MATHS SET-1

MATHS SAMPLE PAPER(SOLUTION)

CLASS- X^{TH} **SESSION-2024-25**

TIME ALLOWED: 3 HRS **MAXIMUM MARKS:-80**

General Instructions

1. *This Question paper has 5 Section A, B, C, D and E.*
2. *Section A has 20 Multiple Choice Question (MCQs) carrying 1 marks each.*
3. *Section B has 5 short Answer-I (SA-I) type question carrying 2 marks each.*
4. *Section C has 6 short Answer-II(SA-II) type question carrying 3 marks each.*
5. *Section D has 4 Long Answer (LA) type question carrying 5 marks each.*
6. *Section E has 3 Case Based integrated units of assessment (4 marks each).*
7. *All Questions are compulsory. However, an internal choice in 2 Questions of 2 marks, 2 Question of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks question of Section E.*
8. *Draw neat figures wherever required. Take* $\pi = 22/7$ *wherever required if not stated.*

Section A

1. **If two positive integers a and b are expressible in the form $a = pq^2$ and $b = p^3q$; p, q being prime numbers, then LCM (a, b) is**

   (a) pq (b) p^3q^3

   (c) p^3q^2 (d) p^2q^2 **(ANSWER. C)**

2. **If α and β are the zerores of the polynomial $ax^2 - 5x + c$ and $\alpha + \beta = \alpha\beta = 10$, then**

   (a) $a = 5, c = \frac{1}{2}$ (b) $a = 1, c = \frac{5}{2}$

   (c) $a = \frac{5}{2},, c = 1$ (d) $a = \frac{1}{2}, c = 5$ **(ANSWER. D)**

3. **8 chairs and 5 tables cost ₹10,500, while 5 chairs and 3 tables cost ₹ 6,450. The cost of each chair will be**

   (a) ₹ 750 (b) ₹ 600

   (c) ₹ 850 (d) ₹ 900 **(ANSWER. A)**

4. **If the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + c^2 + d^2 = 0$ has equal roots, then**

   (a) $ab = cd$ (b) $ad = bc$

   (c) $ad = \sqrt{bc}$ (d) $ab = \sqrt{cd}$ **(ANSWER. B)**

5. **If the first, second and last term of an A.P. are a, b and $2a$ respectively, its sum is**

   (a) $\frac{ab}{2(b-a)}$ (b) $\frac{ab}{b-a}$

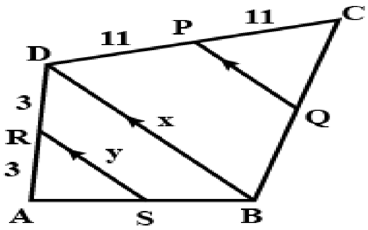
   (c) $\frac{3ab}{2(b-a)}$ (d) none of these **(ANSWER. C)**

6. **The distance between the points $(a \cos 25^0, 0)$ and $(0, a \cos 65^0)$ is**

   (a) a (b) $2a$

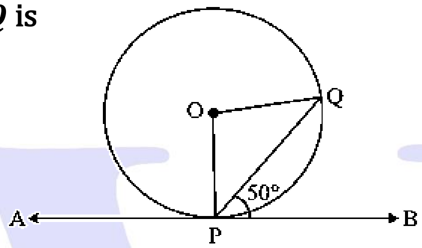
   (c) $3a$ (d) *none of these* **(ANSWER. A)**

7. **In Fig. RS||DB||PQ. If CP=PD=11 cm and DR=RA=3 cm. Then the values of x and y are respectively**



(a) 12, 10 (b) 14, 6
(c) 10, 7 (d) 16, 8 (ANSWER. D)

8. In figure, APB is a tangent to a circle with Centre O at point P. If $\angle QPB = 50^0$, then the measure of $\angle POQ$ is



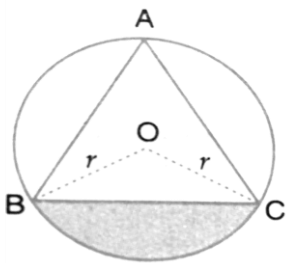
(a) 100^0 (b) 120^0
(c) 140^0 (d) 150^0 (ANSWER. A)

9. If $\sqrt{3}\sec(3x-21)^0 = 2$ then find the value of $\sin^2(x+13)^0 + \cot^2(x+13)^0$.
(a) 21/4 (b) 9/4
(c) 13/4 (d) 25/4 (ANSWER. C)

10. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then $a^2 + b^2 =$
(a) $m^2 - n^2$ (b) m^2n^2
(c) $n^2 - m^2$ (d) $m^2 + n^2$ (ANSWER. D)

11. From a light house the angles of depression of two ships on opposite sides of the light house are observed to be 30^0 and 45^0. If the height of the light house is h metres, the distance between the ships is
(a) $(\sqrt{3}+1)h\ m$ (b) $(\sqrt{3}-1)h\ m$
(c) $\sqrt{3}\ h\ m$ (d) $1+\left(1+\frac{1}{\sqrt{3}}\right)h\ m$ (ANSWER. A)

12. In fig. if ABC is an equilateral triangle, then shaded area is equal to



(a) $\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)r^2$ (b) $\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)r^2$
(c) $\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)r^2$ (d) $\left(\frac{\pi}{3}+\sqrt{3}\right)r^2$ (ANSWER. D)

13. The volumes of two spheres are in the ratio $64:27$. The ratio of their surface areas is
(a) 1 : 2 (b) 2 : 3
(c) 9 : 16 (d) 16 : 9 (ANSWER. D)

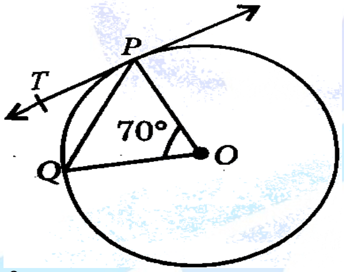
14. **If the arithmetic mean of $x, x+3,\ x+6,\ x+9$, and $x+12$ is 10, the x=**
   (a) 1 (b) 2
   (c) 6 (d) 4 **(ANSWER. D)**

15. **A number is selected from numbers 1 to 25. The probability that it is prime is**
   (a) 2/3 (b) 1/6
   (c) 9/25 (d) 5/6 **(ANSWER. C)**

16. **Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then, the difference between their 4th terms is**
   (a) 1 (b) 8
   (c) 7 (d) 9 **(ANSWER. B)**

17. **A quadrilateral $PQRS$ is drawn to circumscribe a circle. If $PQ = 12$ cm, $QR = 15$ cm and $RS = 14$ cm, then the length of SP is**
   (a) 15 cm (b) 14 cm
   (c) 12 cm (d) 11 cm **(ANSWER. D)**

18. **In the given figure, O is the centre of a circle , PQ is a chord and PT is the tangent at P, $\angle POQ = 70^0$, then $\angle TPQ$ is equal to**



   (a) 55^0 (b) 70^0
   (c) 45^0 (d) 35^0 **(ANSWER. D)**

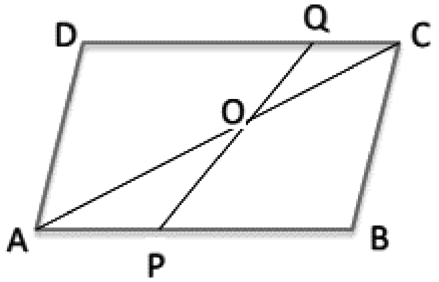
Directions (Q. Nos. 19 and 20) Are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason(R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.
(a) Both **Assertion (A)** and **Reason (R)** are true and **Reason (R)** is the correct explanation of the **Assertion (A).**
(b) Both **Assertion (A)** and Reason (R) are true, but **Reason (R)** is not the correct explanation of the **Assertion (A).**
(c) **Assertion(A)** is true, but **Reason (R)** is false
(d) **Assertion (A)** is false, but **Reason (R)** is true.

19. **Assertion (A)** If the points $A(4,3)$ and $B(x,5)$ lie on a circle with centre $O(2,3)$, then the value of x is 2.
   Reason (R) Centre of a circle is the mid-point of each chord of the circle. **(ANSWER. C)**

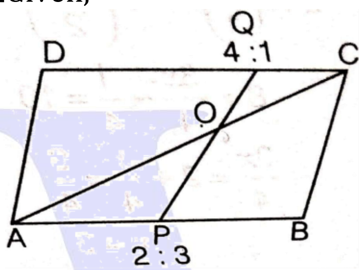
20. **Assertion(A)** The number 5^n cannot end with the digit 0, where n is a natural number.
   Reason(R) Prime factorisation of 5 has only two factors, 1 and 5. **(ANSWER. A)**

SECTION –B

21. **ABCD is a parallelogram. Point P divides AB in the ratio 2 : 3 and point Q divides DC in the ratio 4 : 1. Prove that OC is half of OA.**



Sol. Given,



In ΔOAP and ΔOCQ,

$\therefore \quad \angle AOP = \angle COQ$ [vertically opposite angles]

$\therefore \quad \angle OAP = \angle OCQ$ [alternate angles]

$\Rightarrow \quad \Delta OAP \sim \Delta OCQ$ [by AA similarity criterion]

$\therefore \quad \frac{OA}{OC} = \frac{AP}{CQ}$ ....(i)

Given, $\frac{AP}{PB} = \frac{2}{3}$ and $\frac{DQ}{QC} = \frac{4}{1}$

$\Rightarrow \quad \frac{AP}{AB} = \frac{2}{5}$ and $\frac{QC}{DC} = \frac{1}{5}$

$\Rightarrow \quad AP = \frac{2}{5} AB$ and $QC = \frac{1}{5} DC$

From Eq. (i),

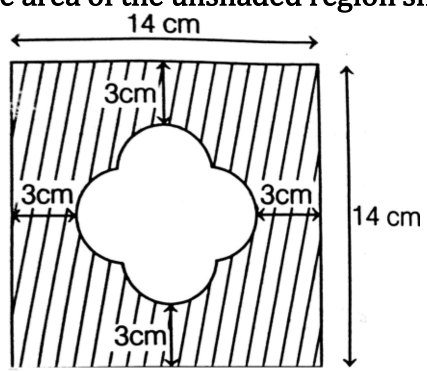
$$\frac{OA}{OC} = \frac{\frac{2}{5}AB}{\frac{1}{5}DC} = 2$$ [∵ABCD is a parallelogram ∴ AB=CD]

$\Rightarrow \quad OC = \frac{1}{2} OA$

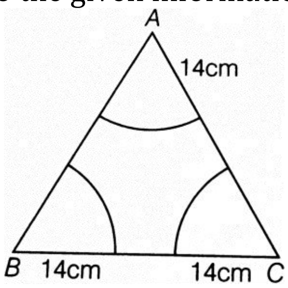
22. **(a)With vertices A, B and C of ΔABC as centres, arcs are drawn with radii 14 cm and the three portions of the triangle, so obtained are removed. Find the total area removed from the triangle.**

Or

(b)Find the area of the unshaded region shown in the given figure.



Sol. According to the given information, the figure is given below.



Area of sector with $\angle A = \frac{\angle A}{360^0} \times \pi \times r^2$

$= \frac{\angle A}{360^0} \times \pi \times (14)^2$

Similarly, other two areas are

$\frac{\angle B}{360^0} \times \pi \times (14)^2$ *and* $\frac{\angle C}{360^0} \times \pi \times (14)^2$

Total area removed $= (\angle A + \angle B + \angle C)\frac{\pi \times (14)^2}{360^0}$

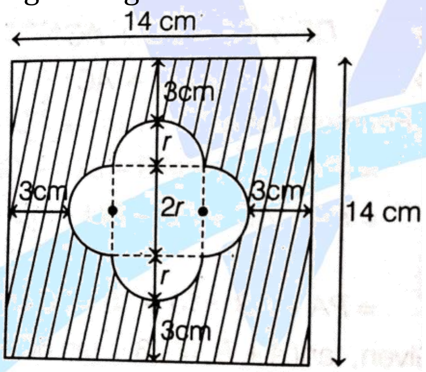
$= \frac{180^0}{360^0} \times \frac{22}{7} \times 14 \times 14$

$[\because \angle A + \angle B + \angle C = 180^0]$

$= 11 \times 2 \times 14 = 308\ cm^2$

Or

The given figure is



Let the radius of each semi-circle$= r$

$\therefore$ From figure,

$3 + r + 2r + r + 3 = 14$

$4r + 6 = 14$

$r = \frac{8}{4} = 2\ cm$

Area of unshaded region

$= 4 \times$ Area of each semi-circle $+$ Area of square with length of each side, 2r

$= 4 \times \frac{\pi r^2}{2} + (2r)^2$

$= 2\pi r^2 + 4r^2 = 2r^2(\pi + 2)cm^2$

23. **(a)If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0^0 < A + B < 90^0$; $A > B$, find A *and* B.**

Or

(b)Find the value of x

$$2cosec^2 30^0 + x\sin^2 60^0 - \frac{3}{4}\tan^2 30^0 = 10$$

Sol. Given, $\tan(A + B) = \sqrt{3} = \tan 60^0$

and $\tan(A - B) = \frac{1}{\sqrt{3}} = \tan 30^0$

⇨ $A + B = 60^0$ *and* $A - B = 30^0$

⇨ $2A = 90^0$

⇨ $A = 45^0$

and $B = 15^0$

Or

We have,

$$2\,cosec^2 30^0 + x\sin^2 60^0 - \frac{3}{4}\tan^2 30^0 = 10$$

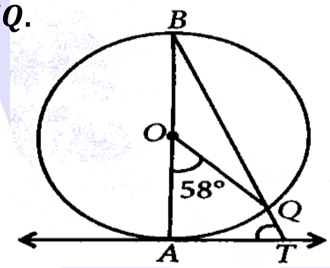
⇨ $$2 \times \left(\frac{2}{1}\right)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$$

⇨ $$8 + \frac{3x}{4} - \frac{1}{4} = 10$$

⇨ $$\frac{3x}{4} = 2 + \frac{1}{4} = \frac{9}{4}$$

⇨ $$x = 3$$

24. **In the given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^0$, find $\angle ATQ$.**



Sol.

$\angle ABQ = \frac{1}{2}\angle AOQ$

$\Rightarrow \frac{1}{2} \times 58 = 29$

$\angle$ A =90 (AT is a tangent)

$\angle$ BAT+ $\angle$ ABT + $\angle$ ATQ = 180 (angle sum property of triangle)

90 +29 + $\angle$ ATQ = 180^0

$\angle$ ATQ = 180 - 119

$\angle$ ATQ = 61^0

25. **Check whether the lines $x + y = 1$ and $2x + y = x + 2$ are either parallel or perpendicular.**

Sol. We have, $x + y = 1$

and $2x + y = x + 2 \Rightarrow x + y = 2$

On comparing both equations with $ax + by + c = 0$,

we get,

$a_1 = 1, b_1 = 1, c_1 = -1, a_2 = 1, b_2 = 1$ and $c_2 = -2$

Here, $\frac{a_1}{a_2} = \frac{1}{1}, \frac{b_1}{b_2} = \frac{1}{1}, and \frac{c_1}{c_2} = \frac{1}{2}$

∴ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given pair of linear equations are parallel.

SECTION –C

26. **If α, β are zeroes of quadratic polynomial x^2+5x+1, find the value of**

(i) $\alpha^2 + \beta^2$ **(ii) $\alpha^{-1} + \beta^{-1}$**

Sol. Given, α *and* β are the roots of the quadratic polynomial $x^2 + 5x + 1$.

Sum of roots $= \alpha + \beta = \frac{-5}{1} = -5$

Product of roots $\alpha\beta = 1$

(i)$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25 - 2 = 23$

(ii) $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = -\frac{5}{1} = -5$

27. **(a)The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?**

Or

(b)Solve the following pair of linear equation by using substitution method

$\mathbf{2x + 3y = 11}$

$\mathbf{2x - 4y = -24}$

Sol. Let the digit at unit place be x

and the digit at tens place be y.

$\therefore$ The two-digit number is $10y + x$.

Now, the number obtained by reversing the digits

$=10x + y$

According to the question,

$(10y + x) + (10x + y) = 66$

$\Rightarrow \quad 11(x + y) = 66$

$\Rightarrow \quad x + y=6 \quad$ (i)

Also, the digits of the number differ by 2

$\Rightarrow \quad x - y = 2 \quad$ (ii)

$\Rightarrow \quad y - x = 2 \quad$ (iii)

On solving Eqs. (i) and (ii), we get

$x = 4, y = 2$

On solving Eqs. (i) and (iii), we get

$y = 4, x = 2$

$\therefore$ Two such numbers are there

They are $\quad 10(4) + 2 = 42$

and $\quad 10(2) + 4 = 24$

Or

Given, $\quad 2x + 3y = 11 \quad$ (i)

$2x - 4y = -24 \quad$ (ii)

From Eq. (i),

$2x = 11 - 3y$

Substituting the value of $2x$ in Eq. (ii), we get

$11 - 3y - 4y = -24$

$\Rightarrow \quad 11 - 7y = -24$

$\Rightarrow \quad 7y = 35$

$\Rightarrow \quad y = 5$

Now, from $2x = 11 - 3y$

Substituting $y = 5$ in above relation, we get

$2x = 11 - 3 \times 5$

$= \quad 11 - 15$

$= \quad -4$

$\Rightarrow \quad x = -2$

Hence, $x = -2$ and $y = 5$

28. **The length of 40 leaves of a plant are measured correct to nearest millimetre and the data obtained is represented in the following table.**

Length [in mm]	Number of leaves
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

Sol. Let us make the table for given data.

Length(in mm)	Number of leaves (f_1)	Class mark (x_1)	$f_i x_i$
117.5-126.5	3	122	366
126.5-135.5	5	131	655
135.5-144.5	9	140	1260
144.5-153.5	12	149	1178
153.5-162.5	5	158	790
162.5-171.5	4	167	668
171.5-180.5	2	176	352
	$\sum f_i = 40$		$\sum f_i x_i = 5269$

$\therefore$ Average length of the leaves $= \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\frac{5269}{40} = 131.72$

29. **Show graphically that the following system of equations is inconsistent i.e. it has no solution $3x - 4y - 1 = 0$ *and* $2x - \frac{8}{3}y + 5 = 0$.**

Sol. We have, $3x - 4y - 1 = 0$

and $2x - \frac{8}{3}y + 5 = 0$

Table for equation $3x - 4y - 1 = 0 \Rightarrow y = \frac{3x-1}{4}$

x	0	3	-1
y	$-1/4$	2	-1
Points	$A\left(0, \frac{-1}{4}\right)$	$B(3,2)$	$C(-1\ 1)$

Now, we plot all these points on a graph paper and join them.

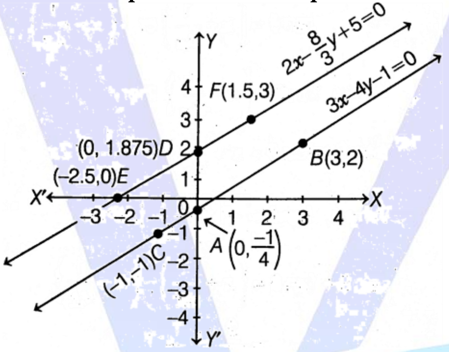
Table for equation $2x - \frac{8}{3}y + 5 = 0$

⇨ $6x - 8y + 15 = 0$

⇨ $y = \frac{6x+15}{5}$

x	0	-2.5	1.5
y	1.875	0	3
Points	$D(0,1.875)$	$E(-2.5,0)$	$F(1.5,3)$

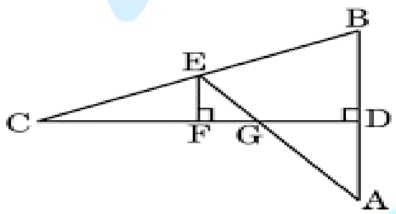
Now, we plot all these points on a graph paper and join them.



We find the line represented by equations $3x - 4y - 1 = 0$ and $2x - \frac{8}{3}y + 5 = 0$ are parallel.

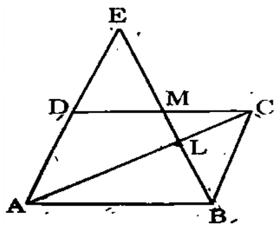
So, the two lines have no common point. Hence, the given system of equations is inconsistent.

30. **(a) In the given figure, CD is the perpendicular bisector of AB. EF is perpendicular to CD. AE intersects CD at G. Prove $\frac{CF}{CD} = \frac{FG}{DG}$.**



Or

(b) In the given figure, $ABCD$ is parallelogram. BE bisects CD at M and intersects AC at L. Prove that $EL = 2BL$



Sol. (a) In ΔEFC and ΔBDC,

$\angle EFC = \angle BDC$ [90^0 *each*]

$\angle ECF = \angle BCD$ [common]

By AA similarity criterion,

$\Delta EFC \sim \Delta BDC$

$\Rightarrow \quad \frac{EF}{BD} = \frac{FC}{DC}$ [by CPCT]……….(i)

Now, in ΔEFG and ΔADG

$\angle EGF = \angle DGA$ [vertically opposite angles]

$\angle EFG = \angle GDA$ [90^0 each]

By AA similarity criterion,

$\Delta EFG \sim \Delta ADG$

$\Rightarrow \quad \frac{EF}{AD} = \frac{FG}{DG}$ [by CPCT]

As $AD = BD \Rightarrow \frac{EF}{BD} = \frac{FG}{DG}$

From Eqs. (i) and (ii), we get

$\frac{CF}{CD} = \frac{FG}{DG}$ **Hence proved**

(b) In ΔBMC and ΔEMD,

$\angle BMC = \angle EMD$ [vertically opposite angles]

$MC = DM$ [given]

$\angle BCM = \angle EMD$ [alternate angles]

$\therefore \quad \Delta BMC \cong \Delta EMD$ [by ASA congruency criteria]

$\Rightarrow \quad BC = DE$ [by CPCT] …….(i)

Now, $AE = AD + DE$

$= BC + BC = 2BC$ (ii) (1/2)

In ΔALE and ΔCLB

$\angle LAF = \angle LCB$ [alternate angles]

$\angle LEA = \angle LBC$ [alternate angles]

$\therefore \quad \Delta ALE \sim \Delta CLB$ [by AA similarity criterion]

$\Rightarrow \quad \frac{EL}{BL} = \frac{AE}{CB}$ [by CPCT]

$\Rightarrow \quad \frac{EL}{BL} = \frac{2BC}{BC}$ [from Eq. (ii)]

$\Rightarrow \quad \frac{EL}{BL} = 2 \Rightarrow EL = 2BL$ Hence proved.

31. **Prove that**

$$\boldsymbol{\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\mathrm{cosec}\theta}$$

Sol. We have, LHS. $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta} = \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta(1-\tan\theta)}$

$= \frac{1-\tan^3\theta}{\tan\theta(1-\tan\theta)}$

$= \frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta(1-\tan\theta)}$

$= \frac{\sec^2\theta+\tan\theta}{\tan\theta}$ $[\because 1+\tan^2 A = \sec^2 A]$

$= \frac{\sec^2\theta}{\tan\theta} + 1 = \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta} + 1$

$[\because \cos A = \frac{1}{\sec A}, \tan A = \frac{\sin A}{\cos A}]$

$[\because \sin A = \frac{1}{\mathrm{cosec} A}]$

$= \sec\theta\,\mathrm{cosec}\theta + 1$

= RHS.

SECTION-D

32. **(a)A motor boat whose speed is 18 km/h in still water takes 1 h more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream.**

Or

(b)Two water taps together can fill a tank in $9\frac{3}{8}$ h. The tap of larger diameter takes 10 h less than the smaller one to fill the tank separately.
Find the time in which each tap can separately fill the tank.

Sol. Let the speed of the stream be x km/h.
Then, speed of the boat downstream = $(18+x)km/h$
and speed of the boat upstream= $(18-x)km/h$
Then, according to the question,
$$\frac{24}{18-x}-\frac{24}{18+x}=1$$
$\Rightarrow 24(18+x)-24(18-x)=(18-x)(18+x)$
$\Rightarrow \quad 24x+24x=324-x^2$
$\Rightarrow \quad x^2+48x-324=0$
$\Rightarrow x^2+54x-6x-324=0$
$\Rightarrow x(x+54)-6(x+54)=0$
$\Rightarrow \quad (x+54)(x-6)=0$
$\therefore \quad x=6$
$[\because x \text{ cannot be negative}]$
$\therefore$ Speed of stream = $x=6\ km/h$

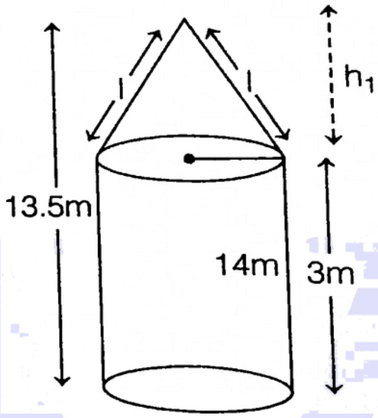
Or

Let time taken by tap of smaller diameter be t h.
$\therefore$ Time taken by tap of larger diameter = $(t-10)h$
Given, time taken by both the taps of smaller diameter in
$=9\frac{3}{8}=\frac{75}{8}h$
Portion of the tank filled by tap of smaller diameter in
$1h=\frac{1}{t}$
Portion of the tank filled by tap of larger diameter in
$1h=\frac{1}{t-10}$
Portion of the tank filled by both the taps in 1 h
$$=\frac{1}{\frac{75}{8}}=\frac{8}{75}$$
$\therefore \quad \frac{1}{t}+\frac{1}{t-10}=\frac{8}{75}$
$\Rightarrow \quad \frac{(t-10)+t}{t(t-10)}=\frac{8}{75}$
$\Rightarrow \quad \frac{2t-10}{t^2-10t}=\frac{8}{75}$
$\Rightarrow \quad 150t-750=8t^2-80t$
$\Rightarrow \quad 8t^2-230t+750=0$
$\Rightarrow \quad 4t^2-115t+375=0$
$\Rightarrow \quad 4t^2-100t-15t+375=0$
$\Rightarrow \quad 4t(t-25)-15t(t-25)=0$
$\Rightarrow \quad (t-25)(4t-15)=0$
$\Rightarrow \quad t=25 \text{ or } t=\frac{15}{4}$
If $t=25$, then time taken by tap of larger diameter
$=25-10=15\ h$
If $t=\frac{15}{4}$, then time taken by tap of larger diameter
$=\frac{15}{4}-10=-\frac{25}{4}$ which is not possible.
$\therefore$ Time taken by taps of smaller and larger diameter are 25 h and 15 h, respectively.

33. **A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 3 m and 14 m, respectively and the total height of the tent is 13.5 m, find the area**

of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of ₹ 500 per m^2.

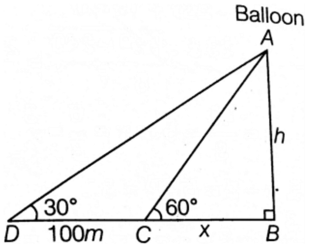
Sol. Given, total height of the tent = 13.5 m
and radius of the base of the tent = 14 m
As shown in figure,
Height of the cone, h_1 = Total height − Height of cylinder
$= 13.5 - 3 = 10.5$ m



Radius of cone and radius of cylinder $r = 14$ m
Slant height l of cone $= \sqrt{h_1^2 + r^2}$
$= \sqrt{(10.5)^2 + (14)^2} = 17.5$ m
Curved surface area of cone $= \pi r l$
$= \pi \times 14 \times 17.5 = 245\,\pi\, m^2$
Curved surface area of cylinder $= 2\pi r h_2$,
where h_2 is height of cylinder
$= 2\pi \times 14 \times 3 = 84\pi\, m^2$
∴ Area of canvas required $= 245\pi + 84\pi + 26$
$= 245 \times \frac{22}{7} + 84 \times \frac{22}{7} + 26$
$= 770 + 264 + 26 = 1060\, m^2$
∴ Cost of canvas = ₹(500 × 1060) = ₹530000

34. **One observer estimates the angle of elevation to the basket of a hot air balloon to be 60°, while another observer 100 m away estimates the angle of elevation to be 30°. Find**
(a) the height of the basket from the ground.
(b) the distance of the basket from the first observer's eye.
(c) the horizontal distance of the second observer from the basket.

Sol. (a) In ΔABC, $\tan 60^0 = \frac{AB}{BC}$



$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$

$\Rightarrow \quad h = \sqrt{3}x \qquad$ (i)

InΔABD, $\tan 30^0 = \frac{AD}{BD}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{100+x}$

$\Rightarrow \quad 100 + x = \sqrt{3}h$

$\Rightarrow \quad 100 + x = \sqrt{3}(\sqrt{3}x)$ [from Eq.(i)]

$\Rightarrow \quad 100 = 2x \Rightarrow x = 50\ cm$

$\therefore$ Height of basket

$= \quad h = \sqrt{3}x = \sqrt{3} \times 50$

$= \quad 50 \times 1.732 = 86.60$ m

(b) In $\Delta ABC, \cos 60^0 = \frac{BC}{AC}$

$\Rightarrow \quad \frac{1}{2} = \frac{50}{AC} \Rightarrow AC = 100\ m$

$\therefore$ Distance of the basket from first observer's eye =100 m

(c) Horizontal distance of the second observer from the basekt

$= \quad BD = BD + CD$

$=$ 50+100=150 cm

35. **(a)A right angled triangle whose sides other than hypotenuse are 15 cm and 20 cm, is made to revolve about its hypotenuse. Find the volume and surface area of the double cone, so formed. $[\boldsymbol{take, \pi = 3.14}]$**

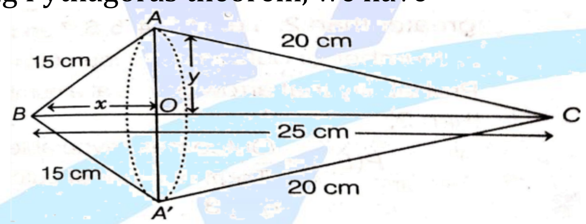
Or

(b)A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 4 cm and the diameter of the base is 8 cm. If a right circular cylinder circumscribes the solid. Find how much more space it will cover?

Sol. Let BAC be a right angled triangle such that

$AB = 15\ cm\ and\ AC = 20\ cm.$

Using Pythagoras theorem, we have



$(BC)^2 = (AB)^2 + (AC)^2$

$\Rightarrow \quad (BC)^2 = 15^2 + 20^0$

$\Rightarrow \quad (BC)^2 = 225 + 400 = 625$

$\therefore$ BC= 25 cm [taking positive square root]

Let $OB = x\ and\ OA = y$

Again, using Pythagoras theorem in ΔOAB and ΔOAC, we have

$AB^2 = OB^2 + OA^2$

$\Rightarrow \quad 15^2 = x^2 + y^2$

$\Rightarrow \quad x^2 + y^2 = 225$ ......(i)

and $\quad AC^2 = OA^2 + OC^2$

$\Rightarrow \quad 20^2 = y^2 + (25 - x)^2$

$\Rightarrow \quad (25 - x)^2 + y^2 = 400$ ......(ii)

On subtracting Eq. (i) from Eq. (ii) we get

$\Rightarrow \quad \{(25 - x)^2 + y^2\} - \{x^2 + y^2\} = 400 - 225$

$\Rightarrow \quad (25 - x)^2 - x^2 = 175$

$\Rightarrow \quad (25 - x - x)(25 - x + x) = 175$ $[\because a^2 - b^2 = (a - b)(a + b)]$

$\Rightarrow \quad (25 - 2x) \times 25 = 175$

$\Rightarrow \quad 2x = 18 \Rightarrow x = 9$

On putting $x = 9$ in $Eq. (i)$, we get

$81 + y^2 = 225$

⇨ $y^2 = 144$

⇨ $y = 12$ [taking positive square root]

Thus, we have OA=12 cm and OB=9 cm

∴ Volume of the double cone = Volume of cone CAA'+ Volume of cone BAA'

$= \frac{1}{3}\pi(OA)^2 \times OC + \frac{1}{3}\pi(OA)^2 \times OB$

$= \frac{1}{3}\pi \times 12^2 \times 16 + \frac{1}{3}\pi \times 12^2 \times 9$

$= \frac{1}{3}\pi \times 144(16+9)$

$= \frac{1}{3} \times 3.14 \times 144 \times 25 = 3768\ cm^3$

∴ Total surface area of the double cone = Curved surface area of cone CAA'+ Curved surface

area of cone BAA'

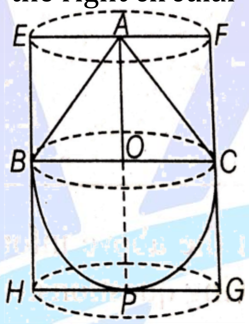
$= \pi \times OA \times AC + \pi \times OA \times AB$

$= \pi \times 12 \times 20 + \pi \times 12 \times 15 = 420\pi\ cm^2$

$= 420 \times 3.14 = 1318.8\ cm^2$

Or

Let BPC be the hemisphere and ABC be the cone mounted on the base of the hemisphere.
Let EFGH be the right circular cylinder circumscribing the given toy.



We have, height of cone, OA= 4 cm
Diameter of the base of the cone, d=8 cm
∴ Radius of the base of cone, $r = \frac{d}{2} = \frac{8}{2} = 4\ cm$

Here, $AP = AO + OP = 4 + 4 = 8\ cm$

∴ Required space = volume of cylinder – (Volume of cone + Volume of hemisphere)

$= \pi r^2 H - \left[\frac{1}{3}\pi r^2 h + \frac{2}{3}\ \pi r^3\right]$

$= \pi r^2 \left[H - \frac{1}{3}h - \frac{2}{3}r\right]$

$= \pi(4)^2 \left[8 - \frac{1}{3} \times 4 - \frac{2}{3} \times 4\right]$ [here, H=AP =8 cm and h h= AO= 4cm]

$= 16\pi \left[8 - \frac{4}{3} - \frac{8}{3}\right]$

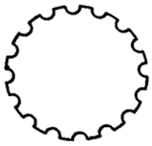
$= 16\pi \left[\frac{24-4-8}{3}\right] = 16\pi \times \frac{12}{3}$

$= 64\pi$

Hence, the right circular cylinder covers $64\pi\ cm^3$ more space than the solid toy.

SECTION-E

36. **A golf ball is spherical with about 300-500 dimples that help increase its velocity while in play. Golf balls are traditionally white but available in colours also. In the given figure, a golf ball has diameter 4.2 cm and the surface has 315 dimples (hemi-spherical) of radius 2 mm.**

Based on the above information, answer the following questions.

(i) Find the surface area of one such dimple. (1)

(ii) Find the volume of the material dug out to make one dimple. (1)

(iii)(a)Find the total surface area exposed to the surroundings. (2)

Or

(b)Find the volume of the golf ball. (2)

Sol. Given, diameter of golf ball $= 4.2\ cm$

Radius (R) = 2.1 cm

Radius of dimple (r)=2 mm=0.2 cm

(i) Surface area of each dimple $= 2\pi r^2$

$= 2 \times \pi \times 0.2 \times 0.2 = 0.08\ \pi\ cm^2$

(ii) Volume of material dug out to make 1 dimple

= Volume of 1dimple (hemisphere)

$= \frac{2}{3}\pi r^3 = \frac{0.016\pi}{3}\ cm^3$

(iii) Total surface area exposed to surrounding = Surface area of gold ball – Area of circle due to 315 dimples +Surface area of 315 dimples

$= 4\pi R^2 - 315 \times \pi r^2 + 315 \times 2\pi r^2$

$=4\pi R^2 + 315 \times \pi r^2$

$= 4\pi(2.1)^2 + 315 \times \pi(0.2)^2$

$= \pi[17.64 + 12.6]$

$= 30.24\ \pi\ cm^2$

Or

Volume of golf ball = Volume of sphere− Volume of 315 dimples

$= \frac{4}{3}\pi R^2 - 315 \times \frac{2}{3}\pi r^3$

$=\frac{\pi}{3}(37.044 - 5.04)\ cm^3$

$= 10.668\ \pi\ cm^3$

37. **Ram and shyam are very close friends. Both of them decided to go to Ranchi by their own vehicles. Ram travels at speed of x km/hr slower than Ram. Shyam took 3 hours more than ram to complete the total journey of 480 km.**

Based on the given information, answer the following questions.

(i) Describe the speed of Ram's car in the form of quadratic equation.

(ii) (a) What is speed of ram's car?

Or

(b)If total journey travelled is 400 km, then express the speed of Ram's car in quadratic equation.

(iii) If shyam took 4 hours more than Ram to complete the same journey, then what will be the speed of Ram's car in the form of quadratic equation?

Sol. Since, the speed of Ram is $x\ km/hr$.

Time taken to cover a distance 480 km by

Ram $\frac{480}{x}\ hr$

Also, time taken by Shy am to cover a distance 480 km with the speed of $(x-8)$ km/hr= $\frac{480}{x-8}$ hr

(i) According to question,

⇨ $\frac{480}{x-8}=\frac{480}{x}+3 \Rightarrow \frac{480}{x-8}-\frac{480}{x}=3$

⇨ $\frac{480(x-x+8)}{x(x-8)}=3 \Longrightarrow 3x(x-8)=480\times 8$

⇨ $x^2-8x-1280=0$

(ii) (a) To find the speed of Ram's car, we will solve the quadratic equation $x^2-8x-1280=0$

$\therefore x=\frac{8\pm\sqrt{(-8)^2-4\times1\times(-1280)}}{2\times1}$

$=\frac{8\pm\sqrt{64+5120}}{2}=\frac{8\pm72}{2} \Rightarrow x=40\ and\ x=-32$

Since, speed can not be negative, therefore $x=40\ km/hr$.

Or

(c) Distance travelled = 400 km

According to question, $\frac{400}{x-8}=\frac{400}{x}+3 \Rightarrow \frac{400}{x-8}=3$

$\Rightarrow 3200=3x(x-8) \Rightarrow 3x^2-24x-3200=0$

(iii) According to question, $\frac{480}{x-8}-\frac{480}{x}=4$

$\Rightarrow 3840=4x(x-8) \Rightarrow x^2-8x-960=0$

38. **Coconut oil is used by various people to cook food. Its demand has immensely increased because of the growing awareness about naturally prepared oils among people. The production of coconut oil in the month of January in a particular year was 12000 metric tons and increasing uniformly by a fixed number every month. It produced 27000 metric tone in the month of June in that particular year.**

Based on the given information, answer the following questions.

(i) Find the fixed increased in production of coconut oil in every month.

(ii) What will be the production of coconut oil in the month of October?

(iii) (a) Find the production of coconut oil in first 6 months.

Or

(b) Find the sum of the production of coconut oil in a complete year.

Sol. (i) The production of coconut oil in the month of January (a)=12000

∵ June is the 6th month of the year

∴ The production of coconut oil in the month of june.

$(a_6)=27000$

$\therefore a_6=a+5d=27000$ $[\therefore a_n=a+(n-1)d]$

⇨ 12000+5d=27000 $\Rightarrow 5d=27000-12000$

⇨ $d=\frac{15000}{5} \Rightarrow d=3000$

The production of coconut oil is increased by 3000 metric tons in every month.

(ii) Now, October is the 10th month of the year.

$\therefore a_{10}=a+9d$

$=12000+9(3000)=12000+27000=393000$

The production of the coconut oil in the month of October is 39000 metric tons.

(iii) (a) $a=12000; a_6=27000=l$

$S_n=\frac{n}{2}(a+l); S_6=\frac{6}{2}(12000+27000)=3\times39000$

$S_6=117000$

Hence, production in first six months is 117000 metric tones.

Or

(b) $a = 120000; d = 3000$

$n = 12$ (∵Number of months in a year=12)

We know, $S_n = \frac{n}{2}\ [2a + (n-1)d]$

$S_n = \frac{12}{2}[2a + (12-1)d] = 6[2 \times 12000 + (11)3000]$

$=6[24000 + 33000] = 6 \times 57000 = 342000$

Hence, total production in a year is 242000 metric tons.