MATHS SAMPLE PAPER(SOLUTION)

CLASS- XTH SESSION-2024-25

TIME ALLOWED: 3 HRS. MAXIMUM MARKS:-80

General Instructions

1. *This Question paper has 5 Section A, B, C, D and E.*
2. *Section A has 20 Multiple Choice Question (MCQs) carrying 1 marks each.*
3. *Section B has 5 short Answer-I (SA-I) type question carrying 2 marks each.*
4. *Section C has 6 short Answer-II(SA-II) type question carrying 3 marks each.*
5. *Section D has 4 Long Answer (LA) type question carrying 5 marks each.*
6. *Section E has 3 Case Based integrated units of assessment (4 marks each).*
7. *All Questions are compulsory. However, an internal choice in 2 Questions of 2 marks, 2 Question of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks question of Section E.*
8. *Draw neat figures wherever required. Take* $\pi = 22/7$ *wherever required if not stated.*

SECTION A

1. The LCM and HCF of two rational numbers are equal, then the numbers must be
   (a) Prime (b) co-prime
   (c) composite (d) equal **(ANSWER- D)**

2. If α, β are the zeros of polynomial $f(x) = x^2 - p(x+1) - c$, then $(\alpha+1)(\beta+1)=$
   (a) $c - 1$ (b) $1 - c$
   (c) c (d) $1 + c$ **(ANSWER- B)**

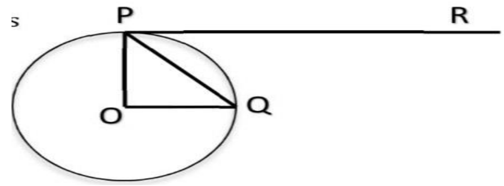
3. The area of the triangle formed by the lines $y = x, x = 6$ *and* $y = 0$ is
   (a) 36 sq. units (b) 18 sq. units
   (c) 9 sq. units (d)72 sq. units **(ANSWER- B)**

4. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then
   (a) $k < 4$ (b) $k > 4$
   (c) $k \geq 4$ (d) $k \leq 4$ **(ANSWER- A)**

5. Two A.P.'s have the same common difference. The first term of one of these is 8 and that of the other is 3. The difference between their 30th terms is
   (a) 11 (b) 3
   (c) 8 (d)5 **(ANSWER- D)**

6. The distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$ is
   (a) $a^2 + b^2$ (b) $a + b$
   (c) $a^2 - b^2$ (d) $\sqrt{a^2 + b^2}$ **(ANSWER- D)**

7. If O is centre of a circle and chord PQ makes and angle 50^0 with the tangent PR at the point of contact P, find the angle made by the chord at the centre.



(a) 130^0 (b) 100^0
(c) 50^0 (d) 30^0 **(ANSWER- B)**

8. If $\tan\theta = \frac{1}{\sqrt{7}}, then \frac{cosec^2\theta - \sec^2\theta}{cosec^2\theta + \sec^2\theta} =$

(a) $\frac{5}{7}$ (b) $\frac{3}{7}$
(c) $\frac{1}{12}$ (d) $\frac{3}{4}$ **(ANSWER- D)**

9. The angle of elevation of the top of a tower standing on a horizontal plane from a point $A\ is\ \alpha$. After walking a distance d towards the foot of the tower the angle of elevation is found to be β. The height of the tower is

(a) $\frac{d}{\cot\alpha + \cot\beta}$ (b) $\frac{d}{\cot\alpha - \cot\beta}$
(c) $\frac{d}{\tan\beta - \tan\alpha}$ (d) $\frac{d}{\tan\beta + \tan\alpha}$ **(ANSWER- B)**

10. If the areas of two circles are in the ratio 4 : 9, then the ratio of the perimeters of their semi-circles is

(a) 2 : 3 (b) 3 : 2
(c) 1 : 2 (d) 1 : 3 **(ANSWER- B)**

11. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is

(a) $60\pi\ cm^2$ (b) $68\pi\ cm^2$
(c) $120\pi\ cm^2$ (d) $136\pi\ cm^2$ **(ANSWER- D)**

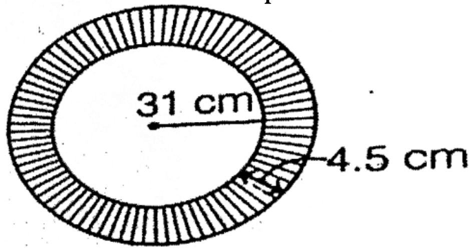
12. If the mean of a frequency distribution is 8.1 and $\sum f_i x_i = 132 + 5k,\ \sum f_i = 20$, then k=

(a) 3 (b) 4
(c) 5 (d)6 **(ANSWER- D)**

13. From the letters of the word "MOBILE ", a letter is selected. The probability that the letter is a vowel, is

(a) $\frac{1}{3}$ (b) $\frac{3}{7}$
(c) $\frac{1}{6}$ (d) $\frac{1}{2}$ **(ANSWER- D)**

14. The area of the shaded portion is



(a) $940.5\ cm^2$ (b) $930.5\ cm^2$
(c) $400.5\ cm^2$ (d) $510.5\ cm^2$ **(ANSWER- A)**

15. In two triangle ABC and DEF, $\angle A = \angle E\ and\ \angle B = \angle B = \angle F$. Then, $\frac{AB}{AC}$ is equal to
(a) $\frac{DE}{DF}$ (b) $\frac{ED}{EF}$
(c) $\frac{EF}{ED}$ (d) $\frac{EF}{DF}$ **(ANSWER- C)**

16. Two numbers are in the ratio of 15: 11. If their HCF is 13, then number will be
(a) 195 and 143 (b) $190\ and\ 140$
(c) $185\ and\ 163$ (d) $185\ and\ 143$ **(ANSWER- A)**

17. Write the value of k for which the system of equations $x + ky = 0$ and $2x - y = 0$ has unique solution.
(a) $k = \frac{1}{2}$ (b) $k = -\frac{1}{2}$
(c) $k \neq \frac{-1}{2}$ (d) $k \neq \frac{1}{2}$ **(ANSWER- C)**

18. A parallelogram has vertices $P(1,4), Q(7,11), R(a,4)\ and\ S(1,-3)$. Then, the value of a is
(a) 6 (b) 7
(c) 5 (d) 4 **(ANSWER- B)**

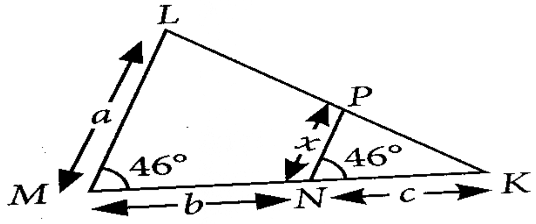
DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R).**
Choose the correct option.
(a) Both **assertion (A)** and **reason (R)** are true and **reason (R)** is the correct explanation of **assertion (A)**
(b) Both **assertion (A)** and **reason (R)** are true and **reason (R)** is not the correct explanation of **assertion (A)**
(c) **Assertion (A)** is true but **reason (R)** is false.
(d) **Assertion (A)** is false but **reason (R)** is true.

19. **Assertion (A):** Point $P\left(1,\frac{5}{2}\right)$ is equidistant from the points A(−5, 3) and B(7, 2).
Reason (R): If a point P is equidistant from the points A and B, then AP = BP. **(ANSWER- A)**

20. **Assertion (A):** Two players, Sania and Ashnam play a tennis match. The probability of Sania winning the match is 0.79 and that of Ashnam winning the match is 0.21.
Reason (R): The sum of probabilities of two complementary events is 1. **(ANSWER- A)**

SECTION –B

21. **In figure, $\angle LMK = \angle PNK = 46^0$. Express x in terms of $a, b\ and\ c$, where $a, b\ and\ c$ are length of $LM, MN\ and\ NK$ respectively.**



Or

In $\Delta ABC, \angle CAB = 90^0$ and $AD \perp BC$. If $AC = 75\ mm, AB = 1m$ and $BD = 80\ cm, find\ AD$.

Sol. (a) In $\Delta LMK\ and\ \Delta PNK$,

$\angle LMK = \angle PNK$ [Each equal 46^0]

$\angle K = \angle K$ [Common]

$\therefore \Delta LMK \sim \Delta PNK$ [By AA similarity criterion]

⇨ $\frac{LM}{PN} = \frac{MK}{NK} = \frac{LK}{PK} \Rightarrow \frac{a}{x} = \frac{b+c}{c} = x = \frac{ac}{b+c}$

Or

(b) In ΔABD

$AB^2 = AD^2 + BD^2$

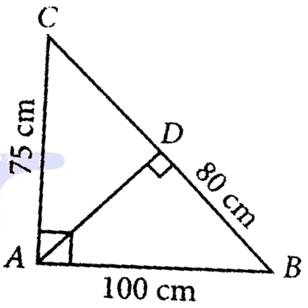
⇨ $(100)^2 = AD^2 + 80^2$

⇨ $AD^2 = 100^2 - 80^2$

$= (100-80)(100+80)$

$= 20 \times 180 = 3600$

$\therefore$ AD=60 cm



22. **(a)Prove that**

$$(\boldsymbol{cosecA - \sin A})\ (\boldsymbol{\sec A - \cos A}) = \frac{1}{\tan A + \cot A}$$

Or

(b)If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ then prove that $\cos\theta - sin\theta = \sqrt{2}\sin\theta$

Sol. LHS $= (cosec\ A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$

$= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} = \sin A\cos A$ .... (i) [1]

RHS $= \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A\cos A}}$

$= \frac{\sin A\cos A}{1}$ [$\because \sin^2\theta + \cos^2\theta = 1$] [1/2]

$= \sin A\cos A$ ....(ii)

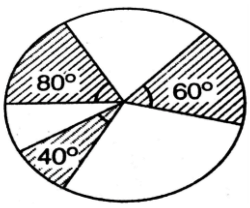
From Eqs. (i) and (ii) , we have

LHS = RHS

Hence, $(cosec\ A - \sin A)(\sec A - \cos A)$ [1/2]

$= \frac{1}{\tan A + \cot A}$ **Hence proved.**

23. **In the given figure, three sectors of a circle of radius 7 cm, making angles of $60^0, 80^0, 40^0$ at the centre are shown. Find the area ($in\ cm^2$) of the shaded region.**



Sol. Given, radius of circle, $r = 7\ cm$

Now, area of shaded region = Area of three sectors

$= \frac{\theta_1}{360^0}\pi r^2 + \frac{\theta_2}{360^0}\pi r^2 + \frac{\theta_3}{360^0}\pi r^2$ $\left[\because area\ of\ sector = \frac{\theta}{360^0}\pi r^2\right]$ (1)

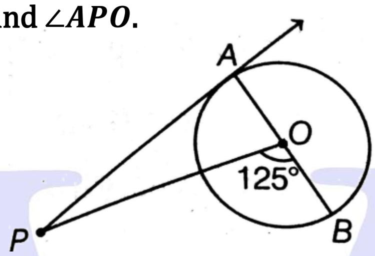
$= \frac{\pi r^2}{360^0}(\theta_1 + \theta_2 + \theta_3)$

$= \frac{22}{7} \times \frac{1}{360^0} \times 7 \times 7(60^0 + 80^0 + 40^0)$

$= 11 \times \frac{1}{180^0} \times 7 \times 180^0$

$= 77\ cm^2$

24. **In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 125^0$, then find $\angle APO$.**



Sol. $\angle OAP = 90^0, AO \perp PA$

Now, $\angle AOP + \angle BOP = 180^0$

⇨ $\angle AOP + 125^0 = 180^0$

⇨ $\angle AOP = 180^0 - 125^0 = 55^0$ (1)

Also, in ΔAPO,

$\angle APO + 55^0 + 90^0 = 180^0$

⇨ $\angle APO + 145^0 = 180^0$

∴ $\angle APO = 180^0 = -145^0 = 35^0$ (1)

25. **The students of a class are made to stand in rows. If 4 students are extra in each row, there would be 2 rows less. If 4 students are less in each row, there would be 4 rows more, then find the number of students in the class.**

Sol. Let the number of rows $= x$

and the number of students in each row= y

Then, the total number of students $= xy$

When there are 4 more students in each row,

Number of students in each row $= y + 4$

and number of rows $= x - 2$

Now, total number of students $= (x - 2)(x + 4)$

Given, $(x - 2)(y + 4) = xy$

⇨ $4x - 2y = 8$

⇨ $2x - y = 4$ .....(i)

When 4 students are removed from each row, number of students in each row= $(y - 4)$

and number of rows= $(x + 4)$

Total number of students $= (x + 4)(y - 4)$

Given, $(x + 4)(y - 4) = xy$

⇨ $4y - 4x = 16$

⇨ $4(y - x) = 16$

⇨ $y - x = 4$ ....(ii) [1/2]

Adding Eqs. (i) and (ii), we get $x = 8$

On putting $x = 8$ in Eq. (ii), we get

$y - 8 = 4 \Rightarrow y = 12$

$x = 8\ and\ y = 12$

Total number of students in the class = $(12 \times 8) = 96$ [1/2]

SECTION –C

26. **Prove that $5\sqrt{2}$ is irrational.**

Sol. Let us assume that $5\sqrt{2}$ is irrational number.

Then, there exist co-prime positive integers a and b such that

$5\sqrt{2} = \frac{a}{b}$

⇨ $\sqrt{2} = a/5b$

∵ 5, a and b are integers, so $\frac{a}{5b}$ is a rational number.

⇨ $\sqrt{2}$ is a rational number.

But this contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is not correct.

Hence, $5\sqrt{2}$ is an irrational number.

27. **(a) If the 3rd and 9th terms of an AP are 4 and −8 respectively, then which term of this AP is zero?**

Or

(b) Find the common difference of an AP, whose firs term is 1/2 and the 8th term is 17/6. Also, find the ratio of 4th term and 50th term.

Sol. Let a be the first term and d be the common difference of an AP.

∵ The nth term of an AP is $a_n = a + (n-1)d$

∴ $a_3 = a + 2d = 4$ [∵ $a_3 = 4$, given] ...(i)

and $a_9 = a + 8d = -8$ [∵ $a_9 = -8$, given] .....(ii) (1)

On subtracting Eq. (i) from Eq. (ii), we get

$6d = -12$

$d = \frac{-12}{6} = -2$

On putting the value of d inEq. (i) we get

$a + 2 \times (-2) = 4$

⇨ $a - 4 = 4$

⇨ $a = 4 + 4 = 8$

Let the nth term of this AP be zero.

i.e. $a_n = 0$

➔ $a + (n-1)d = 0$

⇨ $8 + (n-1)(-2) = 0$ [∵ $a = 8, d = -2$]

⇨ $(n-1)(-2) = -8$

⇨ $n - 1 = \frac{-8}{-2} = 4$

∴ $n = 4 + 1 = 5$

Hence, 5th term of this AP is zero. (1)

Or

Let a be the first term and d be the common difference of an AP.

Then, nth term, $T_n = a + (n-1)d$

Given, 8th term, $T_8 = \frac{17}{6} \Rightarrow a + 7d = \frac{17}{6}$ (1)

⇨ $\frac{1}{2} + 7d = \frac{17}{6}$ $\left[\because a = \frac{1}{2}\right]$

⇨ $7d = \frac{17}{6} - \frac{1}{2}$

⇨ $7d = \frac{14}{6}$

⇨ $d = \frac{1}{3}$ (1)

Now, 4th term, $T_4 = a + 3d$

$= \frac{1}{2} + 3\left(\frac{1}{3}\right) = \frac{1}{2} + 1 = \frac{3}{2}$

and 50th term, $T_{50} = a + 49d$

$= \frac{1}{2} + 49 \times \frac{1}{3} = \frac{101}{6}$

∴ Required ratio $= \frac{3/2}{101/6} = \frac{3}{2} \times \frac{6}{101}$

$= 9 : 101$ (1)

28. **(a) If α and β are zeroes of the polynomial $x^2 - 2x - 15$, then form a quadratic polynomial whose zeroes are 2α and 2β.**

Or

(b) The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Sol. Let $p(x) = x^2 - 2x - 15$

On comparing with $ax^2 + bx + c$, we get

$a = 1, b = -2$ and $c = -15$

Given, α and β are the zeroes of $p(x)$.

∴ Sum of zroes,

$(\alpha + \beta) = -\frac{b}{a}$

⇨ $\alpha + \beta = -\frac{(-2)}{1}$

⇨ $\alpha + \beta = 2$

and product of zeroes, $(\alpha\beta) = \frac{c}{a}$

⇨ $\alpha\beta = \frac{-15}{1}$

⇨ $\alpha\beta = -15$

We have to form a polynomial whose zeroes are 2α and 2β.

∴ Sum of zeroes $= 2\alpha + 2\beta = 2(\alpha + \beta)$

$= 2 \times 2 = 4$ [using Eq. (i)]

and product of zeroes $= 2\alpha . 2\beta$

$= 4\alpha\beta$

$= 4 \times (-15)$

$= -60$ [using Eq. (ii)] (1/2)

∴ Required polynomial $= x^2 - (Sum\ of\ zeroes)x + (product\ pf\ zeroes)$

$= x^2 - 4 + (-60) = x^2 - 4x - 60$

$= x^2 - 4x + (-60) = x^2 - 4x - 60$

Or

Let the required numbers x and y, where $x > y$

Given, difference of squares of two numbers = 180

We have , $x^2 - y^2 = 180$

and also it is given that the square of smaller number = 8 × larger number

We have, $y^2 = 8x$

From Eqs. (i) and (ii), we get

$x^2 - 180 = 8x$

⇨ $x^2 - 8x - 180 = 0$

⇨ $x^2 - 18x + 10x - 180 = 0$ [by factorization]

⇨ $x(x - 18) + 10(x - 18) = 0$

⇨ $(x - 18)(x - 18) = 0$

⇨ $x - 18 = 0$ or $x + 10 = 0$

⇨ $x = 18 \; or \; x = -10$

Now, it $x = 18$, then square of smaller number

$= 8 \times 18 = 144$ [from Eq. (ii)] (1 1/2)

⇨ Smaller number $= \pm 12$

⇨ Smaller number $= 12$ or -12

and if $x = -10$, then square of smaller number $= [8 \times (-10)] = -80$, which is not possible as square of a number cannot be negative.

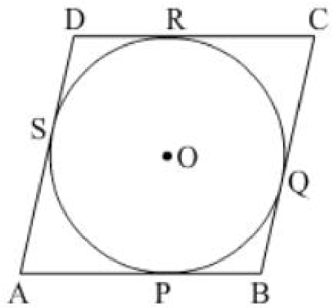
Hence, the required numbers are 18 and 12 or 18 and −12. (1)

29. **Prove that the parallelogram circumscribing a circle is a rhombus.**

Sol. ABCD is a parallelogram. Therefore, opposite sides are equal.

AB = CD

BC = AD



According to Theorem 10.2: The lengths of tangents drawn from an external point to a circle are equal.

Therefore,

BP = BQ (Tangents from point B)…… (1)

CR = CQ (Tangents from point C)…… (2)

DR = DS (Tangents from point D)…… (3)

AP = AS (Tangents from point A)……. (4)

Adding (1) + (2) + (3) + (4)

BP + CR + DR + AP = BQ + CQ + DS + AS

On re-grouping,

BP + AP + CR + DR = BQ + CQ + DS + AS

AB + CD = BC + AD

Substitute CD = AB and AD = BC since ABCD is a parallelogram, then

AB + AB = BC + BC

2AB = 2BC

AB = BC

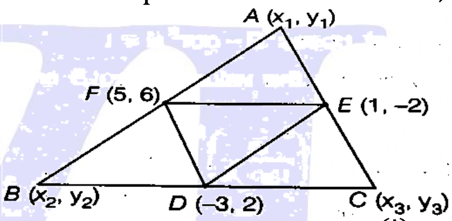
∴ AB = BC = CD = DA

This implies that all the four sides are equal.

Therefore, the parallelogram circumscribing a circle is a rhombus.

30. **If $(-3,2),(1,-2)$ *and* $(5,6)$ are the mid-point of the sides of a triangle, then find the coordinates of the vertices of the triangle.**

Sol. Let $A \equiv (x_1, y_1)$, B$\equiv (x_2, y_2)$ *and* $C \equiv (x_3, y_3)$ *be the* coordinates of vertices of the triangle.
Let $D(-3,2), E(1,-2)$ *and* $F(5,6)$ be the mid-points of the sides BC, CA *and* AB respectively.
Sine, $D(-3,2)$ is the mid-point of BC



∴ $\frac{x_2+x_3}{2} = -3$ *and* $\frac{y_2+y_3}{2} = 2$

⇒ $x_2 + x_3 = -6$ ....(i)

and $y_2 + y_3 = 4$ ......(ii)

As, $E(1,-2)$ is the mid-point of AC

∴ $\frac{x_1+x_3}{2} = 1$ *and* $\frac{y_1+y_3}{2} = -2$

⇨ $x_1 + x_2 = 2$ .....(iii)

and $y_1 + y_3 = -4$ ...(iv) (1)

Also, $F(5,6)$ is the mid-point of AB

∴ $\frac{x_1+x_2}{2} = 5$ *and* $\frac{y_1+y_2}{2} = 6$

⇨ $x_1 + x_2 = 10$ ......(v)

and $y_1 + y_2 = 12$ ....(vi)

On adding Eqs. (i), (iii) and (v), we get

$2(x_1 + x_2 + x_3) = 6$

⇨ $x_1 + x_2 + x_3 = 3$ .....(vii) (1)

On subtracting Eqs. (i), (iii) and (v) from Eq. (viii) in turn, we get

$x_1 = 9, x_2 = 1$ *and* $x_3 = -7$

On adding Eqs. (ii), (iv) and (vi), we get

$2(y_1 + y_2 + y_3) = 12$

⇨ $y_1 + y_2 + y_3 = 6$ .....(viii)

Subtracting Eqs. (ii), (iv) and (vi) from Eq. (viii)in turn, we get

$y_1 = 2, y_2 = 10$ *and* $y_3 = -6$

Hence, the vertices of ΔABC are $A(9,2), B(1,10)$ and $C(-7,-6)$.

31. **Following table gives marks scored by students in an examination**

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Number of Students	3	7	15	24	16	8	5	2

Calculate the mean mark correct to 2 decimal places.

Sol. We shall use step-deviation method. Construct the table as under, taking assumed mean a=17.5.

Here, c (width of each class)=5

Classes	Class mark y_i	$u_i = \frac{y_1 - a}{c}$	Frequency f_i	$f_1 u_i$
0-5	2.5	-3	3	-9
5-10	7.5	-2	7	-14
10-15	12.5	-1	15	-15
15-20	17.5	0	24	0
20-25	22.5	1	16	16
22-30	27.5	2	8	16
30-35	32.5	3	5	15
35-40	37.5	4	2	8
Total			80	17

$\therefore$ Mean $a + c \times \frac{\Sigma f_i u_i}{\Sigma f_i} = 17.5 + 5 \times \frac{17}{80}$ [2]

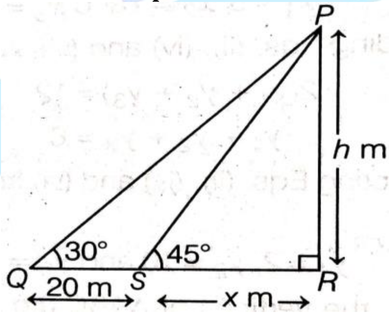
$= 17.5 + \frac{17}{16} = 17.5 + 1.0625$

$= 18.56$ (correct to 2 decimal places) (1)

SECTION-D

32. The angle of elevation of the top of a tower from certain point is 30^0. If the observer moves 20 m towards the tower, the angle of elevation of the top increases by 15^0, then find the height of the tower.

Sol. Let the height of the tower PR be $h\ m$, the angle of elevation at point Q is 30^0 $ie. \angle PQR = 30^0$ and S be the position of observer after moving 20 m towards the tower.



According to the question

$\angle PSR = \angle PQR + 15^0$

$\Rightarrow \angle PSR = 30^0 + 15^0$

$\Rightarrow \angle PSR = 45^0$

Now, in right angled ΔPRS

$\tan \angle PSR = \frac{PR}{SR} = \frac{h}{x}$ $\left[\because \tan\theta = \frac{P}{B}\right]$

$\Rightarrow \tan 45^0 = \frac{h}{x}$ $[from\ Eq.(i)]$

$\Rightarrow 1 = \frac{h}{x}$ $[\because \tan 45^0 = 1]$

$\Rightarrow x = h\ m$ ....(ii) (1/2)

and in right angled ΔPRQ,

$\tan 30^0 = \frac{PR}{QR} = \frac{PR}{QS+SR}$ $[\because QR = QS + SR]$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$ $\left[\because \tan 30^0 = \frac{1}{\sqrt{3}}\right]$

$\Rightarrow 20 + x = \sqrt{3}h$ (1/2)

⇨ $20+h=\sqrt{3}h$ $[from\ Eq.(ii)]$

⇨ $\sqrt{3}h-h=20 \Rightarrow h(\sqrt{3}-1)=20$

⇨ $h=\frac{20}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$ [by rationalising] (1/2)

⇨ $h=\frac{20(\sqrt{3}+1)}{3-1}$

$=\frac{20(\sqrt{3}+1)}{3-1}$

$=\frac{20(\sqrt{3}+1)}{2}$

$=10(\sqrt{3}-1)$m

Hence, the required height of the tower is $10(\sqrt{3}+1)$m.

33. **(a)Draw the graphs of $2x+y=6$ and $2x-y+2=0$. Shade the region bounded by these lines and X-axis. Find the area of the shaded region.**

Or

(b) Places $P_1 and\ P_2$ are 250 km apart from each other on a national highway. A car starts from P_1 and another from P_2 at the same time. If they go in the same direction, then they meet in 5 h and if they go in opposite directions they meet in $\frac{25}{13}h$, then find their speeds.

Sol. We have, $2x+y=6\ and\ 2x-y+2=0$

Table for equation $y=6-2x$ is

X	0	3
y	6	0

On plotting the points $A(0,6)\ and\ B(3,0)$ on a graph paper and join them, we obtain the graph of line represented by the equation $2x+y=6$ as shown in the figure.

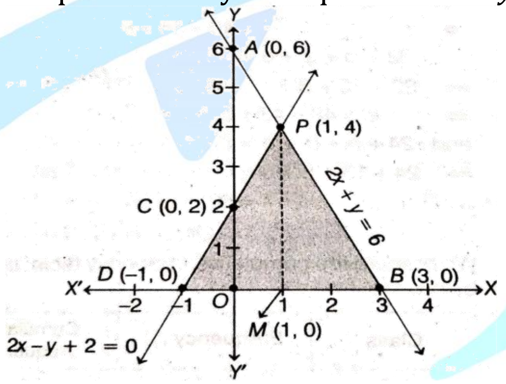


Table for equation $y=2x+2$ is

X	0	−1
y	2	0

On plotting the points $C(0,2)\ abd\ D(-1,0)$ on the same graph paper and join them, we obtain the graph of line represented by the equation $2x-y+2=0$ as shown in the figure.

The two line intersect at point $P(1,4)$

Thus, $x=1$ and $y=4$ is the solution of the given system of equations. The area enclosed by the lines and X-axis is shaded part in the figure. Draw PM perpendicular fro P on X-axis.

Clearly, we have

PM= y-coordinate of point $P(1,4)$

⇨ PM= 4 and DB=4

Area of the shaded region = Area of ΔPBD

$=\frac{1}{2}\times Base\times Height$

$= \frac{1}{2}(DM \times PM)$

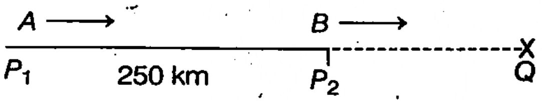
$= \frac{1}{2} \times 4 \times 4$

$= 8$ sq. units (1)

Or

Let A and B be the two cars. A starts from P_1 with constant speed of $x\ km/h$ and B starts from P_2 with constant speed of $y\ km/h$.

Case I When the two cars move in same directions as shown in figure, the cars meet at the position Q.



Here, $P_1Q = 5x\ km$, i.e. the distance travelled by car a in 5 h with $x\ km/h$ speed.

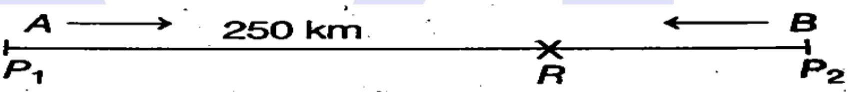
$P_2Q = 5\ y\ km$ i.e. the distance travelled by car B in 5 h with $y\ km/h$ speed.

We have, $P_1Q - P_2Q = 250$

⇨ $5x - 5y = 250$

⇨ $x - y = 50$ .....(i) (2)

Case II When two cars move in opposite directions as shown in figure, the cars meet at the position R.



Here, $P_1R = \frac{25}{13}x\ km$ and $P_2R = \frac{25}{13}y\ km$

So, $P_1R + P_2R = 250$

⇨ $x = y = 130$ ......(ii) (2)

On adding Eq. (i) and Eq. (ii), we get

$2x = 180$

⇨ $x = 90$

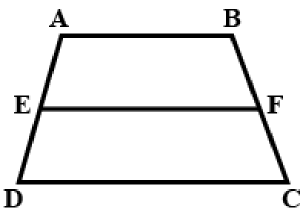
On adding Eq. (i) and Eq. (ii), we get

$2y = 80$

⇨ $y = 40$

∴ Their speeds are 90 km/h and 40 km/h. (1)

34. ***ABCD* is a trapezium with $AB||DC$. *E and F* are two points on non-parallel sides *AD and BC* respectively, such that *EF* is parallel to *AB*. Show that $\frac{AE}{ED} = \frac{BF}{FC}$.**

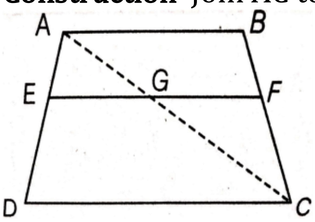


Sol. **Given** In trapezium $ABCD$

$AB||DC$ and $EF||AB$

To prove $\frac{AE}{ED} = \frac{BF}{FC}$

Construction Join AC to intersect EF at G.



Proof Since, $AB||DC$ and $EF||AB$

$\therefore \quad EF||DF$ [since, lines parallel to the same line are also parallel to each other]

In ΔADC, $EG||DC$ [$\because EF||DC$]

By using basic proportionally theorem,

$$\frac{CG}{AG} = \frac{CF}{BF} \text{ or } \frac{AG}{GC} = \frac{BF}{CF} \quad \text{....(ii)}$$

[on taking reciprocal of the terms]

From Eqs. (i) and (ii), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$ **Hence proved.** (2)

35. **(a) From a solid cylinder whose height is 12 cm and diameter is 10 cm, a conical cavity of same height and same diameter is hollowed out. Find the volume and total surface area of the remaining solid.**

Or

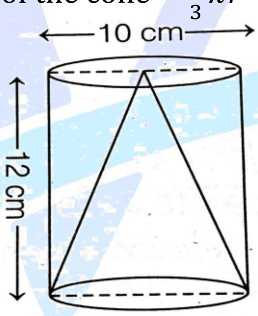
(b) A right angled triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone, so formed. [choose the value of π as found appropriate]

Sol. Given, diameter of the cylinder, $d = 10$ cm $\Rightarrow r = 5\ cm$ and height of the cylinder, $h = 12\ cm$

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 5 \times 5 \times 12 = \frac{6600}{7}\ cm^3 \quad (1)$$

and volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 = \frac{2200}{7}\ cm^3$ (1/2)



Now, volume of remaining solid = Volume of the cylinder – Volume of the cone

$= \frac{6600}{7} - \frac{2200}{7} = \frac{4400}{7} = 628.57\ cm^3$ (1)

Since, slant height of the cone,

$l = \sqrt{r^2 + h^2} = \sqrt{(5)^2 + (12)^2} = \sqrt{169} = 13\ cm$

$\therefore$ Curved surface area of the cone $= \pi r l$

$$= \frac{22}{7} \times 5 \times 3 = \frac{1430}{7}\ cm^2 \quad (1/2)$$

Clearly, curved surface, area of the cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 5 \times 12 = \frac{2640}{7}\ cm^2$$

and area of upper base of the cylinder $= \pi r^2$

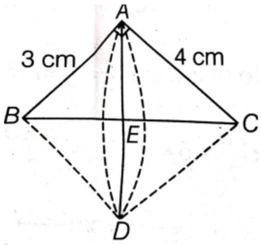
$$= \frac{22}{7} \times 5 \times 5 = \frac{550}{7}\ cm^2 \quad (1)$$

Now, total surface area of the remaining solid= Curved surface area of the cylinder + Curved surface area of the cone +Area of upper base of the cylinder

$= \frac{2640}{7} + \frac{1430}{7} + \frac{550}{7}$

$= \frac{4620}{7} = 660\ cm^2$

Let ABC be a right angled triangle, right angled at A and BC is the hypotenuse.

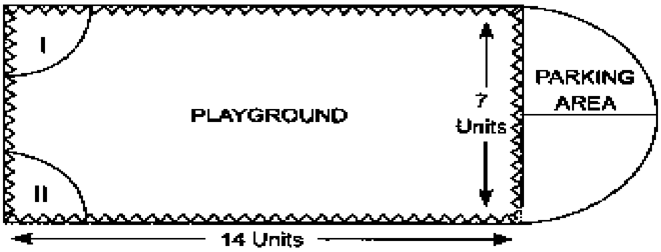


Also, let $AB = 3\ cm$
and $AC = 4\ cm,$
Then, $BC = \sqrt{3^2 + 4^2}$ [by Pythagoras theorem]
$\sqrt{9 + 16} = \sqrt{25} = 5\ cm$ (1)
As, ΔABC revolves about the hypotenuse BC. It forms two cones ABD and ACD.
In $\Delta ABC\ and\ \Delta CAB, \angle AEB = \angle CAB$ [each 90^0]
$\angle ABE = \angle ABC$ [common]
∴ $\Delta AEB \sim \Delta CAB$ [by AA similarity criterion]
∴ $\frac{AE}{CA} = \frac{AB}{BC}$ [∵ in similar triangles, corresponding sides are proportional]
⇨ $\frac{AE}{4} = \frac{3}{5} \Rightarrow AE = \frac{12}{5} = 2.4$ (1)
So, radius of the base of each cone, $AE = 2.4\ cm$
Now, in right angled ΔAEB,
$BE = \sqrt{AB^2 - AE^2}$ [by Pythagoras theorem]
$= \sqrt{(3)^2 - (2.4)^2}$
$= \sqrt{9 - 5.76} = \sqrt{3.24} = 1.8$
So, height of the cone $ABD = BE = 1.8\ cm$
∴ Height of the cone, $ACD = CE = BC - BE$
$= 5 - 1.8 = 3.2\ cm$ (1)
Now, volume of he cone ACD
$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 \times 1.8$
$= \frac{22}{21} \times 10.368 = 10.86\ cm^3$
and volume of the cone ACD
$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 \times 3.2$ [∵ radius will be same as AD is common]
$= \frac{404.504}{21} = 19.31\ cm^3$ (1)
∴ Required volume of double cone = 10.86+19.31
$= 30.17\ cm^3$
Now, surface area of cone $ABD = \pi rl$
$= \frac{22}{7} \times 2.4 \times 3 = \frac{158.4}{7}$
$= 22.63\ cm^3$
and surface area of cone $ACD = \pi rl$
$= \frac{22}{7} \times 2.4 \times 4 = \frac{211.2}{7} = 30.17\ cm^2$
∴ Required surface area of double cone
$= 22.63 + 30.17 = 52.8\ cm^2.$ (1)

SECTION-E

Case Study Based.

36. **Governing council of a local public development 2 authority of Dehradun decided Sa adventurous playground on to build an the top of a hill, which 36 will have adequate space for parking.**



After survey, it was decided to build rectangular playground, with a semi-circular area allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.

Based on the given information, answer the following questions.

(i) What is the total perimeter of the parking area?

(ii) (a) What is the total area of parking and the two quadrants?

Or

(b) What is the ratio of area of playground to the area of parking area?

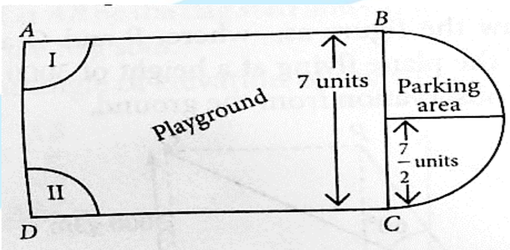
(iii) Find the cost of fencing the playground and parking area at the rate ₹2 per unit.

Sol. (i) Length of playground, $AB = 14\ units$

Breadth of playground, $AD = 7\ units$

Radius of semi-circular part is 7/2 units

Total perimeter of parking area= $\pi r + 2r$



$= \frac{22}{7} \times \frac{7}{2} + 2 \times \frac{7}{2} = 11 + 7 = 18\ units$

(ii) $(a) Area\ of\ parking = \frac{\pi r^2}{2} = \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 19.25\ sq.units.$

Area of two quadrants (I) and (II) $= 2 \times \frac{1}{4} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 2 \times 2 = 6.29\ sq.units$

Total area of parking and two quadrant

$= 19.25 + 6.29 = 25.54\ sq.units$

Or

(c) Area of playground = length $\times\ breadth = 14 \times 7 = 98\ sq.units.$

Area of parking $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

Required ratio$= \frac{98}{\frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}} = \frac{98\times4}{77} = \frac{56}{11} = 56 : 11$

(iii) Perimeter of parking area =18 units.

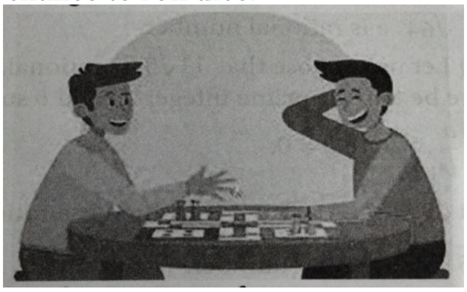
So, the cost of fencing the parking area=₹(18 × 2)=₹36

Length of remaining three sides of playground = 14+14+7=35 units.

Now, the cost of fencing three sides = ₹2× 35 = ₹70

Total cost = ₹36+₹70=₹106.

37. **Rohan and his friend Akash planned to play ludo Set in weekend Rohan got first chance to roll two dice, then Akash got next chance to roll dice.**



Based on the given information, answer the following questions.

(i) What is the probability that Rohan got the product of two numbers appearing on the top of the dice is 12?

(ii) (a) Find the probability that Akash got the odd number on the top of both the dice.

Or

(b) What is the probability that Rohan got the sum of two numbers appearing on the top of dice is 8?

(iii) Find the probability that Akash got the prime numbers on the top of both the dice.

Sol. (i) Total number of outcomes= $6 \times 6 = 36$

Favourable outcomes are {(2,6), (3,4), (4,3), (6,2)}

Total number of favourable outcomes =4

$\therefore\ P(Product\ on\ both\ dice\ is\ 12) = \frac{4}{36}\ \frac{1}{9}$

(ii) (a) total number of outcomes = $6 \times 6 = 36$

Favourable outcomes are {(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5)}

Total number of favourable outcome = 9

∴ P(Odd number on both dice)=$\frac{9}{36} = \frac{1}{4}$

Or

(b) Total number of outcomes = $6 \times 6 = 36$

According to the question, number of favourable outcomes are {(2,6), (3,5), (4,4), (5,3), (6,2)}

∴ Total number of favourable outcome=5

So, probability that Rohan got the sum of two numbers appearing on the top of dice is 8 = $\frac{5}{36}$

(iii) Total number of outcomes = $6 \times 6 = 36$

Prime number included in dice = {2,3,5}

Favourable outcomes are [(2,2), (2,3), (2,5), (3,2), (3,3), (3,5), (5,2), (5,3), (5,5)]

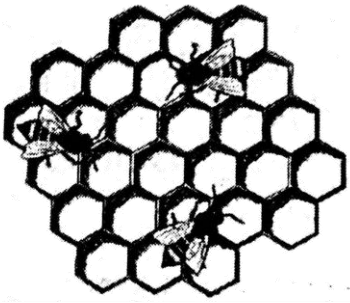
∴ Total number of favourable outcomes = 9

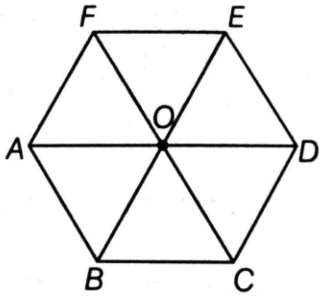
Hence the probability that Akash got prime numbers on the top of both dice= $\frac{9}{36} = \frac{1}{4}$

38. **Beehive**

A beehive is an enclosed cell structure in which some honeybee species of the subgenus apis live and raise their young. Each cell is in the shape of a hexagon.

In a regular hexagon, there are six edges of equal lengths, Take *O* as centre and join all the vertices from the centre.





Similarity of Triangle

Two triangles are said to be similar, if their all corresponding angles are equal and all corresponding sides are proportional.

Based on the above information, answer the following questions.

(i) Find the number of equilateral triangles in the given figure. (1)

(ii) If area of two triangles are equal, then they are always congruent or not. (1)

(iii) (a)How many triangles are similar in the given figure? (2)

Or

(b)Find the area of the hexagon, if each edge is of length a. (2)

Sol. (i) Total number of equilateral triangles in the given figure is 6.

(ii) If area of two triangles are equal, then they are always congruent.

(iii) As we known that there are six equilateral triangle all having equal sides. Hence, we get six similar triangles.

Or

Area of hexagon $= 6 \times Area\ of\ one\ equilateral\ triangle$ having side a

$= 6 \times \frac{\sqrt{3}}{4} \times (a)^2$

$= \frac{3\sqrt{3}}{2}\ (a)^2$ sq. units