2 (a) Express $3\sin x - 2\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving your value of α to the nearest $0.1°$.

[3 marks]

(b) Hence find the minimum value of $3\sin x - 2\cos x$ and the value of x in the interval $0° < x < 360°$ where the minimum occurs. Give your value of x to the nearest $0.1°$.

[3 marks]

2 By forming and solving a suitable quadratic equation, find the solutions of the equation

$$3\cos 2\theta - 5\cos\theta + 2 = 0$$

in the interval $0° < \theta < 360°$, giving your answers to the nearest $0.1°$.

[5 marks]

Q 2	Solution	Mark	Total	Comment
(a)	$R = \sqrt{13}$ $\cos\alpha = \frac{3}{their\sqrt{13}}$ or $\sin\alpha = \frac{2}{their\sqrt{13}}$ $\alpha = 33.7$	**B1** **M1** **A1**	**3**	Accept 3.61 or better rounded correctly. or $\tan\alpha = \frac{2}{3}$ **CAO** - **must** be to 1 d.p.
	Award **B1** for $R = \sqrt{13}$ even if it comes from $\sqrt{3^2 + (-2)^2}$ If $\tan\alpha = \frac{2}{3}$ comes from **wrong** work e.g. $\cos\alpha = 3$ and $\sin\alpha = 2$ or using $\tan\alpha = \frac{\cos\alpha}{\sin\alpha}$ then **M0 A0** but full marks can be earned in (ii). If $\tan\alpha = -\frac{2}{3}$ leads to $\alpha = 33.7^0$ then **M0 A0** but again marks can be earned in (ii). R and α must be found in part (a) to earn these marks.			
(b)	Minimum value is $-\sqrt{13}$ (comes from) $\sin(x - \alpha) = -1$ $x - 33.7 = 270$ $x = 303.7$	**B1ft** **M1** **A1**	**3**	**ft** on $-R$ from (i). Allow -3.61 or better PI by later correct work – e.g. 270^0 or correct final answer **CAO** - **must** be to 1 d.p.
			6	

Q2	Solution	Mark	Total	Comment
	$\cos 2\theta = 2\cos^2\theta - 1$ **used**	B1		PI: Correct expression in terms of $\cos\theta$ used.
	$3(2\cos^2\theta - 1) - 5\cos\theta + 2 \quad (= 0)$	M1		Attempt to use identity for $\cos 2\theta$ of the form $a\cos^2\theta + b$ to obtain a quadratic in $\cos\theta$.
	$6\cos^2\theta - 5\cos\theta - 1 = 0$ $(\cos\theta - 1)(6\cos\theta + 1) = 0$	m1		Attempt to factorise their quadratic or correct use of quadratic formula.
	$(\cos\theta = 1) \quad \cos\theta = -\frac{1}{6}$			
	$\theta = 99.6^0 \quad , 260.4^0$	A1 A1		**Either** correct – CAO **Both** correct and no extra values in the interval but ignore any values outside of the interval including 0^0 and 360^0.
	Total		5	

3 (a) Show that

$$\sin 2x - \tan x = \tan x \cos 2x$$

[3 marks]

(b) Hence solve the equation

$$\sin 2x = \tan x$$

in the interval $0° \leqslant x \leqslant 180°$.

[3 marks]

(a)	LHS $= \sin 2x - \tan x = 2\sin x\cos x - \frac{\sin x}{\cos x}$	B1		**Correct** in terms of sin x and cos x. Condone any letter provided consistent
	$= \sin x\left(2\cos x - \frac{1}{\cos x}\right)$	B1		or $\frac{2\sin x\cos^2 x - \sin x}{\cos x}$
	$= \frac{\sin x}{\cos x}(2cos^2 x - 1)$			or $\frac{\sin x}{\cos x}(1 - 2\sin^2 x)$
	$= \tan x\cos 2x$	B1	3	AG!!! - be convinced – expect to see intermediate line.

<u>**Some Possible Alternatives**</u>

LHS $= 2\sin x\cos x - \tan x$ **B1** $= 2\frac{\sin x}{\cos x}.cos^2 x - \tan x$ **B1** $= \tan x\,(2cos^2 x - 1) = \tan x\cos 2x$ **B1**

LHS $= 2\sin x\cos x - \frac{\sin x}{\cos x}$ **B1** $= \sin x\left(2\cos x - \frac{1}{\cos x}\right)$ **B1** $= \tan x\cos 2x$ **B0** **not** convinced.

RHS $= \tan x\cos 2x = \frac{\sin x}{\cos x}(2cos^2 x - 1)$ **B1** $= 2\sin x\cos x - \frac{\sin x}{\cos x}$ **B1** $= \sin 2x - \tan x$ **B1**

RHS $= \tan x\cos 2x = \tan x\,(2cos^2 x - 1)$ **B1** $= 2\sin x\cos x - \tan x$ **B1** $= \sin 2x - \tan x$ **B1**

There may be other ways – be reasonably generous with first two marks that could lead to the result – but be more rigorous with the final mark.

Candidates who work from both sides and meet can score **B1** for any useful identity then **B1** when in a position to equate the two sides but then **B0**.

(b)	<u>**Hence**</u>			
	$\sin 2x = \tan x \rightarrow \tan x = 0$ or $\cos 2x = 0$	M1		PI by one pair of solutions.
	$\tan x = 0 \rightarrow x = 0 \quad x = 180$	A1		**Both** solutions.
	$\cos 2x = 0 \rightarrow x = 45 \quad x = 135$	A1	3	**Both** solutions

<u>**Otherwise**</u>

Although question says 'Hence…' we will allow

$\sin 2x = \tan x \rightarrow 2\sin x\cos x = \frac{\sin x}{\cos x} \rightarrow \sin x = 0$ **or** $2\cos x = \frac{1}{\cos x}$ (OE) **M1** (either of these).

$\sin x = 0 \rightarrow x = 0$ or 180 **A1** Both solutions $2\cos x = \frac{1}{\cos x} \rightarrow x = 45$ or 135 **A1** Both solutions

In either method, ignore any values (even if wrong) outside $0 \leq x \leq 180$

Penalise more than 4 solutions inside $0 \leq x \leq 180$ just once.

<u>**Answers only**</u>

Award **B2** for one pair $(0$ **and** $180)$ **or** $(45$ **and** $135)$ and **B1** for second pair

2 The angle α is **acute** and $\cos\alpha = \frac{\sqrt{3}}{3}$. The angle β is **obtuse** and $\sin\beta = \frac{1}{3}$.

(a) Show that $\tan\alpha = \sqrt{2}$ and find an exact value for $\tan\beta$.

[3 marks]

(b) Hence show that $\tan(\alpha - \beta)$ can be written as $p\sqrt{2}$, where p is a rational number.

[2 marks]

4 The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 18x^3 - 3x^2 - 28x - 12$.

(a) (i) Use the Factor Theorem to show that $(3x + 2)$ is a factor of $\mathrm{f}(x)$.

[2 marks]

(ii) Express $\mathrm{f}(x)$ as a product of linear factors.

[3 marks]

(b) The function g is defined for all real values of θ by

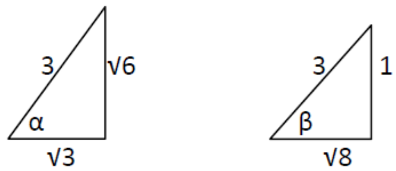
$$\mathrm{g}(\theta) = 18\sin 2\theta\cos\theta - 3\cos 2\theta + 20\sin\theta + 27$$

(i) Show that the equation $\mathrm{g}(\theta) = 0$ can be written as $\mathrm{f}(x) = 0$, where $x = \sin\theta$.

[4 marks]

(ii) Hence solve the equation $\mathrm{g}(\theta) = 0$, giving your answers, in radians, to three significant figures in the interval $0 \leqslant \theta \leqslant 2\pi$.

[2 marks]

(a)				or Pythagoras
	$\tan\alpha = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$	B1		AG - must see √6 in this approach
(b)	$\tan\beta = (\pm)\frac{1}{\sqrt{8}}$	M1		Either $\frac{1}{\sqrt{8}}$ or $-\frac{1}{\sqrt{8}}$
	$\tan\beta = -\frac{1}{\sqrt{8}}$	A1	3	ACF: e.g. $-\frac{1}{2\sqrt{2}}$ or $-\frac{\sqrt{2}}{4}$ etc.
	$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$			
	$= \frac{\sqrt{2} - \left(-\frac{1}{\sqrt{8}}\right)}{1+\sqrt{2}\left(-\frac{1}{\sqrt{8}}\right)}$	M1		**Correct** identity with $\tan\alpha = \sqrt{2}$ and **their** $\tan\beta$ value **correctly** substituted.
	$= \frac{5}{2}\sqrt{2}$	A1	2	OE – accept if written as $\frac{5\sqrt{2}}{2}$ etc. NMS scores 0/2.
			5	

Q4	Solution	Mark	Total	Comment
(a)(i)	$18\left(-\frac{2}{3}\right)^3 - 3\left(-\frac{2}{3}\right)^2 - 28\left(-\frac{2}{3}\right) - 12$	M1		Correct substitution of $x = -\frac{2}{3}$
	$= 18 \times \left(-\frac{8}{27}\right) - 3\left(\frac{4}{9}\right) - 28\left(-\frac{2}{3}\right) - 12$			or better
	$= 0$ (hence) factor	A1	2	**Correct** arithmetic **and** conclusion.
(a)(ii)	By factors $6x^2 + bx - 6$	M1		'Spotting' $a = 6$ and $c = -6$.
	$= 6x^2 - 5x - 6$	A1		
	$(f(x)) = (3x+2)(3x+2)(2x-3)$ OE	A1	3	**NMS** scores 3/3 if correct
(b)(i)	$18\sin2\theta\cos\theta - 3\cos2\theta + 20\sin\theta + 27$			
	$= 18 \times 2\sin\theta\cos\theta\cos\theta$	B1		**Correct** identity **used** for $\sin2\theta$.
	$-3(1-2\sin^2\theta) + 20\sin\theta + 27$	B1		**Any correct** identity **used** for $\cos2\theta$.
	$= 36\sin\theta(1-\sin^2\theta) - 3 + 6\sin^2\theta + 20\sin\theta + 27$	M1		Use $\cos^2\theta = 1 - \sin^2\theta$ to obtain a cubic expression in $\sin\theta$ only.
	$= -36\sin^3\theta + 6\sin^2\theta + 56\sin\theta + 24$ $(= -36x^3 + 6x^2 + 56x + 24)$			
	Equate to 0 and cancel down to the **equation** $18x^3 - 3x^2 - 28x - 12 = 0$	A1	4	Do not award final **A1** if division or changing signs occurs before equating to 0 or any error seen. Accept in terms of $\sin\theta$.
(b)(ii)	$(\theta =)$ 3.87 , 5.55 CAO	B1B1	2	-1 for **each** extra sol^n in $0 \le \theta \le 2\pi$

2 (a) Express $7\cos x + 3\sin x$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving your value of α to the nearest $0.1°$.

[3 marks]

(b) Use your answer to part **(a)** to solve the equation $7\cos 2\theta + 3\sin 2\theta = 5$ in the interval $0° < \theta < 180°$, giving your solutions to the nearest $0.1°$.

[3 marks]

5 (a) By replacing 3θ by $(2\theta + \theta)$ show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

[4 marks]

(b) By using the result from part **(a)** and assuming that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, find the exact value of

$$\int_0^{\frac{\pi}{6}} \left(2\sin^3\theta + 3\right)\,\mathrm{d}\theta$$

[6 marks]

Q2	Solution	Mark	Total	Comment
(a)	$R = \sqrt{58}$	B1		**Must see** $R = \sqrt{58}$, $R = \pm\sqrt{58}$ is **B0**
	$\sqrt{58}\cos\alpha = 7$ or $\sqrt{58}\sin\alpha = 3$ or $\tan\alpha = \frac{3}{7}$	M1		**ft** on their value of R.
	$\alpha = 23.2^0$	A1	3	**Must see** $\alpha = 23.2^0$ Allow AWRT 22.9^0 to 23.3^0.
	Accept any decimal equivalent to $\sqrt{58}$ to at least 3 SF provided it is rounded **correctly** – e.g. 7.62, 7.616 etc. e.g. using $R = 7.61$ to get $\alpha = 23.1^0$ would score **B0 M1 A1**. Explicit use of $\cos\alpha = 7$ and $\sin\alpha = 3$ to get to $\tan\alpha = \frac{3}{7}$ is **M0 A0** but marks in (b) are available. Candidates who write $R\cos\alpha = 7$ and $R\sin\alpha = 3$ without finding R but reach a correct value for α score **M1 A1**. An expression of the form $R\cos(x - \alpha)$ can score the **B1** and/or **A1** if R and/or α are correct.			
(b)	$\cos^{-1}\left(\frac{5}{\sqrt{58}}\right)$ or 48.964 ...	M1		Finding an angle from $\cos^{-1}\left(\frac{5}{R}\right)$.
	and 311.0359…	dM1		For $360 - \cos^{-1}\left(\frac{5}{R}\right)$ **only** between 0^0 and 360^0
	36.1^0 and 167.1^0	A1	3	CAO. These two values **only**.
	Question says 'Use your answer to part (a)' so using a different method or **NMS** is 0/3. For **M1** and **dM1** marks accept 2 SF or better. **dM1** is for correct ft fourth quadrant solution (360^0 – first solution) and **NO** others between 0^0 and 360^0. The **dM1** mark could be PI if candidate goes straight to the two correct answers from the **M1** mark. Ignore any solutions outside the interval $0^0 \leq \theta \leq 180^0$ for final **A1**. Condone omission of degree symbol or other letter in place of θ.			
	Total		6	

(a)	$\sin 3\theta = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$	B1		**Use** of the **correct** $\sin(A + B)$ identity. PI by next two B marks
	$= 2\sin\theta\cos\theta\cos\theta$	B1		**Use** of the **correct** sin *2A* identity
	$+ (1 - 2\sin^2\theta)\sin\theta$	B1		**Use** of a **correct** cos *2A* identity in ACF
	$= 3\sin\theta - 4\sin^3\theta$	B1	4	AG – be convinced
(b)	From (a) $2\sin^3\theta = \frac{1}{2}(3\sin\theta - \sin 3\theta)$	B1		OE: ACF to enable them to replace $2\sin^3\theta$.
	$\int 2\sin^3\theta \, d\theta = p\cos\theta + q\cos 3\theta$	M1		p and q are any constants
	$= -\frac{3}{2}\cos\theta + \frac{1}{6}\cos 3\theta$	A1		Both integrated correctly
	$\int 3 \, d\theta = 3\theta$	B1		Must be 3θ
	$\int \quad = \left[-\frac{3}{2}\cos\frac{\pi}{6} + \frac{1}{6}\cos\frac{3\pi}{6} + 3\frac{\pi}{6}\right]$ $-\left[-\frac{3}{2}\cos 0 + \frac{1}{6}\cos 0 + (0)\right]$	dM1		$F\left(\frac{\pi}{6}\right) - F(0)$ **correct** for their integrated function - no MC allowed here.
	$= \left[-\frac{3\sqrt{3}}{4} + (0) + \frac{\pi}{2}\right] - \left[-\frac{3}{2} + \frac{1}{6} + (0)\right]$			
	$= -\frac{3\sqrt{3}}{4} + \frac{4}{3} + \frac{\pi}{2}$	A1	6	ACF but must be exact. Apply ISW (if necessary) after correct answer seen

5 It is given that $\sin A = \frac{\sqrt{5}}{3}$ and $\sin B = \frac{1}{\sqrt{5}}$, where the angles A and B are both acute.

(a) (i) Show that the exact value of $\cos B = \frac{2}{\sqrt{5}}$.

[1 mark]

(ii) Hence show that the exact value of $\sin 2B$ is $\frac{4}{5}$.

[2 marks]

(b) (i) Show that the exact value of $\sin(A - B)$ can be written as $p(5 - \sqrt{5})$, where p is a rational number.

[4 marks]

(ii) Find the exact value of $\cos(A - B)$ in the form $r + s\sqrt{5}$, where r and s are rational numbers.

[3 marks]

(a)(i)	Use of $\sin^2 B + \cos^2 B = 1$ $\left(\frac{1}{\sqrt{5}}\right)^2 + \cos^2 B = 1$ $\cos B = \frac{2}{\sqrt{5}}$	**B1**	**1**	Or use of right-angled triangle with opp = 1 and hyp = $\sqrt{5}$ to get adj = $\sqrt{4}$ or 2. $\cos\left(\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) = \frac{2}{\sqrt{5}}$ is **B0** **AG** ; must see evidence of working
(a)(ii)	$(\sin 2B = 2\sin B\cos B)$ $= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$ $= \frac{4}{5}$	**M1** **A1**	**2**	Correct identity (PI) **and** substitution **AG** so line above must be seen.
(b)(i)	$\cos A = \frac{2}{3}$ **exact** value $\sin(A - B) = \sin A\cos B - \cos A\sin B$ $= \frac{\sqrt{5}}{3} \times \frac{2}{\sqrt{5}} - \frac{2}{3} \times \frac{1}{\sqrt{5}}$ Use of $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ or $\frac{10-2\sqrt{5}}{15}$ OE seen $\frac{2}{15}(5 - \sqrt{5})$	**B1** **M1** **m1** **A1**	**4**	$\cos A = \frac{2}{3}$ seen or used (**not** 0.667 etc.) ft on their value of cosA $\frac{2}{3\sqrt{5}}$ term becoming $\frac{2\sqrt{5}}{15}$ **before** final answer $\frac{2}{15}$ OE seen and be convinced
	You must see justification between the use of the identity and the final answer to earn the **m1 A1**.			
(b)(ii)	$\cos(A - B) = \cos A\cos B + \sin A\sin B$ $= \frac{2}{3} \times \frac{2}{\sqrt{5}} + \frac{\sqrt{5}}{3} \times \frac{1}{\sqrt{5}}$ $= \frac{1}{3} + \frac{4}{15}\sqrt{5}$	**M1** **A1** **A1**	**3**	ft on their value of $\cos A$ fully correct OE for $\frac{1}{3}$ and $\frac{4}{15}$ but not left as $\frac{5+4\sqrt{5}}{15}$
	Total		**10**	

2 (a) Express $2\cos x - 5\sin x$ in the form $R\cos(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving your value of α, in radians, to three significant figures.

[3 marks]

(b) (i) Hence find the value of x in the interval $0 < x < 2\pi$ for which $2\cos x - 5\sin x$ has its maximum value. Give your value of x to three significant figures.

[2 marks]

(ii) Use your answer to part **(a)** to solve the equation $2\cos x - 5\sin x + 1 = 0$ in the interval $0 < x < 2\pi$, giving your solutions to three significant figures.

[3 marks]

3. (a) Write $\cos\theta - 8\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha \leqslant 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

The temperature of a cellar is modelled by the equation

$$f(t) = 13 + \frac{\cos(15t)° - 8\sin(15t)°}{10} \qquad 0 \leqslant t < 24$$

where $f(t)$ is the temperature of the cellar in degrees Celsius and t is the time measured in hours after midnight.

Find, according to the model,

(b) the maximum temperature of the cellar, giving your answer to 2 decimal places

(2)

(c) the times, after midnight, when the temperature of the cellar is 12.5 °C

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Q2	Solution	Mark	Total	Comment
(a)	$R=\sqrt{29}$	B1		Allow 5.4 or better
	$\sqrt{29}\cos\alpha=2, \sqrt{29}\sin\alpha=5$ or $\tan\alpha=\frac{5}{2}$	M1		Their $\sqrt{29}$ Note $\cos\alpha=2$ or $\sin\alpha=5$ is M0
	$\alpha=1.19$	A1	3	Must be exactly this
(b)(i)	$R\cos(x+\alpha)=R$ or $\cos(x+\alpha)=1$ or $x+\alpha=2\pi$ or $x+\alpha=0$ or $x=-\alpha$	M1		Candidate's R and α
	$(x=)$ 5.09	A1	2	Must be exactly this
(ii)	$\cos(x+\alpha)=-\frac{1}{R}$	M1		Candidate's R and α; PI
	$(x+\alpha=)$ 1.75757... and 4.52560...	A1		Rounded or truncated to at least 2 dp; Ignore 'extra' solutions
	$x=0.567$ and $x=3.34$	A1	3	Condone $x=0.568$; $x=3.34$ must be correct NMS is 0/3 **A0** if extra values in interval $0<x<2\pi$
	Total		8	

3.(a)	$R=\sqrt{65}$	B1
	$\tan\alpha=\frac{8}{1}\Rightarrow\alpha=\text{awrt } 82.87°$	M1A1
		(3)
(b)	$13+\frac{'R'}{10}=13.81(°C)$	M1 A1
		(2)
(c)	$\cos(15t+82.87)°=-\frac{5}{\sqrt{65}}$	M1
	$15t+82.87=128.33\Rightarrow t=3.03$	A1
	$15t+82.87=(360-128.33)\Rightarrow t=...(9.92)$	dM1
	Both times correct 03:02 and 09:55	A1
		(4) **(9 marks)**

2. (i)

$$y = \frac{(2x-1)^3}{(3x-2)} \qquad x \neq \frac{2}{3}$$

(a) Find $\frac{dy}{dx}$ writing your answer as a single fraction in simplest form.

(4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} \geqslant 0$

(2)

(ii) Given

$$y = \ln(1 + \cos 2x) \qquad x \neq (2n+1)\frac{\pi}{2} \quad n \in \mathbb{Z}$$

show that $\frac{dy}{dx} = C\tan x$, where C is a constant to be determined.

(You may assume the double angle formulae.)

(4)

2(i)(a)	$y=\dfrac{(2x-1)^3}{(3x-2)} \Rightarrow \dfrac{dy}{dx}=\dfrac{\alpha(3x-2)(2x-1)^2-\beta(2x-1)^3}{(3x-2)^2}$ where $\alpha>0$ and $\beta>0$ OR $y=(2x-1)^3(3x-2)^{-1} \Rightarrow \dfrac{dy}{dx}=\alpha(2x-1)^2(3x-2)^{-1}-\beta(3x-2)^{-2}(2x-1)^3$ where $\alpha>0$ and $\beta>0$	M1
	$y=\dfrac{(2x-1)^3}{(3x-2)} \Rightarrow \dfrac{dy}{dx}=\dfrac{(3x-2)\times 6(2x-1)^2-(2x-1)^3\times 3}{(3x-2)^2}$ OR $y=(2x-1)^3(3x-2)^{-1} \Rightarrow \dfrac{dy}{dx}=3\times 2\times(2x-1)^2(3x-2)^{-1}-3(3x-2)^{-2}(2x-1)^3$	A1
	$\dfrac{dy}{dx}=\dfrac{(2x-1)^2(18x-12-6x+3)}{(3x-2)^2}$	M1
	$\dfrac{dy}{dx}=\dfrac{(2x-1)^2(12x-9)}{(3x-2)^2}$ or $\dfrac{3(2x-1)^2(4x-3)}{(3x-2)^2}$ or $(2x-1)^2(12x-9)(3x-2)^{-2}$ o.e	A1
		(4)
(i)(b)	$\dfrac{dy}{dx}\geqslant 0 \Rightarrow$ either their $(12x-9)\geqslant 0 \Rightarrow x\geqslant \ldots$ or $2x-1=0 \Rightarrow x=\ldots$	M1
	$x=\dfrac{1}{2},\ x\geqslant\dfrac{3}{4}$	
		(2)
(ii)	**<u>Way 1</u>**	
	$y=\ln(1+\cos 2x) \Rightarrow \dfrac{dy}{dx}=\dfrac{\pm\lambda\sin 2x}{1+\cos 2x}$	M1
	$\dfrac{dy}{dx}=\dfrac{-2\sin 2x}{1+\cos 2x}$	A1
	$\dfrac{dy}{dx}=\dfrac{-4\sin x\cos x}{2\cos^2 x}$	M1
	$\dfrac{dy}{dx}=-2\tan x$	A1
		(4)

7. (a) Given $-90° < A < 90°$, prove that

$$2\cos(A - 30°)\sec A \equiv \tan A + k$$

where k is a constant to be determined.

(3)

(b) Hence or otherwise, solve, for $-90° < x < 90°$, the equation

$$2\cos(x - 30°) = \sec x$$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

7(a) Way 1	$2\cos(A-30°)\sec A \equiv 2(\cos A\cos 30° + \sin A\sin 30°)\times \sec A$	M1
	$\frac{2(\cos A\cos 30° + \sin A\sin 30°)}{\cos A} \Rightarrow ... \Rightarrow \tan x + k$	dM1
	$\equiv \tan A + \sqrt{3}$ **cso**	A1*
Way 2	$2\cos(A-30°)\sec A \equiv \tan A + k \Rightarrow 2\cos(A-30°) \equiv \sin A + k\cos A$	
	$\Rightarrow 2(\cos A\cos 30° + \sin A\sin 30°) \equiv \sin A + k\cos A$	M1
	$\Rightarrow \sqrt{3}\cos A + \sin A \equiv \sin A + k\cos A$	dM1
	Hence true and $k = \sqrt{3}$	A1
		(3)
(b)	$2\cos(x-30°) = \sec x$ and $2\cos(x-30°)\sec x \equiv \tan A + \sqrt{3}$	
Way 1	**For example**	
	1) $2\cos(x-30°)\sec x = \sec^2 x \Rightarrow \tan x + '\sqrt{3}' = \sec^2 x$	
	OR	
	2) $\frac{\tan x + '\sqrt{3}'}{\sec x} = \sec x \Rightarrow \tan x + '\sqrt{3}' = \sec^2 x$	
	OR	
	3) $2\cos(x-30°) \equiv (\tan A + '\sqrt{3}')\cos x \Rightarrow \sec x = (\tan A + '\sqrt{3}')\cos x$	M1
	$\Rightarrow \tan x + '\sqrt{3}' = \sec^2 x$	
	OR	
	4) $2\cos(x-30°) = \sec x \Rightarrow 2(\cos x\cos 30° + \sin x\sin 30°) = \sec x$	
	$\Rightarrow 2\cos^2 x\cos 30° + 2\sin x\cos x\sin 30° = 1$	
	$\Rightarrow \cos^2 x\sqrt{5} + \sin x\cos x = 1 \Rightarrow \sqrt{5} + \tan x = \sec^2 x$	
	$\Rightarrow \tan^2 x - \tan x + 1 - \sqrt{3} = 0$	M1A1
	$\tan x = \frac{1 \pm \sqrt{1-4(1-\sqrt{3})}}{2} = \text{awrt } 1.49, \quad -0.49 \Rightarrow x = ...$	M 1
	$x = \text{awrt } 56.2°, -26.2°$	A1
		(5)
		(8 marks)

6. (i) Using the identity for $\tan(A \pm B)$, solve, for $-90° < x < 90°$,

$$\frac{\tan 2x + \tan 32°}{1 - \tan 2x \tan 32°} = 5$$

Give your answers, in degrees, to 2 decimal places.

(4)

(ii) (a) Using the identity for $\tan(A \pm B)$, show that

$$\tan(3\theta - 45°) \equiv \frac{\tan 3\theta - 1}{1 + \tan 3\theta}, \qquad \theta \neq (60n + 45)°,\ n \in \mathbb{Z}$$

(2)

(b) Hence solve, for $0 < \theta < 180°$,

$$(1 + \tan 3\theta)\tan(\theta + 28°) = \tan 3\theta - 1$$

(5)

9. (a) Express $\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α, in radians, to 3 decimal places.

(3)

$$\mathrm{M}(\theta) = 40 + (3\sin\theta - 6\cos\theta)^2$$

(b) Find

(i) the maximum value of $\mathrm{M}(\theta)$,

(ii) the smallest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $\mathrm{M}(\theta)$ occurs.

(3)

$$\mathrm{N}(\theta) = \frac{30}{5 + 2(\sin 2\theta - 2\cos 2\theta)^2}$$

(c) Find

(i) the maximum value of $\mathrm{N}(\theta)$,

(ii) the largest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $\mathrm{N}(\theta)$ occurs.

(3)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

6(i)	$\dfrac{\tan 2x + \tan 32°}{1 - \tan 2x \tan 32°} = 5 \Rightarrow \tan(2x + 32°) = 5$	B1
	$\Rightarrow x = \dfrac{\arctan 5 - 32°}{2}$	M1
	$\Rightarrow x = \text{awrt } 23.35°, -66.65°$	A1A1
		(4)
(ii)(a)	$\tan(3\theta - 45°) = \dfrac{\tan 3\theta - \tan 45°}{1 + \tan 45° \tan 3\theta} = \dfrac{\tan 3\theta - 1}{1 + \tan 3\theta}$	M1A1*
		(2)
(b)	$(1 + \tan 3\theta)\tan(\theta + 28°) = \tan 3\theta - 1$	
	$\Rightarrow \tan(\theta + 28°) = \tan(3\theta - 45°)$	B1
	$\theta + 28° = 3\theta - 45° \Rightarrow \theta = 36.5°$	M1A1
	$\theta + 28° + 180° = 3\theta - 45° \Rightarrow \theta = 126.5°$	dM1A1
		(5)
		(11 marks)

9.(a)	$R = \sqrt{5}$	B1
	$\tan\alpha = 2 \Rightarrow \alpha = \text{awrt } 1.107$	M1A1
		(3)
(b)(i)	$'40 + 9R^2' = 85$	M1A1
(ii)	$\theta = \dfrac{\pi}{2} + 1.107 \Rightarrow \theta = \text{awrt } 2.68$	B1ft
		(3)
(c)(i)	6	B1
(ii)	$2\theta - '1.107' = 3\pi \Rightarrow \theta = \text{awrt } 5.27$	M1A1
		(3)
		(9 marks)

4. (a) Write $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 \leqslant \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(b) Show that the equation

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined.

(2)

(c) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

9. (a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq (2n+1)90°, \qquad n \in \mathbb{Z}$$

(4)

(b) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leqslant x < 360°$,

$$\sin 2x - \tan x = 3\tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

4.(a)	$R=\sqrt{29}$	B1	
	$\tan\alpha=\frac{2}{5}\Rightarrow\alpha=\text{awrt } 0.381$	M1A1	
			(3)
(b)	$5\cot 2x-3\operatorname{cosec}2x=2\Rightarrow 5\frac{\cos 2x}{\sin 2x}-\frac{3}{\sin 2x}=2$	M1	
	$\Rightarrow 5\cos 2x-2\sin 2x=3$	A1	
			(2)
(c)	$5\cos 2x-2\sin 2x=3\Rightarrow\cos(2x+0.381)=\frac{3}{\sqrt{29}}$	M1	
	$2x+0.381=\arccos\left(\frac{3}{\sqrt{29}}\right)\Rightarrow x=...$	dM1	
	$x=\text{awrt } 0.30,\ 2.46$	A1A1	
			(4)

9(a)	$\sin 2x-\tan x=2\sin x\cos x-\tan x$	M1	
	$=\frac{2\sin x\cos^2 x}{\cos x}-\frac{\sin x}{\cos x}$	M1	
	$=\frac{\sin x}{\cos x}\times(2\cos^2 x-1)$		
	$=\tan x\cos 2x$	dM1 A1*	
			(4)
(b)	$\tan x\cos 2x=3\tan x\sin x\Rightarrow\tan x(\cos 2x-3\sin x)=0$		
	$\cos 2x-3\sin x=0$	M1	
	$\Rightarrow 1-2\sin^2 x-3\sin x=0$	M1	
	$\Rightarrow 2\sin^2 x+3\sin x-1=0\Rightarrow\sin x=\frac{-3\pm\sqrt{17}}{4}\Rightarrow x=...$	M1	
	Two of $x=16.3°,163.7°,0,180°$	A1	
	All four of $x=16.3°,163.7°,0,180°$	A1	
			(5)

3. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

(b) Hence solve, for $0 \leqslant \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15$$

Give your answers to one decimal place.

(5)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15$$

Give your answer to one decimal place.

(2)

5. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x}\cos 4x, \quad \frac{\pi}{4} \leqslant x < \frac{\pi}{2}$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y$, $\quad 0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p\,\text{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined.

(5)

3.(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow\alpha=26.57°$	M1A1
		(3)
(b)	$\frac{2}{2\cos\theta-\sin\theta-1}=15\Rightarrow\frac{2}{\sqrt{5}\cos(\theta+26.6°)-1}=15$	
	$\Rightarrow\cos(\theta+26.6°)=\frac{17}{15\sqrt{5}}=(\textit{awrt } 0.507)$	M1A1
	$\theta+26.57°=59.54°$	
	$\Rightarrow\theta=\textit{awrt } 33.0°$ or $\textit{awrt } 273.9°$	A1
	$\theta+26.6°=360°-\text{their '}59.5°\text{'}$	dM1
	$\Rightarrow\theta=\textit{awrt } 273.9°$ and $\textit{awrt } 33.0°$	A1
		(5)
(c)	$\theta-\text{their } 26.57°=\text{their } 59.54°\Rightarrow\theta=\ldots$	M1
	$\theta=\text{awrt } 86.1°$	A1
		(2)
		(10 marks)

5 (i)	$y=e^{3x}\cos 4x\Rightarrow\left(\frac{dy}{dx}\right)=\cos 4x\times 3e^{3x}+e^{3x}\times -4\sin 4x$	M1A1
	Sets $\cos 4x\times 3e^{3x}+e^{3x}\times -4\sin 4x=0\Rightarrow 3\cos 4x-4\sin 4x=0$	M1
	$\Rightarrow x=\frac{1}{4}\arctan\frac{3}{4}$	M1
	$\Rightarrow x=\text{awrt } 0.9463$ 4dp	A1
		(5)
(ii)	$x=\sin^2 2y\Rightarrow\frac{dx}{dy}=2\sin 2y\times 2\cos 2y$	M1A1
	Uses $\sin 4y=2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dx}{dy}=2\sin 4y\Rightarrow\frac{dy}{dx}=\frac{1}{2\sin 4y}=\frac{1}{2}\text{cosec}\,4y$	M1A1
		(5)

8. (a) Prove that

$$2\cot 2x + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leqslant x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

1. Given that

$$\tan\theta° = p, \text{ where } p \text{ is a constant, } p \neq \pm 1$$

use standard trigonometric identities, to find in terms of p,

(a) $\tan 2\theta°$

(2)

(b) $\cos\theta°$

(2)

(c) $\cot(\theta - 45)°$

(2)

Write each answer in its simplest form.

8 (a)	$2\cot 2x + \tan x \equiv \dfrac{2}{\tan 2x} + \tan x$	B1
	$\equiv \dfrac{(1-\tan^2 x)}{\tan x} + \dfrac{\tan^2 x}{\tan x}$	M1
	$\equiv \dfrac{1}{\tan x}$	M1
	$\equiv \cot x$	A1*
		(4)
(b)	$6\cot 2x + 3\tan x = \text{cosec}^2 x - 2 \Rightarrow 3\cot x = \text{cosec}^2 x - 2$	
	$\Rightarrow 3\cot x = 1 + \cot^2 x - 2$	M1
	$\Rightarrow 0 = \cot^2 x - 3\cot x - 1$	A1
	$\Rightarrow \cot x = \dfrac{3 \pm \sqrt{13}}{2}$	M1
	$\Rightarrow \tan x = \dfrac{2}{3 \pm \sqrt{13}} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6)
		(10 marks)

1.(a)	$\tan 2\theta° = \dfrac{2\tan\theta°}{1-\tan^2\theta°} = \dfrac{2p}{1-p^2}$ Final answer	M1A1
		(2)
(b)	$\cos\theta° = \dfrac{1}{\sec\theta°} = \dfrac{1}{\sqrt{1+\tan^2\theta°}} = \dfrac{1}{\sqrt{1+p^2}}$ Final answer	M1A1
		(2)
(c)	$\cot(\theta-45)° = \dfrac{1}{\tan(\theta-45)°} = \dfrac{1+\tan\theta°\tan 45°}{\tan\theta° - \tan 45°} = \dfrac{1+p}{p-1}$ Final answer	M1A1
		(2)
		(6 marks)

3. $$\mathrm{g}(\theta) = 4\cos 2\theta + 2\sin 2\theta$$

Given that $\mathrm{g}(\theta) = R\cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(a) find the exact value of R and the value of α to 2 decimal places.

(3)

(b) Hence solve, for $-90° < \theta < 90°$,

$$4\cos 2\theta + 2\sin 2\theta = 1$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $\mathrm{g}(\theta) = k$ has no solutions,

(c) state the range of possible values of k.

(2)

8. (a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \; n \in \mathbb{Z}$$

(5)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places.

(4)

3(a)	$4\cos 2\theta + 2\sin 2\theta = R\cos(2\theta - \alpha)$	
	$R = \sqrt{4^2 + 2^2} = \sqrt{20} = \left(2\sqrt{5}\right)$	B1
	$\alpha = \arctan\left(\frac{1}{2}\right) = 26.565°... = \text{awrt } 26.57°$	M1A1
		(3)
(b)	$\sqrt{20}\cos(2\theta - 26.6) = 1 \Rightarrow \cos(2\theta - 26.57) = \frac{1}{\sqrt{20}}$	M1
	$\Rightarrow (2\theta - 26.57) = +77.1.. \Rightarrow \theta = ...$	dM1
	$\theta = \text{awrt } 51.8°$	A1
	$2\theta - 26.57 = '-77.1...' \Rightarrow \theta = -\text{awrt } 25.3°$	ddM1A1
		(5)
(c)	$k < -\sqrt{20}, k > \sqrt{20}$	B1ft either B1ft both
		(2)

8(a)	$\sec 2A + \tan 2A = \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A}$	B1
	$= \frac{1 + \sin 2A}{\cos 2A}$	M1
	$= \frac{1 + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \frac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	
	$= \frac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}$	M1
	$= \frac{\cos A + \sin A}{\cos A - \sin A}$	A1*
		(5)
(b)	$\sec 2\theta + \tan 2\theta = \frac{1}{2} \Rightarrow \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1}{2}$	
	$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$	
	$\Rightarrow \tan\theta = -\frac{1}{3}$	M1 A1
	$\Rightarrow \theta = awrt\ 2.820, 5.961$	dM1A1 **(4)**