Statistics Sector 2: Hypothesis Tests Normal Population Mean μ, σ known z test

Aims:

- Be able to carry out a hypothesis test about a sample mean from a normal distribution with known variance.
- Interpret the findings of a hypothesis test in context.
- Understand the use of a p-value.

Recall from estimation section that

$$\text{If } X \sim N(\mu, \sigma^2) \text{ then that } \bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

hence we can use the Normal distribution to test a specified value of μ using:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

This test assumes:

1) Random sample
2) Data can be modelled by the normal distribution.

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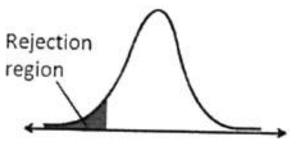
1) Let X be.....

2) State the hypothesis
   $H_o: \mu = value$ (Population mean = given value)
   $H_1: \mu < value, \ \mu > value \ or \ \mu \neq value$

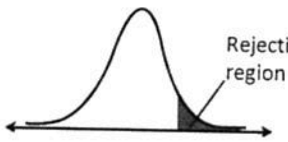
3) Test Statistic

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

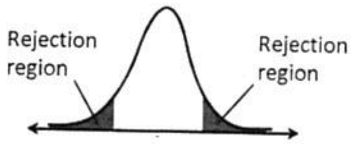
4) Find the critical value for z from Normal distribution using InvN $\sigma = 1$ and $\mu = 0$ and mark on diagram



H_1: $\mu < value$



H_1 : $\mu > value$



H_1: $\mu \neq value$

5) Conclude in the context of the question, if z is in the critical region then there is evidence to reject H_o

Example 1

A light bulb manufacturer has established that the life of a bulb has a mean of 95.2 days with standard deviation 10.4 days. Following a change in the manufacturing process, which is intended to increase the life of a bulb, a random sample of 16 bulbs has a mean of 96.6 days.

Assuming that the life of a bulb follows a normal distribution and the population standard deviation is unchanged, test whether there is significant evidence, at the 5% level of an increase in life.

Hypothesis

$H_0: \mu = 95.2$

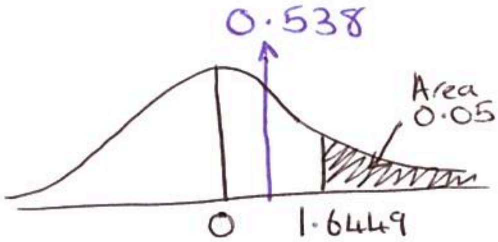
$H_1: \mu > 95.2$ (One tailed)

Test Statistic

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{96.6 - 95.2}{\frac{10.4}{\sqrt{16}}} = 0.538$$

Critical Value

InvN
Tail: Right
Area: 0.05
σ : 1
μ : 0



Conclusion

Accept H_0, there is no significant evidence to suggest a change in the manufacturing process has increased the average life of a bulb.

Example 2

A forestry worker decided to keep records of the first year's growth of pine seedlings. Over several years, she found that the growth followed a normal distribution with a mean of 11.5 and standard deviation of 2.5cm.

Last year she used an experimental soil preparation for the pine seedlings and the first year's growth of eight of the seedlings was

7 22 19 15 11 18 17 15 $\bar{x} = 15.5$

Investigate at the 1% significance level, whether there has been a change in the mean growth. Assume the standard deviation has not changed.

Hypothesis

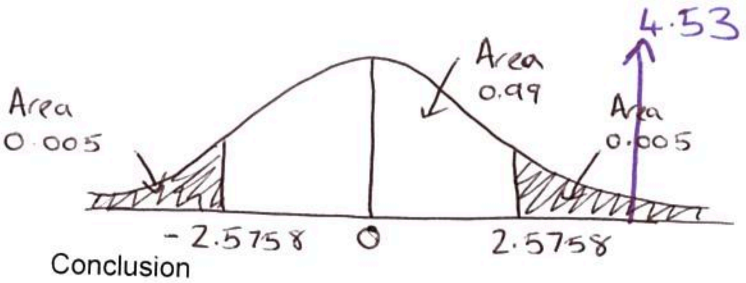
$H_0: \mu = 11.5$

$H_1: \mu \neq 11.5$ (two tailed)

Test Statistic

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{15.5 - 11.5}{\frac{2.5}{\sqrt{8}}} = 4.53$$

Critical Value



Conclusion

Reject H_0, there is significant evidence to suggest that there has been a change in the mean growth of pine seedlings.

Example 3

The keepers of a lighthouse were required to keep records of weather conditions. The lighthouse is no longer manned, but a coast-guard has a theory that the air has become clearer and so visibility has increased. To test this theory he decides to record the visibility from the lighthouse around mid-day, in nautical miles, for a sample of 20 days over the following 3 months.

a) State and describe a suitable sampling method the coast-guard could use to collect the data.

Simple Random Sampling

- Number the days from 1 to n, where n is the number of days in 3 months.
- Use a random number generator to select 20 numbers from 1 to n, ignoring repeats.
- Select the corresponding days and record the visibility at mid-day on those days.

b) Give an advantage and a disadvantage of your chosen sampling method.

Advantage

- The sample is random and unbiased.

Disadvantage

- It may not be possible/practical to travel to the lighthouse on those days.

c) Other than a change in the sampling method, give one other way the coastguard could reduce bias in his sample.

The readings should be recorded by the same person at exactly the same time each day.

After collecting his data, the results, in nautical miles, were as follows:

35	21	12	7	2	14	18	20	16	11
8	8	5	11	28	35	16	35	9	17

d) Calculate the mean and standard deviation of this sample.

$\bar{x} = 16.4$

$S = 10.0389$

Analysis of data over many years showed that the visibility at mid-day had a mean value of 14 nautical miles with standard deviation 9.4 nautical miles.

e) Assuming that the visibility can be modelled by a normal distribution with standard deviation 9.4 nautical miles, test the coast-guard's theory at the 5% significance level.

Hypothesis

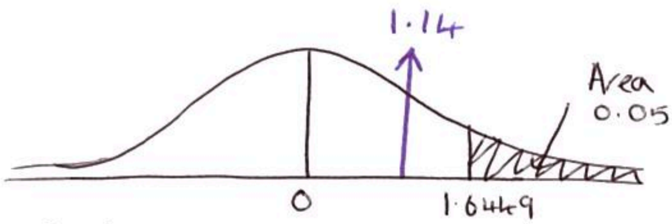
$H_0: \mu = 14$

$H_1: \mu > 14$ (one tailed)

Test Statistic

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{16.4 - 14}{\frac{9.4}{\sqrt{20}}} = 1.14$$

Critical Value



Conclusion

Accept Ho, there is no significant evidence to suggest the average visibility has increased at the lighthouse.

Example 4

In an experiment on people's perception, each student in a sample of 100 university students was given a piece of paper which was blank except for a line 120 mm long. The students were asked to judge by eye the centre point of the line, and to mark it. Each student then measured the distance between the left hand end of the line and the mark made. Their mean distance was 58.9 mm. It is known from previous research that $\sigma = 3.71$ mm.

a) If there were no bias in the students' perception of the centre of the line, the mean distance would be 60 mm. Determine whether there is significant evidence, at the 5% level, of any overall bias in the students' perception of the centre of a line. State any assumptions you have made about the population.

b) Is your conclusion affected by whether or not the sample was taken at random? If so, how?

a) Assumptions

1. The sample of university students was random.
2. The distances are normally distributed.

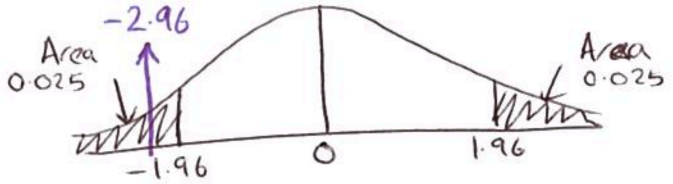
$H_0: \mu = 60$

$H_1: \mu \neq 60$ (two tailed)

Test Statistic

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{58.9 - 60}{\frac{3.71}{\sqrt{100}}} = -2.96$$

Critical Values



Reject H_0, there is significant evidence to suggest the average distance is not 60mm and that there is bias in the students' perception of the centre of the line.

b) If the sample was not taken at random the conclusion may not be valid.

Exam Questions

A machine fills paper bags with flour. Before maintenance on the machine, the weight of the flour in a bag could be modelled by a normal distribution with mean 1005 grams and standard deviation 2.1 grams. Following this maintenance, the flour in each of a random sample of 8 bags was weighed. The weights, in grams, were as follows.

$\bar{x} = 1006.2875$

1006.1 1004.9 1005.8 1007.9 1004.7 1006.3 1007.4 1007.2

Carry out a test, at the 10% significance level, to decide whether the mean weight of flour in a bag filled by the machine had **changed**. Assume that the distribution of weights was still normal with standard deviation 2.1 grams. *(7 marks)*

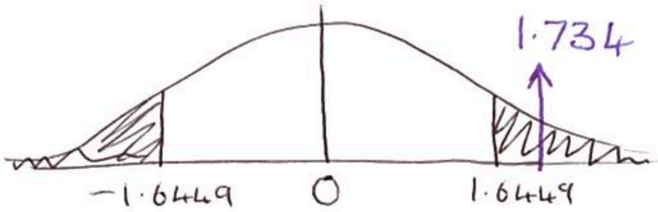
$H_0: \mu = 1005$

$H_1: \mu \neq 1005$ (two tailed)

Test Statistic

$$Z = \frac{1006.2875 - 1005}{\frac{2.1}{\sqrt{8}}} = 1.734$$

Critical Values



Reject H_0, there is significant evidence to suggest the mean weight of flour in a bag has changed.

A consumer organisation is investigating the service offered by companies supplying household gas.

(a) The waiting times, in seconds, between a telephone call connecting to a gas company, Northgas, and the caller actually speaking to one of its employees were recorded for nine telephone calls as follows:

76 157 62 56 193 34 89 185 134 $\bar{x} = 109.56$

Test, using the 5% significance level, whether the mean waiting time for calls made to Northgas exceeds 90 seconds. Assume that this sample is a random sample from a normal distribution with standard deviation 55 seconds. *(8 marks)*

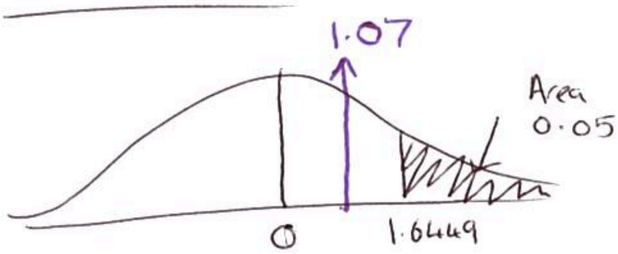
$H_0: \mu = 90$

$H_1: \mu > 90$ (one tailed)

Test Statistic

$$Z = \frac{109.56 - 90}{\frac{55}{\sqrt{9}}} = 1.07$$

Critical Value



Accept Ho, there is no significant evidence to suggest the mean waiting time for calls made to Northgas exceeds 90 secs.