximum number of winning coalitions of a voting game is 2^{n-1}. Also, find a voting game for which the number of winning coalitions is 2^{n-1}.

184. Let S be the set of all sequences $(a_1, a_2, \dots)$ of non-negative integers such that
(i) $a_1 \geq a_2 \geq \dots$; and
(ii) there exists a positive integer N such that $a_n = 0$ for all $n \geq N$.
Define the dual of the sequence $(a_1, a_2, \dots)$ belonging to S to be the sequence $(b_1, b_2, \dots)$, where, for $m \geq 1$, b_m is the number of a_n's which are greater than or equal to m.

(i) Show that the dual of a sequence in S belongs to S.

(ii) Show that the dual of the dual of a sequence in S is the original sequence itself.

(iii) Show that the duals of distinct sequences in S are distinct.

185. An operation $*$ on a set G is a mapping that associates with every pair of elements a and b of the set G, a unique element $a * b$ of G. G is said to be a *group under the operation* $*$, if the following conditions hold:
(i) $(a * b) * c = a * (b * c)$ for all elements a, b and c of G;
(ii) there is an element e of G such that $a * e = e * a = a$ for all elements a of G; and
(iii) for each element a of G, there is an element a' of G such that $a * a' = a' * a = e$.

If G is the set whose elements are all subsets of a set X, and, if $*$ is the operation on G defined as $A * B = (A \cup B) \setminus (A \cap B)$, show that G is a group under $*$.

(For any two subsets C and D of X, $C \setminus D$ denotes the set of all those elements which are in C but not in D).

186. At time 0, a particle is at the point 0 on the real line. At time 1, the particle divides into two and instantaneously after division, one particle moves 1 unit to the left and the other moves one unit to the right. At time 2, each of these particles divides into two, and one of the two new particles moves one unit to the left and the other moves one unit to the right. Whenever two particles meet, they destroy each other leaving nothing behind. How many particles will be there after time $2^{11} + 1$?

187. Suppose S={0,1} with the following addition and multiplication rules:

$$0+0=1+1=0 \qquad 0\cdot 0=0\cdot 1=1\cdot 0=0$$
$$0+1=1+0=1 \qquad 1\cdot 1=1$$

A system of polynomials is defined with coefficients in S. The sum and product of two polynomials in the system are the usual sum and product, respectively, where for the addition and multiplication of coefficients the above mentioned rules apply. For example, in the system,

$$(x+1)\cdot(x^2+x+1)=x^3+(1+1)x^2+(1+1)x+1=x^3+0x^2+0x+1=x^3+1.$$

Show that in this system x^3+x+1 is not factorizable, that is, one cannot write

$$x^3+x+1=(ax+b)\cdot(cx^2+dx+e),$$

where a, b, c, d and e are elements of S.

188. Consider the squares of an 8×8 chessboard filled with the numbers 1 to 64 as in the figure below. If we choose 8 squares with the property that there is exactly one from each row and exactly one from each column, and add up the numbers in the chosen squares, show that the sum obtained is always 260.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

189. **Let $a_1, a_2, \ldots, a_{100}$ be real numbers, each less than one, satisfy**

$$a_1+a_2+\cdots+a_{100}>1.$$

(i) Let n_0 be the smallest integer n such that

$$a_1 + a_2 + \cdots + a_n > 1.$$

Show that all the sums a_{n_0}, $a_{n_0} + a_{n_0-1}, \ldots, a_{n_0} + \cdots + a_1$ are positive.

(ii) Show that there exist two integers p and q, $p < q$, such that the numbers

$$a_q, a_q + a_{q-1}, \ldots, a_q + \cdots + a_p,$$
$$a_p, a_p + a_{p+1}, \ldots, a_p + \cdots + a_q$$

are all positive.

B.Stat.(Hons.) Admission Test: 2007

Short-Answer Type Test
Time: 2 hours

1. Suppose a is a complex number such that

$$a^2 + a + \frac{1}{a} + \frac{1}{a^2} + 1 = 0.$$

If m is a positive integer, find the value of

$$a^{2m} + a^m + \frac{1}{a^m} + \frac{1}{a^{2m}}.$$

2. Use calculus to find the behaviour of the function

$$y = e^x \sin x \qquad -\infty < x < +\infty$$

and sketch the graph of the function for $-2\pi \leq x \leq 2\pi$. Show clearly the locations of the maxima, minima and points of inflection in your graph.

3. Let $f(u)$ be a continuous function and, for any real number u, let $[u]$ denote the greatest integer less than or equal to u. Show that for any $x > 1$,

$$\int_1^x [u]([u]+1)f(u)du = 2\sum_{i=1}^{[x]} i \int_i^x f(u)du.$$

4. Show that it is not possible to have a triangle with sides a, b and c whose medians have lengths $\frac{2}{3}a$, $\frac{2}{3}b$ and $\frac{4}{5}c$.

5. Show that

$$-2 \;\leq\; \cos\theta(\sin\theta + \sqrt{\sin^2\theta + 3}) \;\leq\; 2$$

for all values of θ.

6. Let $S = \{1, 2, \cdots, n\}$ where n is an odd integer. Let f be a function defined on $\{(i,j) : i \in S, j \in S\}$ taking values in S such that

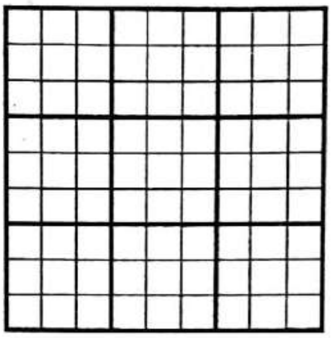
(i) $f(s,r) = f(r,s)$ for all $r, s \in S$

(ii) $\{f(r,s) : s \in S\} = S$ for all $r \in S$.

Show that $\{f(r,r) : r \in S\} = S$.

7. Consider a prism with triangular base. The total area of the three faces containing a particular vertex A is K. Show that the maximum possible volume of the prism is $\sqrt{K^3/54}$ and find the height of this largest prism.

8. The following figure shows a $3^2 \times 3^2$ grid divided into 3^2 subgrids of size 3×3. This grid has 81 cells, 9 in each subgrid.



Now consider an $n^2 \times n^2$ grid divided into n^2 subgrids of size $n \times n$. Find the number of ways in which you can select n^2 cells from this grid such that there is exactly one cell coming from each subgrid, one from each row and one from each column.

9. Let $X \subset \mathbb{R}^2$ be a set satisfying the following properties:

(i) if (x_1, y_1) and (x_2, y_2) are any two distinct elements in X, then

$$\textbf{either} \quad x_1 > x_2 \text{ and } y_1 > y_2$$
$$\textbf{or} \quad x_2 > x_1 \text{ and } y_2 > y_1;$$

(ii) there are two elements (a_1, b_1) and (a_2, b_2) in X such that for any (x, y) in X,

$$a_1 \leq x \leq a_2 \quad \text{and} \quad b_1 \leq y \leq b_2;$$

(iii) if (x_1, y_1) and (x_2, y_2) are two elements of X, then for all $\lambda \in [0, 1]$,

$$\Big(\lambda x_1 + (1-\lambda)x_2 \ , \ \lambda y_1 + (1-\lambda)y_2\Big) \in X.$$

Show that if (x, y) is in X, then for some $\lambda \in [0, 1]$,

$$x = \lambda a_1 + (1-\lambda)a_2, \quad y = \lambda b_1 + (1-\lambda)b_2.$$

10. Let A be a set of positive integers satisfying the following properties:

(i) if m and n belong to A, then $m + n$ also belongs to A;

(ii) there is no prime number that divides all elements of A.

(a) Suppose n_1 and n_2 are two integers belonging to A such that $n_2 - n_1 > 1$. Show that you can find two integers m_1 and m_2 in A such that $0 < m_2 - m_1 < n_2 - n_1$.

(b) Hence show that there are two consecutive integers belonging to A.

(c) Let n_0 and $n_0 + 1$ be two consecutive integers belonging to A. Show that if $n \geq n_0^2$ then n belongs to A.

B.Math.(Hons.) Admission Test: 2007

Short-Answer Type Test
Time: 2 hours

1. Let n be a positive integer. If n has odd number of divisors (other than 1 and n), then show that n is a perfect square.

2. Let a and b be two non-zero rational numbers such that the equation $ax^2+by^2=0$ has a non-zero solution in rational numbers. Prove that for any rational number t, there is a rational solution of the equation $ax^2+by^2=t$.

3. For a natural number $n>1$, consider the $n-1$ points on the unit circle $e^{2\pi ik/n}(k=1,2,\ldots,n-1)$. Show that the sum of the distances of these points from 1 is n.

4. Let ABC be an isosceles triangle with $AB=AC=20$. Let P be a point inside the triangle ABC such that the sum of the distances of P to AB and AC is 1. Describe the locus of all such points inside ABC.

5. Let $P(X)$ be a polynomial with integer coefficients of degree $d>0$.

(a) If α and β are two integers such that $P(\alpha)=1$ and $P(\beta)=-1$, then prove that $|\beta-\alpha|$ divides 2.

(b) Prove that the number of distinct integer roots of $P^2(x)-1$ is at most $d+2$.

6. In ISI Club each member is on two committees and any two committees have exactly one member in common. There are five committees. How many members does ISI Club have?

7. Let $0\leq\theta\leq\frac{\pi}{2}$. Prove that $\sin\theta\geq\frac{2\theta}{\pi}$.

8. Let $P:\mathbb{R}\rightarrow\mathbb{R}$ be a continuous function such that $P(X)=X$ has no real solution. Prove that $P(P(X))=X$ has no real solution.

9. In a group of five people any two are either friends or enemies, no three of them are friends of each other and no three of them are enemies of each other. Prove that every person in this group has exactly two friends.

10. The eleven members of a cricket team are numbered $1,2,\ldots,11$. In how many ways can the entire cricket team sit on the eleven chairs arranged around a circular table so that the numbers of any two adjacent players differ by one or two?

B.Stat.(Hons.) Admission Test: 2008

Short-Answer Type Test
Time: 2 hours

1. Of all triangles with a given perimeter, find the triangle with the maximum area. Justify your answer.

2. A 40 feet high screen is put on a vertical wall 10 feet above your eye-level. How far should you stand to maximize the angle subtended by the screen (from top to bottom) at your eye?

3. Study the derivatives of the function

$$y = \sqrt{x^3 - 4x}$$

and sketch its graph on the real line.

4. Suppose P and Q are the centres of two disjoint circles C_1 and C_2 respectively, such that P lies outside C_2 and Q lies outside C_1. Two tangents are drawn from the point P to the circle C_2, which intersect the circle C_1 at points A and B. Similarly, two tangents are drawn from the point Q to the circle C_1, which intersect the circle C_2 at points M and N. Show that $AB = MN$.

5. Suppose ABC is a triangle with inradius r. The incircle touches the sides BC, CA and AB at D, E and F respectively. If $BD = x$, $CE = y$ and $AF = z$, then show that

$$r^2 = \frac{xyz}{x+y+z}.$$

6. Evaluate: $\lim_{n\to\infty} \frac{1}{2n} \log \binom{2n}{n}$.

7. Consider the equation $x^5 + x = 10$. Show that

(a) the equation has only one real root;

(b) this root lies between 1 and 2;

(c) this root must be irrational.

8. In how many ways can you divide the set of eight numbers $\{2, 3, \ldots, 9\}$ into 4 pairs such that no pair of numbers has g.c.d. equal to 2?

9. Suppose S is the set of all positive integers. For $a, b \in S$, define

$$a * b = \frac{\text{l.c.m.}(a,b)}{\text{g.c.d.}(a,b)}$$

For example, $8 * 12 = 6$.

Show that ***exactly two*** of the following three properties are satisfied :

(i) If $a, b \in S$ then $a * b \in S$.

(ii) $(a * b) * c = a * (b * c)$ for all $a, b, c \in S$.

(iii) There exists an element $i \in S$ such that $a * i = a$ for all $a \in S$.

10. Two subsets A and B of the (x, y)-plane are said to be *equivalent* if there exists a function $f : A \to B$ which is both one-to-one and onto.

(i) Show that any two line segments in the plane are equivalent.

(ii) Show that any two circles in the plane are equivalent.

B.Math.(Hons.) Admission Test: 2008

Short-Answer Type Test
Time: 2 hours

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Suppose

$$f(x) = \frac{1}{t}\int_0^t (f(x+y) - f(y))dy$$

for all $x \in \mathbb{R}$ and all $t > 0$. Then show that there exists a constant c such that $f(x) = cx$ for all x.

2. Suppose that $P(x)$ is a polynomial with real coefficients such that for some positive real numbers c, d and for all natural numbers n, we have

$$c|n|^3 \leq |P(n)| \leq d|n|^3.$$

Prove that $P(x)$ has a real zero.

3. Let z be a complex number such that z, z^2, z^3 are collinear in the complex plane. Show that z is a real number.

4. Let $a_1, \ldots, a_n$ be integers. Show that there exists integers k and r such that the sum

$$a_k + a_{k+1} + \cdots + a_{k+r}$$

is divisible by n.

5. If a polynomial P with integer coefficients has three distinct integer zeroes, then show that $P(n) \neq 1$ for any integer n.

6. Let $\binom{n}{k}$ denote the binomial coefficient $\frac{n!}{k!(n-k)!}$, and F_m be the mth Fibonacci number given by $F_1 = F_2 = 1$ and $F_{m+2} = F_m + F_{m+1}$ for all $m \geq 1$. Show that

$$\sum \binom{n}{k} = F_{m+1} \quad \text{for all } m \geq 1.$$

Here, the above sum is over all pairs of integers $n \geq k \geq 0$ with $n + k = m$.

7. Let

$$C = \{(i,j) \mid i, j \text{ integers such that } 0 \leq i, j \leq 24\}.$$

How many squares can be formed in the plane all of whose vertices are in C and whose sides are parallel to the x-axis and y-axis?

8. Let

$$a^2 + b^2 = 1, \quad c^2 + d^2 = 1, \quad ac + bd = 0.$$

Prove that

$$a^2 + c^2 = 1, \quad b^2 + d^2 = 1, \quad ab + cd = 0.$$

9. For $n \geq 3$, determine all real solutions of the system of n equations:

$$x_1 + x_2 + \cdots + x_{n-1} = \frac{1}{x_n}$$

$$\cdots\cdots\cdots$$

$$x_1 + x_2 + \cdots + x_{i-1} + x_{i+1} + \cdots + x_n = \frac{1}{x_i}$$

$$\cdots\cdots\cdots$$

$$x_2 + \cdots + x_{n-1} + x_n = \frac{1}{x_1}.$$

10. If p is a prime number and $a > 1$ is a natural number, then show that the greatest common divisor of the two numbers $a - 1$ and $\frac{a^p - 1}{a-1}$ is either 1 or p.

B.Stat.(Hons.) Admission Test: 2009

Short-Answer Type Test
Time: 2 hours

1. Two train lines intersect each other at a junction at an acute angle θ. A train is passing along one of the two lines. When the front of the train is at the junction, the train subtends an angle α at a station on the other line. It subtends an angle β ($< \alpha$) at the same station, when its rear is at the junction. Show that

$$\tan\theta = \frac{2\sin\alpha\sin\beta}{\sin(\alpha-\beta)}.$$

2. Let $f(x)$ be a continuous function, whose first and second derivatives are continuous on $[0, 2\pi]$ and $f''(x) \geq 0$ for all x in $[0, 2\pi]$. Show that

$$\int_0^{2\pi} f(x)\cos x\, dx \geq 0.$$

3. Let ABC be a right-angled triangle with $BC = AC = 1$. Let P be any point on AB. Draw perpendiculars PQ and PR on AC and BC respectively from P. Define M to be the maximum of the areas of BPR, APQ and $PQCR$. Find the minimum possible value of M.

4. A sequence is called an *arithmetic progression of the first order* if the differences of the successive terms are constant. It is called an *arithmetic progression of the second order* if the differences of the successive terms form an arithmetic progression of the first order. In general, for $k \geq 2$, a sequence is called an *arithmetic progression of the k-th order* if the differences of the successive terms form an arithmetic progression of the $(k-1)$-th order.

The numbers

$$4, 6, 13, 27, 50, 84$$

are the first six terms of an arithmetic progression of some order. What is its least possible order? Find a formula for the n-th term of this progression.

5. A cardboard box in the shape of a rectangular parallelopiped is to be enclosed in a cylindrical container with a hemispherical lid. If the total height of the container from the base to the top of the lid is 60 centimetres and its base has radius 30 centimetres, find the volume of the largest box that can be completely enclosed inside the container with the lid on.

6. Let $f(x)$ be the function satisfying

$$xf(x) = \log x, \qquad \text{for } x > 0.$$

Show that $f^{(n)}(1) = (-1)^{n+1} n! \left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right)$, where $f^{(n)}(x)$ denotes the n-th derivative of the function f evaluated at x.

7. Show that the vertices of a regular pentagon are concyclic. If the length of each side of the pentagon is x, show that the radius of the circumcircle is $\frac{x}{2}$ cosec36°.

8. Find the number of ways in which three numbers can be selected from the set $\{1, 2, \ldots, 4n\}$, such that the sum of the three selected numbers is divisible by 4.

9. Consider 6 points located at $P_0 = (0, 0)$, $P_1 = (0, 4)$, $P_2 = (4, 0)$, $P_3 = (-2, -2)$, $P_4 = (3, 3)$ and $P_5 = (5, 5)$. Let R be the region consisting of *all* points in the plane whose distance from P_0 is smaller than that from any other P_i, $i = 1, 2, 3, 4, 5$. Find the perimeter of the region R.

10. Let x_n be the n-th non-square positive integer. Thus, $x_1 = 2$, $x_2 = 3$, $x_3 = 5$, $x_4 = 6$, etc. For a positive real number x, denote the integer closest to it by $\langle x \rangle$. If $x = m + 0.5$, where m is an integer, then define $\langle x \rangle = m$. For example, $\langle 1.2 \rangle = 1$, $\langle 2.8 \rangle = 3$, $\langle 3.5 \rangle = 3$. Show that $x_n = n + \langle \sqrt{n} \rangle$.

B.Math.(Hons.) Admission Test: 2009

Short-Answer Type Test
Time: 2 hours

1. Let x, y, z be non-zero real numbers. Suppose α, β, γ are complex numbers such that $|\alpha| = |\beta| = |\gamma| = 1$. If $x + y + z = 0 = \alpha x + \beta y + \gamma z$, then prove that $\alpha = \beta = \gamma$.

2. Let c be a fixed real number. Show that a root of the equation

$$x(x+1)(x+2)\cdots(x+2009) = c$$

can have multiplicity at most 2. Determine the number of values of c for which the equation has a root of multiplicity 2.

3. Let $1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, \ldots$ be the sequence of all the positive integers integers which do not contain the digit zero. Write $\{a_n\}$ for this sequence. By comparing with a geometric series, show that $\sum_n \frac{1}{a_n} < 90$.

4. Find the values of x, y for which $x^2 + y^2$ takes the minimum value where $(x + 5)^2 + (y - 12)^2 = 14$.

5. Let p be a prime number bigger than 5. Suppose, the decimal expansion of $1/p$ looks like $0.\overline{a_1 a_2 \cdots a_r}$ where the line denotes a recurring decimal. Prove that 10^r leaves a remainder of 1 on dividing by p.

6. Let a, b, c, d be integers such that $ad - bc$ is non-zero. Suppose b_1, b_2 are integers both of which are multiples of $ad - bc$. Prove that there exist integers simultaneously satisfying both the equalities $ax + by = b_1, cx + dy = b_2$.

7. Compute the maximum area of a rectangle which can be inscribed in a triangle of area M.

8. Suppose you are given six colours and, are asked to colour each face of a cube by a different colour. Determine the different number of colourings possible.

9. Let $f(x) = ax^2 + bx + c$ where a, b, c are real numbers. Suppose $f(-1), f(0), f(1) \in [-1, 1]$. Prove that $|f(x)| \leq 3/2$ for all $x \in [-1, 1]$.

10. Given odd integers a, b, c, prove that the equation $ax^2 + bx + c = 0$ cannot have a solution x which is a rational number.

B.Stat.(Hons.) Admission Test: 2010

Short-Answer Type Test
Time: 2 hours

1. Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ be two permutations of the numbers $1, 2, \ldots, n$. Show that

$$\sum_{i=1}^{n} i(n+1-i) \leq \sum_{i=1}^{n} a_i b_i \leq \sum_{i=1}^{n} i^2.$$

2. Let a, b, c, d be distinct digits such that the product of the 2-digit numbers ab and cb is of the form ddd. Find all possible values of $a+b+c+d$.

3. Let I_1, I_2, I_3 be three open intervals of $\mathbb{R}$ such that none is contained in another. If $I_1 \cap I_2 \cap I_3$ is non-empty, then show that at least one of these intervals is contained in the union of the other two.

4. A real valued function f is defined on the interval $(-1, 2)$. A point x_0 is said to be a fixed point of f if $f(x_0) = x_0$. Suppose that f is a differentiable function such that $f(0) > 0$ and $f(1) = 1$. Show that if $f'(1) > 1$, then f has a fixed point in the interval $(0, 1)$.

5. Let A be the set of all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(xy) = xf(y)$ for all $x, y \in \mathbb{R}$.

(a) If $f \in A$, then show that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

(b) For $g, h \in A$, define a function $g \circ h$ by $(g \circ h)(x) = g(h(x))$ for $x \in \mathbb{R}$. Prove that $g \circ h$ is in A and is equal to $h \circ g$.

6. Consider the equation $n^2 + (n+1)^4 = 5(n+2)^3$.

(a) Show that any integer of the form $3m+1$ or $3m+2$ can not be a solution of this equation.

(b) Does the equation have a solution in positive integers?

7. Consider a rectangular sheet of paper $ABCD$ such that the lengths of AB and AD are respectively 7 and 3 centimetres. Suppose that B' and D' are two points on AB and AD respectively such that if the paper is folded along $B'D'$ then A falls on A' on the side DC. Determine the maximum possible area of the triangle $AB'D'$.

8. Let $1 \leq r \leq n$. Consider all subsets of $\{1, 2, \ldots, n\}$ consisting of r elements. Let $F(n, r)$ denote the arithmetic mean of the smallest elements of these subsets. (For example, when $n = 4$ and $r = 2$, the subsets are $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$ and $F(n, r) = \frac{3\times1+2\times2+1\times3}{3+2+1} = 5/3$.) Prove that $F(n, r) = \frac{n+1}{r+1}$.

9. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a function having the following property: For any two points A and B in $\mathbb{R}^2$, the distance between A and B is the same as the distance between the points $f(A)$ and $f(B)$.

Denote the unique straight line passing through A and B by $\ell(A, B)$.

(a) Suppose that C, D are two fixed points in $\mathbb{R}^2$. If X is a point on the line $\ell(C, D)$, then show that $f(X)$ is a point on the line $\ell(f(C), f(D))$.

(b) Consider two more points E and F in $\mathbb{R}^2$ and suppose that $\ell(E, F)$ intersects $\ell(C, D)$ at an angle α. Show that $\ell(f(C), f(D))$ intersects $\ell(f(E), f(F))$ at an angle α. What happens if the two lines $\ell(C, D)$ and $\ell(E, F)$ do not intersect? Justify your answer.

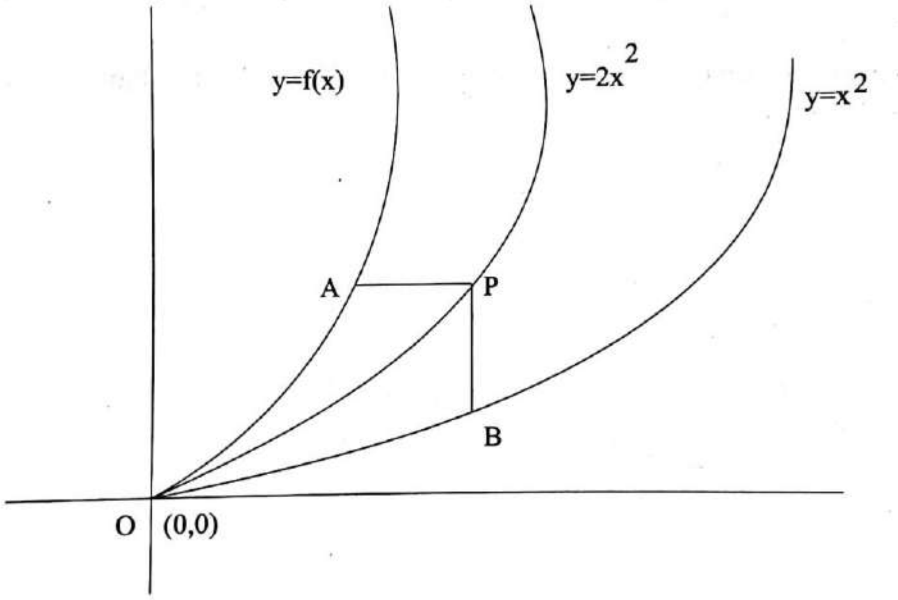
10. There are 100 people in a queue waiting to enter a hall. The hall has exactly 100 seats numbered from 1 to 100. The first person in the queue enters the hall, chooses any seat and sits there. The n-th person in the queue, where n can be $2, \ldots, 100$, enters the hall after the $(n-1)$-th person is seated. He sits in seat number n if he finds it vacant; otherwise he takes any unoccupied seat. Find the total number of ways in which 100 seats can be filled up, provided the 100-th person occupies seat number 100.

B.Math.(Hons.) Admission Test: 2010

Short-Answer Type Test
Time: 2 hours

1. Prove that in each year, the 13th day of some month occurs on a Friday.

2. In the accompanying figure, $y = f(x)$ is the graph of a one-to-one continuous function f. At each point P on the graph of $y = 2x^2$, assume that the areas OAP and OBP are equal. Here PA, PB are the horizontal and vertical segments. Determine the function f.



3. Show that, for any positive integer n, the sum of $8n+4$ consecutive positive integers cannot be a perfect square.

4. If $a, b, c \in (0,1)$ satisfy $a+b+c=2$, prove that $\frac{abc}{(1-a)(1-b)(1-c)} \geq 8$.

5. Let $a_1 > a_2 > \cdots > a_r$ be positive real numbers.
Compute $\lim_{n\to\infty}(a_1^n + a_2^n + \cdots + a_r^n)^{1/n}$.

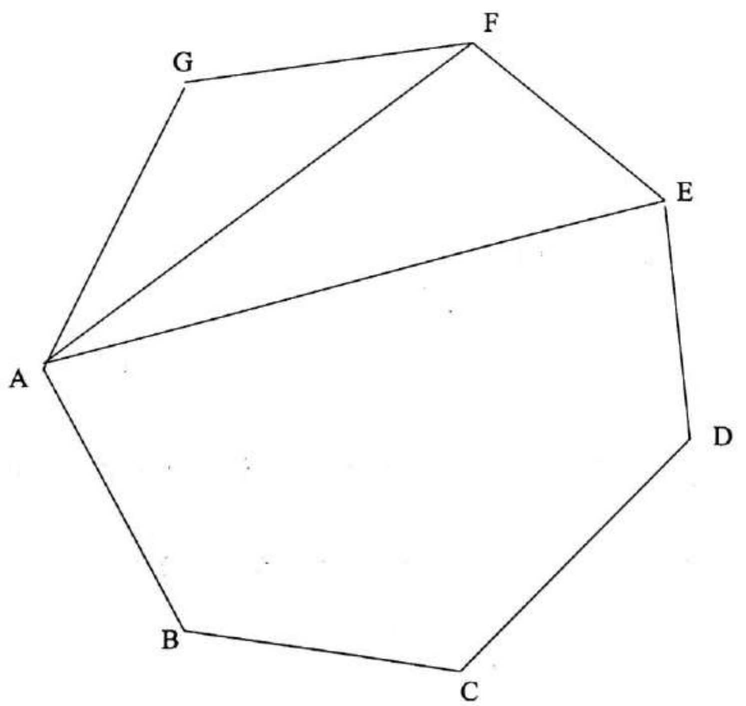
6. Let each of the vertices of a regular 9-gon (polygon of 9 equal sides and equal angles) be coloured black or white.
(a) Show that there are two adjacent vertices of the same colour.
(b) Show there are 3 vertices of the same colour forming an isosceles triangle.

7. Let a, b, c be real numbers and, assume that all the roots of $x^3+ax^2+bx+c=0$ have the same absolute value. Show that $a=0$ if, and only if, $b=0$.

8. Let f be a real-valued differentiable function on the real line $\mathbb{R}$ such that $\lim_{x\to 0} \frac{f(x)}{x^2}$ exists, and is finite. Prove that $f'(0)=0$.

9. Let $f(x)$ be a polynomial with integer coefficients. Assume that 3 divides the value $f(n)$ for each integer n. Prove that when $f(x)$ is divided by x^3-x, the remainder is of the form $3r(x)$, where $r(x)$ is a polynomial with integer coefficients.

10. Consider a regular heptagon (polygon of 7 equal sides and equal angles) ABCDEFG.
(a) Prove $\frac{1}{\sin\frac{\pi}{7}} = \frac{1}{\sin\frac{2\pi}{7}} + \frac{1}{\sin\frac{3\pi}{7}}$.
(b) Using (a) or otherwise, show that $\frac{1}{AG} = \frac{1}{AF} + \frac{1}{AE}$. (See the figure appearing in the next page.)



B.Stat.(Hons.) Admission Test: 2011

Short-Answer Type Test
Time: 2 hours

1. Let $x_1, x_2, \ldots, x_n$ be positive real numbers with $x_1 + \cdots + x_n = 1$. Then show that

$$\sum_{i=1}^{n} \frac{x_i}{2-x_i} \geq \frac{n}{2n-1}.$$

2. Consider three positive real numbers a, b and c. Show that there cannot exist two distinct positive integers m and n such that both $a^m + b^m = c^m$ and $a^n + b^n = c^n$ hold.

3. Let $\mathbb{R}$ denote the set of real numbers. Suppose a function $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(f(f(x))) = x$, for all $x \in \mathbb{R}$. Show that

(a) f is one-to-one,

(b) f cannot be strictly decreasing, and

(c) if f is strictly increasing, then $f(x) = x$ for all $x \in \mathbb{R}$.

4. Let f be a twice differentiable function on the open interval $(-1, 1)$ such that $f(0) = 1$. Suppose f also satisfies $f(x) \geq 0$, $f'(x) \leq 0$ and $f''(x) \leq f(x)$, for all $x \geq 0$. Show that $f'(0) \geq -\sqrt{2}$.

5. $ABCD$ is a trapezium such that AB is parallel to DC and $\frac{AB}{DC} = \alpha > 1$. Suppose P and Q are points on AC and BD respectively, such that

$$\frac{AP}{AC} = \frac{BQ}{BD} = \frac{\alpha-1}{\alpha+1}.$$

Show that $PQCD$ is a parallelogram.

6. Let α be a complex number such that both α and $\alpha + 1$ have modulus 1. If for a positive integer n, $1 + \alpha$ is an n-th root of unity, then show that α is also an n-th root of unity and n is a multiple of 6.

7. (a) Show that there cannot exist three prime numbers, each greater than 3, which are in arithmetic progression with a common difference less than 5.

(b) Let $k > 3$ be an integer. Show that it is not possible for k prime numbers, each greater than k, to be in in arithmetic progression with a common difference less than or equal to $k + 1$.

8. Let $I_n = \int_0^{n\pi} \frac{\sin x}{1+x}\, dx$, $n = 1, 2, 3, 4$. Arrange I_1, I_2, I_3, I_4 in increasing order of magnitude. Justify your answer.

9. Consider all non-empty subsets of the set $\{1, 2, \ldots, n\}$. For every such subset, we find the product of the reciprocals of each of its elements. Denote the sum of all these products as S_n. For example,

$$S_3 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{1.2} + \frac{1}{1.3} + \frac{1}{2.3} + \frac{1}{1.2.3}.$$

(a) Show that $S_n = \frac{1}{n} + (1 + \frac{1}{n})S_{n-1}$.

(b) Hence or otherwise, deduce that $S_n = n$.

10. Show that the triangle whose angles satisfy the equality

$$\frac{\sin^2 A + \sin^2 B + \sin^2 C}{\cos^2 A + \cos^2 B + \cos^2 C} = 2$$

is right-angled.

B.Math.(Hons.) Admission Test: 2011

Short-Answer Type Test
Time: 2 hours

1. Let $a \geq 0$ be a constant such that $\sin(\sqrt{x+a}) = \sin(\sqrt{x})$ for all $x \geq 0$. What can you say about a? Justify your answer.

2. Let $f(x) = e^{-x}$ for $x \geq 0$. Define a function g on the nonnegative real numbers as follows: for each integer $k \geq 0$, the graph of the function g on the interval $[k, k+1]$ is the straight line segment connecting the points $(k, f(k))$ and $(k+1, f(k+1))$. Find the total area of the region which lies between the curves of f and g.

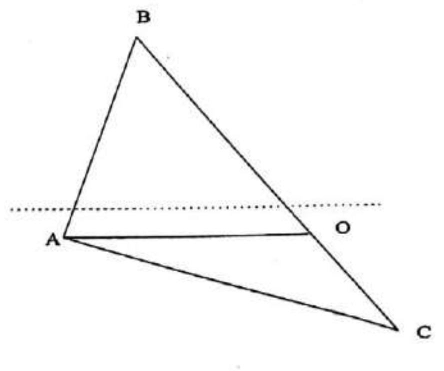
3. For any positive integer n, show that $\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \cdots \frac{(2n-1)}{2n} < \frac{1}{\sqrt{2n+1}}$.

4. If $a_1, \ldots, a_7$ are not necessarily distinct real numbers such that $1 < a_i < 13$ for all i, then show that we can choose three of them such that they are the lengths of the sides of a triangle.

5. For any real number x, let $[x]$ denote the largest integer which is less than or equal to x. Let $N_1 = 2$, $N_2 = 3$, $N_3 = 5, \ldots$ be the sequence of non-square positive integers. If the nth non-square positive integer satisfies $m^2 < N_n < (m+1)^2$, then show that $m = [\sqrt{n} + \frac{1}{2}]$.

6. Let R and S be two cubes with sides of lengths r and s respectively, where r and s are positive integers. Show that the difference of their volumes equals the difference of their surface areas, if and only if $r = s$.

7. Let ABC be any triangle and let O be a point on the line segment BC. Show that there exists a line parallel to AO which divides the triangle ABC into two equal parts of equal area. (See the figure in the next page.)



8. Let $t_1 < t_2 < \cdots < t_{99}$ be real numbers, and consider the function $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = | x - t_1 | + | x - t_2 | + \cdots + | x - t_{99} |$. Show that $\min_{x \in \mathbb{R}} f(x) = f(t_{50})$.

B.Stat. (Hons.) & B.Math. (Hons.) Admission Test: 2012

Short-Answer Type Test
Time: 2 hours

1. Let X, Y, Z be the angles of a triangle.

(i) Prove that

$$\tan\frac{X}{2}\tan\frac{Y}{2}+\tan\frac{X}{2}\tan\frac{Z}{2}+\tan\frac{Z}{2}\tan\frac{Y}{2}=1.$$

(ii) Using (i) or otherwise prove that

$$\tan\frac{X}{2}\tan\frac{Y}{2}\tan\frac{Z}{2}\leq\frac{1}{3\sqrt{3}}.$$

2. Let α be s real number. Consider the function

$$g(x)=(\alpha+\mid x\mid)^2e^{(5-|x|)^2},\ -\infty<x<\infty\cdot\infty.$$

(i) Determine the values of α for which g is continuous at all x.

(ii) Determine the values of α for which g is differentiable at all x.

3. Write the set of all positive integers in triangular array as

$$\begin{array}{cccccccc}
1 & 3 & 6 & 10 & 15 & \cdot & \cdot \\
2 & 5 & 9 & 14 & \cdot & \cdot & \cdot \\
4 & 8 & 13 & \cdot & \cdot & \cdot & \cdot \\
7 & 12 & \cdot & \cdot & \cdot & \cdot & \cdot \\
11 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}$$

Find the row number and column number where 20096 occurs. For example 8 appears in the third row and second column.

4. Show that the polynomial $x^8-x^7+x^2-x+15$ has no real root.

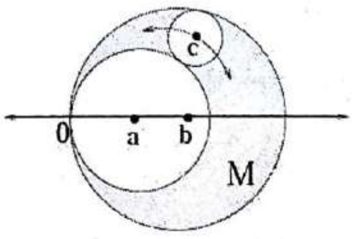
5. Let m be a natural number with digits consisting entirely of 6's and 0's. Prove that m is not the square of a natural number.

6. Let $0<a<b$.

(i) Show that amongst the triangles with base a and perimeter $a+b$ the maximum area is obtained when the other two sides have equal length $\frac{b}{2}$.

(ii) Using the result (i) or otherwise show that amongst the quadrilateral of given perimeter the square has maximum area.

7. Let $0 < a < b$. Consider two circles with radii a and b and centers $(a, 0)$ and $(0, b)$ respectively with $0 < a < b$. Let c be the center of any circle in the crescent shaped region M between the two circles and tangent to both (See figure below). Determine the locus of c as its circle traverses through region M maintaining tangency.



8. Let $n \geq 1$, and $S = \{1, 2, \ldots, n\}$.For a function $f : S \to S$, a subset $D \subset S$ is said t be invariant under f, if $f(x) \in D$ for all $x \in D$. Note that the emptyset and S are invariant for all f. Let deg(f) be the number of subsets of S invariant under f.

   (i) Show that there is a function $f : S \to S$ such that $\deg(f) = 2$.

   (ii) Further show that for any k such that $1 \leq k \leq n$ there is a function $f : S \to S$ such that $\deg(f) = 2^k$.

B.Stat. (Hons.) & B.Math. (Hons.) Admission Test: 2013

Short-Answer Type Test
Time: 2 hours

1. Let a, b, c be real numbers greater than 1. Let S denote the sum

$$S = \log_a bc + \log_b ca + \log_c ab.$$

Find the smallest possible value of S.

2. For $x \geq 0$ define

$$f(x) = \frac{1}{x + 2\cos(x)}.$$

Determine the set $\{y \in \mathbb{R} : y = f(x), x \geq 0\}$.

3. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function satisfying

$$|f(x+y) - f(x-y) - y| \leq y^2$$

for all $x, y \in \mathbb{R}$. Show that $f(x) = \frac{x}{2} + c$, where c is a constant.

4. In a badminton singles tournament, each player played against all the others exactly once and each game had a winner. After all the games, each player listed the names of all the players she defeated as well as the names of all the players defeated by the players defeated by her. For instance, if A defeats B and B defeats C, then in the list of A both B and C are included. Prove that at least one player listed the names of all other players.

5. Let AD be a diameter of a circle of radius r. Let B, C be points on the semi-circle (with C distinct from A) so that $AB = BC = \frac{r}{2}$. Determine the ratio of the length of the chord CD to the radius.

6. Let $p(x), q(x)$ be distinct polynomials with real coefficients such that the sum of the coefficients of each of the polynomials equals s. If $(p(x))^3 - (q(x))^3 = p(x^3) - q(x^3)$, then prove the following:

1. $p(x) - q(x) = (x-1)^a r(x)$ for some integer $a \geq 1$ and a polynomial $r(x)$ with $r(1) \neq 0$.
2. $s^2 = 3^{a-1}$ where a is as given in (a).

7. Let N be a positive integer such that $N(N - 101)$ is the square of a positive integer. Then determine all possible values of N. (Note that 101 is a prime number).

8. Let $ABCD$ be a square with the side AB lying on the line $y = x + 8$. Suppose C, D lie on the parabola $x^2 = y$. Find the possible values of the length of the side of the square.

B.Stat. (Hons.) & B.Math. (Hons.) Admission Test: 2014

Short-Answer Type Test

Time: 2 hours

1. A class has 100 students. Let a_i, $1 \leq i \leq 100$, denote the number of friends the i-th student has in the class. For each $0 \leq j \leq 99$, let c_j denote the number of students having *at least* j friends. Show that

$$\sum_{i=1}^{100} a_i = \sum_{j=0}^{99} c_j .$$

2. It is given that the graph of $y = x^4 + ax^3 + bx^2 + cx + d$ (where a, b, c, d are real) has at least 3 points of intersection with the x-axis. Prove that either there are *exactly* 4 distinct points of intersection, or one of those 3 points of intersection is a local minimum or maximum.

3. Consider a triangle PQR in $\mathbb{R}^2$. Let A be a point lying on $\triangle PQR$ or in the region enclosed by it. Prove that, for any function $f(x,y) = ax + by + c$ on $\mathbb{R}^2$,

$$f(A) \leq \max\{f(P), f(Q), f(R)\} .$$

4. Let f and g be two non-decreasing twice differentiable functions defined on an interval (a,b) such that for each $x \in (a,b)$, $f''(x) = g(x)$ and $g''(x) = f(x)$. Suppose also that $f(x)g(x)$ is linear in x on (a,b). Show that we must have $f(x) = g(x) = 0$ for all $x \in (a,b)$.

5. Show that the sum of 12 consecutive integers can never be a perfect square. Give an example of 11 consecutive integers whose sum is a perfect square.

6. Let A be the region in the xy-plane given by

$$A = \{(x,y) : x = u + v, y = v, u^2 + v^2 \leq 1\} .$$

Derive the length of the longest line segment that can be enclosed inside the region A.

7. Let $f : [0,\infty) \to \mathbb{R}$ be a non-decreasing continuous function. Show then that the inequality

$$(z-x)\int_y^z f(u)du \geq (z-y)\int_x^z f(u)du$$

holds for any $0 \leq x < y < z$.

8. Consider $n\,(> 1)$ lotus leaves placed around a circle. A frog jumps from one leaf to another in the following manner. It starts from some selected leaf. From there, it skips exactly one leaf in the clockwise direction and jumps to the next one. Then it skips exactly two leaves in the clockwise direction and jumps to the next one. Then it skips three leaves again in the clockwise direction and jumps to the next one, and so on. Notice that the frog may visit the same leaf more than once. Suppose it turns out that if the frog continues this way, then all the leaves are visited by the frog sometime or the other. Show that n *cannot* be odd.

B.Stat. (Hons.) & B.Math. (Hons.) Admission Test: 2015

Short-Answer Type Test

Time: 2 hours

1. Let $0 < a_1 < a_2 < \cdots < a_n$ be real numbers. Show that the equation

$$\frac{a_1}{a_1 - x} + \frac{a_2}{a_2 - x} + \cdots + \frac{a_n}{a_n - x} = 2015$$

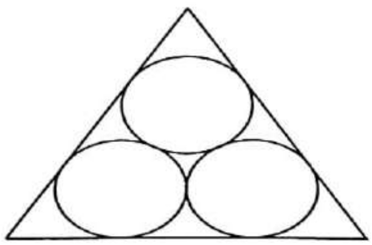
has exactly n real roots.

2. Let $\mathbb{R}$ denote the set of real numbers. Find all functions $f : \mathbb{R} \to \mathbb{R}$, satisfying

$$|f(x) - f(y)| = 2|x - y|$$

for all $x, y \in \mathbb{R}$. Justify your answer.

3. Three circles of unit radius tangentially touch each other in the plane. Consider the triangle enclosing them such that each side of the triangle is tangential to two of these three circles. See picture below:



Find the length of each side of the triangle.

4. Let a and b be real numbers. Define a function $f : \mathbb{R} \to \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers, by the formula $f(x) = x^2 + ax + b$. Assume that the graph of f intersects the co-ordinate axes in three distinct points. Prove that the circle passing through these three points also passes through the point $(0, 1)$.

5. Find all positive integers n for which $5^n + 1$ is divisible by 7. Justify your answer.

6. Let $p(x) = x^7 + x^6 + b_5x^5 + \cdots + b_1x + b_0$ and $q(x) = x^5 + c_4x^4 + \cdots + c_1x + c_0$ be polynomials with integer coefficients. Assume that $p(i) = q(i)$ for integers $i = 1, 2, \ldots, 6$. Then, show that there exists a negative integer r such that $p(r) = q(r)$.

7. Let $S = \{1, 2, \ldots, l\}$. For every non-empty subset A of S, let $m(A)$ denote the maximum element of A. Then, show that

$$\sum m(A) = (l-1)2^l + 1$$

where the summation in the left hand side of the above equation is taken over all non-empty subsets A of S.

8. 1. Let $m_1 < m_2 < \cdots < m_k$ be positive integers such that $\frac{1}{m_1}, \frac{1}{m_2}, \cdots, \frac{1}{m_k}$ are in arithmetic progression. Then prove that $k < m_1 + 2$.

   2. For any integer $k > 0$, give an example of a sequence of k positive integers whose reciprocals are in arithmetic progression.

B.Stat. (Hons.) & B.Math. (Hons.) Admission Test: 2016

Short-Answer Type Test

Time: 2 hours

1. Suppose that in a sports tournament featuring n players, each pair plays one game and there is always a winner and a loser (no draws). Show that the players can be arranged in an order $P_1, P_2, \cdots, P_n$ such that player P_i has beaten P_{i+1} for all $i = 1, 2, \cdots, n-1$.

2. Consider a cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ where a, b, c, d are integers such that ad is odd and bc is even. Prove that not all roots of $p(x)$ can be rational.

3. Given the polynomial
$$f(x) = x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n,$$
with real coefficients, and $a_1^2 < a_2$, show that not all roots of $f(x)$ can be real.

4. Let d be a positive integer. Prove that there exists a right-angled triangle with rational sides and area equal to d if and only if there exists an arithmetic progression x^2, y^2, z^2 of squares of rational numbers whose common difference is d.

5. Let $ABCD$ be a square two of whose adjacent vertices, say A, B, are on the positive X-axis and the positive Y-axis, respectively. If C has co-ordinates (u, v) in the first quadrant, determine the area of $ABCD$ in terms of u and v.

6. Let a, b, c be the sides of a triangle and A, B, C be the angles opposite to these sides respectively. If
$$\sin(A - B) = \frac{a}{a+b}\sin A\cos B - \frac{b}{a+b}\cos A\sin B,$$
then prove that the triangle is isosceles.

7. Let f be a differentiable function such that $f(f(x)) = x$ for $x \in [0, 1]$. Suppose $f(0) = 1$. Determine the value of $\int_0^1 (x - f(x))^{2016}\, dx$.

8. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers defined recursively by
$$a_{n+1} = \frac{3a_n}{2 + a_n},$$
for all $n \geq 1$.

   (i) If $0 < a_1 < 1$, then prove that $\{a_n\}_{n\geq 1}$ is increasing and $\lim_{n\to\infty} a_n = 1$.

   (ii) If $a_1 > 1$, then prove that $\{a_n\}_{n\geq 1}$ is decreasing and $\lim_{n\to\infty} a_n = 1$.