5 A curve is defined by the parametric equations

$$x = 3e^{t}, \quad y = e^{2t} - e^{-2t}$$

(a) (i) Find $\dfrac{dy}{dx}$ in terms of t.

[2 marks]

(ii) The point P, where $t = 0$, lies on the curve. The tangent at P crosses the y-axis at the point Q. Find the coordinates of Q.

[3 marks]

(b) Show that the Cartesian equation of the curve can be written in the form

$$kx^2y = (x^2 - k)(x^2 + k)$$

where k is an integer.

[3 marks]

1 A curve is defined by the parametric equations

$$x = (t-1)^3 \quad , \quad y = 3t - \frac{8}{t^2} \qquad t \neq 0$$

(a) Find $\dfrac{dy}{dx}$ in terms of t.

[3 marks]

(b) Find the equation of the normal at the point on the curve where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[3 marks]

Q 5	Solution	Mark	Total	Comment
(a)(i)	$x = 3e^t \quad y = e^{2t} - e^{-2t}$ When t = 0, P is $(3,0)$ $\frac{dx}{dt} = 3e^t \quad \frac{dy}{dt} = 2e^{2t} + 2e^{-2t}$	B1		**Both** correct
	$\frac{dy}{dx} = \frac{2e^{2t} + 2e^{-2t}}{3e^t}$	B1ft	2	From $\frac{dx}{dt} = ae^t$ and $\frac{dy}{dt} = be^{2t} + ce^{-2t}$
(ii)	When $t = 0$, $\frac{dy}{dx} = \frac{4}{3}$	B1		From correct work
	Tangent at P is $y - 0 = \frac{4}{3}(x - 3)$	M1		Using their $\frac{4}{3}$ and their co-ordinates of P
	Q is $(0, -4))$	A1	3	CSO but allow $x = 0, y = -4$ seen
	Candidates may get a form of the Cartesian equation in either (a)(i) or (ii). In this case they could earn marks for part (b) if **no** work attempted there.			
(b)	From $x = 3e^t \quad e^{2t} = \frac{x^2}{9}$ or $\left(\frac{x}{3}\right)^2$ OE	B1		or $t = \ln\left(\frac{x}{3}\right)$ PI later
	$y = \left(\frac{x}{3}\right)^2 - \left(\frac{x}{3}\right)^{-2}$	M1		Finding a **correct** Cartesian equation. ACF e.g. $y = e^{2\ln\left(\frac{x}{3}\right)} - e^{-2\ln\frac{x}{3}}$
	$y = \frac{x^2}{9} - \frac{9}{x^2}$ $9x^2y = x^4 - 81$ $9x^2y = (x^2 - 9)(x^2 + 9)$	A1	3	Format given – be convinced.

(a)	$\left(\frac{dt}{dt} =\right) \quad 3(t-1)^2$	B1		ACF e.g. $3t^2 - 6t + 3$
	$\left(\frac{dy}{dt} =\right) \quad 3 + 16t^{-3}$	B1		ACF
	$\frac{dy}{dx} = \frac{3 + 16t^{-3}}{3(t-1)^2}$	B1ft	3	ACF: must see $\frac{dy}{dx} = \cdots$ but **ft** on their $\frac{dy/dt}{dx/dt}$ **provided** numerator is of the form $3 \pm kt^{-3}$ **and** denominator is a quadratic in t.
	Accept missing $\frac{dx}{dt}$ and/or $\frac{dy}{dt}$ or poor notation but must see $\frac{dy}{dx} = \ldots$ on final line If answer left as the product $\frac{1}{3(t-1)^2} \times 3 + 16t^{-3}$ with missing brackets then **B0(ft)** but can score the next **B1** in part (b).			
(b)	At $t = 2$, $\quad m = \frac{3+\frac{16}{8}}{3\times 1^2} = \frac{5}{3}$	B1		From a **correct** $\frac{dy}{dx}$. PI by next line.
	Gradient of normal $= -\frac{3}{5}$ (Normal is) $y - 4 = -\frac{3}{5}(x - 1)$	M1		Use of their $-\frac{1}{m}$ with $x = 1$ **and** $y = 4$.
	$3x + 5y - 23 = 0$	A1	3	Integer coefficients with all terms on one side (in any order) and = 0 on the other.
	If $y = mx + c$ is used they must use $x = 1$ and $y = 4$ to find a value for c to earn the **M1** mark. An answer such as $0 = -10y - 6x + 46$ would score **A1** but $5y = -3x + 23$ or $3x + 5y = 23$ is **A0**.			

7 A curve C is defined by the parametric equations

$$x = \frac{4 - e^{2-6t}}{4}, \qquad y = \frac{e^{3t}}{3t}, \qquad t \neq 0$$

(a) Find the exact value of $\frac{dy}{dx}$ at the point on C where $t = \frac{2}{3}$.

[5 marks]

(b) Show that $x = \frac{4 - e^{2-6t}}{4}$ can be rearranged into the form $e^{3t} = \frac{e}{2\sqrt{(1-x)}}$.

[2 marks]

(c) Hence find the Cartesian equation of C, giving your answer in the form

$$y = \frac{e}{f(x)[1 - \ln(f(x))]}$$

[2 marks]

Q7	Solution	Mark	Total	Comment
(a)	$\left(\frac{dx}{dt}\right) = -(-6)\frac{e^{2-6t}}{4}$	B1		ACF : $\frac{3}{2}e^{2-6t}$
	$\left(\frac{dy}{dt}\right) = \frac{pe^{3t}.t + qe^{3t}}{(3t)^2}$	M1		From quotient rule
	$= \frac{3e^{3t}.3t - e^{3t}.3}{(3t)^2}$	A1		ACF : $\frac{9te^{3t}-3e^{3t}}{9t^2}$, $\frac{e^{3t}}{t} - \frac{e^{3t}}{3t^2}$ etc.
	$\frac{dy}{dx} = \frac{(9.\frac{2}{3}.e^2 - 3e^2)/4}{\frac{3}{2}e^{-2}}$	m1		Using $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ **and** clear evidence of an attempt to substitute $t = \frac{2}{3}$ (**must** be this) done in either order.
	$= \frac{1}{2}e^4$	A1	5	**CAO : Accept this or $\frac{e^4}{2}$ or $0.5e^4$ only**
(b)	$x = \frac{4-e^{2-6t}}{4} \Rightarrow 4x = 4 - e^{2-6t}$ leading to			
	$e^{-6t} = \frac{4-4x}{e^2}$	M1		Any correct expression for e^{-6t} or e^{6t}.
	$e^{6t} = \frac{e^2}{4(1-x)}$			
	$e^{3t} = \frac{e}{2\sqrt{1-x}}$	A1	2	**AG** Must see inversion step and be convinced they haven't worked backwards
(c)	From (b) $e^{3t} = \frac{e}{2\sqrt{1-x}}$			
	$\ln(e^{3t}) = \ln\left(\frac{e}{2\sqrt{1-x}}\right)$			
	$3t = \ln e - \ln(2\sqrt{1-x})$	M1		Find $3t$ (or t) in terms of x; must have used laws of logs correctly on both sides (possibly $\ln e = 1$ at this stage)
	$y = \frac{e}{2\sqrt{1-x}\,[1 - \ln 2\sqrt{1-x}\,]}$	A1	2	From $y = \frac{e^{3t}}{3t}$; must be in this form

5 A curve is defined by the parametric equations $x = \cos 2t$, $y = \sin t$.

The point P on the curve is where $t = \frac{\pi}{6}$.

(a) Find the gradient at P.

[3 marks]

(b) Find the equation of the normal to the curve at P in the form $y = mx + c$.

[3 marks]

(c) The normal at P intersects the curve again at the point $Q(\cos 2q,\ \sin q)$.

Use the equation of the normal to form a quadratic equation in $\sin q$ and hence find the x-coordinate of Q.

[5 marks]

Q5	Solution	Mark	Total	Comment
(a)	$\left(\frac{dx}{dt}=\right)-2\sin 2t \qquad \left(\frac{dy}{dt}=\right)\cos t$	B1		Both correct
	$\left(\frac{dy}{dx}=\right)\frac{\cos t}{-2\sin 2t}$	M1		Correct use of chain rule with their derivatives of form $a\sin 2t,\ \ b\cos t$
	At $t=\frac{\pi}{6}$ gradient $m_T=-\frac{1}{2}$	A1	3	
(b)	Gradient of normal $m_N=2$	B1ft		ft gradient of tangent; $m_N=\frac{-1}{m_T}$
	$\left(y-\cos\left(\frac{2\pi}{6}\right)\right)=m_N\left(x-\sin\left(\frac{\pi}{6}\right)\right)$	M1		For m_N, allow their m_T with a change of sign or the reciprocal at $\left(\sin\frac{\pi}{6},\cos\frac{2\pi}{6}\right)$ or $\left(\frac{1}{2},\frac{1}{2}\right)$
	$y=2x-\frac{1}{2}$	A1	3	Must be in this $y=mx+c$ form
	Alternative for M1 $\sin\left(\frac{\pi}{6}\right)=2\cos\left(\frac{2\pi}{6}\right)+c$			Use $y=mx+c$ to find c with their gradient m_N at $\left(\sin\frac{\pi}{6},\cos\frac{2\pi}{6}\right)$ or $\left(\frac{1}{2},\frac{1}{2}\right)$
(c)	$\cos 2q=1-2\sin^2 q$	B1		Seen or used in this form
	$\sin q=2\left(1-2\sin^2 q\right)-\frac{1}{2}$	M1		Use parametric equations and candidate's $\cos 2q$ in the form $\pm1+k\sin^2 q$
	$8\sin^2 q+2\sin q-3=0$ **OE**	A1		Collect like terms; must be a quadratic equation
	$\left(\sin q=\frac{1}{2}\right) \qquad \sin q=-\frac{3}{4}$	A1		Must come from a correct quadratic equation with the previous 3 marks awarded
	$(x=)\ -\frac{1}{8}$	A1	5	Previous 4 marks must have been awarded
	Total		**11**	

7. The curve C has parametric equations

$$x = -3 + 6\sin\theta \qquad y = 4\sqrt{3}\cos 2\theta \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

where θ is a parameter.

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ.

(2)

The curve C cuts the y-axis at the point A.

The line l is the normal to C at the point A.

(b) Show that an equation for l is

$$\sqrt{3}x - 4y + 8\sqrt{3} = 0$$

(6)

The line l intersects the curve C again at the point B.

(c) Find the coordinates of B. Give your answer in the form $(p, q\sqrt{3})$, where p and q are rational constants.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(a)	$\frac{dy}{dx} = \frac{-(2)(4)\sqrt{3}\sin 2\theta}{6\cos\theta}$ $\left\{= \frac{-4\sqrt{3}\sin 2\theta}{3\cos\theta} = -\frac{8}{3}\sqrt{3}\sin\theta\right\}$	their $\frac{dy}{d\theta} \div$ their $\frac{dx}{d\theta}$	M1
		Correct simplified or un-simplified result	A1 isw
			(2)
(b)	$\{x = 0 \Rightarrow\}$ $0 = -3 + 6\sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$	Sets $x = 0$ to find θ or $\sin\theta$ and uses their θ or $\sin\theta$ to find y	M1
	$y_A = 4\sqrt{3}\cos\left(\frac{\pi}{3}\right) = 2\sqrt{3}$ $\{\Rightarrow A(0, 2\sqrt{3})\}$	$y_A = 2\sqrt{3}$ or $\sqrt{12}$ or awrt 3.46	A1
	$m_T = \frac{dy}{dx} = \frac{-4\sqrt{3}\sin(2(\frac{\pi}{6}))}{3\cos(\frac{\pi}{6})} = \frac{-4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)}{3\left(\frac{\sqrt{3}}{2}\right)} = -\frac{4\sqrt{3}}{3} = -\frac{4}{\sqrt{3}}$	Substitutes $\theta = \frac{\pi}{6}$ or $30°$ or $\sin\theta = \frac{1}{2}$ into their $\frac{dy}{dx}$. Can be implied.	M1
	So, $m_N = \frac{3}{4\sqrt{3}}$ or $\frac{\sqrt{3}}{4}$	Correctly applies $m_N = -\frac{1}{\text{their } m_T}$	M1
	• $y - 2\sqrt{3} = \frac{\sqrt{3}}{4}(x - 0)$ • $y = \frac{\sqrt{3}}{4}x + 2\sqrt{3}$	$y - (\text{their } y_A) = (\text{their } m_N)(x - 0)$ or $y = (\text{their } m_N)x + (\text{their } y_A)$ with a numerical $m_N (\neq m_T)$	M1
	$4y - 8\sqrt{3} = \sqrt{3}x \Rightarrow \sqrt{3}x - 4y + 8\sqrt{3} = 0$ *	Correct proof	A1*
			(6)
(c) **Way 1**	$\sqrt{3}(-3 + 6\sin\theta) - 4(4\sqrt{3}\cos 2\theta) + 8\sqrt{3} = 0$	Substitutes $x = -3 + 6\sin\theta$ and $y = 4\sqrt{3}\cos 2\theta$ into the normal equation to form an equation in θ only	M1
	$-3 + 6\sin\theta - 16\cos 2\theta + 8 = 0$		
	$-3 + 6\sin\theta - 16(1 - 2\sin^2\theta) + 8 = 0$	**dependent on the previous M mark** Applies $\cos 2\theta \equiv 1 - 2\sin^2\theta$	dM1
	$32\sin^2\theta + 6\sin\theta - 11 = 0$ or $32\sin^2\theta + 6\sin\theta = 11$	Correct 3TQ in $\sin\theta$ e.g. $32\sin^2\theta + 6\sin\theta - 11 \{= 0\}$	A1
	$(2\sin\theta - 1)(16\sin\theta + 11) = 0 \Rightarrow \sin\theta = ...$	**dependent on the first M mark** Correct method (e.g. factorising, applying the quadratic formula, completing the square or calculator) of solving a 3TQ to give $\sin\theta = ...$	dM1
	$\left\{\sin\theta = \frac{1}{2}\right\}$ $\sin\theta = -\frac{11}{16}$		
	So, $B(x, y) = B\left(-\frac{57}{8}, \frac{7}{32}\sqrt{3}\right)$	Either x or y is correct	A1 o.e.
		Both x and y are correct	A1 o.e.
			(6)

5.

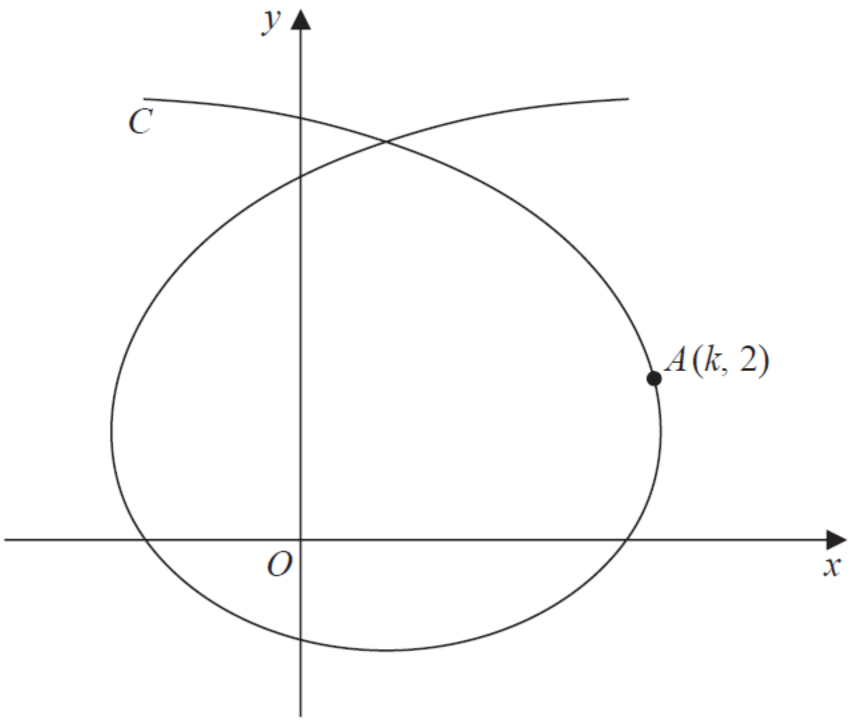

Diagram not drawn to scale

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 1 + t - 5\sin t, \quad y = 2 - 4\cos t, \quad -\pi \leqslant t \leqslant \pi$$

The point A lies on the curve C.

Given that the coordinates of A are $(k, 2)$, where $k > 0$

(a) find the exact value of k, giving your answer in a fully simplified form.

(2)

(b) Find the equation of the tangent to C at the point A.
Give your answer in the form $y = px + q$, where p and q are exact real values.

(5)

5.	$x=1+t-5\sin t,\ y=2-4\cos t,\ -\pi \leqslant t \leqslant \pi;\ A(k,2),\ k>0$, lies on C		
(a)	{When $y=2$,} $2=2-4\cos t \Rightarrow t=-\frac{\pi}{2}, \frac{\pi}{2}$ $k(\text{or } x)=1+\frac{\pi}{2}-5\sin\left(\frac{\pi}{2}\right)$ **or** $k(\text{or } x)=1-\frac{\pi}{2}-5\sin\left(-\frac{\pi}{2}\right)$	Sets $y=2$ to find t and some evidence of using their t to find $x=...$	M1
	$\left\{\text{When } t=-\frac{\pi}{2}, k>0,\right\}$ so $k=6-\frac{\pi}{2}$ or $\frac{12-\pi}{2}$	k (or x) $=6-\frac{\pi}{2}$ or $\frac{12-\pi}{2}$	A1
			[2]
(b)	$\frac{dx}{dt}=1-5\cos t,\quad \frac{dy}{dt}=4\sin t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct (Can be implied)	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct (Can be implied)	B1
	$\frac{dy}{dx}=\frac{4\sin t}{1-5\cos t}$ at $t=-\frac{\pi}{2},\ \frac{dy}{dx}=\frac{4\sin\left(-\frac{\pi}{2}\right)}{1-5\cos\left(-\frac{\pi}{2}\right)}$ $\{=-4\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their t into their $\frac{dy}{dx}$ **Note:** their t can lie outside $-\pi \leqslant t \leqslant \pi$ for this mark	M1
	• $y-2=-4\left(x-\left(6-\frac{\pi}{2}\right)\right)$ • $2=(-4)\left(6-\frac{\pi}{2}\right)+c \Rightarrow y=-4x+2+4\left(6-\frac{\pi}{2}\right)$	Correct straight line method for an equation of a tangent where m_T ($\neq m_N$) is found using calculus **Note:** their k (or x) must be in terms of π and correct bracketing must be used or implied	M1
	$\{y-2=-4x+24-2\pi \Rightarrow\}$ $y=-4x+26-2\pi$	**dependent on all previous marks in part (b)** $y=-4x+26-2\pi$	A1 cso
		$(p=-4,\ q=26-2\pi)$	**[5]**
			7

1. The curve C has parametric equations

$$x = 3t - 4, \quad y = 5 - \frac{6}{t}, \qquad t > 0$$

(a) Find $\frac{dy}{dx}$ in terms of t

(2)

The point P lies on C where $t = \frac{1}{2}$

(b) Find the equation of the tangent to C at the point P. Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c) Show that the cartesian equation for C can be written in the form

$$y = \frac{ax + b}{x + 4}, \qquad x > -4$$

where a and b are integers to be determined.

(3)

1.	$x = 3t - 4,\ y = 5 - \dfrac{6}{t},\quad t > 0$		
(a)	$\dfrac{dx}{dt} = 3,\ \dfrac{dy}{dt} = 6t^{-2}$		
	$\dfrac{dy}{dx} = \dfrac{6t^{-2}}{3}\ \left\{= \dfrac{6}{3t^2} = 2t^{-2} = \dfrac{2}{t^2}\right\}$	their $\dfrac{dy}{dt}$ divided by their $\dfrac{dx}{dt}$ to give $\dfrac{dy}{dx}$ in terms of t **or** their $\dfrac{dy}{dt}$ multiplied by their $\dfrac{dt}{dx}$ to give $\dfrac{dy}{dx}$ in terms of t	M1
		$\dfrac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t. **See note.**	A1 isw
	Award **Special Case 1st M1** if **both** $\dfrac{dx}{dt}$ **and** $\dfrac{dy}{dt}$ are stated **correctly** and **explicitly**.		**[2]**
(b)	$\left\{t = \dfrac{1}{2} \Rightarrow\right\} P\left(-\dfrac{5}{2}, -7\right)$	$x = -\dfrac{5}{2},\ y = -7$ **or** $P\left(-\dfrac{5}{2}, -7\right)$ seen or implied.	B1
	$\dfrac{dy}{dx} = \dfrac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y - \text{"}-7\text{"} = \text{"}8\text{"}\left(x - \text{"}-\frac{5}{2}\text{"}\right)$ • $\text{"}-7\text{"} = (\text{"}8\text{"})(\text{"}-\frac{5}{2}\text{"}) + c$ So, $y = (\text{their } m_T)x + \text{"}c\text{"}$	**Some** attempt to substitute $t = 0.5$ into their $\dfrac{dy}{dx}$ which contains t in order to find m_T **and either** applies $y - (\text{their } y_P) = (\text{their } m_T)(x - \text{their } x_P)$ **or** finds c from $(\text{their } y_P) = (\text{their } m_T)(\text{their } x_P) + c$ and uses their numerical c in $y = (\text{their } m_T)x + c$	M1
	T: $y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 **cso**
	Note: their x_P, their y_P and their m_T must be numerical values in order to award M1		**[3]**
(c) **Way 1**	$\left\{t = \dfrac{x+4}{3} \Rightarrow\right\}\ y = 5 - \dfrac{6}{\left(\dfrac{x+4}{3}\right)}$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow\ y = 5 - \dfrac{18}{x+4}\ \Rightarrow\ y = \dfrac{5(x+4) - 18}{x+4}$		
	So, $y = \dfrac{5x+2}{x+4},\ \{x > -4\}$	$y = \dfrac{5x+2}{x+4}$ (or implied equation)	A1 **cso**
			[3]

8.

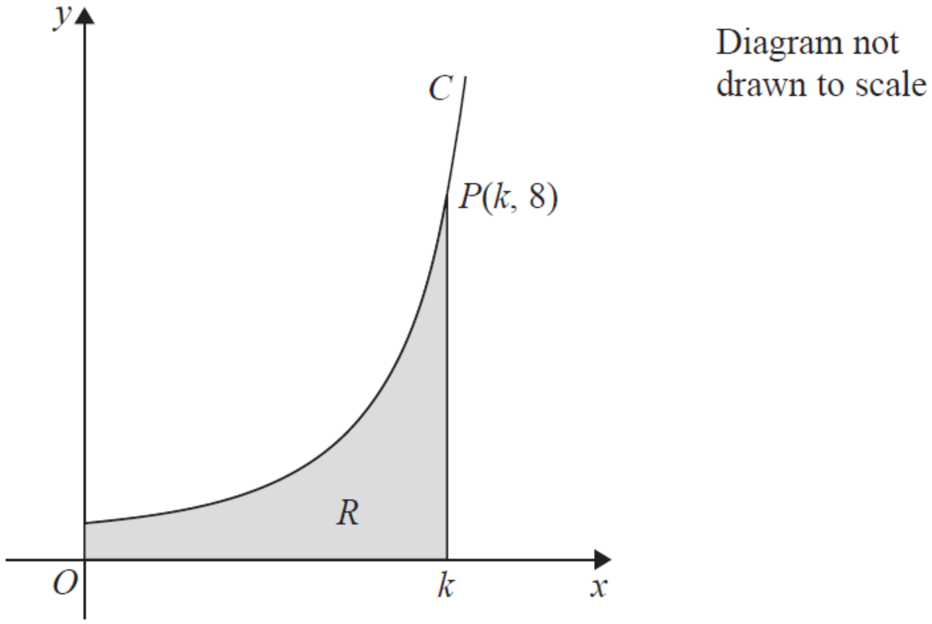

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\theta\sin\theta, \qquad y = \sec^3\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point $P(k, 8)$ lies on C, where k is a constant.

(a) Find the exact value of k.

(2)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the y-axis, the x-axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda\int_{\alpha}^{\beta}\left(\theta\sec^2\theta + \tan\theta\sec^2\theta\right)\mathrm{d}\theta$$

where λ, α and β are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R.

(6)

(a)	$\{\text{When } y=8,\}\ 8=\sec^3\theta \Rightarrow \cos^3\theta=\frac{1}{8} \Rightarrow \cos\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}$ $k \text{ (or } x) = 3\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)$	Sets $y=8$ to find θ **and** attempts to substitute their θ into $x=3\theta\sin\theta$	M1
	so $k \text{ (or } x) = \frac{\sqrt{3}\pi}{2}$	$\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	A1
	Note: Obtaining two value for k without accepting the correct value is final A0		**[2]**
(b)	$\frac{dx}{d\theta}=3\sin\theta+3\theta\cos\theta$	$3\theta\sin\theta \to 3\sin\theta+3\theta\cos\theta$ Can be implied by later working	B1
	$\left\{\int y\frac{dx}{d\theta}\{d\theta\}\right\}=\int(\sec^3\theta)(3\sin\theta+3\theta\cos\theta)\{d\theta\}$	Applies $(\pm K\sec^3\theta)\left(\text{their } \frac{dx}{d\theta}\right)$ Ignore integral sign and $d\theta$; $K \neq 0$	M1
	$=3\int \theta\sec^2\theta+\tan\theta\sec^2\theta\, d\theta$	Achieves the correct result no errors in their working, e.g. bracketing or manipulation errors. **Must have** integral sign and $d\theta$ in their final answer.	A1 *
	$x=0$ and $x=k \Rightarrow \underline{\alpha=0}$ and $\underline{\beta=\frac{\pi}{3}}$	$\alpha=0$ and $\beta=\frac{\pi}{3}$ or evidence of $0\to 0$ and $k\to\frac{\pi}{3}$	B1
	Note: The work for the final B1 mark must be seen in part (b) only.		**[4]**
(c) **Way 1**	$\left\{\int\theta\sec^2\theta d\theta\right\}=\theta\tan\theta-\int\tan\theta\{d\theta\}$	$g(\theta)=$ their $\int\sec^2\theta d\theta$. [Note: $g(\theta)\neq\sec^2\theta$] **dependent on the previous M mark** **Either** $\lambda\theta\sec^2\theta\to A\theta\tan\theta-B\int\tan\theta,\ A>0,\ B>0$ **or** $\theta\sec^2\theta\to\theta\tan\theta-\int\tan\theta$	dM1
	$=\theta\tan\theta-\ln(\sec\theta)$ **or** $=\theta\tan\theta+\ln(\cos\theta)$	$\theta\sec^2\theta\to\theta\tan\theta-\ln(\sec\theta)$ or $\theta\tan\theta+\ln(\cos\theta)$ **or** $\lambda\theta\sec^2\theta\to\lambda\theta\tan\theta-\lambda\ln(\sec\theta)$ or $\lambda\theta\tan\theta+\lambda\ln(\cos\theta)$	A1
	Note: Condone $\theta\sec^2\theta\to\theta\tan\theta-\ln(\sec x)$ or $\theta\tan\theta+\ln(\cos x)$ for A1		
	$\left\{\int\tan\theta\sec^2\theta d\theta\right\}$	$\tan\theta\sec^2\theta$ or $\lambda\tan\theta\sec^2\theta\to\pm C\tan^2\theta$ or $\pm C\sec^2\theta$ or $\pm Cu^{-2}$, where $u=\cos\theta$	M1
	$=\frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2u^2}$ where $u=\cos\theta$ or $\frac{1}{2}u^2$ where $u=\tan\theta$	$\tan\theta\sec^2\theta\to\frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2\cos^2\theta}$ or $\tan^2\theta-\frac{1}{2}\sec^2\theta$ or $0.5u^{-2}$, where $u=\cos\theta$ or $0.5u^2$, where $u=\tan\theta$ or $\lambda\tan\theta\sec^2\theta\to\frac{\lambda}{2}\tan^2\theta$ or $\frac{\lambda}{2}\sec^2\theta$ or $\frac{\lambda}{2\cos^2\theta}$ or $0.5\lambda u^{-2}$, where $u=\cos\theta$ or $0.5\lambda u^2$, where $u=\tan\theta$	A1
	$\{\text{Area}(R)\}=\left[3\theta\tan\theta-3\ln(\sec\theta)+\frac{3}{2}\tan^2\theta\right]_0^{\frac{\pi}{3}}$ **or** $\left[3\theta\tan\theta-3\ln(\sec\theta)+\frac{3}{2}\sec^2\theta\right]_0^{\frac{\pi}{3}}$		
	$=\left(3\left(\frac{\pi}{3}\right)\sqrt{3}-3\ln 2+\frac{3}{2}(3)\right)-(0)$ **or** $\left(3\left(\frac{\pi}{3}\right)\sqrt{3}-3\ln 2+\frac{3}{2}(4)\right)-\left(\frac{3}{2}\right)$		
	$=\frac{9}{2}+\sqrt{3}\pi-3\ln 2$ **or** $\frac{9}{2}+\sqrt{3}\pi+3\ln\left(\frac{1}{2}\right)$ **or** $\frac{9}{2}+\sqrt{3}\pi-\ln 8$ **or** $\ln\left(\frac{1}{8}e^{\frac{9}{2}+\sqrt{3}\pi}\right)$		A1 o.e.

5.

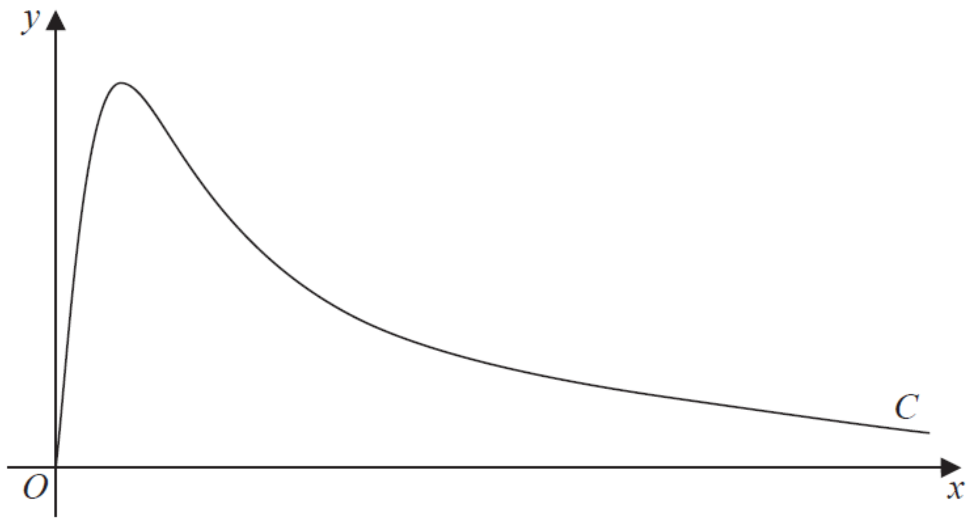

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point P.

Give your answer as a simplified surd.

(4)

The point Q lies on the curve C, where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(b) Find the exact coordinates of the point Q.

(2)

(a) **Way 1**	$\frac{dx}{dt} = 4\sec^2 t,\ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t}\ \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	**Either both** x and y are differentiated correctly with respect to t **or** their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
		Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** ***Some evidence*** of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**

(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right),\ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y = \text{awrt } 8.7$	M1
	Coordinates are $\left(4, 5\sqrt{3}\right)$	$\left(4, 5\sqrt{3}\right)$ **or** $x = 4,\ y = 5\sqrt{3}$	A1
			[2]

(ii) (a) **Way 1**	$\{x = 4\sin^2\theta \Rightarrow\}\ \frac{dx}{d\theta} = 8\sin\theta\cos\theta$ or $\frac{dx}{d\theta} = 4\sin 2\theta$ or $dx = 8\sin\theta\cos\theta\, d\theta$		B1
	$\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}.\,8\sin\theta\cos\theta\,\{d\theta\}$ or $\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}.\,4\sin 2\theta\,\{d\theta\}$		M1
	$= \int \underline{\underline{\tan\theta}}.\,8\sin\theta\cos\theta\,\{d\theta\}$ or $\int \underline{\underline{\tan\theta}}.\,4\sin 2\theta\,\{d\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \to \pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$	$\underline{\underline{\text{M1}}}$
	$= \int 8\sin^2\theta\, d\theta$	$\int 8\sin^2\theta\, d\theta$ including $d\theta$	A1
	$3 = 4\sin^2\theta$ **or** $\frac{3}{4} = \sin^2\theta$ **or** $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\{x = 0 \to \theta = 0\}$	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits	B1
			[5]
(ii) (b)	$= \{8\}\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta\ \left\{= \int(4 - 4\cos 2\theta)\,d\theta\right\}$	Applies $\cos 2\theta = 1 - 2\sin^2\theta$ to their integral. (See notes)	M1
	$= \{8\}\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\ \{= 4\theta - 2\sin 2\theta\}$	For $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha,\beta \neq 0$	M1
		$\sin^2\theta \to \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)$	A1
	$\left\{\int_0^{\frac{\pi}{3}} 8\sin^2\theta\, d\theta = 8\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}}\right\} = 8\left(\left(\frac{\pi}{6} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right) - (0+0)\right)$		
	$= \frac{4}{3}\pi - \sqrt{3}$	"two term" exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{1}{3}\left(4\pi - 3\sqrt{3}\right)$	A1 o.e.

5. A curve C has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

(a) Find the value of $\frac{dy}{dx}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

(b) Show that the cartesian equation of the curve C can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where a and b are integers to be determined.

(3)

5.	**Note: You can mark parts (a) and (b) together.**		
(a)	$x = 4t + 3,\ y = 4t + 8 + \frac{5}{2t}$		
	$\frac{dx}{dt} = 4,\ \frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$	Both $\frac{dx}{dt} = 4$ or $\frac{dt}{dx} = \frac{1}{4}$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$	B1
	So, $\frac{dy}{dx} = \frac{4 - \frac{5}{2}t^{-2}}{4}$ $\left\{= 1 - \frac{5}{8}t^{-2} = 1 - \frac{5}{8t^2}\right\}$	Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$	M1 **o.e.**
	$\{\text{When } t = 2,\}\ \frac{dy}{dx} = \frac{27}{32}$	$\frac{27}{32}$ or 0.84375 **cao**	A1
			[3]
(b)	$\left\{t = \frac{x-3}{4} \Rightarrow\right\} y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \frac{10}{x-3}$		
	$y = \frac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ ***or*** $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ ***or*** $y = \frac{(x+5)(x-3) + 10}{x-3}$ ***or*** $y = \frac{(x+5)(x-3)}{x-3} + \frac{10}{x-3}$	See notes	dM1
	$\Rightarrow y = \frac{x^2 + 2x - 5}{x-3}$, $\{a = 2 \text{ and } b = -5\}$	Correct algebra leading to $y = \frac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 **cso**
			[3]