A bag contains 64 coloured beads. There are r red beads, y yellow beads and 1 green bead and $r + y + 1 = 64$

Two beads are selected at random, one at a time without replacement.

(a) Find the probability that the green bead is one of the beads selected.

(4)

The probability that both of the beads are red is $\frac{5}{84}$

(b) Show that r satisfies the equation $r^2 - r - 240 = 0$

(3)

(c) Hence show that the only possible value of r is 16

(2)

(d) Given that at least one of the beads is red, find the probability that they are both red.

(4)

A and B are two events such that

$$\mathrm{P}(B) = \frac{1}{2} \qquad \mathrm{P}(A \mid B) = \frac{2}{5} \qquad \mathrm{P}(A \cup B) = \frac{13}{20}$$

(a) Find $\mathrm{P}(A \cap B)$.

(2)

(b) Draw a Venn diagram to show the events A, B and all the associated probabilities.

(3)

Find

(c) $\mathrm{P}(A)$

(1)

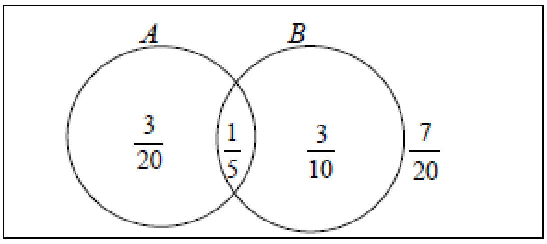
(d) $\mathrm{P}(B \mid A)$

(2)

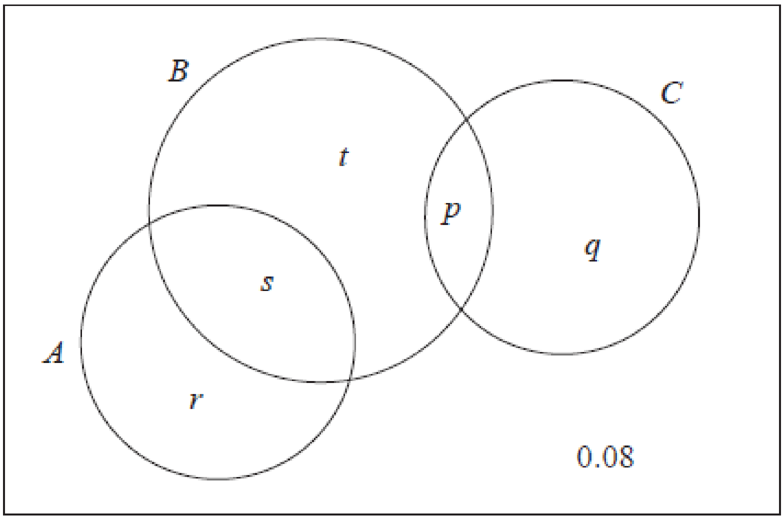
(e) $\mathrm{P}(A' \cap B)$

(1)

4.	**(a)**	$P(G_1)+P(R_1\cap G_2)+P(Y_1\cap G_2)$ or $P(GY)+P(GR)+P(RG)+P(YG)$ (o.e.)	M1
		$=\frac{1}{64}+\frac{r}{64}\times\frac{1}{63}+\frac{y}{64}\times\frac{1}{63}=\frac{1}{64}+\frac{r+y}{64\times 63}$ or $2\times\frac{r+y}{64\times 63}$	A1
		$=\frac{1}{64}+\frac{63}{64\times 63}$ or $\frac{2\times 63}{64\times 63}$ or $\frac{1}{64}+\frac{1}{64}$ or	M1
		$=\underline{\frac{1}{32}}$ or 0.03125	A1
			(4)
	(b)	$P(R_1\cap R_2)=\frac{r}{64}\times\frac{r-1}{63}=\frac{5}{84}$	M1A1
		$r(r-1)=5\times 64\times 63\div 84=240$ hence $r^2-r-240=0$ or $r^2-r=240$ (*)	A1cso
			(3)
	(c)	$r^2-r-240=(r-16)(r+15)\{=0\}$ or $16^2-16-240=256-256$ or $\frac{16}{64}\times\frac{15}{63}=\frac{5}{84}$	M1
		so $r=16$ and rejecting -15 (*)	A1cso
			(2)
	(d)	$P(\geqslant 1\text{red})=P(RG)+P(GR)+P(RY)+P(YR)+P(RR)$ or $\frac{2}{252}+\frac{2y}{252}+\frac{15}{252}$ (o.e.)	M1,
		or $P(R_1)+P(R_1'\cap R_2)$ or $\frac{16}{64}+\frac{48}{64}\times\frac{16}{63}$ or $1-\frac{48}{64}\times\frac{47}{63}$, $=\underline{\frac{37}{84}}$	A1
		Require: $\frac{P(R_1\cap R_2)}{P(\text{at least one red})}=\frac{\frac{5}{84}}{"\frac{37}{84}"}$, $=\underline{\frac{5}{37}}$ or $0.\dot{1}3\dot{5}$	M1, A1
			(4)
			[Total 13]

4.	**(a)**	$P(A\cap B)=P(A\mid B)\times P(B)$ $P(A\cap B)=\frac{2}{5}\times\frac{1}{2}=\frac{1}{5}$	M1 A1
			(2)
	(b)	 2 intersecting circles and ‘$P(A\cap B)$’	B1ft
		$\frac{3}{20}$ and $\frac{3}{10}$	B1
		Box and $\frac{7}{20}$	B1
			(3)
	(c)	$\left[P(A)=\frac{3}{20}+\frac{1}{5}\right]=\frac{7}{20}$ or 0.35	B1ft
			(1)
	(d)	$P(B\mid A)=\frac{P(A\cap B)}{P(A)}=\frac{\frac{1}{5}}{\frac{7}{20}}$	M1
		$=\frac{4}{7}$	A1 cao
			(2)
	(e)	0.3	B1ft
			(1)
			[Total 9]

The Venn diagram shows three events A, B and C, where p, q, r, s and t are probabilities.



P(A) = 0.5, P(B) = 0.6 and P(C) = 0.25 and the events B and C are independent.

(a) Find the value of p and the value of q.

(2)

(b) Find the value of r.

(2)

(c) Hence write down the value of s and the value of t.

(2)

(d) State, giving a reason, whether or not the events A and B are independent.

(2)

(e) Find P($B \mid A \cup C$).

(3)

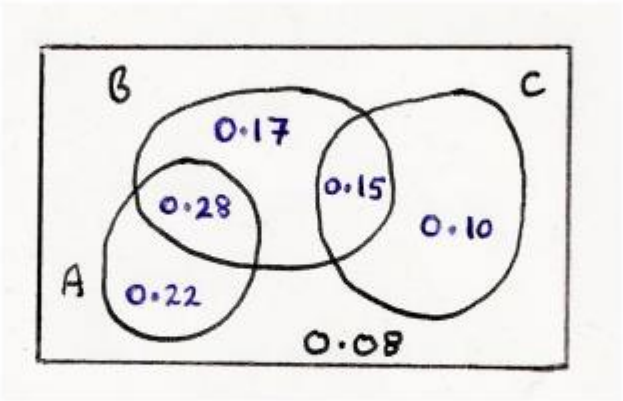
4 A supermarket has a large stock of eggs. 40% of the stock are from a firm called Eggzact. 12% of the stock are brown eggs from Eggzact.

An egg is chosen at random from the stock. Calculate the probability that

(i) this egg is brown, given that it is from Eggzact, [2]

(ii) this egg is from Eggzact and is not brown. [2]

3.	(a)	$p = \mathrm{P}(B \cap C) = \mathrm{P}(B)\times \mathrm{P}(C) = 0.6\times 0.25 = \underline{0.15}$	M1
		$q = [\mathrm{P}(C) - p\,] = \underline{0.10}$	A1 (2)
	(b)	$r = 1 - 0.08 - [\mathrm{P}(B) + q] = 1 - 0.08 - 0.6 - 0.1$ (o.e.) <u>or</u> $1 - 0.08 - (0.6 + 0.25 - p)$	M1
		$= \underline{0.22}$	A1cao (2)
	(c)	$s = [\mathrm{P}(A) - r\,] = \underline{0.28}$	B1ft
		$t = [\ \mathrm{P}(B) - p - s$ <u>or</u> use $\mathrm{P}(B \cap C') - s = 0.6\times 0.75 - "0.28"\ ] = \underline{0.17}$	B1ft (2)
	(d)	$\mathrm{P}(A)\times \mathrm{P}(B) = 0.5\times 0.6 = 0.3$ which is <u>not</u> equal to s (= 0.28)	M1
		So A and B are <u>not</u> independent	A1 (2)
	(e)	$\dfrac{(s+p) \text{ or } (0.6-t)}{\mathrm{P}(A\cup C) \text{ or } [\mathrm{P}(A)+\mathrm{P}(C)] \text{ or } (r+s+p+q)}, = \dfrac{("0.28"+"0.15") \text{ or } (0.6-"0.17")}{0.5+0.25}$	M1, A1ft
		$= \underline{\dfrac{43}{75}}$	A1 (3)
			[11]



Fully correct Venn diagram will score the first 6 marks

4i	0.4 x $p = 0.12$ or $^{0.12}/_{0.4}$ or $^{12}/_{40}$ oe $p = 0.3$ oe	M1 A1 2	
ii	0.4 x (1 – their 0.3) oe eg $^{40}/_{100} \times {}^{28}/_{40}$ 0.28 or 28% oe	M1 A1ft 2	or 0.4 – 0.12 or 0.28 or 28 seen Not 0.4×0.88 unless ans to (i) is 0.12
Total		4	

When a patient takes the painkilling drug *PD1*, the patient may have no side effects (event A), slight side effects (event B) or severe side effects (event C).

It has been established that $\mathrm{P}(A) = 0.85$, $\mathrm{P}(B) = 0.10$ and $\mathrm{P}(C) = 0.05$.

(a) A doctor prescribes *PD1* to three unrelated patients.

Calculate the probability that:

(i) all three patients have no side effects;

(ii) two patients have no side effects and one patient has slight side effects;

(iii) one patient has no side effects, one patient has slight side effects and one patient has severe side effects.

[7 marks]

(b) Other painkilling drugs are available.

Of patients taking *PD1*, none of those who suffer no side effects will change to another drug, 25 per cent of those who suffer slight side effects will change to another drug, and 90 per cent of those who suffer severe side effects will change to another drug.

A second doctor prescribes *PD1* to a patient.

Calculate the probability that:

(i) the patient does not change to another drug;

(ii) the patient changes to another drug, given that the patient experienced side effects from taking *PD1*.

[6 marks]

6	**Accept the equivalent percentage answers with %-sign (see GN5)**			
(a) **(i)**	$P(A_1 \cap A_2 \cap A_3) = 0.85^3$ $= \underline{0.614}$	B1	**(1)**	AWRT (0.614125)
(ii)	$P(A_1 \cap A_2 \cap B) = 0.85^2 \times 0.10$ **or** (0.0722 to 0.0723) **or** 289/4000	M1		OE; do **not** accept additional terms (0.07225)
	$\underline{\times\ 3}$	A1		OE
	$= \underline{0.216 \text{ to } 0.217}$	A1	**(3)**	AWFW (0.21675)
(iii)	$P(A \cap B \cap C) = 0.85 \times 0.10 \times 0.05$ **or** (0.0042 to 0.0043) **or** 17/4000	M1		OE; do **not** accept additional terms (0.00425)
	$\underline{\times\ 6}$	A1		OE
	$= \underline{0.025 \text{ to } 0.026}$	A1	**(3)**	AWFW (0.0255)
(b) **(i)**	(α) P(OD′) = $0.10 \times 0.75 + 0.05 \times 0.10$	M1		
	PLUS 0.85 (× 1)	A1		CAO
	$= 0.85 + 0.075 + 0.005 = \underline{0.93}$	A1		CAO
	or			
	(β) P(OD) = $0.10 \times 0.25 + 0.05 \times 0.90 + 0.85 \times 0$	(M1)		See SC 3 below
	$= 0.025 + 0.045 = \underline{0.07}$	(A1)		CAO
	P(OD′) = 1 − 0.07 $= \underline{0.93}$	(A1)		CAO
	or			
	(γ)			Accept probabilities rather than percentages
		(B2)		(0, 2.5, 4.5) **or** (85, 7.5, 0.5) CAO
		(B1)	**(3)**	0.93 CAO

	Side Effect			
	None	Slight	Severe	Total
Change	0	2.5	4.5	7.0
No change	85	7.5	0.5	93.0
Total	85	10.0	5.0	100.0

(ii)	$P(\text{OD} \mid B \cup C) = \dfrac{1-0.93}{0.10+0.05}$ or $\dfrac{0.07}{0.10+0.05}$	M1 M1		Numerator; OE Denominator (See Notes 1 & 2 below)
	$= \underline{7/15 \text{ or } 0.466 \text{ to } 0.467 \text{ or } 0.4\dot{6}}$	A1		CAO/AWFW/CAO (0.46667)
	or			
	$P(\text{OD} \mid B \cup C) = \dfrac{2}{3} \times 0.25 + \dfrac{1}{3} \times 0.9$	(M1) (M1)		Either term (OE) **PLUS** other term (OE)
	$= \underline{7/15 \text{ or } 0.466 \text{ to } 0.467 \text{ or } 0.4\dot{6}}$	(A1)		CAO/AWFW/CAO (0.46667)
	or			
	From table, $P(\text{OD} \mid B \cup C)$ $= \underline{7/15 \text{ or } 0.466 \text{ to } 0.467 \text{ or } 0.4\dot{6}}$	(B3)	**(3)**	CAO/AWFW/CAO (0.46667)
Notes	1 A mark of M1 may be available **in a fraction** even if the resultant probability answer is greater than 1 2 Values of (1 – 0.93) or 0.07 or 0.15 seen but **not in a fraction** and with no correct answer ⇒ M0 M0 (A0)			

A school athletics team has 10 members. The table shows which competitions each of the members can take part in.

		Competiton				
		100 m	200 m	110 m hurdles	400 m	Long jump
Athlete	Abel	✓	✓			✓
	Bernoulli		✓		✓	
	Cauchy	✓		✓		✓
	Descartes	✓	✓			
	Einstein		✓		✓	
	Fermat	✓		✓		
	Galois				✓	✓
	Hardy	✓	✓			✓
	Iwasawa		✓		✓	
	Jacobi			✓		

An athlete is selected at random. Events A, B, C, D are defined as follows.

A: the athlete can take part in exactly 2 competitions.
B: the athlete can take part in the 200 m.
C: the athlete can take part in the 110 m hurdles.
D: the athlete can take part in the long jump.

(i) Write down the value of $\mathrm{P}(A \cap B)$. [1]

(ii) Write down the value of $\mathrm{P}(C \cup D)$. [1]

(iii) Which two of the four events A, B, C, D are mutually exclusive? [1]

(iv) Show that events B and D are not independent. [2]

5 **(i)** A bag contains 12 red discs and 10 black discs. Two discs are removed at random, without replacement. Find the probability that both discs are red. [2]

(ii) Another bag contains 7 green discs and 8 blue discs. Three discs are removed at random, without replacement. Find the probability that exactly two of these discs are green. [3]

(iii) A third bag contains 45 discs, each of which is either yellow or brown. Two discs are removed at random, without replacement. The probability that both discs are yellow is $\frac{1}{15}$. Find the number of yellow discs which were in the bag at first. [4]

5 (i)	P(A ∩ B) = 0.4	B1 CAO	**1**
(ii)	P(C U D) = 0.6	B1 CAO	**1**
(iii)	Events B and C are mutually exclusive.	B1 CAO	**1**
(iv)	P(B) = 0.6, P(D) = 0.4 and P(B ∩ D) = 0.2 0.6 x 0.4 ≠ 0.2 (so B and D not independent)	B1 for P(B ∩ D) = 0.2 soi E1	**2**
		TOTAL	**5**

5(i)	${}^{12}/_{22}$ x ${}^{11}/_{21}$ = ${}^{2}/_{7}$ oe or 0.286 (3 sfs)	M1 A1 2	or ${}^{12}C_2$ / ${}^{22}C_2$
(ii)	${}^{7}/_{15}$ x ${}^{6}/_{14}$ x ${}^{8}/_{13}$ or ${}^{8}/_{65}$ oe × 3 oe = ${}^{24}/_{65}$ or 0.369 (3 sfs)	M1 M1 A1 3	Numerators any order ${}^{7}C_2$ x ${}^{8}C_1$:M1 3 x prod any 3 probs (any C or P)/${}^{15}C_3$:M1 (dep <1) 1-(${}^{8}/_{15}$x${}^{7}/_{14}$x${}^{6}/_{13}$+3×${}^{8}/_{15}$x${}^{7}/_{14}$x${}^{7}/_{13}$+${}^{7}/_{15}$x${}^{6}/_{14}$x${}^{5}/_{13}$) : M2 one prod omitted or wrong: M1
(iii)	$\frac{x}{45}\times\frac{x-1}{44}=\frac{1}{15}$ oe $x^2 - x - 132 = 0$ or $x(x - 1) = 132$ $(x - 12)(x + 11) = 0$ or $x = \frac{1 \pm \sqrt{(1^2 - 4\times(-132))}}{2}$ No. of Ys = 12	M1 A1 M1 A1 4	not $\frac{x}{45}\times\frac{x}{44}=\frac{1}{15}$ or $\frac{x}{45}\times\frac{x}{45}=\frac{1}{15}$ or $\frac{x}{45}\times\frac{x-1}{45}=\frac{1}{15}$ oe ft 3-term QE for M1 condone signs interchanged allow one sign error Not x = 12 or –11

Jane buys 5 jam doughnuts, 4 cream doughnuts and 3 plain doughnuts.

On arrival home, each of her three children eats one of the twelve doughnuts. The different kinds of doughnut are indistinguishable by sight and so selection of doughnuts is random.

Calculate the probabilities of the following events.

(i) All 3 doughnuts eaten contain jam. [3]

(ii) All 3 doughnuts are of the same kind. [3]

(iii) The 3 doughnuts are all of a different kind. [3]

(iv) The 3 doughnuts contain jam, given that they are all of the same kind. [3]

On 5 successive Saturdays, Jane buys the same combination of 12 doughnuts and her three children eat one each. Find the probability that all 3 doughnuts eaten contain jam on

(v) exactly 2 Saturdays out of the 5, [3]

(vi) at least 1 Saturday out of the 5. [3]

8 A game uses an unbiased die with faces numbered 1 to 6. The die is thrown once. If it shows 4 or 5 or 6 then this number is the final score. If it shows 1 or 2 or 3 then the die is thrown again and the final score is the sum of the numbers shown on the two throws.

(i) Find the probability that the final score is 4. **[3]**

(ii) Given that the die is thrown only once, find the probability that the final score is 4. **[1]**

(iii) Given that the die is thrown twice, find the probability that the final score is 4. **[3]**

8 **(i)**	P(all jam) $= \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$ $= \frac{1}{22} = 0.04545$	M1 5 x 4 x 3 or $\binom{5}{3}$ in numerator M1 12 x 11 x 10 or $\binom{12}{3}$ in denominator A1 CAO	**3**
(ii)	P(all same) $= \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} + \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} + \frac{3}{12} \times \frac{2}{11} \times \frac{1}{10}$ $= \frac{1}{22} + \frac{1}{55} + \frac{1}{220} = \frac{3}{44} = 0.06818$	M1 Sum of 3 reasonable triples or combinations M1 Triples or combinations correct A1 CAO	**3**
(iii)	P(all different) $= 6 \times \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$ $= \frac{3}{11} = 0.2727$	M1 5,4,3 M1 $6 \times$ three fractions or $\binom{12}{3}$ denom. A1 CAO	**3**
(iv)	P(all jam given all same) = $\frac{1}{22} \Big/ \frac{3}{44} = \frac{2}{3}$	M1 Their (i) in numerator M1 Their (ii) in denominator A1 CAO	**3**
(v)	P(all jam exactly twice) $= \binom{5}{2} \times \left(\frac{1}{22}\right)^2 \times \left(\frac{21}{22}\right)^3 = 0.01797$	M1 for $\binom{5}{2}$x … M1 for their $p^2\ q^3$ A1 CAO	**3**
(vi)	P(all jam at least once) $= 1 - \left(\frac{21}{22}\right)^5 = 0.2075$	M1 for their q^5 M1 indep for 1 – 5th power A1 CAO	**3**
		TOTAL	**18**

8 (i)	$^1/_6 + 3 \times (^1/_6)^2$ $= ^1/_4$	M2 A1 3	or $3 \times (^1/_6)^2$ or $^1/_6 + (^1/_6)^2$ or $^1/_6 + 2(^1/_6)^2$ or $^1/_6 + 4(^1/_6)^2$ M1
(ii)	$^1/_3$	B1 1	
(iii)	3 routes clearly implied out of 18 possible (equiprobable) routes $^1/_6$	M1 M1 A1 3	or $^1/_3 \times {^1/_6} \times 3$ M2 or $^1/_3 \times {^1/_6}$ or $^1/_6 \times {^1/_6} \times 3$ or $^1/_3 \times {^1/_3} \times 3$ or $^1/_4$ - $^1/_6$ M1 but $^1/_6 \times {^1/_6} \times 2$ M0 $\frac{(\frac{1}{6})^2 \times 3}{\frac{1}{2}}$ or $\frac{\frac{1}{4} - \frac{1}{6}}{\frac{1}{2}}$ or $\frac{\frac{1}{2} \times \frac{1}{6}}{\frac{1}{2}}$ oe M2 or $\frac{\text{P(4\&twice)}}{\text{P(twice)}}$ stated or $\frac{\text{prob}}{\frac{1}{2}}$ M1 Whatever 1st, only one possibility on 2nd M2 $^1/_6$, no wking M1M1A1 $^1/_{12}$, no wking M0
Total		7	

Isobel plays football for a local team. Sometimes her parents attend matches to watch her play.

- A is the event that Isobel's parents watch a match.
- B is the event that Isobel scores in a match.

You are given that $\mathrm{P}(B \mid A) = \frac{3}{7}$ and $\mathrm{P}(A) = \frac{7}{10}$.

(i) Calculate $\mathrm{P}(A \cap B)$. [2]

The probability that Isobel does not score and her parents do not attend is 0.1.

(ii) Draw a Venn diagram showing the events A and B, and mark in the probability corresponding to each of the regions of your diagram. [2]

(iii) Are events A and B independent? Give a reason for your answer. [2]

(iv) By comparing $\mathrm{P}(B \mid A)$ with $\mathrm{P}(B)$, explain why Isobel should ask her parents not to attend. [2]

In a large company,

78% of employees are car owners,
30% of these car owners are also bike owners,
85% of those who are not car owners are bike owners.

(a) Draw a tree diagram to represent this information.

(3)

An employee is selected at random.

(b) Find the probability that the employee is a car owner or a bike owner but not both.

(2)

Another employee is selected at random.

Given that this employee is a bike owner,

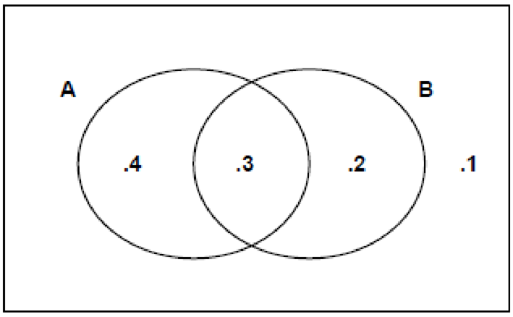
(c) find the probability that the employee is a car owner.

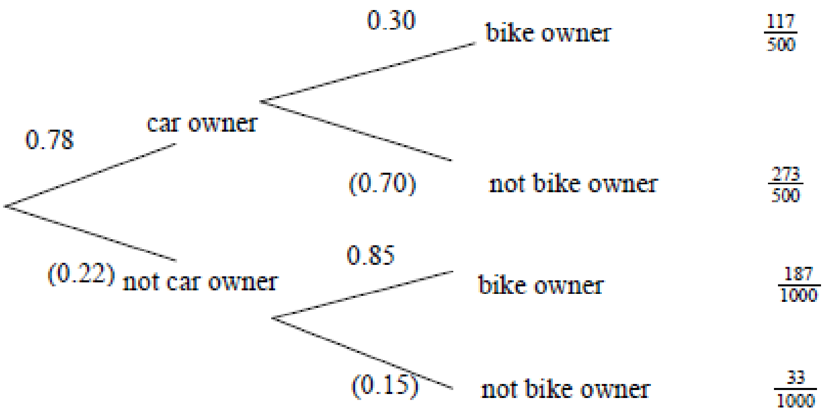
(3)

Two employees are selected at random.

(d) Find the probability that only one of them is a bike owner.

(3)

Q2 **(i)**	$P(A \cap B) = P(A)P(B\mid A) = \frac{7}{10} \times \frac{3}{7}$ $\rightarrow P(A \cap B) = 0.3$ o.e.	M1 Product of these fractions A1	**2**
(ii)		B1FT either 0.4 or 0.2 in correct place B1FT all correct and labelled	**2**
(iii)	P(*B*|*A*) ≠ P(*B*), 3/7 ≠ 0.5 Unequal so not independent	E1 Correct comparison E1*dep* for 'not independent'	**2**
(iv)	3/7 < 0.5 so Isobel is less likely to score when her parents attend	E1 for comparison E1*dep*	**2**
		TOTAL	**8**

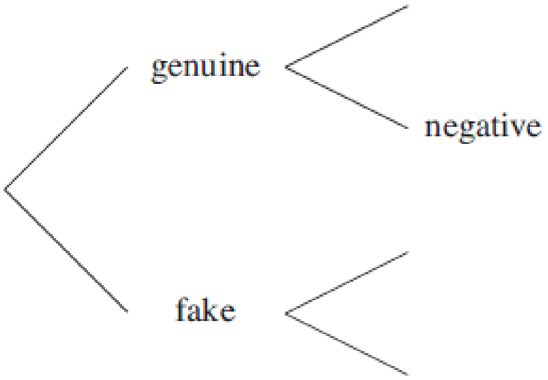
7. (a)		B1 B1 B1 **(3)**
(b)	P(car or bike but not both)$= 0.78 \times 0.70 + 0.22 \times 0.85 = 0.733$	M1 A1 **(2)**
(c)	$[\text{P(car\|bike)}] = \frac{\text{P(car} \cap \text{bike)}}{\text{P(bike)}} = \frac{0.78 \times 0.30}{0.78 \times 0.30 + 0.22 \times 0.85}, = 0.555819....$ **awrt** 0.556	M1A1 A1 **(3)**
(d)	$\text{P(bike)} = 0.78 \times 0.30 + 0.22 \times 0.85 = 0.421$, P(not bike) = 1 – 0.421 $0.421 \times 0.579 + 0.579 \times 0.421$ $= 0.487518$ **awrt** 0.488	M1 dM1 A1 **(3)** **[Total 11]**

It has been estimated that 90% of paintings offered for sale at a particular auction house are genuine, and that the other 10% are fakes. The auction house has a test to determine whether or not a given painting is genuine. If this test gives a positive result, it suggests that the painting is genuine. A negative result suggests that the painting is a fake.

If a painting is genuine, the probability that the test result is positive is 0.95.

If a painting is a fake, the probability that the test result is positive is 0.2.

(i) Copy and complete the probability tree diagram below, to illustrate the information above. [2]



Calculate the probabilities of the following events.

(ii) The test gives a positive result. [2]

(iii) The test gives a correct result. [2]

(iv) The painting is genuine, given a positive result. [3]

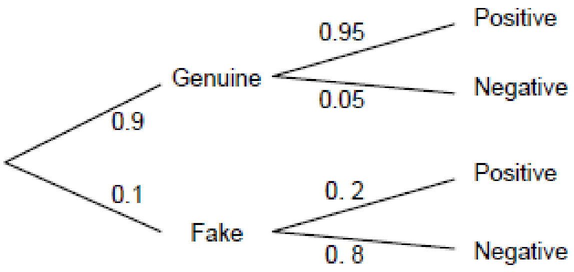
(v) The painting is a fake, given a negative result. [3]

A second test is more accurate, but very expensive. The auction house has a policy of only using this second test on those paintings with a negative result on the original test.

(vi) Using your answers to parts **(iv)** and **(v)**, explain why the auction house has this policy. [2]

The probability that the second test gives a correct result is 0.96 whether the painting is genuine or a fake.

(vii) Three paintings are independently offered for sale at the auction house. Calculate the probability that all three paintings are genuine, are judged to be fakes in the first test, but are judged to be genuine in the second test. [4]

Q6 **(i)**		G1 for left hand set of branches fully correct including labels and probabilities G1 for right hand set of branches fully correct	**2**
(ii)	P (test is positive) = (0.9)(0.95) + (0.1)(0.2) = 0.875	M1 Two correct pairs added A1 CAO	**2**
(iii)	P (test is correct) = (0.9)(0.95) + (0.1)(0.8) = 0.935	M1 Two correct pairs added A1 CAO	**2**
(iv)	P (Genuine|Positive) = 0.855/0.875 = 0.977	M1 Numerator M1 Denominator A1 CAO	**3**
(v)	P (Fake|Negative) = 0.08/0.125 = 0.64	M1 Numerator M1 Denominator A1 CAO	**3**
(vi)	EITHER: A positive test means that the painting is almost certain to be genuine so no need for a further test. However, more than a third of those paintings with a negative result are genuine so a further test is needed. NOTE: Allow sensible alternative answers	E1FT E1FT	**2**
(vii)	P (all 3 genuine) = $(0.9 \times 0.05 \times 0.96)^3$ $= (0.045 \times 0.96)^3$ $= (0.0432)^3$ = 0.0000806	M1 for 0.9 x 0.05 (=0.045) M1 for complete correct triple product M1*indep* for cubing A1 CAO	**4**
		TOTAL	**18**

Each day the probability that Ashwin wears a tie is 0.2. The probability that he wears a jacket is 0.4. If he wears a jacket, the probability that he wears a tie is 0.3.

(i) Find the probability that, on a randomly selected day, Ashwin wears a jacket and a tie. [2]

(ii) Draw a Venn diagram, using one circle for the event 'wears a jacket' and one circle for the event 'wears a tie'. Your diagram should include the probability for each region. [3]

(iii) Using your Venn diagram, or otherwise, find the probability that, on a randomly selected day, Ashwin

(A) wears either a jacket or a tie (or both),

(B) wears no tie or no jacket (or wears neither). [3]

In a factory, three machines, J, K and L, are used to make biscuits.

Machine J makes 25% of the biscuits.

Machine K makes 45% of the biscuits.

The rest of the biscuits are made by machine L.

It is known that 2% of the biscuits made by machine J are broken, 3% of the biscuits made by machine K are broken and 5% of the biscuits made by machine L are broken.

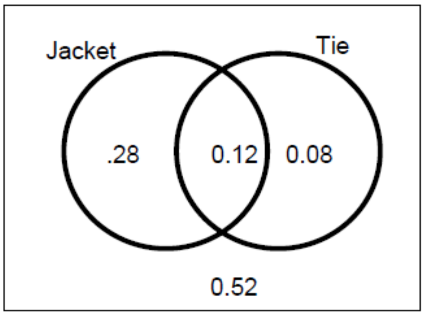
(a) Draw a tree diagram to illustrate all the possible outcomes and associated probabilities.
(2)

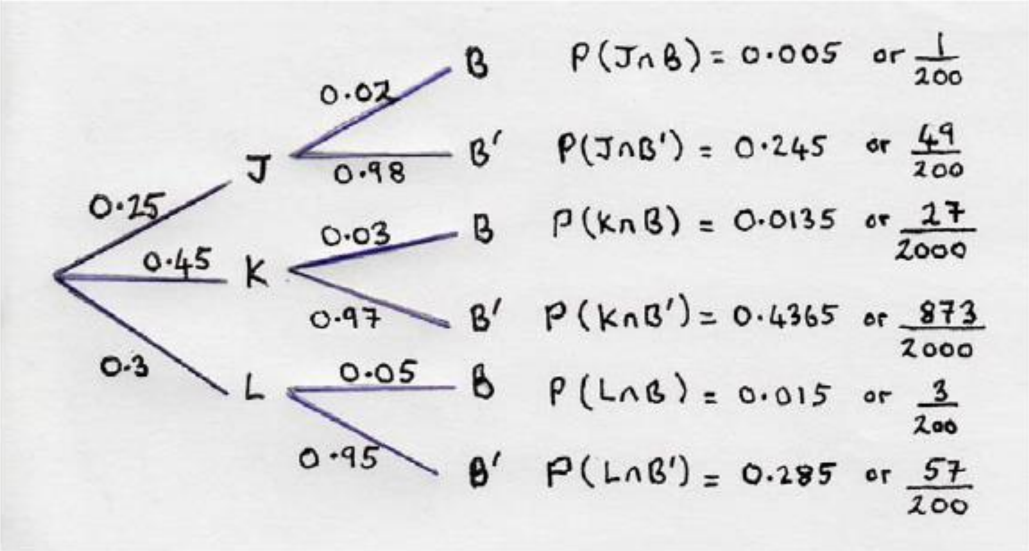
A biscuit is selected at random.

(b) Calculate the probability that the biscuit is made by machine J and is not broken.
(2)

(c) Calculate the probability that the biscuit is broken.
(2)

(d) Given that the biscuit is broken, find the probability that it was not made by machine K.
(3)

Q 5(i)	P(jacket and tie) = 0.4 × 0.3 = 0.12	M1 for multiplying A1 CAO	**2**
(ii)		G1 for two intersecting circles labelled G1 for 0.12 and either 0.28 or 0.08 G1 for remaining probabilities <u>Note</u> FT their 0.12 provided < 0.2	**3**
(iii)	*(A)* P(jacket or tie) =P(J) + P(T) – P(J∩T) = 0.4 + 0.2 – 0.12 = 0.48 *OR* = 0.28 + 0.12 + 0.08 = 0.48 (*B*) P(no jacket or no tie) = 0.52 + 0.28 + 0.08 = 0.88 *OR* 0.6 + 0.8 – 0.52 = 0.88 *OR* 1 – 0.12 = 0.88	B1 FT B2 FT <u>Note</u> FT their 0.12 provided < 0.2	**3**
		TOTAL	**8**

4	(a)	 $P(J\cap B) = 0.005$ or $\frac{1}{200}$ $P(J\cap B') = 0.245$ or $\frac{49}{200}$ $P(K\cap B) = 0.0135$ or $\frac{27}{2000}$ $P(K\cap B') = 0.4365$ or $\frac{873}{2000}$ $P(L\cap B) = 0.015$ or $\frac{3}{200}$ $P(L\cap B') = 0.285$ or $\frac{57}{200}$	M1 A1 **(2)**
	(b)	0.25×0.98, $= \mathbf{0.245}$ (or exact equiv. e.g. $\frac{49}{200}$)	M1A1 **(2)**
	(c)	$0.25\times0.02+0.45\times0.03+0.3\times0.05$, $= \mathbf{0.0335}$ (or exact equiv. e.g. $\frac{67}{2000}$)	M1A1 **(2)**
	(d)	$[P(J\cup L\mid B)] = \frac{0.25\times0.02+0.3\times0.05}{0.0335}$ <u>or</u> $\frac{0.0335-0.45\times0.03}{0.0335}$ $=0.5970...$ awrt **0.597** (or $\frac{40}{67}$ or exact equiv.)	M1A1ft A1 **(3)**
		Notes	**Total 9**

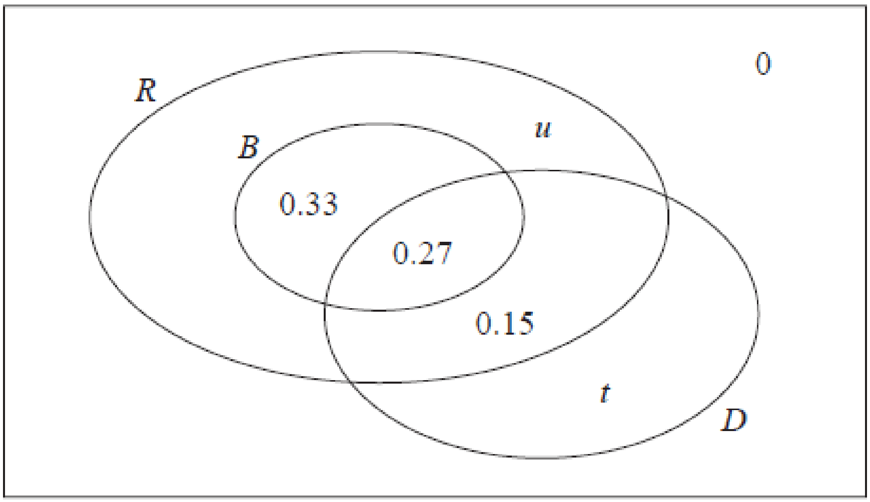
The Venn diagram shows the probabilities of customer bookings at Harry's hotel.

R is the event that a customer books a room

B is the event that a customer books breakfast

D is the event that a customer books dinner

u and t are probabilities.



(a) Write down the probability that a customer books breakfast but does not book a room.
(1)

Given that the events B and D are independent

(b) find the value of t
(4)

(c) hence find the value of u
(2)

(d) Find

(i) $\mathrm{P}(D \mid R \cap B)$

(ii) $\mathrm{P}(D \mid R \cap B')$
(4)

A coach load of 77 customers arrive at Harry's hotel.

Of these 77 customers

40 have booked a room and breakfast

37 have booked a room without breakfast

(e) Estimate how many of these 77 customers will book dinner.
(2)

4.(a)	$[\mathrm{P}(B\cap R')=]$ $\underline{0}$	B1
		(1)
(b)	$\mathrm{P}(B)=0.27+0.33=0.6$, $\mathrm{P}(D)=0.27+0.15+t$, $\mathrm{P}(B\cap D)=0.27$	M1
	$[\mathrm{P}(B)\times\mathrm{P}(D)=\mathrm{P}(B\cap D)$ gives$]$ $0.6\times(0.42+t)=0.27$	M1
	$0.42+t=\frac{0.27}{0.6}$ <u>or</u> $0.6t=0.018$	A1
	$t=\underline{\mathbf{0.03}}$	A1
		(4)
(c)	$[\,u=]\ \ 1-(0.6+0.15+t)$	M1
	$u=\underline{\mathbf{0.22}}$	A1ft
		(2)
(d)(i)	$\left[\frac{\mathrm{P}(D\cap R\cap B)}{\mathrm{P}(R\cap B)}=\right]=\frac{0.27}{0.27+0.33}$ <u>or</u> $\mathrm{P}(D\mid R\cap B)=\mathrm{P}(D\mid B)=\mathrm{P}(D)$	M1
	$=\underline{\mathbf{0.45}}$	A1
(ii)	$\left[\frac{\mathrm{P}(D\cap[R\cap B'])}{\mathrm{P}(R\cap B')}=\right]=\frac{0.15}{0.15+u}$	M1
	$=\frac{15}{37}$	A1
		(4)
(e)	$40\times\text{"}0.45\text{"}$ and $37\times\text{"}\frac{15}{37}\text{"}$	M1
	$=\underline{\mathbf{33}}$	A1
		(2)
		[13 marks]

A college has 80 students in Year 12.

20 students study Biology
28 students study Chemistry
30 students study Physics
7 students study both Biology and Chemistry
11 students study both Chemistry and Physics
5 students study both Physics and Biology
3 students study all 3 of these subjects

(a) Draw a Venn diagram to represent this information.

(5)

A Year 12 student at the college is selected at random.

(b) Find the probability that the student studies Chemistry but not Biology or Physics.

(1)

(c) Find the probability that the student studies Chemistry or Physics or both.

(2)

Given that the student studies Chemistry or Physics or both,

(d) find the probability that the student does not study Biology.

(2)

(e) Determine whether studying Biology and studying Chemistry are statistically independent.

(3)

7 The table shows the numbers of male and female members of a vintage car club who own either a Jaguar or a Bentley. No member owns both makes of car.

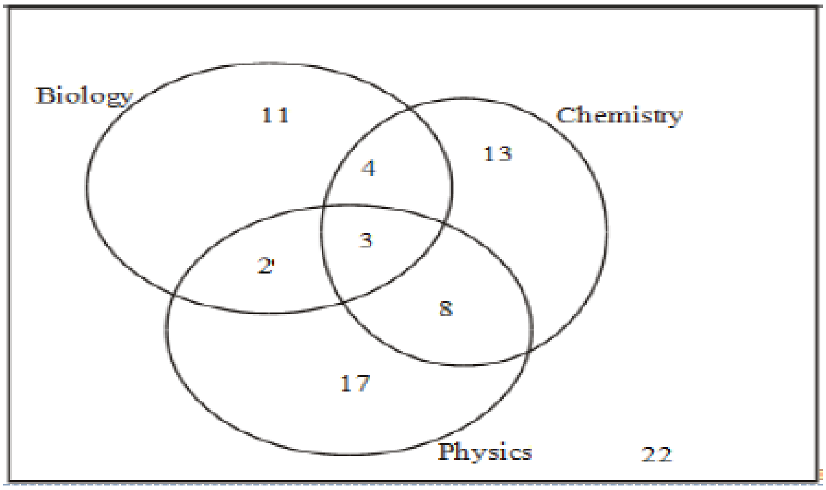
	Male	Female
Jaguar	25	15
Bentley	12	8

One member is chosen at random from these 60 members.

(i) Given that this member is male, find the probability that he owns a Jaguar. **[2]**

Now two members are chosen at random from the 60 members. They are chosen one at a time, without replacement.

(ii) Given that the first one of these members is female, find the probability that both own Jaguars. **[4]**

3.	**(a)**		B1 M1 A1 A1 B1 (5)
	(b)	$\frac{'13'}{80}$ <u>or</u> 0.1625	B1ft (1)
	(c)	$\frac{28+30-11}{80}$ or $\frac{2+3+4+8+13+17}{80}$ or $1-\frac{(11+22)}{80}=\frac{47}{80}$ <u>or</u> 0.5875	M1 A1 (2)
	(d)	$\frac{"17+8+13"}{"47"}$ or $\frac{\frac{"38"}{80}}{\frac{"47"}{80}}$ or $1-\frac{"2+3+4"}{"47"}=\frac{38}{47}$ (condone awrt 0.809)	M1 A1cao (2)
	(e)	$\mathrm{P}(B\mid C)=\frac{7}{28}$, $\mathrm{P}(B)=\frac{20}{80}$	
		$\mathrm{P}(C\mid B)=\frac{7}{20}$, $\mathrm{P}(C)=\frac{28}{80}$	M1
		$\mathrm{P}(B\cap C)=\frac{7}{80}$, $\mathrm{P}(B)\ =\ \frac{20}{80}\mathrm{P}(C)=\frac{28}{80}$	
		$\mathrm{P}(B\mid C)=\mathrm{P}(B)$, $\mathrm{P}(C\mid B)=\mathrm{P}(C)$ these may be implied by correct conclusion	M1
		$\mathrm{P}(B\cap C)=\mathrm{P}(B)\times\mathrm{P}(C)$ this approach requires the product to be seen	
		So, they are independent.	A1 (3) (13 marks)

7 **(i)**	$^{25}/_{37}$	B2 2	B1 num, B1 denom 25/37xp B1
(ii)	$\frac{15}{23}$ seen or implied	M1	
	$\times\frac{39}{59}$ seen or implied	M2	M1 num, M1 denom Allow M1 for 39/59x or + wrong p
	$=\frac{585}{1357}$ or 0.431 (3 sfs) oe	A1 4	
Total		[6]	

For the events A and B,

$$P(A' \cap B) = 0.22 \quad \text{and} \quad P(A' \cap B') = 0.18$$

(a) Find $P(A)$.

(1)

(b) Find $P(A \cup B)$.

(1)

Given that $P(A \mid B) = 0.6$

(c) find $P(A \cap B)$.

(3)

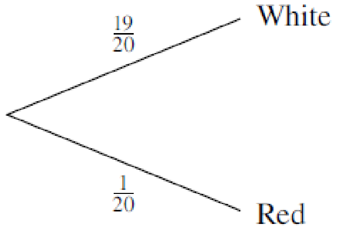
(d) Determine whether or not A and B are independent.

(2)

8 A game at a charity event uses a bag containing 19 white counters and 1 red counter. To play the game once a player takes counters at random from the bag, one at a time, without replacement. If the red counter is taken, the player wins a prize and the game ends. If not, the game ends when 3 white counters have been taken. Niko plays the game once.

(i) **(a)** Copy and complete the tree diagram showing the probabilities for Niko. **[4]**

First counter



(b) Find the probability that Niko will win a prize. **[3]**

(ii) The number of counters that Niko takes is denoted by X.

(a) Find $P(X = 3)$. **[2]**

(b) Find $E(X)$. **[4]**

8	(a)	$[\,\mathrm{P}(A)=1-0.18-0.22\ ] \quad = \quad \mathbf{0.6}$ (or exact equivalent)			B1
					(1)
	(b)	$\mathrm{P}\left(A\cup B\right)=\text{"0.6"}+0.22 \quad = \ \mathbf{0.82}$ (or exact equivalent)			B1ft
					(1)
	(c)	$x=\mathrm{P}\left(A\cap B\right)$	Use $\mathrm{P}(B)\mathrm{P}(A'\mid B)=\mathrm{P}(A'\cap B)$	**Establish** independence before or after 1st M1and score marks for (d) (RH ver)	M1
		$\dfrac{x}{x+0.22}=0.6$	$\mathrm{P}(B)\times[1-0.6]=0.22$	**Find** $\mathrm{P}(B)$	
		$x=0.6x+0.132$	Use $\mathrm{P}(A\cap B)=\mathrm{P}(A\mid B)\mathrm{P}(B)$	Use $\mathrm{P}(B)\mathrm{P}(A)=\mathrm{P}\left(A\cap B\right)$	dM1
		$0.4x=0.132$	$\mathrm{P}(A\cap B)=0.6\times0.55$	$\mathrm{P}(A\cap B)=0.6\times0.55$	
		$x=$ **0.33** (or exact equivalent)			A1cso
					(3)
	(d)	$\mathrm{P}(B)=0.55$			
		$\mathrm{P}(B)\times\mathrm{P}(A)=0.55\times0.6$		or stating $\mathrm{P}(A)=\mathrm{P}(A\mid B)\ [=0.6]$	M1
		$=0.33$			
		$\mathrm{P}(B)\times\mathrm{P}(A)=\mathrm{P}\left(A\cap B\right)$		or $\mathrm{P}(A)=\mathrm{P}(A\mid B)$	A1cso
		therefore (statistically) independent		therefore (statistically) independent	**(2)**
					Total 7

8ia	${}^{18}/_{19}$ or ${}^{1}/_{19}$ seen ${}^{17}/_{18}$ or ${}^{1}/_{18}$ seen structure correct ie 6 branches all correct incl. probs and W & R	B1 B1 B1 B1 4	regardless of probs & labels (or 14 branches with correct 0s & 1s)
b	${}^{1}/_{20}+{}^{19}/_{20}\times{}^{1}/_{19}+{}^{19}/_{20}\times{}^{18}/_{19}\times{}^{1}/_{18}$ $={}^{3}/_{20}$	M2 A1 3	M1 any 2 correct terms added ${}^{19}/_{20}\times{}^{18}/_{19}\times{}^{17}/_{18}$ $1-{}^{19}/_{20}\times{}^{18}/_{19}\times{}^{17}/_{18}$
iia	${}^{19}/_{20}\times{}^{18}/_{19}$ $={}^{9}/_{10}$ oe	M1 A1 2	${}^{19}/_{20}\times{}^{18}/_{19}\times{}^{1}/_{18}+{}^{19}/_{20}\times{}^{18}/_{19}\times{}^{17}/_{18}$ or ${}^{1}/_{20}+{}^{17}/_{20}$
b	$(\mathrm{P}(X=1)={}^{1}/_{20})$ ${}^{19}/_{20}\times{}^{1}/_{19}$ $={}^{1}/_{20}$ Σxp $={}^{57}/_{20}$ or 2.85	M1 A1 M1 A1 4	or $1-({}^{1}/_{20}+{}^{9}/_{10})$ or 2 probs of ${}^{1}/_{20}$ M1A1 ≥ 2 terms, ft their p's if $\Sigma p=1$ NB: ${}^{19}/_{20}\times3=2.85$ no mks

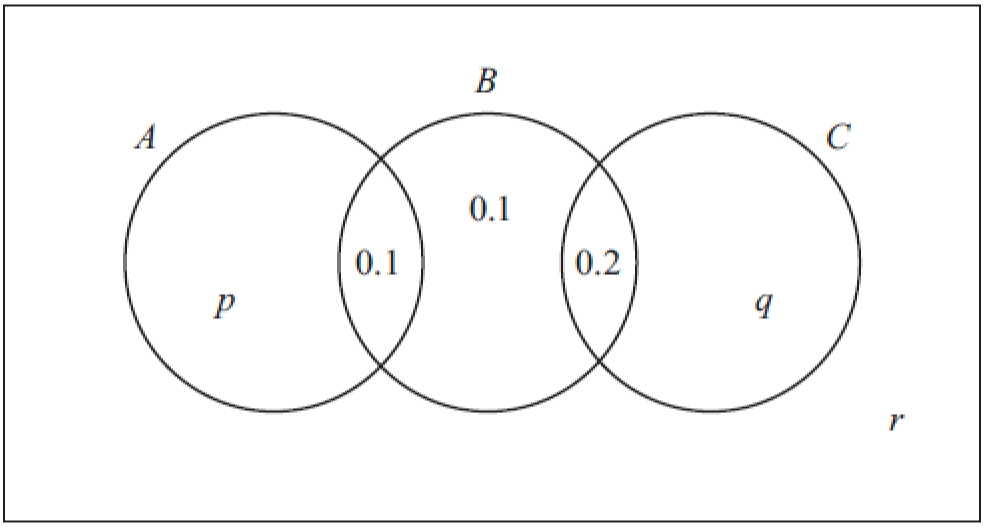


Figure 1

The Venn diagram in Figure 1 shows three events A, B and C and the probabilities associated with each region of B. The constants p, q and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent.

(a) Find the value of p.

(3)

Given that $\mathrm{P}(B|C) = \frac{5}{11}$

(b) find the value of q and the value of r.

(4)

(c) Find $\mathrm{P}(A \cup C|B)$.

(2)

5 A washing-up bowl contains 6 spoons, 5 forks and 3 knives. Three of these 14 items are removed at random, without replacement. Find the probability that

(i) all three items are of different kinds, **[3]**

(ii) all three items are of the same kind. **[3]**

6. (a)	$[\mathrm{P}(B) = 0.4,\ \mathrm{P}(A) = p + 0.1$ so$]$ $\quad 0.4\times(p+0.1)=0.1$ or $\quad 0.4\times\mathrm{P}(A)=0.1$	M1
	$p=\frac{1}{4}-0.1$ $\qquad \underline{\mathbf{p = 0.15}}$	M1A1 (3)
(b)	$\frac{5}{11}=\left[\frac{\mathrm{P}(B\cap C)}{\mathrm{P}(C)}=\right]\frac{0.2}{0.2+q}$ or $\frac{5}{11}=\frac{0.2}{\mathrm{P}(C)}$	M1
	$11\times0.2=5\times(0.2+q)$	dM1
	$\underline{\mathbf{q = 0.24}}$	A1
	$r = 0.6 - (p + q)$ i.e. $\underline{\mathbf{r = 0.21}}$	A1ft (4)
(c)	$\left[\frac{\mathrm{P}((A\cup C)\cap B)}{\mathrm{P}(B)}\right]=\frac{0.3}{0.4}$	M1
	$=\underline{\mathbf{0.75}}$	A1 (2)
		[9]

5 (i)	$\frac{6}{14}\times\frac{5}{13}\times\frac{3}{12}$ $\times\,3!$ oe $=\frac{45}{182}$ or 0.247 (3 sfs)oe	M1 M1 A1 3	${}^6C_1\times{}^5C_1\times{}^3C_1$ $\div\,{}^{14}C_3$ With repl M0M1A0
(ii)	$\frac{6}{14}\times\frac{5}{13}\times\frac{4}{12}+\frac{5}{14}\times\frac{4}{13}\times\frac{3}{12}+\frac{3}{14}\times\frac{2}{13}\times\frac{1}{12}$ $=\frac{31}{364}$ or 0.0852 (3 sf)	M2 A1 3	${}^6C_3+{}^5C_3+{}^3C_3$ M1 for any one $(\div\,{}^{14}C_3)$M1 all 9 numerators correct. With repl M1$(6/14)^3+(5/14)^3+(3/14)^3$
Total		[6]	

In a company the 200 employees are classified as full-time workers, part-time workers or contractors.
The table below shows the number of employees in each category and whether they walk to work or use some form of transport.

	Walk	Transport
Full-time worker	2	8
Part-time worker	35	75
Contractor	30	50

The events F, H and C are that an employee is a full-time worker, part-time worker or contractor respectively. Let W be the event that an employee walks to work.

An employee is selected at random.

Find

(a) $\mathrm{P}(H)$

(2)

(b) $\mathrm{P}\big([F \cap W]'\big)$

(2)

(c) $\mathrm{P}(W \mid C)$

(2)

Let B be the event that an employee uses the bus.
Given that 10% of full-time workers use the bus, 30% of part-time workers use the bus and 20% of contractors use the bus,

(d) draw a Venn diagram to represent the events F, H, C and B,

(4)

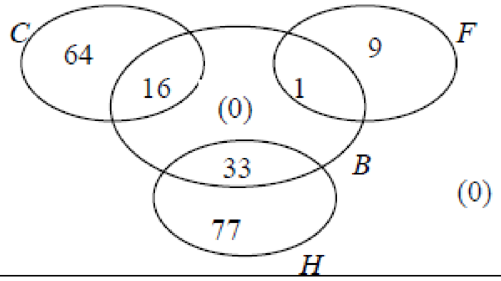
(e) find the probability that a randomly selected employee uses the bus to travel to work.

(2)

5 Events A and B are such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \mid B') = 0.75$.

(i) Find $\mathrm{P}(A \cap B)$ and $\mathrm{P}(A \cup B)$. **[6]**

(ii) Determine, giving a reason in each case,
(a) whether A and B are mutually exclusive,
(b) whether A and B are independent. **[2]**

3. (a)	$\frac{35+75}{200}=0.55$		M1 A1 **(2)**
(b)	$\frac{200-2}{200}=0.99$		M1 A1 **(2)**
(c)	$\left[\mathrm{P}(W\mid C)\right]=\frac{\mathrm{P}(W\cap C)}{\mathrm{P}(C)}=\frac{30/200}{80/200}=\ \frac{30}{80}=0.375$		M1 A1 **(2)**
(d)		Allow diagrams with intersections between *F*, *C* and *H* provided these are marked with 0. If their diagram indicates extra empty regions do not treat a blank as 0	M1 B1 for 9, 1 B1 for 77,33 B1 for 64,16 **(4)**
(e)	$\frac{1+16+33}{200}=0.25$		M1 A1 **(2)** **(12 marks)**

Question			**Answer**	**Marks**	**Guidance**	
5	(i)		$P(A\cap B')=0.75\times0.4=0.3$ $P(A\cap B)=0.5-\text{"}0.3\text{"}=0.2$ $P(A\cup B)=0.5+0.6-\text{"}0.2\text{"}=0.9$	M1A1 M1A1 M1A1 [6]		
	(ii)	(a)	No, $P(A\cap B)\neq0$ oe	B1		
		(b)	No, $0.5\times0.6\neq0.2$ oe	B1 [2]		

Given that

$$P(A) = 0.35\,, \quad P(B) = 0.45 \quad \text{and} \quad P(A \cap B) = 0.13$$

find

(a) $P(A \cup B)$

(2)

(b) $P(A' \mid B')$

(2)

The event C has $P(C) = 0.20$

The events A and C are mutually exclusive and the events B and C are independent.

(c) Find $P(B \cap C)$

(2)

(d) Draw a Venn diagram to illustrate the events A, B and C and the probabilities for each region.

(4)

(e) Find $P\left([B \cup C]'\right)$

(2)

A group of office workers were questioned for a health magazine and $\frac{2}{5}$ were found to take regular exercise. When questioned about their eating habits $\frac{2}{3}$ said they always eat breakfast and, of those who always eat breakfast $\frac{9}{25}$ also took regular exercise.

Find the probability that a randomly selected member of the group

(a) always eats breakfast and takes regular exercise,

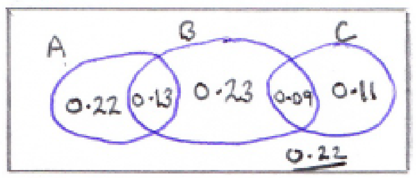
(2)

(b) does not always eat breakfast and does not take regular exercise.

(4)

(c) Determine, giving your reason, whether or not always eating breakfast and taking regular exercise are statistically independent.

(2)

7.		
(a)	$P(A \cup B) = 0.35 + 0.45 - 0.13$ or $0.22 + 0.13 + 0.32$ $= \underline{\mathbf{0.67}}$	M1 A1 (2)
(b)	$P(A' \mid B') = \frac{P(A' \cap B')}{P(B')}$ or $\frac{0.33}{0.55}$ $= \frac{3}{5}$ or 0.6	M1 A1 (2)
(c)	$P(B \cap C) = 0.45 \times 0.2$ $= \underline{\mathbf{0.09}}$	M1 A1 (2)
(d)	 Allow 1st B1 for 3 intersecting circles in a box with zeros in the regions for $A \cap C$ Do not accept "blank" for zero	B1 B1ft B1 B1 (4)
(e)	$P(B \cup C)' = 0.22 + \underline{0.22}$ or $1 - [0.56]$ or $1 - [0.13 + 0.23 + 0.09 + 0.11]$ o.e. $= \underline{\mathbf{0.44}}$	M1 A1 (2) **12**

2	E = take regular exercise B = always eat breakfast	
(a)	$P(E \cap B) = P(E \mid B) \times P(B)$ $= \frac{9}{25} \times \frac{2}{3} = 0.24$ or $\frac{6}{25}$ or $\frac{18}{75}$	M1 A1 (2)
(b)	$P(E \cup B) = \frac{2}{3} + \frac{2}{5} - \frac{6}{25}$ $= \frac{62}{75}$ | or $P(E' \mid B')$ $= \frac{13}{25}$ | or $P(B' \cap E)$ $= \frac{12}{75}$ | or $P(B \cap E')$ $= \frac{32}{75}$ $P(E' \cap B') = 1 - P(E \cup B) = \frac{13}{75}$ or $0.17\dot{3}$	M1 A1 M1 A1 (4)
(c)	$P(E \mid B) = 0.36 \neq 0.40 = P(E)$ or $P(E \cap B) = \frac{6}{25} \neq \frac{2}{5} \times \frac{2}{3} = P(E) \times P(B)$ So E and B are <u>not</u> statistically independent	M1 A1 (2) **[8]**

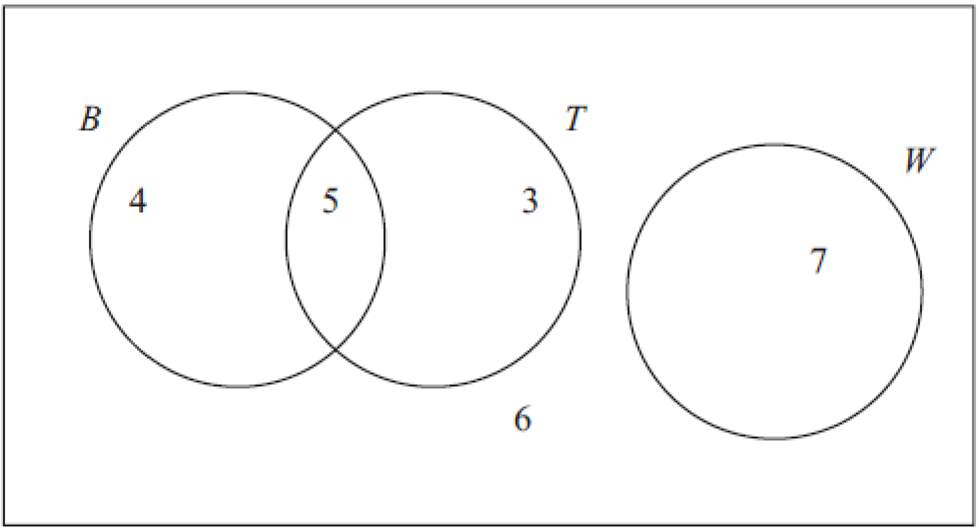


Figure 1

Figure 1 shows how 25 people travelled to work.

Their travel to work is represented by the events

B	bicycle
T	train
W	walk

(a) Write down 2 of these events that are mutually exclusive. Give a reason for your answer.
(2)

(b) Determine whether or not *B* and *T* are independent events.
(3)

One person is chosen at random.

Find the probability that this person

(c) walks to work,
(1)

(d) travels to work by bicycle and train.
(1)

(e) Given that this person travels to work by bicycle, find the probability that they will also take the train.
(2)

4.	**(a)**	B, W <u>or</u> T, W [accept $B \cup T, W$ <u>or</u> $B \cap T, W$] [Condone P(B), P(W) etc]	B1	
		Since there is no <u>overlap</u> between the events <u>or</u> cannot happen together (o.e.) (Accept comment in context e.g. "no one walks and takes the train")	B1	(2)
	(b)	e.g. $\text{P}(B) = \frac{9}{25}$, $\text{P}(T) = \frac{8}{25}$, $\text{P}(B \cap T) = \frac{5}{25}$	M1	
		$\text{P}(B \cap T) \neq \text{P}(B) \times \text{P}(T)$ $[0.2 \neq 0.36 \times 0.32 = 0.1152 \text{ o.e.}]$	M1	
		So B and T are <u>not</u> independent	A1cso	(3)
	(c)	$[\text{P}(W) =]\ \frac{7}{25}$ <u>or</u> 0.28	B1	(1)
	(d)	$[\text{P}(B \cap T) =]\ \frac{5}{25}$ <u>or</u> $\frac{1}{5}$ <u>or</u> 0.2	B1	(1)
	(e)	$[\text{P}(T \mid B) =]\ \frac{\text{P}(T \cap B)}{\text{P}(B)} = \frac{\text{"(d)"}}{(5+4)/25}$	M1	
		$= \frac{5}{9}$ or $0.\dot{5}$	A1	(2)
				[9]

A manufacturer carried out a survey of the defects in their soft toys. It is found that the probability of a toy having poor stitching is 0.03 and that a toy with poor stitching has a probability of 0.7 of splitting open. A toy without poor stitching has a probability of 0.02 of splitting open.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that a randomly chosen soft toy has exactly one of the two defects, poor stitching or splitting open.

(3)

The manufacturer also finds that soft toys can become faded with probability 0.05 and that this defect is independent of poor stitching or splitting open. A soft toy is chosen at random.

(c) Find the probability that the soft toy has none of these 3 defects.

(2)

(d) Find the probability that the soft toy has exactly one of these 3 defects.

(4)

(a) State in words the relationship between two events R and S when $P(R \cap S) = 0$

(1)

The events A and B are independent with $P(A) = \frac{1}{4}$ and $P(A \cup B) = \frac{2}{3}$

Find

(b) $P(B)$

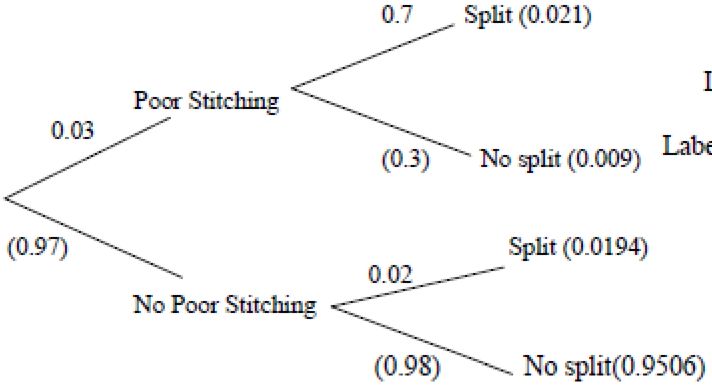
(4)

(c) $P(A' \cap B)$

(2)

(d) $P(B' | A)$

(2)

7. (a)	 Shape Labels & 0.03 Labels & 0.7,0.02	B1 B1 B1 (3)
(b)	P(Exactly one defect) = $0.03\times0.3+0.97\times0.02$ <u>or</u> $\mathrm{P}(PS\cup Split)-2\mathrm{P}(PS\cap Split)$ $=[0.009\ +\ 0.0194\ =]$ **<u>0.0284</u>**	M1A1ft A1 cao (3)
(c)	P(No defects) $=(1-0.03)\times(1-0.02)\times(1-0.05)$ (or better) $=0.90307$ awrt <u>0.903</u>	M1 A1 cao (2)
(d)	P(Exactly one defect) = $(\mathrm{b})\times(1-0.05)\ +\ (1-0.03)\times(1-0.02)\times0.05$ $=$ "0.0284"$\times0.95+0.97\times0.98\times0.05$ $=[0.02698+0.04753]=0.07451$ awrt <u>0.0745</u>	M1 M1 A1ft A1 cao (4) **[12]**

2 (a)	(R and S are mutually) exclusive.	B1 (1)
(b)	$\frac{2}{3}=\frac{1}{4}+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ use of Addition Rule $\frac{2}{3}=\frac{1}{4}+\ \mathrm{P}(B)-\ \frac{1}{4}\times\ \mathrm{P}(B)$ use of independence $\frac{5}{12}=\frac{3}{4}\ \mathrm{P}(B)$ $\mathrm{P}(B)\ =\frac{5}{9}$	M1 M1 A1 A1 (4)
(c)	$\mathrm{P}(A'\cap B)\ =\frac{3}{4}\times\frac{5}{9}=\frac{15}{36}=\frac{5}{12}$	M1A1ft (2)
(d)	$\mathrm{P}(B'\vert A)\ =\frac{(1\text{-}(\mathrm{b}))\times0.25}{0.25}$ or $\mathrm{P}(B')$ or $\frac{\frac{1}{9}}{\frac{1}{4}}$ $=\frac{4}{9}$	M1 A1 (2) **Total 9**

The following shows the results of a survey on the types of exercise taken by a group of 100 people.

65 run
48 swim
60 cycle
40 run and swim
30 swim and cycle
35 run and cycle
25 do all three

(a) Draw a Venn Diagram to represent these data.

(4)

Find the probability that a randomly selected person from the survey

(b) takes none of these types of exercise,

(2)

(c) swims but does not run,

(2)

(d) takes at least two of these types of exercise.

(2)

Jason is one of the above group.
Given that Jason runs,

(e) find the probability that he swims but does not cycle.

(3)

(a) Given that $P(A) = a$ and $P(B) = b$ express $P(A \cup B)$ in terms of a and b when

(i) A and B are mutually exclusive,
(ii) A and B are independent.

(2)

Two events R and Q are such that

$P(R \cap Q') = 0.15$, $P(Q) = 0.35$ and $P(R|Q) = 0.1$

Find the value of

(b) $P(R \cup Q)$,

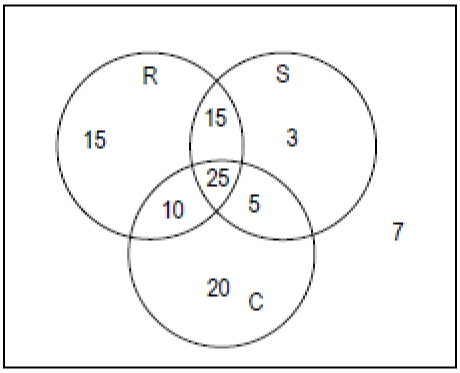
(1)

(c) $P(R \cap Q)$,

(2)

(d) $P(R)$.

(2)

6 (a)			
	3 closed curves and 25 in correct place 15,10,5 15,3,20 Labels R, S, C and box	M1 A1 A1 B1	
	All values/100 or equivalent fractions award accuracy marks.		(4)
(b)	7/100 or 0.07 M1 for ('their 7' in diagram or here)/100	M1 A1	(2)
(c)	(3+5)/100 = 2/25 or 0.08	M1A1	(2)
(d)	(25+15+10+5)/100 = 11/20 or 0.55	M1 A1	(2)
(e)	$\mathrm{P}(S\cap C' \mid R)=\frac{P(S\cap C'\cap R)}{P(R)}$ Require denominator to be 'their 65' or 'their $\frac{65}{100}$'	M1	
	$=\frac{15}{65}$ require 'their 15' and correct denominator of 65	A1	
	$=\frac{3}{13}$ or exact equivalents.	A1	(3)
			Total 13

7. (a) (i)	$\mathrm{P}(A \cup B) = a + b$ cao	B1	
(ii)	$\mathrm{P}(A \cup B) = a + b - ab$ or equivalent	B1	(2)
(b)	$\mathrm{P}(R\cup Q) = 0.15 + 0.35$ $= 0.5$ 0.5	B1	(1)
(c)	$\mathrm{P}(R\cap Q) = \mathrm{P}(R \mid Q) \times \mathrm{P}(Q)$ $= 0.1 \times 0.35$ $= 0.035$	M1 A1	(2)
(d)	$\mathrm{P}(R\cup Q) = \mathrm{P}(R) + \mathrm{P}(Q) - \mathrm{P}(R\cap Q)$ OR $\mathrm{P}(R) = \mathrm{P}(R\cap Q') + \mathrm{P}(R\cap Q)$ $= 0.15 + \text{their (c)}$ $0.5 = \mathrm{P}(R) + 0.35 - 0.035$ $= 0.15 + 0.035$ $\mathrm{P}(R) = 0.185$ $= 0.185$ 0.185	M1 A1	(2)

Jake and Kamil are sometimes late for school.
The events J and K are defined as follows

J = the event that Jake is late for school
K = the event that Kamil is late for school

$\mathrm{P}(J) = 0.25$, $\mathrm{P}(J \cap K) = 0.15$ and $\mathrm{P}(J' \cap K') = 0.7$

On a randomly selected day, find the probability that

(a) at least one of Jake or Kamil are late for school,

(1)

(b) Kamil is late for school.

(2)

Given that Jake is late for school,

(c) find the probability that Kamil is late.

(3)

The teacher suspects that Jake being late for school and Kamil being late for school are linked in some way.

(d) Determine whether or not J and K are statistically independent.

(2)

(e) Comment on the teacher's suspicion in the light of your calculation in (d).

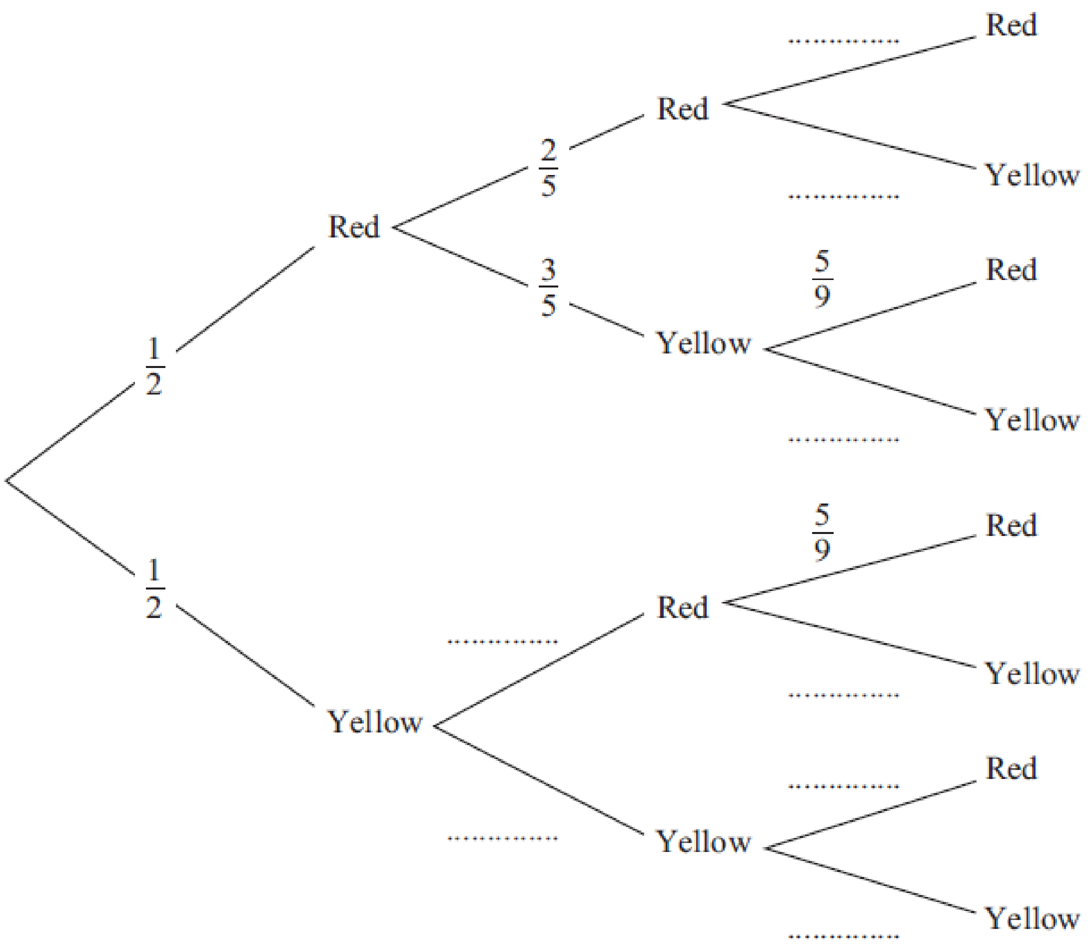
6. **(a)**	$P(J \cup K) = 1 - 0.7$ or $0.1 + 0.15 + 0.05 =$ 0.3	B1 (1)
(b)	$P(K) = 0.05 + 0.15$ or "0.3" $- 0.25 + 0.15$ or "0.3" $= 0.25 + P(K) - 0.15$	M1
	May be seen on Venn diagram $= 0.2$	A1 (2)
(c)	$[P(K \mid J)] = \frac{P(K \cap J)}{P(J)}$	M1
	$= \frac{0.15}{0.25}$	A1
	$= \frac{3}{5}$ or 0.6	A1 (3)
(d)	$P(J) \times P(K) = 0.25 \times 0.2 (= 0.05)$, $P(J \cap K) = 0.15$ or $P(K \mid J) = 0.6$, $P(K) = 0.2$ or may see $P(J\|K) = 0.75$ and $P(J) = 0.25$	M1
	not equal therefore not independent	A1ft (2)
(e)	Not independent so confirms the teacher's suspicion or they are linked (This requires a statement about independence in (d) or in (e))	B1ft (1)
		(9 marks)

The bag P contains 6 balls of which 3 are red and 3 are yellow.
The bag Q contains 7 balls of which 4 are red and 3 are yellow.
A ball is drawn at random from bag P and placed in bag Q. A second ball is drawn at random from bag P and placed in bag Q.
A third ball is then drawn at random from the 9 balls in bag Q.

The event A occurs when the 2 balls drawn from bag P are of the same colour.
The event B occurs when the ball drawn from bag Q is red.

(a) Complete the tree diagram shown below.

(4)



(b) Find P(A)

(3)

(c) Show that $P(B) = \frac{5}{9}$

(3)

(d) Show that $P(A \cap B) = \frac{2}{9}$

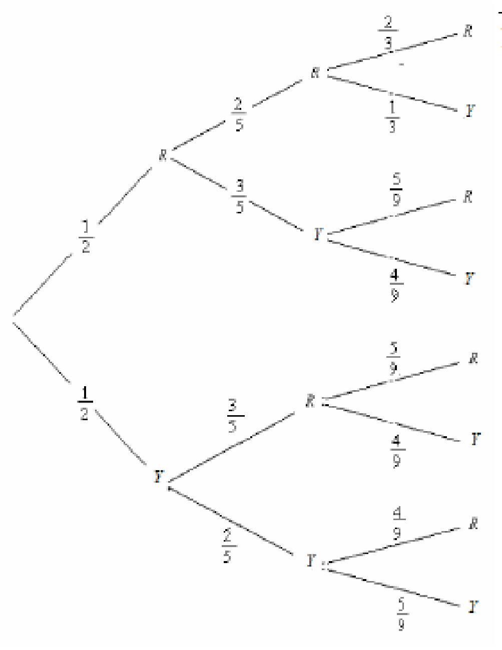
(2)

(e) Hence find $P(A \cup B)$

(2)

(f) Given that all three balls drawn are the same colour, find the probability that they are all red.

(3)

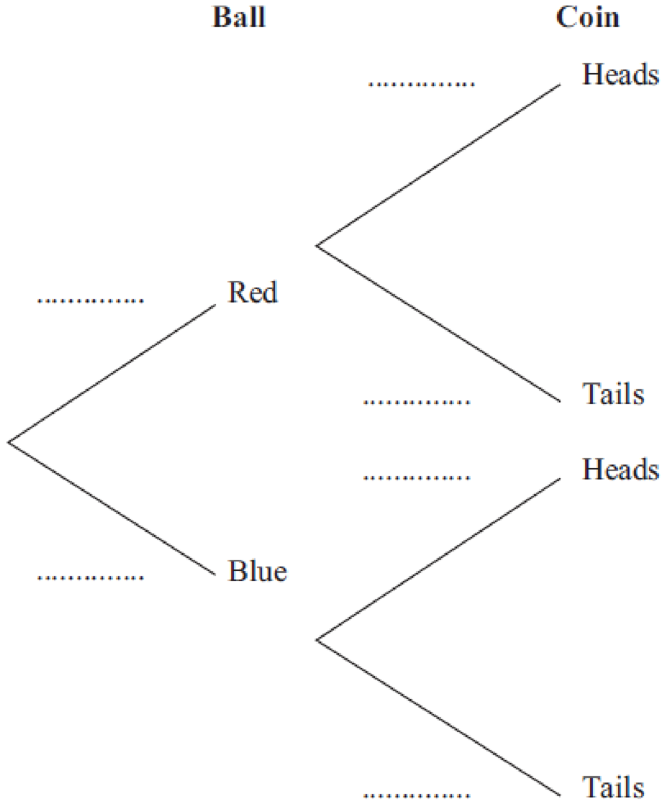
7. **(a)**	 $\frac{2}{15}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{4}{45}$ $\left(\frac{1}{9}\right)$	both $\frac{2}{3},\frac{1}{3}$ $\frac{4}{9}$ both $\frac{3}{5},\frac{2}{5}$ all three of $\frac{4}{9},\frac{4}{9},\frac{5}{9}$	B1 B1 B1 B1 (4)
(b)	$\mathrm{P}(A)=\mathrm{P}(RR)+\mathrm{P}(YY)=\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times"\frac{2}{5}"=\frac{2}{5}$	B1 for $\frac{1}{2}\times\frac{2}{5}$ (oe) seen at least once	B1 M1 A1 (3)
(c)	P(B) = P(RRR) + P(RYR) + P(YRR) + P(YYR)	M1 for at least 1 case of 3 balls identified. (Implied by 2nd M1)	M1
	$\left(\frac{1}{2}\times\frac{2}{5}\times"\frac{2}{3}"\right)+\left(\frac{1}{2}\times\frac{3}{5}\times\frac{5}{9}\right)+\left(\frac{1}{2}\times"\frac{3}{5}"\times\frac{5}{9}\right)+\left(\frac{1}{2}\times"\frac{2}{5}"\times"\frac{4}{9}"\right)$ $=\underline{\frac{5}{9}}$ (*)		M1,A1cso (3)
(d)	$\mathrm{P}(A\cap B)=\mathrm{P}(RRR)+\mathrm{P}(YYR)$	M1 for identifying both cases and + probs. may be implied by correct expressions	M1
	$=\left(\frac{1}{2}\times\frac{2}{5}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{2}{5}\times\frac{4}{9}\right)$ $=\underline{\frac{2}{9}}$ (*)		A1cso (2)
(e)	$\mathrm{P}(A\cup B)=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-\mathrm{P}(A\cap B)$ $="\frac{2}{5}"+\frac{5}{9}-\frac{2}{9}=\underline{\frac{11}{15}}$	Must have some attempt to use	M1 A1cao (2)
(f)	$\frac{\mathrm{P}(RRR)}{\mathrm{P}(RRR)+\mathrm{P}(YYY)}=\frac{\frac{1}{2}\times\frac{2}{5}\times\frac{2}{3}}{\left(\frac{1}{2}\times\frac{2}{5}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{2}{5}\times\frac{5}{9}\right)}=\frac{6}{11}$	Probabilities must come from the product of 3 probs. from their tree diagram.	M1 A1ft A1 cao (3) [17]

An experiment consists of selecting a ball from a bag and spinning a coin. The bag contains 5 red balls and 7 blue balls. A ball is selected at random from the bag, its colour is noted and then the ball is returned to the bag.

When a red ball is selected, a biased coin with probability $\frac{2}{3}$ of landing heads is spun.

When a blue ball is selected a fair coin is spun.

(a) Complete the tree diagram below to show the possible outcomes and associated probabilities.



(2)

Shivani selects a ball and spins the appropriate coin.

(b) Find the probability that she obtains a head.

(2)

Given that Tom selected a ball at random and obtained a head when he spun the appropriate coin,

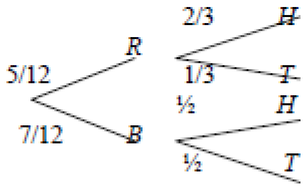
(c) find the probability that Tom selected a red ball.

(3)

Shivani and Tom each repeat this experiment.

(d) Find the probability that the colour of the ball Shivani selects is the same as the colour of the ball Tom selects.

(3)

Q2	(a)		P(R) and P(B)	B1
			2nd set of probabilities	B1
				(2)
	(b)	P(H) = $\frac{5}{12}\times\frac{2}{3}+\frac{7}{12}\times\frac{1}{2}, = \frac{41}{72}$ or awrt 0.569		M1 A1
				(2)
	(c)	P(R|H) = $\frac{\frac{5}{12}\times\frac{2}{3}}{"\frac{41}{72}"}, = \frac{20}{41}$ or awrt 0.488		M1 A1ft A1
				(3)
	(d)	$\left(\frac{5}{12}\right)^2+\left(\frac{7}{12}\right)^2$		M1 A1ft
		$=\frac{25}{144}+\frac{49}{144}=\frac{74}{144}$ or $\frac{37}{72}$ or awrt 0.514		A1
				(3)
				Total 10

The Venn diagram in Figure 1 shows the number of students in a class who read any of 3 popular magazines A, B and C.

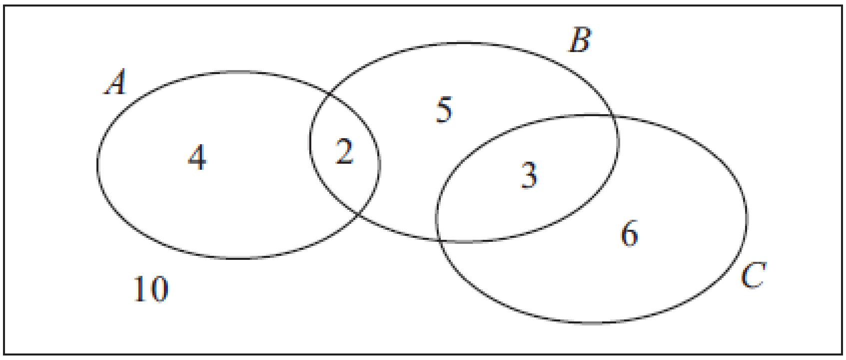


Figure 1

One of these students is selected at random.

(a) Show that the probability that the student reads more than one magazine is $\frac{1}{6}$.
(2)

(b) Find the probability that the student reads A or B (or both).
(2)

(c) Write down the probability that the student reads both A and C.
(1)

Given that the student reads at least one of the magazines,

(d) find the probability that the student reads C.
(2)

(e) Determine whether or not reading magazine B and reading magazine C are statistically independent.
(3)

Q4	(a)	$\frac{2+3}{\text{their total}} = \frac{5}{\text{their total}} = \frac{1}{6}$ (** **given answer****)	M1 A1cso (2)
	(b)	$\frac{4+2+5+3}{\text{total}}$, $= \frac{14}{30}$ or $\frac{7}{15}$ or $0.4\dot{6}$	M1 A1 (2)
	(c)	$\mathrm{P}(A \cap C) = 0$	B1 (1)
	(d)	$\mathrm{P}(\mathrm{C} \mid \text{reads at least one magazine}) = \frac{6+3}{20} = \frac{9}{20}$	M1 A1 (2)
	(e)	$\mathrm{P}(B) = \frac{10}{30} = \frac{1}{3}$, $\mathrm{P}(C) = \frac{9}{30} = \frac{3}{10}$, $\mathrm{P}(B \cap C) = \frac{3}{30} = \frac{1}{10}$ or $\mathrm{P}(B \mid C) = \frac{3}{9}$	M1
		$\mathrm{P}(B) \times \mathrm{P}(C) = \frac{1}{3} \times \frac{3}{10} = \frac{1}{10} = \mathrm{P}(B \cap C)$ or $\mathrm{P}(B \mid C) = \frac{3}{9} = \frac{1}{3} = \mathrm{P}(B)$	M1
		So yes they are statistically independent	A1cso (3)
			Total 10

There are 180 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.

112 take systems support,
70 take developing software,
81 take networking,
35 take developing software and systems support,
28 take networking and developing software,
40 take systems support and networking,
4 take all three extra options.

(a) In the space below, draw a Venn diagram to represent this information.

(5)

A student from the course is chosen at random.

Find the probability that this student takes

(b) none of the three extra options,

(1)

(c) networking only.

(1)

Students who want to become technicians take systems support and networking. Given that a randomly chosen student wants to become a technician,

(d) find the probability that this student takes all three extra options.

(2)

On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that on a randomly chosen day

(i) Bill travels by foot and is late,

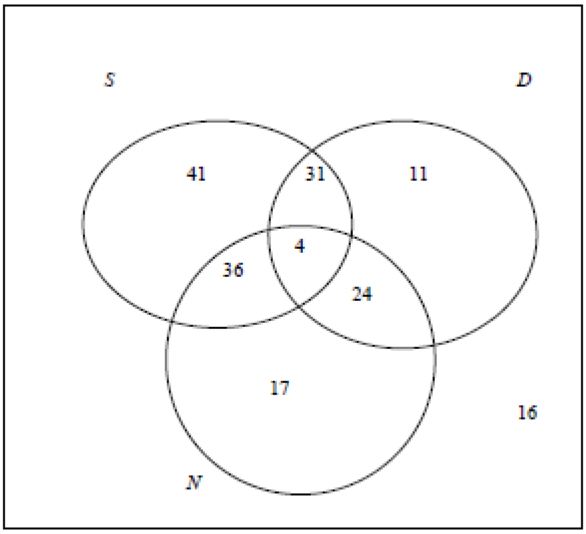
(ii) Bill is not late.

(4)

(c) Given that Bill is late, find the probability that he did not travel on foot.

(4)

4 (a)



3 closed curves and 4 in centre M1
Evidence of subtraction M1

31,36,24 A1
41,17,11 A1
Labels on loops, 16 and box B1

(5)

(b) P(None of the 3 options)=$\frac{16}{180}=\frac{4}{45}$ B1ft

(1)

(c) P(Networking only)=$\frac{17}{180}$ B1ft

(1)

(d) P(All 3 options/technician)=$\frac{4}{40}=\frac{1}{10}$ M1 A1

(2)

Total [9]

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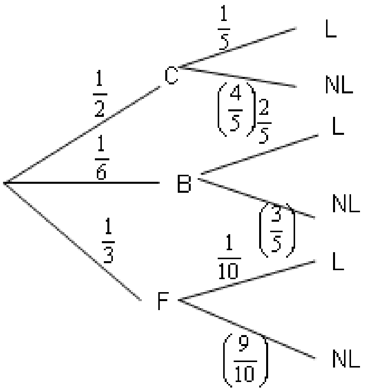
2. (a)



Correct tree B1
All labels B1
Probabilities on correct branches B1 (3)

(b)(i) $\frac{1}{3}\times\frac{1}{10}=\frac{1}{30}$ or equivalent M1 A1 (2)

(ii) CNL + BNL + FNL $=\frac{1}{2}\times\frac{4}{5}+\frac{1}{6}\times\frac{3}{5}+\frac{1}{3}\times\frac{9}{10}$ M1

$=\frac{4}{5}$ or equivalent A1 (2)

(c) $\mathrm{P}(F'/L)=\frac{\mathrm{P}(F'\cap L)}{\mathrm{P}(L)}$ Attempt correct conditional probability M1

$=\frac{\frac{1}{6}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{5}}{1-(ii)}$ $\frac{\text{numerator}}{\text{denominator}}$ $\frac{\text{A1}}{\text{A1ft}}$

$=\frac{\frac{5}{30}}{\frac{1}{5}}=\frac{5}{6}$ or equivalent cao A1 (4)

(a) The events L and M are such that

$$\mathrm{P}(L) = 0.55 \quad \text{and} \quad \mathrm{P}(M) = 0.28 \ .$$

Write down the value of:

(i) $\mathrm{P}(L \cap M)$ if L and M are independent;

(ii) $\mathrm{P}(L \cup M)$ if L and M are mutually exclusive;

(iii) $\mathrm{P}(L \cup M)$ if L and M are independent.

[3 marks]

(b) Rhonda, Samantha and Tracy are members of a club which meets every Wednesday.

At any Wednesday meeting, Rhonda's attendance, event R, has probability 0.94, Samantha's attendance, event S, has probability 0.88, and Tracy's attendance, event T, has probability 0.76. The events R, S and T are independent.

For these three members, calculate the probability that, on a particular Wednesday:

(i) all of them attend the meeting;

[1 mark]

(ii) exactly one of them attends the meeting;

[2 marks]

(iii) at least two of them attend the meeting.

[2 marks]

(iv) Ursula, a neighbour of Tracy, is also a member of the club. At any Wednesday meeting, Ursula's attendance, event U, is independent of events R and S but $\mathrm{P}(U|T) = 0.96$ and $\mathrm{P}(U|T') = 0.48$.

For Rhonda, Samantha, Tracy and Ursula, calculate the probability that, on a particular Wednesday:

(A) all of them attend the meeting;

(B) none of them attend the meeting.

[3 marks]

(a) **(i)**	$P(L \cap M) = 0.55 \times 0.28 = \underline{0.154}$	B1		CAO; accept 154/1000 or 77/500
(ii)	$P(L \cup M) = 0.55 + 0.28 = \underline{0.83}$	B1		CAO; accept 83/100
(iii)	$P(L \cup M) = 0.83 - 0.154 = \underline{0.676}$	B1	**3**	CAO; accept 676/1000 or 338/500 or 169/250
Note	1 For fractional answers, do **not** penalise errors in simplifications; eg 154/1000 = 67/500 ⇒ B1 (for 154/1000)			
(b) **(i)**	$P(A = 3) = 0.94 \times 0.88 \times 0.76$ $= \underline{0.628 \text{ to } 0.629}$	B1	**1**	AWFW (0.628672)
(ii)	$P(A = 1) = (0.94 \times 0.12 \times 0.24) +$ $(0.06 \times 0.88 \times 0.24) +$ $(0.06 \times 0.12 \times 0.76)$	M1		Fully correct; not $(1 - 0.88)$, etc
	or $= 0.027072 + 0.012672 + 0.005472$			Fully correct to 4dp
	$= \underline{0.045 \text{ to } 0.0455}$	A1	**2**	AWFW (0.045216)
(iii)	$P(A \geq 2) = (0.94 \times 0.88 \times 0.76)$ **or** (b)(i) + $(0.94 \times 0.88 \times 0.24) +$ $(0.94 \times 0.12 \times 0.76) +$ $(0.06 \times 0.88 \times 0.76)$	M1		Fully correct (c's (b)(i)); not $(1 - 0.76)$, etc
	or $= 0.628672$ or (b)(i) + $0.198528 + 0.085728 + 0.040128$			Fully correct to 4dp (c's (b)(i))
	$= \underline{0.953 \text{ to } 0.9535}$	A1		AWFW (0.953056)
	or $P(A \geq 2) =$ $1 -$ [0.045216 **or** (b)(ii)] $- (0.06 \times 0.12 \times 0.24)$	(M1)		$1 -$ (b)(ii) $- 0.001728$
	$= \underline{0.953 \text{ to } 0.9535}$	(A1)	**2**	AWFW (0.953056)
(iv) **(A)**	$P(A = 4) = \underline{0.603 \text{ to } 0.604}$	B1	**(1)**	AWFW (0.60352512)
(B)	$P(A = 0) = (0.06 \times 0.12 \times 0.24) \times 0.52$	M1		Fully correct
	$= \underline{0.000898 \text{ to } 0.000899}$	A1	**(2)**	AWFW; (see GN7) (0.00089856)

The table shows, for a random sample of 500 patients attending a dental surgery, the patients' ages, in years, and the NHS charge bands for the patients' courses of treatment. Band 0 denotes the least expensive charge band and band 3 denotes the most expensive charge band.

		Charge band for course of treatment				
		Band 0	**Band 1**	**Band 2**	**Band 3**	**Total**
Age of patient (years)	**Under 19**	32	43	5	0	80
	Between 19 and 40	17	62	22	3	104
	Between 41 and 65	28	82	35	31	176
	66 or over	13	53	68	6	140
	Total	90	240	130	40	500

(a) Calculate, **to three decimal places**, the probability that a patient, selected at random from these 500 patients, was:

(i) aged between 41 and 65;

(ii) aged 66 or over and charged at band 2;

(iii) aged between 19 and 40 and charged **at most** at band 1;

(iv) aged 41 or over, given that the patient was charged at band 2;

(v) charged **at least** at band 2, given that the patient was **not** aged 66 or over.

[9 marks]

(b) Four patients at this dental surgery, **not** included in the above 500 patients, are selected at random.

Estimate, **to three significant figures**, the probability that two of these four patients are aged between 41 and 65 and are **not** charged at band 0, and the other two patients are aged 66 or over and are charged at either band 1 or band 2.

[5 marks]

3(a)	**Accept the equivalent percentage answers with %-sign in parts (a)(i) to (a)(iv) but not in parts (a)(v) and (b) (see GN5)**			
(i)	$P(A_{41\text{-}65}) =$ $\underline{176/500 = 88/250 = 44/125 = 0.352}$	B1	**(1)**	CAO; any one of four listed answers
(ii)	$P(A_{\ge 66} \cap B_2) =$ $\underline{68/500 = 34/250 = 17/125 = 0.136}$	B1	**(1)**	CAO; any one of four listed answers
(iii)	$P(A_{19\text{-}40} \cap B_{\le 1}) = \frac{17+62}{500} = \frac{79}{500}$ $= \underline{0.158}$	M1 A1	**(2)**	Numerator CAO CAO
(iv)	$P(A_{\ge 41} \mid B_2) =$ $\frac{(35+68)/500}{130/500}$ or $\frac{(130-5-22)/500}{130/500}$ or $\frac{103}{130}$ $= \underline{0.792}$	M1 A1	**(2)**	Fraction CAO AWRT (0.79231)
(v)	$P(B_{\ge 2} \mid A_{\le 65}) =$ $\frac{5+(0)+22+3+35+31}{80+104+176}$ or $\frac{96}{360}$ $\frac{48}{180}$ or $\frac{24}{90}$ or $\frac{12}{45}$ or $\frac{4}{15}$ $= \underline{0.267}$	M1 M1 (M2) A1	**(3)**	Numerator CAO (130 – 68 + 40 – 6) Denominator CAO (500 – 140) (Accept numerator and denominator each ÷ 500) **CAO (3 dp only)** (0.26667)
(b)	$P(A_{41\text{-}65} \cap B_{>0}) =$ $\frac{82+35+31}{500}$ or $\frac{176-28}{500}$ or $\frac{148}{500}$ (p_1)	B1		CAO; OE $\left(\frac{74}{250}, \frac{37}{125}, 0.296\right)$ Seen anywhere, even in an incorrect expression
	$P(A_{\ge 66} \cap B_{1 \text{ or } 2}) =$ $\frac{53+68}{500}$ or $\frac{140-13-6}{500}$ or $\frac{121}{500}$ (p_2)	B1		CAO; OE (0.242) Seen anywhere, even in an incorrect expression
	Prob $= (p_1)^2 \times (p_2)^2$ or $(p_1 \times p_2)^2$	M1		Providing $0 < p_1,\ p_2 < 1$ Must be equivalent to product of two squared probabilities with no extra terms
	$\times \binom{4}{2}$ or 6	m1		
	$= \underline{0.0308}$	A1	5	**CAO (3 sf only)** (0.03078686)

A ferry sails once each day from port D to port A. The ferry departs from D on time or late but never early. However, the ferry can arrive at A early, on time or late.

The **probabilities** for some combined events of departing from D and arriving at A are shown in the table below.

(a) Complete the table.

[2 marks]

(b) Write down the probability that, on a particular day, the ferry:

(i) both departs and arrives on time;

(ii) departs late.

[2 marks]

(c) Find the probability that, on a particular day, the ferry:

(i) arrives late, given that it departed late;

(ii) does **not** arrive late, given that it departed on time.

[5 marks]

(d) On three particular days, the ferry departs from port D on time.

Find the probability that, on these three days, the ferry arrives at port A early once, on time once and late once. Give your answer to three decimal places.

[4 marks]

		Arrive at A			
		Early	**On time**	**Late**	**Total**
Depart from D	**On time**	0.16	0.56	0.08	
	Late				
	Total	0.22	0.65		1.00

<table>
<tr><td>3</td><td></td><td></td><td></td><td>In (b) & (c), accept any equivalent fractional answer with den ≤ 100 or the equivalent percentage answer with %- sign (see GN4)</td></tr>
<tr><td>(a)</td><td>
<table>
<tr><td></td><td></td><td colspan="3">Arrive</td><td></td></tr>
<tr><td></td><td></td><td>E</td><td>OT</td><td>L</td><td>Total</td></tr>
<tr><td rowspan="2">Dep</td><td>OT</td><td>0.16</td><td>0.56</td><td>0.08</td><td>0.8(0)</td></tr>
<tr><td>L</td><td>0.06</td><td>0.09</td><td>0.05</td><td>0.2(0)</td></tr>
<tr><td></td><td>Total</td><td>0.22</td><td>0.65</td><td>0.13</td><td>1.00</td></tr>
</table>
</td><td>B2
(B1)</td><td>2</td><td>All 6 correct CAO
Any 3 of 6 correct CAO</td></tr>
<tr><td>(b)
(i)</td><td>$P(OT_D \cap OT_A) = \underline{0.56}$</td><td>B1</td><td>(1)</td><td>CAO/OE; even 0.56/1</td></tr>
<tr><td>(ii)</td><td>$P(L_D) = \underline{0.2}$</td><td>B1</td><td>(1)</td><td>CAO/OE; even 0.2/1</td></tr>
<tr><td></td><td></td><td></td><td>2</td><td></td></tr>
<tr><td>(c)(i)</td><td>$P(L_A \mid L_D) = \frac{0.05}{0.2} =$
$\underline{0.25}$</td><td>M1
A1</td><td>(2)</td><td>(c's 0.05)/(c's (b)(ii))
Can be implied by a correct answer
CAO/OE; not 0.25/1</td></tr>
<tr><td>(ii)</td><td>$P(L'_A \mid OT_D) =$
$\frac{0.16+0.56}{0.8}$ or $\frac{0.8-0.08}{1-0.2}$ or $\frac{0.72}{0.8}$
$\frac{0 < p < 0.8}{0.8}$
$= \underline{0.9}$</td><td>B2
(B1)
B1</td><td>(3)</td><td>Can be implied by a correct answer
CAO/OE; not 0.9/1</td></tr>
<tr><td>(d)</td><td>$P(E_A \cap OT_A \cap L_A \mid OT_D) =$
$\frac{0.16}{0.8} \times \frac{0.56}{0.8} \times \frac{0.08}{0.8}$ or $0.2 \times 0.7 \times 0.1$
$\times$ (3! or 6)
$= \underline{0.084}$</td><td>M2
(M1)
m1
A1</td><td>4</td><td>All three correct (equivalent) fractions or decimals multiplied
At least one correct (equivalent) fraction or decimal
Dependent on M2
CAO</td></tr>
</table>

3 The table shows the colour of hair and the colour of eyes of a sample of 750 people from a particular population.

		Colour of hair					
		Black	**Dark**	**Medium**	**Fair**	**Auburn**	**Total**
Colour of eyes	**Blue**	6	51	68	66	24	215
	Brown	14	92	97	90	47	340
	Green	0	37	55	64	39	195
	Total	20	180	220	220	110	750

(a) Calculate, **to three decimal places**, the probability that a person, selected at random from this sample, has:

(i) fair hair;

(ii) auburn hair and blue eyes;

(iii) either auburn hair or blue eyes but not both;

(iv) green eyes, given that the person has fair hair;

(v) fair hair, given that the person has green eyes.

[8 marks]

(b) Three people are selected at random from the sample.

Calculate, **to three significant figures**, the probability that two of them have dark hair and brown eyes and the other has medium hair and green eyes.

[4 marks]

(a)(i)	P(FH) = **220/750 = 22/75 = 0.293**	B1	(1)	CAO/AWRT (0.29333)
(ii)	$P(AH \cap BE) =$ **24/750 = 8/250 = 4/125 = 0.032**	B1	(1)	CAO
(iii)	$P(AH \cup BE \text{ but not both}) = \frac{110+215-2\times 24}{750}$ = **277/750 = 0.369**	M1 A1	(2)	OE Can be implied by correct answer CAO/AWRT (0.36933)
SC	Award B1 for 301/750 **or** 0.401(33)			
(iv)	$P(GE \mid FH) = \frac{64}{750} \Big/ \frac{220}{750} =$ **64/220 = 32/110 = 16/55 = 0.291**	M1 A1	(2)	OE Can be implied by correct answer CAO/AWRT (0.29091)
(v)	$P(FH \mid GE) = \frac{64}{750} \Big/ \frac{195}{750} =$ **64/195 = 0.328**	M1 A1	(2)	OE Can be implied by correct answer CAO/AWRT (0.32821)
(b)	$P((DH \cap BE) \cap (DH \cap BE) \cap (MH \cap GE)) =$ $\frac{92}{750}\times\frac{91}{749}\times\frac{55}{748}$ Multiplied by 3 or $\binom{92}{2}\binom{55}{1}\div\binom{750}{3}$ = **0.00328 to 0.00329**	M1 M1 m1 (M1 M1) (M1) A1	4	Correct 3 values multiplied in numerator Correct 3 values multiplied in denominator 0.123 × 0.121 × 0.074 (all AWRT) ⇒ M1 M1 (OE products) Dependent on at least one M1 scored Numerator Denominator AWFW (0.00328752)

Alison is a member of a tenpin bowling club which meets at a bowling alley on Wednesday and Thursday evenings.

The probability that she bowls on a Wednesday evening is 0.90. Independently, the probability that she bowls on a Thursday evening is 0.95.

(a) Calculate the probability that, during a particular week, Alison bowls on:

(i) two evenings;

(ii) exactly one evening. *(3 marks)*

(b) David, a friend of Alison, is a member of the same club.

The probability that he bowls on a Wednesday evening, given that Alison bowls on that evening, is 0.80. The probability that he bowls on a Wednesday evening, given that Alison does not bowl on that evening, is 0.15.

The probability that he bowls on a Thursday evening, given that Alison bowls on that evening, is 1. The probability that he bowls on a Thursday evening, given that Alison does not bowl on that evening, is 0.

Calculate the probability that, during a particular week:

(i) Alison and David bowl on a Wednesday evening; *(2 marks)*

(ii) Alison and David bowl on both evenings; *(2 marks)*

(iii) Alison, but not David, bowls on a Thursday evening; *(1 mark)*

(iv) neither bowls on either evening. *(3 marks)*

5(a)(i)	$P(A = 2) = 0.90 \times 0.95 =$ **0.85 to 0.86**	B1		AWFW (0.855 or 171/200 OE)
(ii)	$P(A = 1) = (0.90 \times 0.05) + (0.10 \times 0.95)$ **or** $= 1 - [0.855 + (0.10 \times 0.05)]$	M1		May be implied by a correct answer Do not ignore extra terms
	$= $ **0.14**	A1	3	CAO (7/50 OE)
(b)(i)	$P(A_W \cap D_W) = 0.90 \times 0.80$	M1		May be implied by a correct answer
	$= $ **0.72**	A1	2	CAO (18/25 OE)
(ii)	$P(A_B \cap D_B) = \text{(b)(i)} \times 0.95\ (\times 1)$ **or** $= 0.90 \times 0.80 \times 0.95\ (\times 1)$ **or** $= \text{(a)(i)} \times 0.80$	M1		May be implied by a correct answer
	0.68 to 0.685	A1	2	AWFW (0.684 or 171/250 OE)
(iii)	$P(A_T \cap D'_T) = 0.95 \times 0 =$ **0**	B1	1	CAO; award on value only
(iv)	$P(\text{neither}) = P([A'_W \cap D'_W] \cap [A'_T \cap D'_T])$			
	$(1 - 0.90) \times (1 - 0.15)$	M1		Accept 0.085 or 17/200 OE Award M1 and m1 on value(s) only
	$(1 - 0.95) \times (1 - 0)$	m1		Accept 0.05 or 1/20 OE
	or P(neither) = $P(A'_W \cap A'_T) \cap P(D'_W \mid A'_W) \cap P(D'_T \mid A'_T)$			
	$(1 - 0.90) \times (1 - 0.95)$	(M1)		Accept 0.005 or 1/200 OE Award M1 and m1 on value(s) only
	$(1 - 0.15) \times (1 - 0)$	(m1)		Accept 0.85 or 17/20 OE
	$= 0.085 \times 0.05$ **or** 0.005×0.85			OE
	= 0.0042 to 0.0043	A1	3	AWFW (0.00425 or 17/4000 OE)

Roger is an active retired lecturer. Each day after breakfast, he decides whether the weather for that day is going to be fine (F), dull (D) or wet (W). He then decides on only one of four activities for the day: cycling (C), gardening (G), shopping (S) or relaxing (R). His decisions from day to day may be assumed to be independent.

The table shows Roger's probabilities for each combination of weather and activity.

		Weather		
		Fine (F)	**Dull (D)**	**Wet (W)**
Activity	**Cycling (C)**	0.30	0.10	0
	Gardening (G)	0.25	0.05	0
	Shopping (S)	0	0.10	0.05
	Relaxing (R)	0	0.05	0.10

(a) Find the probability that, on a particular day, Roger decided:

(i) that it was going to be fine and that he would go cycling;

(ii) on either gardening or shopping;

(iii) to go cycling, given that he had decided that it was going to be fine;

(iv) **not** to relax, given that he had decided that it was going to be dull;

(v) that it was going to be fine, given that he did **not** go cycling. *(9 marks)*

(b) Calculate the probability that, on a particular Saturday and Sunday, Roger decided that it was going to be fine and decided on the same activity for both days.

(3 marks)

5 **(a)(i)**	P(F & C) = **0.3 or 3/10 or 30%**	B1	**(1)**	**Ratios (eg 3:10) are only penalised by 1 accuracy mark at first correct answer** CAO (0.3)
(ii)	P(G or S) = **0.45 or 45/100 or 45%**	B1	**(1)**	CAO (0.45)
(iii)	$P(C \mid F) = \dfrac{0.3 \text{ or } (\text{i})}{0.55} =$	M1		
	30/55 or 6/11 **or** **(0.54 to 0.55) or (54% to 55%)**	A1	**(2)**	CAO (6/11) AWFW (0.54545)
(iv)	$P(R' \mid D) = \dfrac{0.25 \text{ or } (0.30 - 0.05)}{0.30}$	M1 M1		Correct numerator Correct denominator
	25/30 or 5/6 **or** **(0.83 to 0.834) or (83% to 83.4%)**	A1	**(3)**	CAO (5/6) AWFW (0.83333)
(v)	$P(F \mid C') = \dfrac{0.25 \text{ or } (0.60 - 0.35)}{0.60}$	M1		Correct expression
	25/60 or 5/12 **or** **(0.416 to 0.42) or (41.6% to 42%)**	A1	**(2, 3)**	CAO (5/12) AWRT (0.41667)
(b)	$P = [P(F \,\&\, C)]^2 + [P(F \,\&\, G)]^2$	M1		Attempt at **sum of at least 2 squared terms; $0 < \text{term} < 1$;** not $(a+b)^2$ May be implied by a correct expression or a correct answer
	$0.30^2 + 0.25^2$ or $0.09 + 0.0625 =$	A1		OE Ignore additional terms or integer multipliers May be implied by a correct answer
	1525/10000 or 305/2000 or 61/400 **or** **(0.152 to 0.153) or (15.2% to 15.3%)**	A1	**3**	CAO (0.1525) AWFW

A survey of the 640 properties on an estate was undertaken. Part of the information collected related to the number of bedrooms and the number of toilets in each property.

This information is shown in the table.

		Number of toilets				
		1	**2**	**3**	**4 or more**	**Total**
Number of bedrooms	**1**	46	14	0	0	60
	2	24	67	23	0	114
	3	7	72	99	16	194
	4	0	19	123	48	190
	5 or more	0	0	11	71	82
	Total	77	172	256	135	640

(a) A property on the estate is selected at random.
Find, giving your answer to three decimal places, the probability that the property has:

(i) exactly 3 bedrooms; *(1 mark)*

(ii) at least 2 toilets; *(2 marks)*

(iii) exactly 3 bedrooms and at least 2 toilets; *(2 marks)*

(iv) at most 3 bedrooms, given that it has exactly 2 toilets. *(3 marks)*

(b) Use relevant answers from part **(a)** to show that the number of toilets is **not** independent of the number of bedrooms. *(2 marks)*

(c) Three properties are selected at random from those on the estate which have exactly 3 bedrooms.

Calculate the probability that one property has 2 toilets, one has 3 toilets and the other has at least 4 toilets. Give your answer to three decimal places. *(4 marks)*

4				**Ratios (eg 194:640) are only penalised by 1 accuracy mark at first correct answer**
(a)(i)	$P(B = 3)$ = 194/640 or 97/320 or 0.303 or 30.3%	B1	1	CAO or AWRT (0.303125)
(ii)	$P(T \geq 2) = \frac{172+256+135}{640}$ or $1-\frac{77}{640}$ or $\frac{563}{640}$	M1		
	= 563/640			CAO
		A1	2	
	or (0.879 to 0.88) or (87.9% to 88%)			AWFW (0.879688)
(iii)	$P(B = 3\ \&\ T \geq 2)$ = $\frac{72+99+16}{640}$ or $\frac{194-7}{640}$ or $\frac{187}{640}$	M1		
	= 187/640 or 0.292 or 29.2%	A1	2	CAO or AWRT (0.292188)
(iv)	$P(B \leq 3 \mid T = 2)$ = $\frac{(14+67+72)}{172}$ or $\frac{172-19}{172}$ or $\frac{153}{172}$	M1 M1		Correct numerator (accept both ÷ 640) Correct denominator
	= 153/172			CAO
		A1	3	
	or (0.888 to 0.89) or (88.8% to 89%)			AWFW (0.889535)
				Answers as fractions, percentages or ratios lose accuracy (A & B) marks in (b) & (c)
(b)	(a)(i) × (a)(ii) ≠ (a)(iii)	M1		Attempted
	since 0.303×0.88 = 0.265 to 0.27 ≠ 0.292	A1	2	AWFW & AWRT
(c)	$P(2T \cap 3T \cap \geq 4T \mid B = 3) = \frac{72}{194} \times \frac{99}{193} \times \frac{16}{192}$	M1 M1		Correct 3 values multiplied in numerator Correct 3 values multiplied in denominator 0.371 × 0.513 × 0.083 (all AWRT) ⇒ M1 M1 (OE products)
	abc multiplied by 6 or 3	M1		$0 < (a, b\ \&\ c) < 1$
	= 0.095 to 0.0952	A1	4	AWFW (0.095187)

Twins Alec and Eric are members of the same local cricket club and play for the club's under 18 team.

The probability that Alec is selected to play in any particular game is 0.85.
The probability that Eric is selected to play in any particular game is 0.60.
The probability that both Alec and Eric are selected to play in any particular game is 0.55.

(a) By using a table, or otherwise:

(i) show that the probability that neither twin is selected for a particular game is 0.10;

(ii) find the probability that at least one of the twins is selected for a particular game;

(iii) find the probability that exactly one of the twins is selected for a particular game.

(5 marks)

(b) The probability that the twins' younger brother, Cedric, is selected for a particular game is:

0.30 given that both of the twins have been selected;
0.75 given that exactly one of the twins has been selected;
0.40 given that neither of the twins has been selected.

Calculate the probability that, for a particular game:

(i) all three brothers are selected;

(ii) at least two of the three brothers are selected. *(6 marks)*

<table>
<tr><td>6</td><td>See supplementary sheet for alternative solutions to parts (a)(i) and (b)(ii)</td><td></td><td></td><td></td></tr>
<tr><td>(a)(i)</td><td>Table Method
(2- way with either R or C totals)
<table>
<tr><th></th><th>A</th><th>A'</th><th>Total</th></tr>
<tr><td>E</td><td>0.55</td><td>0.05</td><td>0.60</td></tr>
<tr><td>E'</td><td>0.30</td><td>0.10</td><td>0.40</td></tr>
<tr><td>Total</td><td>0.85</td><td>0.15</td><td>1.00</td></tr>
</table></td><td>B1
B1
Bdep1</td><td>3</td><td>0.15 or 0.4; CAO; allow fractions
0.05 and 0.3; CAO; allow fractions
0.1; AG so dependent on B1 B1</td></tr>
<tr><td>(ii)</td><td>P(≥1) = 0.9 or 9/10</td><td>B1</td><td>1</td><td>CAO</td></tr>
<tr><td>(iii)</td><td>P(1) = 0.3 + 0.05 = 1 − (0.55 + 0.10)
= 0.35 or 35/100 or 7/20</td><td>B1</td><td>1</td><td>CAO</td></tr>
<tr><td>(b)(i)</td><td>P(3) = 0.55 × 0.30
= 0.165 or 165/1000 or 33/200</td><td>B1
B1</td><td>2</td><td>OE; implied by correct answer
CAO</td></tr>
<tr><td>(ii)</td><td>0.55 × (1 − 0.3) or 0.385
(0.3 × 0.75) or 0.225
or (0.05 × 0.75) or 0.0375
or (0.35 × 0.75) or 0.2625
(0.385 + 0.2625) + 0.165
= 0.812 to 0.813
or $\frac{8125}{10000}$ or $\frac{1625}{2000}$ or $\frac{325}{400}$ or $\frac{65}{80}$ or $\frac{13}{16}$</td><td>M1
M1
B1
A1</td><td>4</td><td>At least one of these expressions or values
OE; implied by correct answer
AWFW (0.8125)
CAO</td></tr>
</table>