8 (a) Solve the equation

$$(2\cos\theta - \sin\theta)(1 + \tan\theta) = 0$$

giving your values of θ to the nearest degree in the interval $0° \leqslant \theta < 270°$.

[5 marks]

(b) (i) Express $5 + 4\cos^2 x$ as a product of two linear factors in terms of $\sin x$.

[2 marks]

(ii) **Hence** find the greatest possible value of $\dfrac{5 + 4\cos^2 x}{3 + 2\sin x}$ **and** state the exact value of x, in radians in the interval $0 \leqslant x < 2\pi$, at which this greatest value occurs.

[3 marks]

5 (a) Solve the equation

$$\operatorname{cosec}(2x - 10°) = 1.5$$

giving all values of x to the nearest $0.1°$ in the interval $0° < x < 180°$.

[3 marks]

(b) Solve the equation

$$4\cot^2(2x - 10°) = 11 - 4\operatorname{cosec}(2x - 10°)$$

giving all values of x to the nearest $0.1°$ in the interval $0° < x < 180°$.

[5 marks]

Q	Solution	Mark	Total	Comment
8(a)	$\sin\theta = 2\cos\theta, \quad 1+\tan\theta = 0$	B1		Either, or better
	$(2=)\dfrac{\sin\theta}{\cos\theta} = \tan\theta$	B1		$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ used at any stage
	$\tan\theta = -1 \Rightarrow \theta = 135^\circ$	B1		135 following the correct value for tan OE
	$(\tan\theta = 2 \Rightarrow)\ \theta = 63^\circ$	B1		63 CAO
	$(\tan\theta = 2 \Rightarrow)\ \theta = 243^\circ$	B1F	5	Ft on 180+c's 63 (need not be given to nearest degree) Deduct 1 mark from relevant B marks for extra solution(s) in given interval. Ignore values outside the given interval.
(b)(i)	$5+4\cos^2 x = 5+4(1-\sin^2 x)$	M1		$4\cos^2 x = 4(1-\sin^2 x)$ seen or used
	$= 9-4\sin^2 x$			
	$= (3+2\sin x)(3-2\sin x)$	A1	2	ACF
(b)(ii)	$\ldots\ldots = \dfrac{(3+2\sin x)(3-2\sin x)}{3+2\sin x}$			
	$=(3-2\sin x)$	M1		$3-2\sin x$ obtained **convincingly**. Must explicitly see the intermediate step with (b)(i) used
	greatest value = 5	A1		5
	(occurs when $x=$) $\dfrac{3\pi}{2}$	B1F	3	$\dfrac{3\pi}{2}$ with no extras in given interval. Cand must be using $a-b\sin x$, a>0, b>0
	Total		**10**	

(b) Maximum loss of 1 accuracy mark if eg θ used instead of x.

(b)(ii) $\dfrac{5+4\cos^2 x}{3+2\sin x} = \dfrac{9-4\sin^2 x}{3+2\sin x} = 3-2\sin x$ scores **M0** so max possible is **M0A0B1**

Accept eg $\dfrac{5+4\cos^2 x}{3+2\sin x} = \dfrac{(5+4\cos^2 x)}{(3+2\sin x)} \times \dfrac{(3-2\sin x)}{(3-2\sin x)} = \dfrac{(5+4\cos^2 x)(3-2\sin x)}{(5+4\cos^2 x)} = 3-2\sin x$

Q5	Solution	Mark	Total	Comment
(a)	$2x-10 = 41.8$	M1		PI by a correct final answer
	$[x=]\ 25.9$	A1		
	$[x=]\ 74.1$	A1		No other answers in range
			3	(ignore answers outside range)
(b)	$4\cot^2 Y = 11-4\operatorname{cosec} Y$			
	$4\operatorname{cosec}^2 Y = 15-4\operatorname{cosec} Y$	M1		Correct use of trig identity
	$4\operatorname{cosec}^2\text{Y}+4\operatorname{cosec}\text{Y}-15=0$			
	$(2\operatorname{cosec}\text{Y}+5)(2\operatorname{cosec}\text{Y}-3)=0$	dM1		
	$\operatorname{cosec} Y = 1.5,\ -2.5$	A1		
	$\operatorname{cosec} Y = -2.5$			
	-23.6	B1		PI by a final answer of 106.8 or 173.2
	$[x=]\ 25.9,\ 74.1,\ 106.8,\ 173.2$	B1		

8 (a) Given that

$$9 \sin^2 \theta - 2 \sin \theta \cos \theta = 8$$

show that

$$(\tan \theta - 4)(\tan \theta + 2) = 0$$

[3 marks]

(b) **Hence** solve the equation

$$9 \sin^2 2x - 2 \sin 2x \cos 2x = 8$$

in the interval $0° \leqslant x \leqslant 180°$, giving your values of x to the nearest degree.

[4 marks]

Q	Solution	Mark	Total	Comment
8(a)	$9\sin^2\theta - 2\sin\theta\cos\theta = 8$ $\dfrac{9\sin^2\theta}{\cos^2\theta} - \dfrac{2\sin\theta\cos\theta}{\cos^2\theta} = \dfrac{8}{\cos^2\theta}$ $9\tan^2\theta - 2\tan\theta = \dfrac{8}{\cos^2\theta}$	M1		Dividing each term by $\cos^2\theta$ and using correct identity to obtain at least two correct terms in different powers of $\tan\theta$
	.... $= 8(\cos^2\theta + \sin^2\theta)$	M1		Replacing 8 by $8(\cos^2\theta + \sin^2\theta)$ or PI by seeing eg $\sin^2\theta - 2\sin\theta\cos\theta =$ $= 8(1-\sin^2\theta) = 8\cos^2\theta$ or PI by seeing $\dfrac{8}{\cos^2\theta} = 8(1+\tan^2\theta)$ [**The two method marks can be awarded in any order**]
	$9\tan^2\theta - 2\tan\theta = \dfrac{8(\cos^2\theta + \sin^2\theta)}{\cos^2\theta}$ $9\tan^2\theta - 2\tan\theta = 8 + 8\tan^2\theta$ $\tan^2\theta - 2\tan\theta - 8 = 0$ $(\tan\theta - 4)(\tan\theta + 2) = 0$	A1	3	AG Be convinced
(b)	$75.96,\ 255.96,\ 116.56,\ 296.56$	M1		Any **two** correct values equal to or rounding to integer values 76, 256, 117, 297 seen
	$(\tan 2x = 4)\quad 2x = 75.96,\ 255.96$ $(\tan 2x = -2)\quad 2x = 116.56,\ 296.56$	A1		$2x$ equal to or rounding to the four integer values 76, 256, 117, 297 **seen or used** Condone eg 2θ for $2x$
	$(x =)\ 38^\circ,\ 128^\circ,\ 58^\circ,\ 148^\circ$	B2,1,0	4	38, 58, 128, 148 with or without working scores **B2**; if **B2** not scored, award **B1** if either four values rounding to the above or three of the above and no more than one incorrect; or 38,128, 59, 149 [Ignore answers outside $0 \le x \le 180$. If more than four answers in interval deduct 1 mark for each extra from **B** marks to a min of 0]
	Total		7	

8 (a) Solve the equation $\cos\theta = \frac{2}{3}$, giving all values of θ to the nearest degree in the interval $0° \leqslant \theta \leqslant 360°$.

[2 marks]

(b) (i) Given that $4\tan\theta\sin\theta = 4 - \cos\theta$, show that $3\cos^2\theta + 4\cos\theta - 4 = 0$.

[3 marks]

(ii) By solving the quadratic equation in part **(b)(i)**, explain why $\cos\theta$ can only take one value.

[2 marks]

(c) Hence solve the equation $4\tan 4x \sin 4x = 4 - \cos 4x$, giving all values of x to the nearest degree in the interval $0° \leqslant x \leqslant 180°$.

[4 marks]

Q8	Solution	Mark	Total	Comment
(a)	$\theta = 48^\circ$,	B1		48 Condone 48.1… , 48.2
	312°	B1	2	312 CAO Ignore values outside the given interval. If more than 2 values in given interval deduct 1 mark for each extra (to min of 0)
(b)(i)	$4\tan\theta\sin\theta = 4\dfrac{\sin\theta}{\cos\theta}\sin\theta$	M1		$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ **used**
	$= 4\dfrac{1-\cos^2\theta}{\cos\theta}$	dM1		Replacing $\sin^2\theta$ by $1-\cos^2\theta$ to either correctly express $4\tan\theta\sin\theta$ in terms of $\cos\theta$ or to obtain $4(1-\cos^2\theta) = \cos\theta(4-\cos\theta)$
	$(\cos\theta \neq 0)$ $4(1-\cos^2\theta) = \cos\theta(4-\cos\theta)$ $4-4\cos^2\theta = 4\cos\theta - \cos^2\theta$ $\Rightarrow 3\cos^2\theta + 4\cos\theta - 4 = 0$	A1	3	AG Be convinced.
(b)(ii)	$(\cos\theta+2)(3\cos\theta-2)$ $(=0)$	B1		Correct factorisation or $\cos\theta = \dfrac{2}{3}, -2$
	Since $-1 \le \cos\theta \le 1$, $\cos\theta \neq -2$ so $\dfrac{2}{3}$ is the only value for $\cos\theta$.	E1	2	Valid explanation that would eliminate one of the c's values, with 'only one value' or an indication of which value is rejected.
(c)	$(\cos 4x \neq 0)$ $\cos 4x = \dfrac{2}{3}$	M1		$\cos 4x = \dfrac{2}{3}$. Ft on c's value in (b)(ii) provided $-1 \le \cos\theta \le 1$. PI eg by finding solns for $\cos\theta = \dfrac{2}{3}$ and clear attempt to divide values by 4
	$4x = 48^\circ, 312^\circ, 408^\circ, 672^\circ$	A1		$4x$ equal to or rounding to OE to the four integer values 48, 312, 408, 672 **seen**
	$(x =)$ 12°, 78°, 102°, 168°	B2,1,0	4	If not **B2** award **B1** if either 2 correct or 3 AWRT three of these values. If more than four values in given interval, deduct 1 mark for each extra, to a min of **B0**. Ignore values outside $0^\circ \le x \le 180^\circ$. NMS Max 2/4.
	Total		**11**	

8 (a) (i) Given that $4\sin x + 5\cos x = 0$, find the value of $\tan x$.

[2 marks]

(ii) Hence solve the equation $(1 - \tan x)(4\sin x + 5\cos x) = 0$ in the interval $0° \leqslant x \leqslant 360°$, giving your values of x to the nearest degree.

[3 marks]

(b) By first showing that $\dfrac{16 + 9\sin^2\theta}{5 - 3\cos\theta}$ can be expressed in the form $p + q\cos\theta$, where p and q are integers, find the least possible value of $\dfrac{16 + 9\sin^2\theta}{5 - 3\cos\theta}$.

State the exact value of θ, in radians in the interval $0 \leqslant \theta < 2\pi$, at which this least value occurs.

[4 marks]

6 (a) Solve the equation $\sin(x + 0.7) = 0.6$ in the interval $-\pi < x < \pi$, giving your answers in radians to two significant figures.

[3 marks]

(b) It is given that $5\cos^2\theta - \cos\theta = \sin^2\theta$.

(i) By forming and solving a suitable quadratic equation, find the possible values of $\cos\theta$.

[4 marks]

(ii) **Hence** show that a possible value of $\tan\theta$ is $2\sqrt{2}$.

[3 marks]

Q8	Solution	Mark	Total	Comment
(a) (i)	$\frac{4\sin x}{\cos x}+\frac{5\cos x}{\cos x}=0$; $4\tan x+5=0$	M1		$\frac{\sin x}{\cos x}=\tan x$ clearly used to obtain a linear equation in $\tan x$.
	$\tan x=-\frac{5}{4}$ $(=-1.25)$	A1	2	-1.25 OE **NMS** mark as **B2** or **B0**
(a)(ii)	$\tan x=1$, $\tan x=-1.25$	B1F		1 and c's answer to **(a)(i)** vals for tan (x) PI by a correct angle for both tan values
	$(x=)$ 45°, 225°, 129°, 309°	B2, 1	3	**B2** 45, 225, AWRT 129, AWRT 309 If not **B2** award **B1** for at least two correct. If more than four values in given interval, deduct 1 mark for each extra to min of B0. Ignore values outside $0^\circ \le x \le 360^\circ$
(b)	$\frac{16+9\sin^2\theta}{5-3\cos\theta}=\frac{16+9(1-\cos^2\theta)}{5-3\cos\theta}$	M1		Replacing $\sin^2\theta$ by $1-\cos^2\theta$ in given expression or replacing $\cos^2\theta$ by $1-\sin^2\theta$ in term $\pm 3q\cos^2\theta$.
	$=\frac{(5-3\cos\theta)(5+3\cos\theta)}{5-3\cos\theta}$	A1		Or any two of $5p-3q=16$, $5q-3p=0$, $3q=9$
	$=5+3\cos\theta$	A1		CSO. Or $q=3$, $p=5$ **and** checking remaining eqn is satisfied.
	Least possible value is 2 and occurs at $\theta=\pi$	B1F	4	Ft on c's p and q non zero values. {If $q>0$, least val=$p-q$ $\theta=\pi$} {If $q<0$, least val=$p+q$ at $\theta=0$} Ignore values of θ outside given interval
	Total		9	

(a)	$\sin^{-1}0.6=0.64(35\ldots)$ $(=\beta)$	B1		PI by one correct value for x to at least 2dp or 2sf
	$x+0.7=\beta$, $x+0.7=\pi-\beta$ (=2.4(98..))	M1		$x+0.7=\beta$ and $x+0.7=\pi-\beta$ where β is the c's value for $\sin^{-1}0.6$
	$x=-0.056$, 1.8 (to 2 sf)	A1	3	Must be correct 2sf values ie -0.056, 1.8 Ignore any values outside given interval. **SC NMS** Condone>2sf and mark as B1 B1 max. {−0.056(498..); 1.7(9809..)}
(b)(i)	$5\cos^2\theta-\cos\theta=1-\cos^2\theta$	M1		Replacing $\sin^2\theta$ by $1-\cos^2\theta$
	$6\cos^2\theta-\cos\theta-1=0$	A1		
	$(2\cos\theta-1)(3\cos\theta+1)$ $(=0)$	m1		$(2\cos\theta\pm1)(3\cos\theta\pm1)$ PI by the two 'correct' roots with correct/incorrect signs
	(Possible values of $\cos\theta=$) $\frac{1}{2}$, $-\frac{1}{3}$	A1	4	The two correct values of $\cos\theta$.
(b)(ii)	When $\cos\theta=-\frac{1}{3}$, $\sin^2\theta=\frac{8}{9}$	B1		
	$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{(\pm)\sqrt{\frac{8}{9}}}{-\frac{1}{3}}$	M1		$\tan\theta=\frac{\sin\theta}{\cos\theta}$ **used**; could be used with either of c's values of $\cos\theta$ from **(b)(i)** and a corresponding value of $\sin\theta$
	So a (+'ve) value for $\tan\theta$ is $-\sqrt{\frac{8}{9}}\div\left(-\frac{1}{3}\right)=\sqrt{8}=2\sqrt{2}$	A1	3	CSO A.G. Be convinced.
	Total		10	

8 (a) Show that the expression $\dfrac{\sec\theta}{\sec\theta - 1} + \dfrac{\sec\theta}{\sec\theta + 1}$ can be written as $2\operatorname{cosec}^2\theta$.

[3 marks]

(b) Hence solve the equation

$$\frac{\sec(2x+0.4)}{\sec(2x+0.4)-1} + \frac{\sec(2x+0.4)}{\sec(2x+0.4)+1} = 8 - \cot(2x+0.4)$$

giving your answers in radians to three significant figures in the interval $0 < x < \pi$.

[7 marks]

Q8	Solution	Mark	Total	Comment
(a)	$\dfrac{\sec\theta(\sec\theta+1)+\sec\theta(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}$ **OE**	B1		Common denominator (could be 2 separate fractions), all in terms of $\sec\theta$
	Use of $\sec^2\theta = 1+\tan^2\theta$	M1		used
	$\dfrac{2\sec^2\theta}{\tan^2\theta}$			**OR** $\dfrac{2+2\tan^2\theta}{\tan^2\theta}$
	$=\dfrac{2}{\sin^2\theta} = 2\mathrm{cosec}^2\theta$	A1	3	$=2+2\cot^2\theta = 2\mathrm{cosec}^2\theta$ **AG be convinced** Penalise poor notation for final **A1** **(consult team leader)**
(b)	$2\mathrm{cosec}^2\theta = 8-\cot\theta$			
	$2(1+\cot^2\theta) = 8-\cot\theta$	M1		Use of $\mathrm{cosec}^2\theta = 1+\cot^2\theta$
	$2\cot^2\theta+\cot\theta-6=0$	A1		Must see = 0, this line or next line
	$(2\cot\theta-3)(\cot\theta+2)\ [=0]$	dM1		Correctly factorising/solving their quadratic in cot (**PI** by next **A1** earned)
	$\tan\theta = \dfrac{2}{3}, -\dfrac{1}{2}$ or $\cot\theta = \dfrac{3}{2}, -2$	A1		**OE**
	0.58800.., 3.72959.., 2.6779.., 5.819537..	B1		sight of **any** of these values correct to at least 3sf rounded or truncated **PI**
	(x =) 0.0940, 1.66, 1.14, 2.71	B1 B1	7	3 correct at least 3sf all 4 correct to 3sf and no others in interval (ignore answers outside interval)
	Total		**10**	

(a) **Alternative 1:** $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta} = \dfrac{1+\cos\theta+1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$ **B1**, $1-\cos^2\theta = \sin^2\theta$ used **M1** **etc**

Alternative 2: $2\mathrm{cosec}^2\theta = \dfrac{2}{\sin^2\theta} = \dfrac{2}{1-\cos^2\theta}$ **B1** $\dfrac{2}{(1-\cos\theta)(1+\cos\theta)} = \dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}$ **M1 etc**

(b) **Alternative:** $2 = 8\sin^2\theta - \sin\theta\cos\theta$ hence $2(1+\tan^2\theta) = 8\tan^2\theta - \tan\theta$ **M1**

with **A1** for a quadratic in $\tan\theta$: $6\tan^2\theta - \tan\theta - 2 = 0$ **;** $(3\tan\theta-2)(2\tan\theta+1)=0$ **dM1** factors **etc**

8 (a) By using a suitable trigonometrical identity, solve the equation

$$\tan^2\left(2x - \frac{\pi}{6}\right) = 11 - \sec\left(2x - \frac{\pi}{6}\right)$$

giving all values of x in radians to two decimal places in the interval $0 \leqslant x \leqslant \pi$.

[7 marks]

(b) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \mathrm{f}\left(2x - \frac{\pi}{6}\right)$ onto the graph of $y = \mathrm{f}(x)$.

[4 marks]

Q8	Solution	Mark	Total	Comment
(a)	$\tan^2 p = \sec^2 p - 1 \quad [= 11 - \sec p]$			
	$\sec^2 p - 1 = 11 - \sec p$	M1		Correct use of trig identity PI
	$\sec^2 p + \sec p - 12 = 0$			
	$(\sec p - 3)(\sec p + 4)[= 0]$	A1		Factorisation or correct use of formula PI
	$\sec p = 3, -4$	A1		Both correct and no errors seen
	$p = 1.23[...], 1.82[...],$	B1		Sight of **any** of these values correct to 2 dp
	$5.05[...], 4.459[...]$	B1		3 of these values correct to 2 dp
	$x = 0.88, 1.17, 2.49, 2.79$	B1		3 correct (must be to 2 dp)
		B1		All 4 correct (must be to 2 dp) and no extras in interval (ignore answers outside interval)
			7	
(b)	Stretch (I)			
	(Parallel to) x-axis (or line $y = 0$) (II)	M1		I and (II or III)
	SF 2 (III)	A1		I + II + III
	(followed by)			
	Translation through $\begin{bmatrix} k \\ 0 \end{bmatrix}$	B1		
	$\begin{bmatrix} -\frac{\pi}{6} \\ 0 \end{bmatrix}$	B1		
	OR			
	Translation through $\begin{bmatrix} k \\ 0 \end{bmatrix}$	(B1)		
	$\begin{bmatrix} -\frac{\pi}{12} \\ 0 \end{bmatrix}$	(B1)		
	(followed by)			
	Stretch (I)	(M1)		As above
	Parallel to x-axis (or line $y = 0$) (II)	(A1)		
	SF 2 (III)			
			4	

8 (a) Show that the equation $4\operatorname{cosec}^2\theta - \cot^2\theta = k$, where $k \neq 4$, can be written in the form

$$\sec^2\theta = \frac{k-1}{k-4}$$

[5 marks]

(b) Hence, or otherwise, solve the equation

$$4\operatorname{cosec}^2(2x + 75°) - \cot^2(2x + 75°) = 5$$

giving all values of x in the interval $0° < x < 180°$.

[5 marks]

Q8	Solution	Mark	Total	Comment
a	$LHS = 4(1+\cot^2\theta) - \cot^2\theta$	M1		Use of a correct trig identity (or identities if using sin/cos) to get an expression/equation in a single trig function
	$4(1+\cot^2\theta) - \cot^2\theta = k$ Or $4\operatorname{cosec}^2\theta - (\operatorname{cosec}^2 - 1) = k$	A1		All correct, including = k
	$\cot^2\theta = \dfrac{k-4}{3}$	m1		Correctly isolating trig function – must be tan or cot or cos or sec, from their **CORRECT** equation
	$\tan^2\theta = \dfrac{3}{k-4}$	m1		Correct inversion (at some stage) from their equation
	$[\sec^2\theta = \dfrac{3}{k-4} + 1]$			Must see at least one line of working, be convinced
	$\sec^2\theta = \dfrac{k-1}{k-4}$	A1	5	AG: no errors seen
b	$\sec^2\theta = 4$ or $\tan^2\theta = 3$ or $\cot^2\theta = \dfrac{1}{3}$ or $\operatorname{cosec}^2\theta = \dfrac{4}{3}$	B1		PI by expression for eg $\sec x = 2$
	$\sec\theta = \pm 2$	M1		or $\cos\theta = \pm 0.5$ or $\tan\theta = \pm\sqrt{3}$ or $\sin\theta = \pm\dfrac{\sqrt{3}}{2}$
	$(\theta =)$ 60, 120, 240, 300, 420	A1		Sight of any four of these answers
	$x = 22.5°, 82.5°, 112.5°, 172.5°$	B1 B1	5	3 correct All correct and no extras in interval (ignore answers outside interval)
	Total		**10**	

7. (a) Show that the equation

$$6\cos^2 x - \sin x - 4 = 0$$

may be written as

$$6\sin^2 x + \sin x - 2 = 0$$

(b) Hence solve, for $-90° \leqslant y < 90°$, the equation

$$6\cos^2(2y) - \sin(2y) - 4 = 0$$

giving your answers to one decimal place where appropriate.

8 *In this question solutions based entirely on graphical or numerical methods are not acceptable.*

(i) Solve for $0 \leqslant x < 360°$,

$$4\cos(x + 70°) = 3$$

giving your answers in degrees to one decimal place.

(4)

(ii) Find, for $0 \leqslant \theta < 2\pi$, all the solutions of

$$6\cos^2\theta - 5 = 6\sin^2\theta + \sin\theta$$

giving your answers in radians to 3 significant figures.

(5)

7. (a)	$6(1-\sin^2 x)-\sin x-4=0$	M1
	$6\sin^2 x+\sin x-2=0$ *	A1 *
		(2)
(b)	$6\sin^2 2y+\sin 2y-2=0$	
	$(2\sin 2y-1)(3\sin 2y+2)=0$ *so* $\sin\theta=$	M1
	$(\sin 2y=)\frac{1}{2}$, $(\sin 2y=)-\frac{2}{3}$	A1
	$2y=30°$ or $150°$ or $-41.8°$ or $-138.2°$ so $y=$	M1
	$y=15°$ or $75°$ or $-20.9°$ or $-69.1°$ (accept awrt)	A1 A1
		(5)
		[7]

8. (i)	$4\cos(x+70°)=3$	
	$\cos(x+70°)=0.75$, so $x+70°=41.4(1)°$	M1A1
	$x=248.6°$ or $331.4°$	M1 A1
		(4)
(ii)	$6\cos^2\theta-5=6\sin^2\theta+\sin\theta$ so $6(1-\sin^2\theta)-5=6\sin^2\theta+\sin\theta$	M1
	$12\sin^2\theta+\sin\theta-1=0$	A1
	$(4\sin\theta-1)(3\sin\theta+1)=0$ *so* $\sin\theta=$	M1
	$\theta=0.253,\ 2.89,\ 3.48,\ 5.94$	A1 A1
		(5)
		[9]

8. (a) Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(3)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

giving your answers to 2 decimal places.

(5)

6. (i) Solve, for $-\pi < \theta \leqslant \pi$,

$$1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0$$

giving your answers in terms of π.

(3)

(ii) Solve, for $0 \leqslant x < 360°$,

$$4\cos^2 x + 7\sin x - 2 = 0$$

giving your answers to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

8. (a)	Way 1	Way 2	
	$1-\sin^2 x = 8\sin^2 x - 6\sin x$	$2=(3\sin x-1)^2$ gives $9\sin^2 x - 6\sin x + 1 = 2$ so $\sin^2 x + 8\sin^2 x - 6\sin x + 1 = 2$	B1
	E.g. $9\sin^2 x - 6\sin x = 1$ or $9\sin^2 x - 6\sin x - 1 = 0$ or $9\sin^2 x - 6\sin x + 1 = 2$	so $8\sin^2 x - 6\sin x = 1 - \sin^2 x$	M1
	So $9\sin^2 x - 6\sin x + 1 = 2$ or $(3\sin x - 1)^2 - 2 = 0$ so $(3\sin x-1)^2 = 2$ or $2=(3\sin x-1)^2$ *	$8\sin^2 x - 6\sin x = \cos^2 x$ *	A1cso*
			(3)
(b)	Way 1: $(3\sin x - 1) = (\pm)\sqrt{2}$	Way 2: Expands $(3\sin x-1)^2 = 2$ and uses quadratic formula on 3TQ	M1
	$\sin x = \dfrac{1\pm\sqrt{2}}{3}$ or awrt 0.8047 **and** awrt − 0.1381		A1
	$x = 53.58$, 126.42 (or 126.41), 352.06, 187.94		dM1A1 A1
			(5) **[8]**

6.	$1-2\cos\left(\theta-\dfrac{\pi}{5}\right)=0$; $-\pi<\theta \leqslant \pi$		
(i)	$\cos\left(\theta-\dfrac{\pi}{5}\right)=\dfrac{1}{2}$	Rearranges to give $\cos\left(\theta-\dfrac{\pi}{5}\right)=\dfrac{1}{2}$ or $-\dfrac{1}{2}$	M1
	$\theta=\left\{-\dfrac{2\pi}{15}, \dfrac{8\pi}{15}\right\}$	At least one of $-\dfrac{2\pi}{15}$ or $\dfrac{8\pi}{15}$ or -24° or 96° or awrt 1.68 or awrt -0.419	A1
		Both $-\dfrac{2\pi}{15}$ and $\dfrac{8\pi}{15}$	A1
			[3]
NB Misread	**Misreading** $\dfrac{\pi}{5}$ as $\dfrac{\pi}{6}$ or $\dfrac{\pi}{3}$ (or anything else)– treat as misread so M1 A0 A0 is maximum mark		
	$4\cos^2 x + 7\sin x - 2 = 0$, $0 \leqslant x < 360^\circ$		
(ii)	$4(1-\sin^2 x) + 7\sin x - 2 = 0$	Applies $\cos^2 x = 1 - \sin^2 x$	M1
	$4 - 4\sin^2 x + 7\sin x - 2 = 0$		
	$4\sin^2 x - 7\sin x - 2 = 0$	Correct 3 term, $4\sin^2 x - 7\sin x - 2\ \{=0\}$	A1 oe
	$(4\sin x + 1)(\sin x - 2)\ \{=0\}$, $\sin x = \ldots$	Valid attempt at solving and $\sin x = \ldots$	M1
	$\sin x = -\dfrac{1}{4}$, $\{\sin x = 2\}$	$\sin x = -\dfrac{1}{4}$ (See notes.)	A1 cso
	$x = \text{awrt}\{194.5, 345.5\}$	At least one of awrt 194.5 **or** awrt 345.5 or awrt 3.4 or awrt 6.0	A1ft
		awrt 194.5 **and** awrt 345.5	A1
			[6]
			9

8. (i) Solve, for $0 \leqslant \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

giving your answers in terms of π.

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \qquad 0 \leqslant k \leqslant 3$$

(a) find $\cos x$ in terms of k.

(3)

(b) When $k = 3$, find the values of x in the range $0 \leqslant x < 360°$

(3)

8. (i)	Way 1: Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so $(3\theta) = \frac{\pi}{3}$ Or Way 2: Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm\frac{1}{2}$ or $\sin 3\theta = \pm\frac{\sqrt{3}}{2}$ so $(3\theta) = \frac{\pi}{3}$	M1
	Adds π or 2π to previous value of angle (to give $\frac{4\pi}{3}$ or $\frac{7\pi}{3}$)	M1
	So $\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$ (all three, no extra in range)	A1 **(3)**
(ii)(a)	$4(1-\cos^2 x) + \cos x = 4 - k$ Applies $\sin^2 x = 1 - \cos^2 x$	M1
	Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$	dM1
	$\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ or $\cos x = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{k}{4}}$ or other correct equivalent	A1 **(3)**
(b)	$\cos x = \frac{1 \pm \sqrt{49}}{8} = 1$ and $-\frac{3}{4}$ (see the note below if errors are made)	M1
	Obtains two solutions from 0 , 139 , 221 (0 or 2.42 or 3.86 in radians)	dM1
	x = 0 and 139 and 221 (allow awrt 139 and 221) must be in degrees	A1 **(3)** **[9]**