Pure Sector 4: Implicit Differentiation

Implicit functions are functions in x and y that aren't written $y = f(x)$ or $x = f(y)$

$x^2 + y^2 = 4$ $\qquad x + y = 10$
$\sin(x + y) = y$ $xy = 10$

> In general, to differentiate a function of y with respect to x we differentiate it with respect to y and multiply by $\frac{dy}{dx}$

Differentiate these functions with respect to x

$x^2 + y^2 = 4$	$x + y = 10$
$\sin(x + y) = y$	$xy = 10$

<u>Example 1</u>
Find $\frac{dy}{dx}$ when $x^3 + 3y^2 = x$

<u>Example 2</u>
Find the gradient of the curve $3x^2 + xy - y^3 = 20$ at the point $(4,2)$.

<u>Example 3</u>
Find the equation of the normal to the curve $x^3 + 2x^2y = y^3 + 15$ at the point $(2,1)$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

Example 4

Find the coordinates of the two stationary points of the curve $x^2 + 4xy + 2y^2 + 18 = 0$

Example 5

A curve is defined by the equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17$$

a) Find an expression for $\frac{dy}{dx}$ in terms of x and y.

The point P with coordinates $\left(3, \frac{1}{2}\right)$ lies on C.

The normal to C at P meets the x-axis at the point A.

(b) Find the x coordinate of A, giving your answer in the form $\frac{a\pi + b}{c\pi + d}$, where a, b, c and d are integers to be determined.

Exam Questions

6 A curve has equation $x^3y + \cos(\pi y) = 7$.

(a) Find the exact value of the x-coordinate at the point on the curve where $y = 1$. *(2 marks)*

(b) Find the gradient of the curve at the point where $y = 1$. *(5 marks)*

5 A curve is defined by the equation

$$y^2 - xy + 3x^2 - 5 = 0$$

(a) Find the y-coordinates of the two points on the curve where $x = 1$. *(3 marks)*

(b) (i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y - 6x}{2y - x}$. *(6 marks)*

(ii) Find the gradient of the curve at each of the points where $x = 1$. *(2 marks)*

(iii) Show that, at the two stationary points on the curve, $33x^2 - 5 = 0$. *(3 marks)*

The curve C has equation

$$4x^2 - y^3 - 4xy + 2^y = 0$$

The point P with coordinates $(-2, 4)$ lies on C.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

(6)

The normal to C at P meets the y-axis at the point A.

(b) Find the y coordinate of A, giving your answer in the form $p + q\ln 2$, where p and q are constants to be determined.

(3)

6 A curve is defined by the equation $2y + e^{2x}y^2 = x^2 + C$, where C is a constant. The point $P\left(1, \frac{1}{e}\right)$ lies on the curve.

(a) Find the exact value of C. *(1 mark)*

(b) Find an expression for $\frac{dy}{dx}$ in terms of x and y. *(7 marks)*

(c) Verify that $P\left(1, \frac{1}{e}\right)$ is a stationary point on the curve. *(2 marks)*

6 A curve is defined by the equation $\sin 2x + \cos y = \sqrt{3}$.

(i) Verify that the point P $\left(\frac{1}{6}\pi, \frac{1}{6}\pi\right)$ lies on the curve. **[1]**

(ii) Find $\frac{dy}{dx}$ in terms of x and y.

Hence find the gradient of the curve at the point P. **[5]**