6 (a) Express $\dfrac{5x^2 - 19x + 50}{(1+3x)(5-x)^2}$ in the form $\dfrac{P}{1+3x} + \dfrac{Q}{5-x} + \dfrac{R}{(5-x)^2}$

where P, Q and R are constants.

[5 marks]

(b) Hence find $\displaystyle\int \frac{5x^2 - 19x + 50}{(1+3x)(5-x)^2}\,\mathrm{d}x$.

[4 marks]

Q 6	Solution	Mark	Total	Comment
(a)	$5x^2 - 19x + 50 = P(5-x)^2 + Q(1+3x)(5-x) + R(1+3x)$	B1		Correctly eliminating denominators If not seen next 4 marks are still possible
	Any correct method that would find P, Q **or** R	M1		Correct substitution, equating coefficients etc. even if **B0** scored
	$P = 2$	A1	5	
	$Q = -1$	A1		
	$R = 5$	A1		

B1 mark can be earned if both sides still include the **correct** common denominator of $(1+3x)(5-x)^2$

The **M1** is scored for a complete method that could find any of the 3 constants.

e.g. letting $x = 5$ to find R (even if wrong) would earn the **M1** mark,

or solving enough simultaneous equations to find a constant would earn the **M1** mark.

After first **B1** mark, if **NMS** or cover up rule used, correct answers can score **B2**, **B1** and then **B1**.

(b)	$\int \frac{P}{1+3x} + \frac{Q}{5-x}\,dx = a\ln(1+3x) + b\ln(5-x)$	M1		Condone missing brackets.
	$= \frac{2}{3}\ln(1+3x)$	A1ft		ft on $\frac{P}{3}\ln(1+3x)$
	$+\ln(5-x)$	A1ft	4	ft on $-Q\ln(5-x)$
	$\int \frac{R}{(5-x)^2}\,dx = \frac{5}{5-x}$	B1ft		ft on $\frac{R}{5-x}$

Full marks can be scored in (b) even if P, Q and R are used or if numerical values for them are 'invented'.

Apply ISW if correct separate logarithms are subsequently combined wrongly.

1 (a) Express $\dfrac{10+24x-12x^2}{(3-x)(1+4x)}$ in the form $A+\dfrac{B}{3-x}+\dfrac{C}{1+4x}$, where A, B and C are integers.

[4 marks]

(b) Hence find $\displaystyle\int_0^2 \frac{10+24x-12x^2}{(3-x)(1+4x)}\,\mathrm{d}x$, giving your answer in the form $p+q\ln 3$.

[5 marks]

1 (a) Express $\dfrac{19x-3}{(1+2x)(3-4x)}$ in the form $\dfrac{A}{1+2x}+\dfrac{B}{3-4x}$.

[3 marks]

(b) (i) Find the binomial expansion of $\dfrac{19x-3}{(1+2x)(3-4x)}$ up to and including the term in x^2.

[7 marks]

(ii) State the range of values of x for which this expansion is valid.

[1 mark]

Q1	Solution	Mark	Total	Comment
(a)	$10 + 24x - 12x^2$ $= A(3-x)(1+4x) + B(1+4x) + C(3-x)$	M1		PI by B or C being correct.
	$A = 3$	B1		Could be spotted
	$B = -2$	A1		
	$C = 1$	A1	4	
(b)	$\int 3 - \frac{2}{3-x} + \frac{1}{1+4x}\,dx = 3x$	B1ft		ft on their value of A.
	$+r\ln(3-x) + s\ln(1+4x)$	M1		
	$+2\ln(3-x) + \frac{1}{4}\ln(1+4x)$	A1ft		ft on their values of B and C.
	$\int_0^2 \mathrm{f}(x)dx = \left(3\times 2 + 2\ln 1 + \frac{1}{4}\ln 9\right)$ $-(3\times 0 + 2\ln 3 + \frac{1}{4}\ln 1)$	M1		Correct use of $F(2) - F(0)$ for **their** $Ax + r\ln(3-x) + s\ln(1+4x)$ form but *0* and ln 1 = *0* can be **PI**
	$= 6 - \frac{3}{2}\ln 3$	A1	5	
			9	

Q1	Solution	Mark	Total	Comment
(a)	$19x - 3 = A(3-4x) + B(1+2x)$ Correct equation and 'attempt' to find A or B	M1		e.g. Using $x = \frac{3}{4}$ or $-\frac{1}{2}$ or simultaneous equation such as $19 = -4A + 2B$ and $-3 = 3A + B$
	$A = -\frac{5}{2}$	A1		
	$B = \frac{9}{2}$	A1	3	
	NMS or cover up rule scores SC2 for $A = -\frac{5}{2}$ or $B = \frac{9}{2}$ or SC3 for both $A = -\frac{5}{2}$ and $B = \frac{9}{2}$			
(b)(i)	$(1+2x)^{-1} = 1 - 2x + kx^2$	M1		provided $k \neq 0$
	$= 1 - 2x + 4x^2$	A1		Accept $+(2x)^2$
	$(3-4x)^{-1} = 3^{-1}\left(1 - \frac{4}{3}x\right)^{-1}$	B1		ACF for 3^{-1} eg $\frac{1}{3}$ or 0.33 ...or $0.\dot{3}$
	$\left(1 - \frac{4}{3}x\right)^{-1}$ $= 1 + (-1)\left(-\frac{4}{3}x\right) + \frac{(-1)(-2)}{2!}\left(-\frac{4}{3}x\right)^2$	M1		Condone poor use of or missing brackets.
	$= \left(1 + \frac{4}{3}x + \frac{16}{9}x^2\right)$	A1		PI by later work
	$\frac{19x-3}{(1+2x)(3-4x)} =$ $-\frac{5}{2}(1 - 2x + 4x^2) + \frac{9}{2}\cdot\frac{1}{3}\left(1 + \frac{4}{3}x + \frac{16}{9}x^2\right)$	M1		PI by correct answer ft on candidate's A and B with their relevant series.
	$= -1 + 7x - \frac{22}{3}x^2$	A1	7	Must have $\frac{22}{3}, 7\frac{1}{3}$ or $7.\dot{3}$

3 (a) Express $\dfrac{3+13x-6x^2}{2x-3}$ in the form $Ax+B+\dfrac{C}{2x-3}$.

[4 marks]

(b) Show that $\displaystyle\int_3^6 \frac{3+13x-6x^2}{2x-3}\,\mathrm{d}x = p+q\ln 3$, where p and q are rational numbers.

[4 marks]

Q3	Solution	Mark	Total	Comment
(a)	$\frac{3+13x-6x^2}{2x-3} = Ax + B + \frac{C}{2x-3}$ $3+13x-6x^2 = Ax(2x-3) + B(2x-3) + C$ Correct above equation and attempt to find one of A, B or C or an attempt at long division	M1		e.g. using $x = \frac{3}{2}$ in an attempt to find C or forming simultaneous equations and attempt to solve.
	$A = -3$	A1		
	$B = 2$	A1		
	$C = 9$	A1	4	
	If long division is used award **M1** once $-3x + \cdots$ has been obtained but only award the **A** marks once the values are clearly identified or it is written in the required form of $Ax + B + \frac{C}{2x-3}$. Alternative method of division $\frac{3+13x-6x^2}{2x-3} = \frac{-3x(2x-3)-9x+13x+3}{2x-3} = -3x + \frac{4x+3}{2x-3}$ **M1** for $-3x + \cdots$. For the **A** marks, A, B and C must be clearly identified or seen in the required form of $Ax + B + \frac{C}{2x-3}$. **NMS** scores **B2** for one correct value, **B3** for 2 correct values and **B4** for all three correct values			
(b)	$\int \frac{3+13x-6x^2}{2x-3}\,dx = \int -3x + 2 + \frac{9}{2x-3}\,dx$ $= px^2 + qx + r\ln(2x-3)$	M1		
	$= -\frac{3}{2}x^2 + 2x + \frac{9}{2}\ln(2x-3)$	A1ft		$\frac{A}{2}x^2 + Bx + \frac{C}{2}\ln(2x-3)$
	Correct use of $F(6) - F(3)$ $= \left[-\frac{3}{2}.6^2 + 2.6 + \frac{9}{2}\ln(12-3)\right]$ $-\left[-\frac{3}{2}.3^2 + 2.3 + \frac{9}{2}\ln(6-3)\right]$	m1		**Correct** substitution of limits for their p, q and r.
	$= -\frac{69}{2} + \frac{9}{2}\ln 3$	A1	4	OE
	The **M1 A1ft** and **m1** can be earned even if left in terms of A, B and C or if 'invented' value(s) for A, B and C are used. Condone missing brackets from the $\ln(2x-3)$ term for the **M1** mark but **only** award the **A1ft** mark if they have clearly recovered; PI by sight of ln9 or ln3 after using the limits or a correct final answer. Treat a decimal answer (should be -29.55 ...) after a correct exact form as ISW but award **A0** if an **exact** answer is not seen.			

1 It is given that $f(x) = \dfrac{19x - 2}{(5 - x)(1 + 6x)}$ can be expressed as $\dfrac{A}{5 - x} + \dfrac{B}{1 + 6x}$, where A and B are integers.

(a) Find the values of A and B.

[3 marks]

(b) Hence show that $\displaystyle\int_0^4 f(x)\,dx = k \ln 5$, where k is a rational number.

[6 marks]

1. Given that

$$\frac{4x^3 - 6x^2 - 18x + 20}{x^2 - 4} \equiv ax + b + \frac{c}{x - 2} \qquad x \neq \pm 2$$

find the values of the constants a, b and c.

(4)

Q1	Solution	Mark	Total	Comment
(a)	$19x-2=A(1+6x)+B(5-x)$	**M1**		Correct equation and attempt to find a value for A or B.
	$A=3$	**A1**		
	$B=-1$	**A1**	**3**	**NMS** or cover up rule; A or B correct **SC2** A **and** B correct **SC3**.
(b)	$\int \frac{3}{5-x}-\frac{1}{1+6x}\,\mathrm{d}x$ $=p\ln(5-x)+q\ln(1+6x)$	**M1**		Condone missing brackets **OE** Either term in a correct form
	$=-3\ln(5-x)$	**A1ft**		ft on their A
	$-\frac{1}{6}\ln(1+6x)$	**A1ft**		ft on their B
	$\int_0^4 =\left[-3\ln 1-\frac{1}{6}\ln 25\right]-\left[-3\ln 5-\frac{1}{6}\ln 1\right]$	**m1**		Substitute limits correctly in their integral; $\mathrm{F}(4)-\mathrm{F}(0)$
	$=-\frac{1}{6}\ln 25+3\ln 5$	**A1**		ACF. $\ln 1=0$ PI
	$=\frac{8}{3}\ln 5$	**A1**	**6**	CSO Condone equivalent fractions or recurring decimal

1	Way 1	Way 2	
	Quotient $4x-6$ $x^2+0x-4\,\big)\,4x^3-6x^2-18x+20$ $4x^3+0x^2-16x$ $-6x^2-2x+20$ $-6x^2+0x+24$ $-2x-4$	Quotient $4x^2-14x+10$ $x+2\,\big)\,4x^3-6x^2-18x+20$ $4x^3+8x^2$ $-14x^2-18x+20$ $-14x^2-28x$ $10x+20$ $10x+20$	M1 A1
	$\frac{4x^3-6x^2-18x+20}{x^2-4}$ $\equiv 4x-6+\frac{-2x-4}{(x+2)(x-2)}$	Quotient $4x-6$ $x-2\,\big)\,4x^2-14x+20$ $4x^2-8x^2$ $-6x+20$ $-6x+12$ -2	M1
	$\equiv 4x-6-\frac{2}{(x-2)}$		A1
			(4)