MATHS(Solution)

INSTUCTIONS

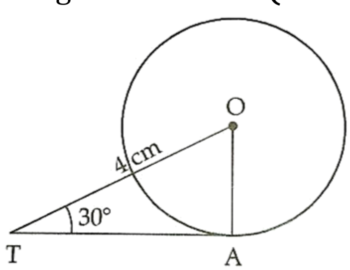
1. This Question paper has 5 Section A, B, C, D and E.
2. Section A has 20 Multiple Choice Question (MCQs) carrying 1 mark each.
3. Section B has 5 short Answer-I (SA-I) type question carrying 2 mark each.
4. Section C has 6 short Answer-II(SA-II) type question carrying 3 mark each.
5. Section D has 4 Long Answer(LA) type question carrying 5 mark each.
6. Section E has 3 Case Based integrated units of assessment (4 marks each).
7. All Questions are compulsory. However, an internal choice in 3 Questions of 2 marks, 3 Question of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks question of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

Section A

1. The Pair of linear equation $2x = 5y + 6$ and $15y = 6x - 18$ represent two lines Which are: **(ANSWER-C)**

(a)intersecting
(b)parallel
(c)coincident
(d)either intersecting or parallel

2. In the given figure, TA is a tangent to the circle with centre O such that OT = 4cm, $\angle OTA = 30^0$, then length of TA is: **(ANSWER-A)**



(a)$2\sqrt{3}cm$ (b)2cm
(c) $2\sqrt{2}cm$ (d) $\sqrt{3}cm$

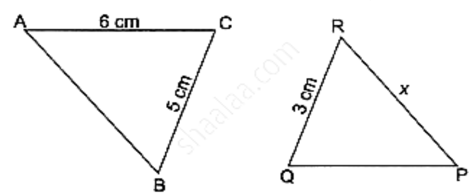
3. The ratio of HCF to LCM of the least composite number and the least prime number is:

(a)1:2 (b)2:1
(c)1:1 (d)1:3 **(ANSWER-A)**

4. If a pole 6m high casts a shadow $2\sqrt{3}m$ long on the ground, then sun's elevation is:

(a)60^0 (b)45^0
(c)30^0 (d)90^0 **(ANSWER-A)**

5. In the given figure, $\Delta ABC \sim \Delta QPR$. If $AC = 6cm, BC = 5cm, QR = 3cm$ and PR =x; then the value of x is: **(ANSWER-B)**



(a)$3.6cm$ (b)$2.5cm$
(c)$10cm$ (d)$3.2cm$

6. The distance of the point (-6, 8) from origin is:

(a)6 (b)-6
(c)8 (d)10 **(ANSWER-D)**

7. The next term of the A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}$ is;

(a)$\sqrt{70}$ (b)$\sqrt{80}$
(c)$\sqrt{97}$ (d)$\sqrt{112}$ **(ANSWER-D)**

8. $(\sec^2\theta - 1)\ (cosec^2\theta - 1)$ is equal to:

(a)-1 (b)1
(c)0 (d)2 **(ANSWER-B)**

9. Two dice are thrown together. The probability of getting the difference of number on their upper faces equals to 3 is:

(a)$\frac{1}{9}$ (b)$\frac{2}{9}$

(c)$\frac{1}{6}$ (d)$\frac{1}{12}$ **(ANSWER-C)**

10. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that the cards drawn is not an ace is: **(ANSWER-D)**

(a)$\frac{1}{13}$ (b)$\frac{9}{13}$

(c)$\frac{4}{13}$ (d)$\frac{12}{13}$

11. The roots of the equation $x^2+3x-10=0$ are:

(a) 2,-5 (b)-2, 5

(c) 2,5 (d)-2,-5 **(ANSWER-A)**

12. If α, β are zeroes of the polynomial $x^2 - 1$, then value of $(\alpha + \beta)$ is:

(a)2 (b)1

(c)-1 (d)0 **(ANSWER-D)**

13. If α, β are the zeroes of the polynomial p(x)= $4x^2-3x-7$, the $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ is equal to:

(a)$\frac{7}{3}$ (b)$-\frac{7}{3}$

(c)$\frac{3}{7}$ (d)$-\frac{3}{7}$ **(ANSWER-D)**

14. What is the area of a semi-circle of diameter \`d\`?

(a)$\frac{1}{16}\pi d^2$ (b)$\frac{1}{4}\pi d^2$

(c)$\frac{1}{8}\pi d^2$ (d)$\frac{1}{2}\pi d^2$ **(ANSWER-C)**

15. For the following distribution:

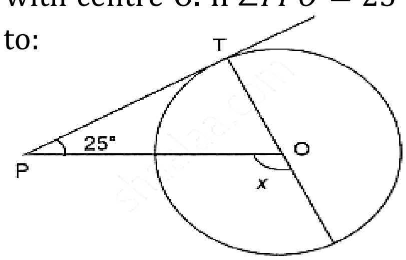
Mark Below	10	20	30	40	50	60
Number of student	3	12	27	57	75	80

The modal class is:

(a)10-20 (b)20-30

(c)30-40 (d)50-60 **(ANSWER-C)**

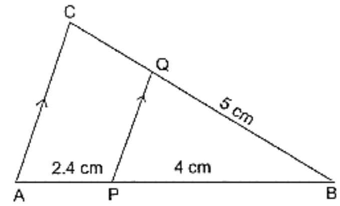
16. In the given figure, PT is a tangent at T to the circle with centre O. If $\angle TPO = 25^0$, then x is equal to:



(a) 25^0 (b)65^0

(c) 90^0 (d)115^0 **(ANSWER-D)**

17. In the given Figure, PQ ∥ AC. If BP = 4cm, AP = 2.4cm and BQ = 5cm, then length of BC is: **(ANSWER-A)**



(a)$8cm$ (b)$3cm$

(c)$0.3cm$ (d)$\frac{25}{3}$ cm

18. The points(-4, 0), (4 , 0) and (0,3) are the vertices of a: **(ANSWER-B)**

(a)right triangle (b)isosceles triangle

(c)equilateral triangle (d)Scalene triangle

Direction: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct option.

19. Assertion (A): The probability that a leap year has 53 Sundays is $\frac{2}{7}$.

Reason (R): The probability that a non-leap year has 53 Sundays is $\frac{5}{7}$. **(ANSWER-C)**

(a)Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b)Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of assertion (A).

(c)Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

20. Assertion (A): a, b, c are in A.P. if and only if 2b=a + c.

Reason (R): The sum of first n odd natural number is n^2. **(ANSWER-B)**

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION-B

Section B consists of 5 questions of 2 marks each

21. (a) The line segment joining the points $A(4,-5)$ and $B(4,5)$ is divided by the point P such that $AP:AB = 2:3$. Find the coordinates of P.

Or

(b) Point $P(x,y)$ is equidistant from points A$(5,1)$ and $B(1,5)$. Prove that $x = y$.

(a) Let point $P(x,y)$ divides the line segment joining the points $A(4,-5)$ and $(4,5)$ in the ratio 2: 3.

2 : 3

A P B

$(4,-5)$ (x,y) $(4,5)$

By using section formula, we get

$x = \frac{(4\times3)+(2\times4)}{2+3}$ and $y = \frac{(2\times5)+(3\times-5)}{2+3}$

$\Rightarrow \quad x = \frac{12+8}{5} = \frac{20}{5}\ and\ y = \frac{10-15}{5} = \frac{-5}{5}$

$\therefore \quad x = 4\ and$ y=-1

So, coordinates of P are $(4,-1)$ (1)

(b) Given, $P(x,y)$ is equidistant from points $A(5,1) and\ B(1,5)$

$\Rightarrow \quad AP = BP$

$\Rightarrow \quad AP^2 = BP^2$

$\Rightarrow \quad (x-5)^2+(y-1)^2 = (x-1)^2+(y-5)^2$

[distance formula]

$\Rightarrow \quad x^2+25-10x+y^2+1-2y$

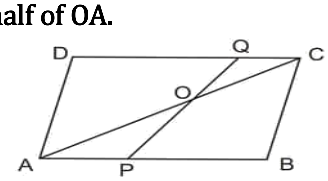
$\Rightarrow \quad x^2+1-2x+y^2+25-10y$

$\Rightarrow \quad -10x-2y = -2x-10y$

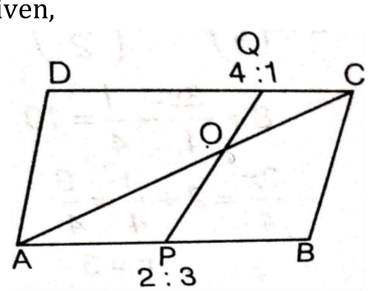
$\Rightarrow \quad x = -8x = -8y$

$\Rightarrow \quad x = y$ **Hence proved.** (1)

22. $ABCD$ is a parallelogram. Point P divides AB in the ratio 2 : 3 and point Q divides DC in the ratio 4 : 1. Prove that OC is half of OA.



Sol. Given,



In $\Delta OAP\ and\ \Delta OCQ$,

$\therefore \quad \angle AOP = \angle COQ$ [vertically opposite angles]

$\therefore \quad \angle OAP = \angle OCQ$ [alternate angles]

$\Rightarrow \quad \Delta OAP \sim \Delta OCQ$ [by AA similarity criterion]

$\therefore \quad \frac{OA}{OC} = \frac{AP}{CQ}$ ....(i)

Given, $\frac{AP}{PB} = \frac{2}{3}$ and $\frac{DQ}{QC} = \frac{4}{1}$

$\Rightarrow \quad \frac{AP}{AB} = \frac{2}{5}$ and $\frac{QC}{DC} = \frac{1}{5}$

$\Rightarrow \quad AP = \frac{2}{5} AB$ and $QC = \frac{1}{5} DC$

From Eq. (i),

$$\frac{OA}{OC} = \frac{\frac{2}{5}AB}{\frac{1}{5}DC} = 2$$ [∵ABCD is a parallelogram ∴ AB=CD]

$\Rightarrow \quad OC = \frac{1}{2} OA$

23. **(a)If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0^0 < A+B < 90^0$; $A > B$, find A and B.**

Or

(b)Find the value of x. $2cosec^2 30^0 + x sin^2 60^0 - \frac{3}{4}\tan^2 30^0 = 10$

Sol. Given, $\tan(A+B) = \sqrt{3} = \tan 60^0$

and $\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^0$

⇨ $A + B = 60^0$ and $A - B = 30^0$

⇨ $2A = 90^0$

⇨ $A = 45^0$

and $B = 15^0$

Or

We have,

$$2\, cosec^2 30^0 + x \sin^2 60^0 - \frac{3}{4}\tan^2 30^0 = 10$$

⇨ $2 \times \left(\frac{2}{1}\right)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$

⇨ $8 + \frac{3x}{4} - \frac{1}{4} = 10$

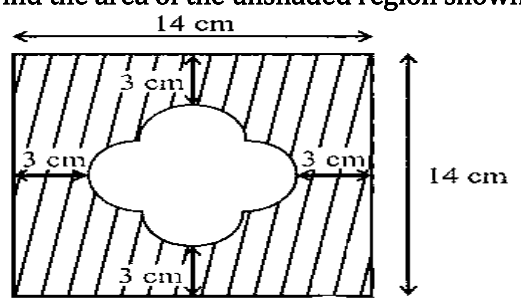
⇨ $\frac{3x}{4} = 2 + \frac{1}{4} = \frac{9}{4}$

⇨ $x = 3$

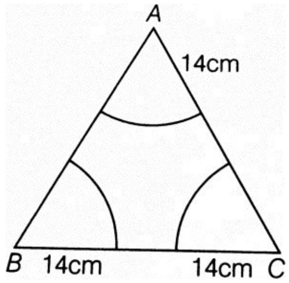
24. **(a)With vertices A, B and C of ΔABC as centres, arcs are drawn with radii 14 cm and the three portions of the triangle, so obtained are removed. Find the total area removed from the triangle.**

Or

(b)Find the area of the unshaded region shown in the given figure.



Sol. According to the given information, the figure is given below.



Area of sector with $\angle A = \frac{\angle A}{360^0} \times \pi \times r^2$

$= \frac{\angle A}{360^0} \times \pi \times (14)^2$

Similarly, other two areas are

$\frac{\angle B}{360^0} \times \pi \times (14)^2$ and $\frac{\angle C}{360^0} \times \pi \times (14)^2$

Total area removed $= (\angle A + \angle B + \angle C)\frac{\pi \times (14)^2}{360^0}$

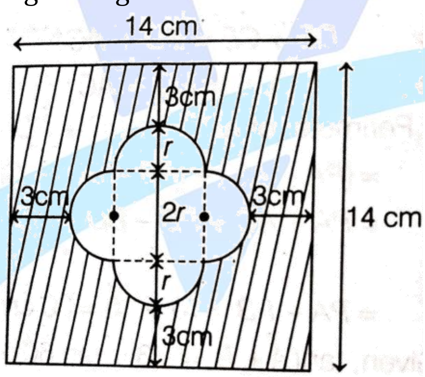
$= \frac{180^0}{360^0} \times \frac{22}{7} \times 14 \times 14$

$[\because \angle A + \angle B + \angle C = 180^0]$

$= 11 \times 2 \times 14 = 308\ cm^2$

Or

The given figure is



Let the radius of each semi-circle$= r$

$\therefore$ From figure,

$3 + r + 2r + r + 3 = 14$

$4r + 6 = 14$

$r = \frac{8}{4} = 2\ cm$

Area of unshaded region

$= 4 \times$ Area of each semi-circle $+$ Area of square with length of each side, 2r

$= 4 \times \frac{\pi r^2}{2} + (2r)^2$

$= 2\pi r^2 + 4r^2 = 2r^2(\pi + 2)cm^2$

25. A bag contains 8 red balls and some blue balls. If the probability of drawing a blue ball is three times of a red ball, then find the number of blue balls in the bag.

Sol.- Let there be x blue balls in the bag.

$\therefore$ Total number of balls in the bag$= (8 + x)$

Now, P_1 =Probability of drawing a blue ball $= \frac{x}{8+x}$ (1)

and $\quad P_2 =$ probability of drawing a red ball $\frac{8}{8+x}$ (1)

It is given that

$$P_1 = 3P_2$$

➔ $\frac{x}{8+x} = 3 \times \frac{8}{(8+x)}$

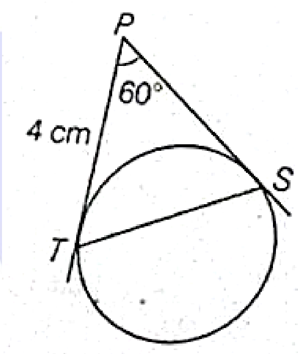
➔ $\frac{x}{8+x} = \frac{24}{8+x} = x \Rightarrow 24$

Hence, there are 24 blue balls in the bag. (1)

SECTION C

Section C consists of 6 questions of 3 marks each

26. (a) In the given figure, PT and PS are tangents to a circle from a point P such that $PT = 4cm$ and $\angle TPS = 60^0$.



Find the length of chord TS. How many lines of same length TS can be drawn in the circle?

Or

(b)AB is a diameter and AC is a chord of a circle such that $\angle BAC = 30^0$. If the tangent at C intersects AB produced at D, then prove that BC=BD.

Sol. We know that tangents drawn from external point to the circle are equal in length.

Here, P is an external point.

$\therefore \quad PS = PT = 4cm$

So, $\quad \angle PTS = \angle PST \quad$ [$\because$ *angles opposite to equal sides are equal*]

In $\quad \Delta PTS$, we have

$\angle PTS + \angle PST + \angle TPS = 180^0 \quad [\because \angle PST = \angle PTS \text{ and } \angle TPS = 60^0]$

⇨ $2\angle PTS = 180^0 - 60^0$

⇨ $2\angle PTS = 120^0$

⇨ $\angle PTS = \frac{120^0}{2} = 60^0$

$\therefore \Delta PTS$, is an equilateral triangle.

Hence, $\quad TS = 4\ cm$

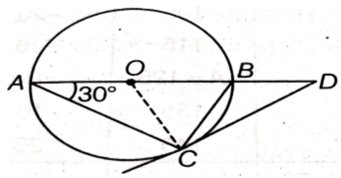
Here, infinite lines of same length TS can be drawn in a circle.

Or

Given AB is a diameter of the circle with centre O and DC is the tangent of circle and $\angle BAC = 30^0$

To prove $BC = BD$

Construction Join O to C.



Proof Since, $OC \perp CD$

[$\because$ The tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore\ \angle OCB + \angle BCD = 90^0$

27. If α and β are the zeroes of the quadratic polynomial $f(x) = 3x^2 - 5x - 2$, then find the value of $\alpha^3 + \beta^3$.

Sol. Given α and β are the zeroes of the quadratic polynomial.

$f(x) = 3x^2 - 5x - 2$

On comparing with $f(x) = ax^2 + bx + c$, we get

$a = 3, b = -5$ and $C = -2$

We know that

$\alpha + \beta = -\frac{b}{a} = \frac{-(-5)}{3} = \frac{5}{3}$ and $\alpha.\beta = \frac{c}{a} = -\frac{2}{3}$

$\therefore \alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha.\beta + \beta^2)$

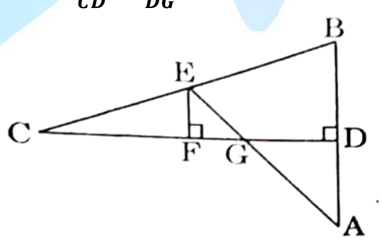
$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha.\beta]$

$= \frac{5}{3}\left[\left(\frac{5}{3}\right)^2 - 3 \times \left(-\frac{2}{3}\right)\right]$

$= \frac{125}{27} + 2 \times \frac{5}{3}$

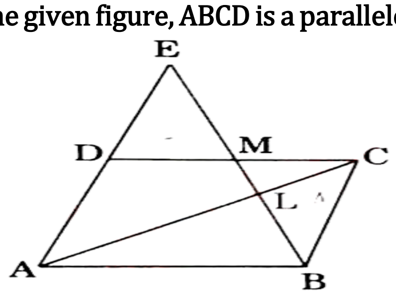
$= \frac{125+90}{27} = \frac{215}{27}$

28. (a) In the given figure CD is the perpendicular bisector of AB. EF is perpendicular to CD. AE intersects CD at G. Prove that $\frac{CF}{CD} = \frac{FG}{DG}$.



Or

(b) In the given figure, ABCD is a parallelogram. BE bisects CD at M and intersects AC at L. Prove that $EL = 2BL$.



Sol. (a) In ΔEFC and ΔBDC,

$\angle EFC = \angle BDC$ [90^0 each]

$\angle ECF = \angle BCD$ [common]

By AA similarity criterion,

$\Delta EFC \sim \Delta BDC$

$\Rightarrow \frac{EF}{BD} = \frac{FC}{DC}$ [by CPCT]..........(i)

Now, in ΔEFG and ΔADG

$\angle EGF = \angle DGA$ [vertically opposite angles]

$\angle EFG = \angle GDA$ [90^0 each]

By AA similarity criterion,

$\Delta EFG \sim \Delta ADG$

$\Rightarrow \frac{EF}{AD} = \frac{FG}{DG}$ [by CPCT]

As $AD = BD \Rightarrow \frac{EF}{BD} = \frac{FG}{DG}$

From Eqs. (i) and (ii), we get

$\frac{CF}{CD} = \frac{FG}{DG}$ **Hence proved**

(b) In ΔBMC and ΔEMD,

$\angle BMC = \angle EMD$ [vertically opposite angles]

$MC = DM$ [given]

$\angle BCM = \angle EMD$ [alternate angles]

$\therefore \Delta BMC \cong \Delta EMD$ [by ASA congruency criteria]

$\Rightarrow BC = DE$ [by CPCT] ....(i)

Now, $AE = AD + DE$

$= BC + BC = 2BC$ (ii) (1/2)

In ΔALE and ΔCLB

$\angle LAF = \angle LCB$ [alternate angles]

$\angle LEA = \angle LBC$ [alternate angles]

$\therefore \Delta ALE \sim \Delta CLB$ [by AA similarity criterion]

$\Rightarrow \frac{EL}{BL} = \frac{AE}{CB}$ [by CPCT]

$\Rightarrow \frac{EL}{BL} = \frac{2BC}{BC}$ [from Eq. (ii)]

$\Rightarrow \frac{EL}{BL} = 2 \Rightarrow EL = 2BL$ Hence proved.

29. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

Sol. We have, LHS. $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta} = \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta(1-\tan\theta)}$

$= \frac{1-\tan^3\theta}{\tan\theta(1-\tan\theta)}$

$= \frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta(1-\tan\theta)}$

$= \frac{\sec^2\theta+\tan\theta}{\tan\theta}$ [$\because 1 + \tan^2 A = \sec^2 A$]

$= \frac{\sec^2\theta}{\tan\theta} + 1 = \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta} + 1$

[$\because cosA = \frac{1}{\sec A}, \tan A = \frac{\sin A}{\cos A}$]

$= \sec\theta cosec\theta + 1$ [$\because \sin A = \frac{1}{cosecA}$]

$=$ RHS.

30. **(a) Prove that $\sqrt{3}$ is an irrational number.**

Or

(b) The traffic lights at three different road crossing change after every 48s, 72s and 108s, respectively. If they change simultaneously at 7 am, at what time will they change together next?

Sol. (a) Let us assume to the contrary that $\sqrt{3}$ is rational i.e.

we can find integers a and $b(\neq 0)$ such that $\sqrt{3} = \frac{a}{b}$.

Suppose $a\ and\ b$ have a common factor other than 1, then we can divide by the common factor and assume that $a\ and\ b$ are co-prime.

So, $b\sqrt{3} = a$

Squaring on both sides and rearranging, we get

$3b^2 = a^2$

$\therefore a^2$ is divisible by 3

➔ a is divisible by 3

So, we can write a $=3c$ for some integer c

Substituting for a, we get $3b^2 = 9c^2$

i.e. $b^2 = 3c^2$

➔ b^2 is divisible by 3

➔ b is dividable by 3

$\therefore$ $a\ and\ b$ have atleast 3 as a common factor.

But this contradicts the fact that a and b are co-prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.

So, we conclude that $\sqrt{3}$ is an irrational number

(b) Given, the traffic light at three different road crossing change after 48 s, 72 s and 108 s.

Using prime factorization, we get

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

and $108 = 2 \times 2 \times 3 \times 3 \times 3$

LCM of 48, 72 and 108

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 432$

Therefore, after 432 s, they will change

Simultaneously.

We know that 60 s = 1 min

⇨ $1s = \frac{1}{60}\ min$

⇨ $432s = \frac{432}{60}$ min

$= 7$ min 12 s

Hence, the lights change simultaneously at 7 : 07 : 12 am.

31. If pth term of an AP is q and qth term is p, then prove that its nth term is $(p + q - n)$.

Sol. Given, pth term of AP$=q$

and qth term of $AP = p$

To prove, nth term of $AP = p + q - n$

Let a be the first term and d be the common difference of the AP.

Since, p the term is q

$\Rightarrow \qquad a_p = q$

$\Rightarrow \qquad a + (p-1)d = q$ ....(i) 1

Similarly, qth term is =p

$\Rightarrow \qquad a_q = p$

$\Rightarrow \qquad a + (q-1)d = p$ ....(ii)

Subtracting Eq. (ii) from Eq. (i)

$(p - 1 - q + 1)d = q - p$

$(p - d)d = q - p$

$d = -1$ (1)

From Eq. (i).

$a + (p-1)(-1) = q$

$a - p + 1 = q$

$a = p + q - 1$

$\therefore \quad nth$ term, $a_n = a + (n-1)d$

$= p + q - 1 + (n-1)(-1)$

$= p + q - 1 - n + 1$

$= p + q - n$ **Hence proved. (1)**

▪ SECTION D consists of 4 questions of 5 marks each

32. Draw the graphs of $2x + y = 6$ and $2x - y + 2 = 0$. Shade the region bounded by these lines and X-axis. Find the area of the shaded region.

Or

Places P_1 and P_2 are 250 km apart from each other on a national highway. A car starts from P_1 and another from P_2 at the same time. If they go in the same direction, then they meet in 5 h and if they go in opposite directions they meet in $\frac{25}{13}$ h. then find their speeds.

Sol. We have, $2x + y = 6$ and $2x - y + 2 = 0$

Table for equation $y = 6 - 2x$ is

X	0	3
y	6	0

On plotting the points $A(0,6)$ and $B(3,0)$ on a graph paper and join them, we obtain the graph of line represented by the equation $2x + y = 6$ as shown in the figure.

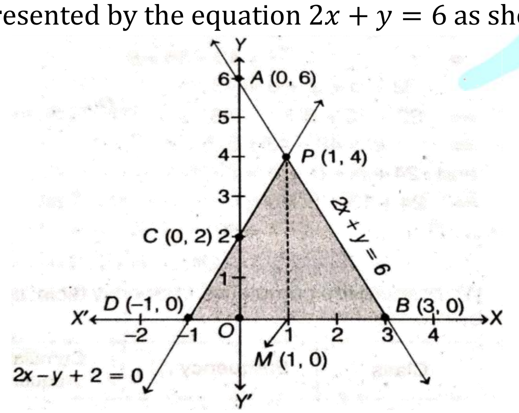


Table for equation $y = 2x + 2$ is

X	0	-1
y	2	0

On plotting the points $C(0, 2)$ abd $D(-1, 0)$ on the same graph paper and join them, we obtain the graph of line represented by the equation $2x - y + 2 = 0$ as shown in the figure.

The two line intersect at point $P(1, 4)$

Thus, $x = 1$ and $y = 4$ is the solution of the given system of equations. The area enclosed by the lines and X-axis is shaded part in the figure. Draw PM perpendicular fro P on X-axis.

Clearly, we have

PM= y-coordinate of point $P(1, 4)$

⇨ PM= 4 and DB=4

Area of the shaded region = Area of ΔPBD

$$= \frac{1}{2} \times Base \times Height$$

$$= \frac{1}{2}(DM \times PM)$$

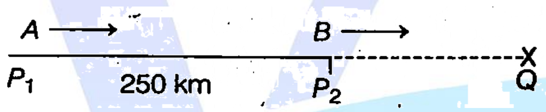
$$= \frac{1}{2} \times 4 \times 4$$

= 8 sq. units (1)

Or

Let A and B be the two cars. A starts from P_1 with constant speed of $x\ km/h$ and B starts from P_2 with constant speed of $y\ km/h$.

Case I When the two cars move in same directions as shown in figure, the cars meet at the position Q.



Here, $P_1Q = 5x\ km$, i.e. the distance travelled by car a in 5 h with $x\ km/h$ speed.

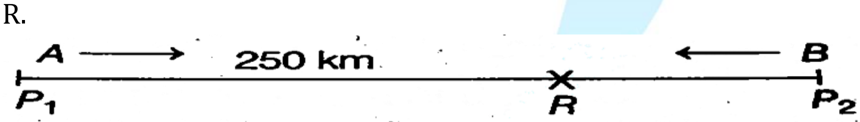
$P_2Q = 5\ y\ km$ i.e. the distance travelled by car B in 5 h with $y\ km/h$ speed.

We have, $P_1Q - P_2Q = 250$

⇨ $5x - 5y = 250$

⇨ $x - y = 50$ .....(i) (2)

Case II When two cars move in opposite directions as shown in figure, the cars meet at the position R.



Here, $P_1R = \frac{25}{13}x\ km$ and $P_2R = \frac{25}{13}y\ km$

So, $P_1R + P_2R = 250$

⇨ $x = y = 130$ ......(ii) (2)

On adding Eq. (i) and Eq. (ii), we get

$2x = 180$

⇨ $x = 90$

On adding Eq. (i) and Eq. (ii), we get

$2y = 80$

⇨ $y = 40$

∴ Their speeds are 90 km/h and 40 km/h. (1)

33. (a)Find the unknown entries a, b, c, d, e and f in the following distribution of heights of students in a class

Height (in cm)	Frequency	Cumulative frequency
150-155	12	a
155-160	b	25
160-165	10	c
165-170	d	43
170-175	e	48
175-180	2	f
Total	50	

Or

(b)Find the median for the following frequency distribution.

Class	0-8	8-16	16-24	24-32	32-40	40-48
Frequency	8	10	16	24	15	7

Sol.- The cumulative frequency table for the given continuous distribution is given below.

Height (in cm)	Frequency	Cumulative frequency (given)	Cumulative frequency (cf)
150-155	12	a	12
155-160	b	25	12+b
160-165	10	c	22+b
165-170	d	43	22+b+d
170-175	e	48	22+b+d+e
175-180	2	f	24+b+d+e
Total	50		

On comparing last two tables, we get

a=12

∴ 12+b=25

⇨ B=25−12=13

22+b=c

⇨ c=22+12=35

22+b+d=43

⇨ 22+13+d=43

⇨ D=43−35=8

22+b+d+e=48

⇨ 22+13+8+e=48

⇨ e=48−43=5

and 24+b+d+e=f

⇨ 24+13+8+5=f

∴ $f = 50$

Or

We prepare the cumulative frequency table, as given below.

Class	Frequency (f_i)	Cumulative frequency
0-8	8	8
8-16	10	18
16-24	16	$34(cf)$
24-32	24(f)	58

32-40	15	73
40-48	7	80
Total	$N = \sum f_i = 80$	

Now, $N = 80$

⇨ $\left(\frac{N}{2}\right) = 40$

The cumulative frequency just greater than 40 is 58 and the corresponding class is 24-32.

∴ $l = 24, f = 24\ cf = 34\ and\ h = 8$

∴ Median$= l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

$= 24 + \left\{8 \times \frac{(40-34)}{24}\right\}$

$= (24 + 2) = 26$

Hence, the median is 26.

34. Find the nature of roots of the following quadratic equations. In case real roots exist, find them

(i) $\boldsymbol{4x^2 + 12x + 9 = 0}$

(ii) $\boldsymbol{3x^2 + 5x - 7 = 0}$

Sol. (i) Given, quadratic equation is

$$4x^2 + 12x + 9 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 4, b = 12\ and\ c = 9$$

Now, D$=b^2 - 4ac$

$= (12)^2 - 4(4)(9)$

Since, D=0 so given quadratic equation has two equal and real roots which are given by

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-12 \pm 0}{2(4)}$$

⇨ $x = \frac{-12+0}{8}$

Or $x = \frac{-12-0}{8}$

⇨ $x = -\frac{3}{2}\ or\ x = -\frac{3}{2}$

Hence, the roots are $\frac{-3}{2}\ and - \frac{3}{2}$.

(ii) Given, quadratic equation is

$3x^2 + 5x - 7 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 3, b = 5\ and\ c = -7$

= 25+84=109

Since, D>0, so given quadratic equation has two distinct real roots which are given by

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{109}}{2(3)}$

⇨ $x = \frac{-5 \pm \sqrt{109}}{6}$ [taking positive sign]

Or $x = \frac{-5 - \sqrt{109}}{6}$ [taking negative sign]

Hence, the root are $\frac{-5+\sqrt{109}}{6}$ and $\frac{-5-\sqrt{109}}{6}$

35. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also, find the increase in grazing area if length of rope is increased to 10 m. ($use\ \pi = 3.14$)

Sol. Given, side of square = 15 m

and length of rope = 5 m

$\therefore$ Radius of arc = 5 m

$\therefore$ Area of part of field in which horse can graze = Area of sector QBP

$= \frac{\theta}{360^0} \times \pi r^2$

$= \frac{90^0}{360^0} \times 3.14 \times 5 \times 5$ [$\because$ each angle of square is 90^0]

$= 19.625\ m^2$ (2 1/2)

Area of part of field in which horse can graze when rope is 10 m long.

= Area of sector HGB

$= \frac{\theta}{360^0} \times \pi r^2$

$= \frac{90^0}{360^0} \times 3.14 \times 10 \times 10$

$= 78.5\ m^2$

Increase in grazing area

= Area of sector HGB – Area of sector QBP

$= 78.5 - 19.625$

$= 58.875\ m^2$

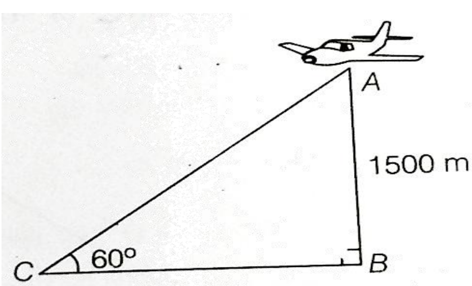
■SECTION E

Section E consists of 3 Case based questions of 4 marks each.

36. Flight Features

The aviation technology has evolved many up gradations in the last few years. It has taken in account, speed, direction, and distance as well as other features of the flight. Even the wind plays a vital role, when a plane travels.

Angle of elevation The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level.



Based on the above information, answer the following questions.

(i) If the point C moves towards the point$B$, then how does the angle of elevation vary? (1)

(ii) Find the distance of point C form the object. (2)

Or

What is the value of distance BC? (2)

Sol. (i) If the point C moves towards the point B, then angle of elevation increases.

(ii) In right angled ΔACB,

$$\sin 60^0 = \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{1500}{AC}$$

⇨ $$AC=1500 \times \frac{2}{\sqrt{3}} = \frac{3000}{3} \times \sqrt{3}$$

$$= 1000\sqrt{3}m$$

Or

In ΔABC, $\tan 60^0 = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{1500}{BC}$

⇨ $BC = \frac{1500}{\sqrt{3}} = 500\sqrt{3}m$

(iii) If angle of elevation changes from 60^0 to 45, then in $\Delta ABC0$

$$\tan 45^0 = \frac{AB}{BC}$$

⇨ $$1=\frac{1500}{BC}$$

⇨ $$BC = 1500\ m$$

37. **Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of ₹118000 by paying every month starting with the first instalment of ₹1000. If he increase the instalment by ₹100 every month.**

On the basis of above information, answer the following questions.

(i) **What amount does he still have to pay after 30th instalment?**

(ii) **Find the amount paid by him in 30th instalment?**

Or

Find the amount paid by him in the 30 instalments.

(iii) **If total instalments are 40, then what amount paid in the last instalment.**

Sol.- (i) After 30th instalment, he still have to pay

$$= 118000 - 73500 = ₹44500$$

(ii) Since, he pays first instalments of ₹ 1000 and next consecutive months he pay the instalment are 1100, 1200, 1300,…………….

Thus, we get the AP sequence,

1000, 1100, 1200,……………..

Here, a = 1000 and d=1100−1000=100

Now, $T_{30} = a + (30-1)d$ $\quad [\because T_n = a + (n-1)d]$

= 1000+29×100

= 1000+2900=3900

Hence, the amount paid by him in 30th instalment is ₹3900.

Or

$\therefore\ S_{30} = \frac{30}{2}\ [2a + (30-1)d]$ $\quad\left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$

$= 15(2 \times 1000 + 29 \times 100)$

$= 15(2000 + 2900)$

$= 15 \times 4900$

$=$ ₹73500

(iii) The amount in last 40th instalment is

$T_{40} = a + (40-1)d$

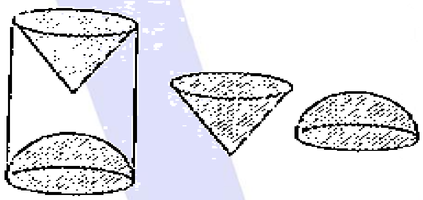
$= 1000 + 39 \times 100$

$= 1000+3900$

$=$ ₹4900

38. Cylindrical Wooden Article

A wooden article was made by scooping out a hemisphere from one end of a cylinder and cone from the other end as shown in the figure. If the height of the cylinder is 40 cm, radius of cylinder is 7 cm and height of the cone is 24 cm.



On the basis of above information, answer the following questions.

(i) Find the slant height of the cone and volume of hemisphere. (2)

Or

Find the total volume of the article. (2)

(ii) Find the curve surface area of cone. (1)

(iii) Find the surface area of the article. (1)

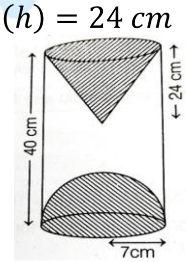
Sol. Given, height of the cylinder $(H) = 40\ cm$

Radius of the cylinder $(r) = 7cm$

Radius of the hemisphere $(r) = 7cm$

Radius of the cone$(r) = 7\ cm$

Height of the cone$(h) = 24\ cm$



(i) Slant height of the cone $(l) = \sqrt{h^2 + r^2}$

$= \sqrt{(24)^2 + (7)^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625} = 25\ cm$

and volume of hemisphere

$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (7)^3$

$= \frac{44}{3} \times 49$

$= \frac{2156}{3} = 718.67\ cm^3$

Or

Volume of the article

= Volume of the cylinder – Volume of the cone –Volume of the cone

$\left[\pi r^2 H - \frac{1}{3}\pi r^2 h - \frac{2}{3}\pi r^3\right]$

$= \pi r^2\left[H - \frac{1}{3}h - \frac{2}{3}r\right] = \pi r^2\left[40 - \frac{1}{3}(24) - \frac{2}{3}(7)\right]$

$= \frac{22}{7} \times 7 \times 7 \times \left[40 - 8 - \frac{14}{3}\right] = 154 \times \left[\frac{96-14}{3}\right]$

$= 4209.33\ cm^3$

(ii) Curve surface area of cone $= \pi r l$

$= \frac{22}{7} \times 7 \times 25 = 22 \times 25 = 550\ cm^2$

(iii) Total surface area of the article= Curved surface area of the cylinder + Curved surface area of the cone +Surface area of the hemisphere

$= 2\pi r H + \pi r l + 2\pi r^2 = \pi r[2H + l + 2r]$

$= \frac{22}{7} \times 7 \times [2 \times 40 + 25 + 2 \times 7]$

$= 22 \times [80 + 25 + 14] = 22 \times 119 = 2618\ cm^2$