Pure Sector 4: Trigonometry 4

Aims:

- Understand and use double angle formulae
- Use of formulae for sin(*A* ± *B)* , cos(*A* ± *B)* and tan(*A* ± *B)*
- Understand geometrical proofs of these formulae
- Understand and use expressions for a cos θ + b sin θ in the equivalent forms of Rcos($\theta \pm \alpha$) *or R*sin($\theta \pm \alpha$)
- Construct proofs involving trigonometric functions and identities
- Apply trigonometric identities to find integrals

Addition Formulae

The addition formaulae are given in the formula booklet:

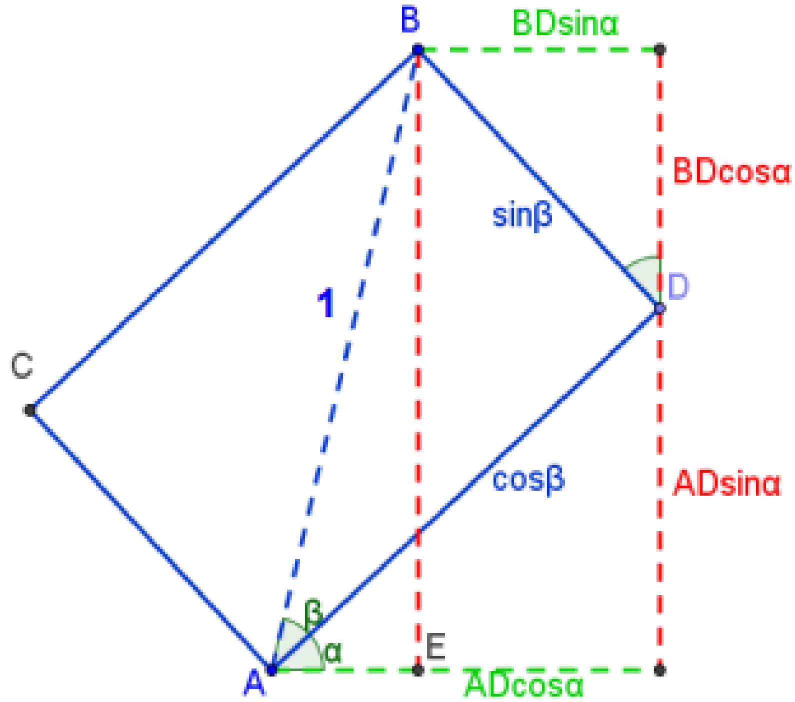
Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

Geometric Proofs



cos(α+β)=AE

=ADcosα-BDsinα

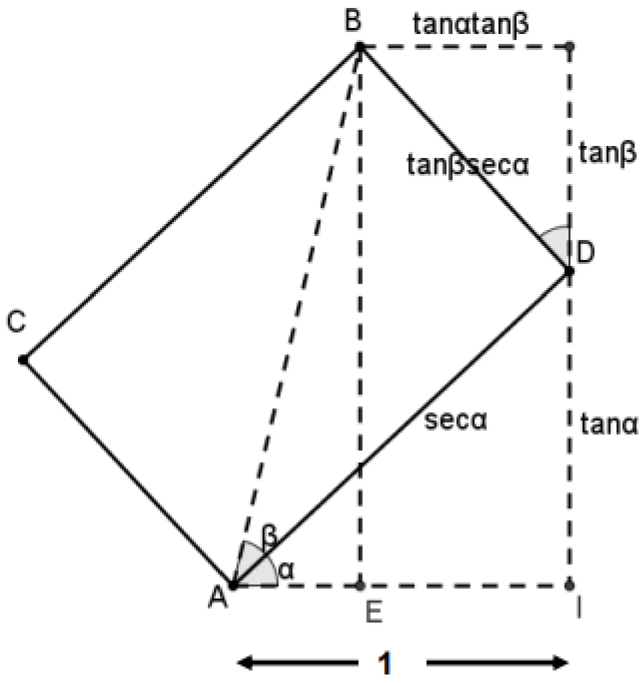
=cosαcosβ-sinαsinβ

sin(α+β)=BE

=ADsinα+BDcosα

=sinαcosβ+cosαsinβ

Use this diagram to prove that $\tan(\alpha+\beta) = \frac{BE}{AE} = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$



<u>Example 1</u>

Prove that $\frac{\sin(A-B)}{\cos A \cos B} = \tan A - \tan B$

<u>Example 2</u>

Solve $\cos(\theta + 60) = \sin\theta$ for $0 \leq \theta \leq 360$ (to 3sf)

Example 3

Given that $\tan 60 = \sqrt{3}$ show that $\tan 15 = 2 - \sqrt{3}$

Example 4

Angle α is acute and $\alpha = \frac{8}{17}$. Angle β is obtuse and $\sin\beta = \frac{12}{13}$. Find the value of $\tan(\alpha + \beta)$.

Double Angle Formulae

Use the addition formulae with B=A to find:

$\sin 2A =$

$\tan 2A =$

$\cos 2A =$

The $cos2A$ formula can be written in terms of sin or cos by using $\sin^2 A + \cos^2 A \equiv 1$

So, $\cos 2A =$

Or $\cos 2A =$

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \begin{cases} \cos^2 A - \sin^2 A \\ 2\cos^2 A - 1 \\ 1 - 2\sin^2 A \end{cases}$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

These are **not** given in the formula booklet.

Example 5

Solve $3\cos 2\theta - 7\cos\theta - 2 = 0$ for $0 \leq \theta \leq 360$

Example 6

Show that:

a) $2\, cosec 2\theta \equiv sec\theta cosec\theta$

b) $\sin 3x = 3\sin x - 4\sin^3 x$

Exam Question – AQA C4 Jun 06

4 (a) (i) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(ii) Express $\cos 2x$ in terms of $\cos x$. *(1 mark)*

(b) Show that

$$\sin 2x - \tan x = \tan x \cos 2x$$

for all values of x. *(3 marks)*

(c) Solve the equation $\sin 2x - \tan x = 0$, giving all solutions in degrees in the interval $0° < x < 360°$. *(4 marks)*

Application to Integration

Example 7

Find $\int \sin^2 x \, dx$

Example 8

Find $\int \sin 3x \cos 3x \, dx$

Example 9

Find $\int 6 \cos^2 \frac{\theta}{2} \, d\theta$

$R\sin(\theta+\alpha)$ and $R\cos(\theta+\alpha)$ form

This is useful when solving equations

Example 10

a) Express $3\sin x + 4\cos x$ in the form $R\sin(x+\alpha)$
b) What is the maximum value of the expression $3\sin x + 4\cos x$?
c) What is the smallest positive value of x for which this value occurs?

Example 11

Express $\cos\theta - \sqrt{3}\sin\theta$ in the form $R\cos(\theta+\alpha)$ and hence find its min and max values.

Example 12

Write $2\cos x - 3\sin x$ in the form $R\cos(x+\alpha)$ and hence solve $2\cos x - 3\sin x = 3$ for $0 \leq x \leq 2\pi$

AQA C4 June 10

5 (a) (i) Show that the equation $3\cos 2x + 2\sin x + 1 = 0$ can be written in the form

$$3\sin^2 x - \sin x - 2 = 0$$ *(3 marks)*

(ii) Hence, given that $3\cos 2x + 2\sin x + 1 = 0$, find the possible values of $\sin x$.
(2 marks)

(b) (i) Express $3\cos 2x + 2\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving α to the nearest $0.1°$. *(3 marks)*

(ii) Hence solve the equation

$$3\cos 2x + 2\sin 2x + 1 = 0$$

for all solutions in the interval $0° < x < 180°$, giving x to the nearest $0.1°$.
(3 marks)

AQA Specimen Paper 2

5 (a) Determine a sequence of transformations which maps the graph of $y = \cos\theta$ onto the graph of $y = 3\cos\theta + 3\sin\theta$

Fully justify your answer.

[6 marks]

5 (b) Hence or otherwise find the least value and greatest value of

$$4 + (3\cos\theta + 3\sin\theta)^2$$

Fully justify your answer.

[3 marks]