Pure Sector 3: Partial Fractions

Aims:

- To be able to simplify algebraic fractions.
- To be able to express algebraic fractions in Partial Fraction form.
- Utilise Partial Fractions to solve problems.

Simplifying Algebraic Fractions

Example 1

Simplify:

$$\frac{2x^2 - 8}{x^2 + 3x + 2}$$

Example 2

Simplify:

$$\frac{36 - 4x^2}{x^2 + x - 12}$$

Improper Fractions

$Ax^2 + Bx + C$ is the **quotient**

D is the **remainder**

Example 3

Write $\frac{2x^3-3x^2+x-3}{x-2}$ in the form $\boldsymbol{Ax^2 + Bx + C + \frac{D}{x-2}}$

Example 4

Write $\frac{x^3-3x^2+1}{x^2-x-1}$ in the form $\boldsymbol{Ax + B + \frac{Cx+D}{x^2-x-1}}$

Partial Fractions

Example 5

Write $\frac{3}{(2x+1)(x+2)}$ in the form $\frac{A}{(2x+1)} + \frac{B}{(x+2)}$

Example 6

Express $\frac{3x+1}{2x^2-4x}$ in partial fractions.

Example 7

Express $\frac{\boldsymbol{x}}{(\boldsymbol{2x+1})^{\boldsymbol{2}}(\boldsymbol{x-1})}$ in the form $\frac{\boldsymbol{A}}{(\boldsymbol{2x+1})}+\frac{\boldsymbol{B}}{(\boldsymbol{2x+1})^{\boldsymbol{2}}}+\frac{\boldsymbol{C}}{(\boldsymbol{x-1})}$, where $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are constants.

Example 8

Express in partial fractions

$$\frac{5x+3}{(2x+1)(x+1)^2}$$

General Forms:

$$\frac{px+q}{(ax+b)(cx+d)(ex+f)} \equiv \frac{A}{(ax+b)}+\frac{B}{(cx+d)}+\frac{C}{(ex+f)}$$

$$\frac{px+q}{(ax+b)(cx+d)^2} \equiv \frac{A}{(ax+b)}+\frac{B}{(cx+d)}+\frac{C}{(cx+d)^2}$$

Applications of Partial Fractions

Example 9

Show that $\int_1^2 \frac{2x+2}{2x+1}\,\mathrm{d}x = 1 + \frac{1}{2}\ln\frac{5}{3}$.

Example 10

Expand $\frac{5-x}{(1+x)(1-2x)}$ up to the term in x^3.

Example 11

Find the value of $\int_3^4 \frac{x+4}{(x+1)(x-2)}\,\mathrm{d}x$ leaving your answer in the form $p\ln 4 - q\ln 5$ where p and q are integers to be found.

Exam Questions

It is given that $\mathrm{f}(x) = \dfrac{19x-2}{(5-x)(1+6x)}$ can be expressed as $\dfrac{A}{5-x} + \dfrac{B}{1+6x}$, where A and B are integers.

(a) Find the values of A and B.

[3 marks]

(b) Hence show that $\displaystyle\int_0^4 \mathrm{f}(x)\,\mathrm{d}x = k\ln 5$, where k is a rational number.

[6 marks]

(a) Express $\dfrac{19x-3}{(1+2x)(3-4x)}$ in the form $\dfrac{A}{1+2x}+\dfrac{B}{3-4x}$.

[3 marks]

(b) (i) Find the binomial expansion of $\dfrac{19x-3}{(1+2x)(3-4x)}$ up to and including the term in x^2.

[7 marks]

(ii) State the range of values of x for which this expansion is valid.

[1 mark]

(a) Given that $\dfrac{4x^3-2x^2+16x-3}{2x^2-x+2}$ can be expressed as $Ax+\dfrac{B(4x-1)}{2x^2-x+2}$, find the values of the constants A and B.

[3 marks]

(b) The gradient of a curve is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4x^3-2x^2+16x-3}{2x^2-x+2}$$

The point $(-1,\,2)$ lies on the curve. Find the equation of the curve.

[4 marks]

5 (a) (i) Obtain the binomial expansion of $(1 - x)^{-1}$ up to and including the term in x^2.

(2 marks)

(ii) Hence, or otherwise, show that

$$\frac{1}{3 - 2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of x. *(3 marks)*

(b) Obtain the binomial expansion of $\dfrac{1}{(1 - x)^2}$ up to and including the term in x^2.

(2 marks)

(c) Given that $\dfrac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$ can be written in the form $\dfrac{A}{(3 - 2x)} + \dfrac{B}{(1 - x)} + \dfrac{C}{(1 - x)^2}$,

find the values of A, B and C. *(5 marks)*

(d) Hence find the binomial expansion of $\dfrac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$ up to and including the term in x^2. *(3 marks)*