The random variable Y has a normal distribution with mean μ and standard deviation σ
The $P(Y > 17) = 0.4$

Find

(a) $P(\mu < Y < 17)$

(1)

(b) $P(\mu - \sigma < Y < 17)$

(4)

Farmer Adam grows potatoes. The weights of potatoes, in grams, grown by Adam are normally distributed with a mean of 140 g and a standard deviation of 40 g.

Adam cannot sell potatoes with a weight of less than 92 g.

(a) Find the percentage of potatoes that Adam grows but cannot sell.

(3)

The upper quartile of the weight of potatoes **sold** by Adam is q_3

(b) Find the probability that the weight of a randomly selected potato **grown** by Adam is more than q_3

(2)

(c) Find the lower quartile, q_1, of the weight of potatoes **sold** by Adam.

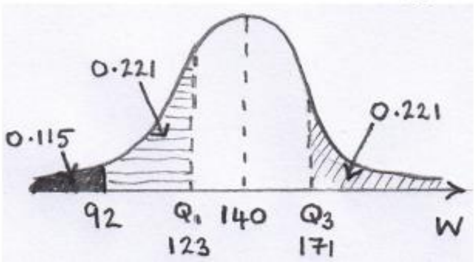
(5)

Betty selects a random sample of 3 potatoes **sold** by Adam.

(d) Find the probability that one weighs less than q_1, one weighs more than q_3 and one has a weight between q_1 and q_3

(3)

Question Number	Scheme	Marks
3. (a)	$[\text{P}(\mu < Y < 17) =]\ 0.5 - 0.4 \quad = \underline{\mathbf{0.1}}$	B1 (1)
(b)	$\text{P}(Y > \mu - \sigma) = \text{P}(Z > -1)$	M1
	$= 0.841(3)$	A1
	$\text{P}(\mu - \sigma < Y < 17) = 0.8413 - 0.4$	dM1
	$= \underline{\mathbf{0.441}}(3)$	A1 (4)

7.	$[W \sim \text{N}(140, 40^2)]$	
(a)	$\text{P}(W < 92) = \text{P}\left(Z < \dfrac{92-140}{40}\right) = [\text{P}(Z < -1.2)]$	M1
	$= 1 - 0.8849 \qquad = \text{awrt } \underline{\mathbf{11.5}}\ (\%) \text{ or } \underline{\mathbf{0.115}}$	dM1,A1 (3)
(b)	$[\text{P}(W > q_3) = \text{P}(W > 92) \times \text{P}(W > q_3 \mid W > 92) =] \quad (1-(\text{a})) \times 0.25 = 0.8849 \times 0.25$	M1
	$= 0.221225 = \text{awrt } \underline{\mathbf{0.221}}$	A1 (2)
(c)	$\text{P}(W < q_1 \mid W > 92) = 0.25$ <u>or</u> $\text{P}(W > q_1 \mid W > 92) = 0.75$	M1
	$\text{P}(92 < W < q_1) = 0.25 \times 0.8849 = \text{"}0.221..\text{"}$ <u>or</u> $\text{P}(W > q_1) = 0.75 \times 0.8849 = 0.663675$	M1
	$\text{P}(W < q_1) = 0.221225 + 0.115 = \text{awrt } \mathbf{0.336}$ <u>or</u> $\text{P}(W > q_1) = 0.663675 = \text{awrt } \mathbf{0.664}$	A1
	$\dfrac{q_1 - 140}{40} = -0.42$ (calculator gives $-0.422513 \sim -0.423404$)	M1
	so $q_1 = 123.2 \quad = \text{awrt } \underline{\mathbf{123}}$ (g)	A1 (5)
(d)	 $\dfrac{1}{4} \times \dfrac{1}{4} \times \dfrac{1}{2} \times 3!$	M1M1
	$= \underline{\dfrac{3}{16}}$ or 0.1875	A1 (3)
		[Tot 13]

Yuto works in the quality control department of a large company. The time, T minutes, it takes Yuto to analyse a sample is normally distributed with mean 18 minutes and standard deviation 5 minutes.

(a) Find the probability that Yuto takes longer than 20 minutes to analyse the next sample.
(3)

The company has a large store of samples analysed by Yuto with the time taken for each analysis recorded. Serena is investigating the samples that took Yuto longer than 15 minutes to analyse.

She selects, at random, one of the samples that took Yuto longer than 15 minutes to analyse.

(b) Find the probability that this sample took Yuto more than 20 minutes to analyse.
(4)

Serena can identify, in advance, the samples that Yuto can analyse in under 15 minutes and in future she will assign these to someone else.

(c) Estimate the median time taken by Yuto to analyse samples in future.
(5)

The time, in minutes, taken by men to run a marathon is modelled by a normal distribution with mean 240 minutes and standard deviation 40 minutes.

(a) Find the proportion of men that take longer than 300 minutes to run a marathon.
(3)

Nathaniel is preparing to run a marathon. He aims to finish in the first 20% of male runners.

(b) Using the above model estimate the longest time that Nathaniel can take to run the marathon and achieve his aim.
(3)

The time, W minutes, taken by women to run a marathon is modelled by a normal distribution with mean μ minutes.

Given that $\mathrm{P}(W < \mu + 30) = 0.82$

(c) find $\mathrm{P}(W < \mu - 30 \mid W < \mu)$
(3)

5. (a)	$[\text{P}(T > 20) =]\ \text{P}\left(Z > \frac{20-18}{5}\right)$	M1
	$\text{P}(Z > 0.4) = 1 - 0.6554$	M1
	$= \underline{\mathbf{0.3446}}$ <u>or</u> awrt $\underline{\mathbf{0.345}}$	A1
		(3)
(b)	Require $\text{P}(T > 20 \mid T > 15)$ <u>or</u> $\frac{\text{P}(T > 20)}{\text{P}(T > 15)}$	M1
	$\frac{\text{"(a)"}}{\text{P}(Z > \frac{15-18}{5})} = \frac{\text{"(a)"}}{\text{P}(Z > -0.6)}, = \frac{\text{"0.3446"}}{0.7257}$ <u>or</u> $\frac{\text{"0.345"}}{0.726}$	M1, A1ft
	$= 0.47485\ldots =$ awrt $\underline{\mathbf{0.475}}$	A1
		(4)
(c)	$\text{P}(T > d \mid T > 15) = 0.5$ <u>or</u> $\text{P}(T < d \mid T > 15) = 0.5$	M1
	$\text{P}(T > d)$ <u>or</u> $\text{P}(15 < T < d) = 0.5 \times$ "0.7257" $= [0.36285]$	A1ft
	$\text{P}(T < d) =$ "0.63715"	M1
	So $\frac{d-18}{5} = 0.35$ (calculator gives 0.35085…)	A1
	$d = 19.754\ldots$ = awrt $\underline{\mathbf{19.8}}$	A1cso
	(Accept 19 mins 45(secs) or 19:45 but 19.45 is A0)	(5)
		[12]

6.(a)	[$T \sim \text{N}(240, 40^2)$…require $\text{P}(T > 300)$]	
	$\text{P}\left(Z > \frac{300-240}{40}\right)$	M1
	$= 1 - \text{P}(Z < 1.5)$ or $1 - 0.9332$	M1
	= awrt $\underline{\mathbf{0.0668}}$ or 6.68%	A1
		(3)
(b)	$[\text{P}(T < n) = 0.20 \Rightarrow]$ $\frac{n-240}{40} = -0.8416$	M1 B1
	$n =$ awrt $\underline{\mathbf{206}}$ minutes	A1
		(3)
(c)	$[\text{P}(W < \mu - 30 \mid W < \mu) =]\ \frac{\text{P}(W < \mu - 30)}{\text{P}(W < \mu)}$	M1
	$= \frac{1 - 0.82}{0.50}$	A1
	$= \underline{\mathbf{0.36}}$	A1cao
		(3)
		[9 marks]

The weight, X grams, of a bar of *PureAV* soap may be modelled by a normal distribution with mean 105 grams and standard deviation 4 grams.

(a) Determine the probability that the weight of a randomly selected bar is:

(i) less than 105 grams;

(ii) **not** exactly 100 grams;

(iii) more than 110 grams;

(iv) between 102 grams and 108 grams.

[8 marks]

(b) The weight, Y grams, of a bar of *RichAV* soap may be modelled by a normal distribution with mean 160 grams, unknown standard deviation σ grams and $\mathrm{P}(Y < 150) = 0.05$.

(i) Determine the value of σ. Give your answer to two decimal places.

[3 marks]

(ii) *RichAV* soap is sold in packs of 3 bars. The bars in a pack may be assumed to be a random sample.

Determine the probability that:

(A) the weight of **each** of the 3 bars in a randomly selected pack is more than 150 grams;

5(a)	**Accept the equivalent percentage answers with %-sign (see GN5)**			
(i)	P(X < 105) = **0.5 or 1/2 or half or 50%**	B1	(1)	CAO; accept nothing else but ignore zeros after 0.5 (eg 0.50) Ignore additional words providing that they are not contradictory
(ii)	P($X \neq$ 100) = **1 or one or unity or 100%**	B1	(1)	CAO; accept nothing else but ignore zeros after decimal point (eg 1.00) Ignore additional words providing that they are not contradictory (eg certain so = 1)
(iii)	P(X > 110) = $P\left(Z > \frac{110-105}{4}\right)$ =	M1		Standardising **110** with **105** and **4** but allow (105 – 110)
	P(Z > 1.25) = 1 – 0.89435 = **0.105 to 0.106**	A2	(3)	AWFW (0.10565)
SCs	1 Answer of 0.894 to 0.895 ⇒ M1 A1 2 **Correct seen** standardisation with 0 < **incorrect** answer < 0.5 ⇒ M1 A1 3 **Incorrect or no seen** standardisation with 0 < **incorrect** answer < 0.5 ⇒ M1 A0			
(iv)	P(102 < X < 108) = P(**–0.75** < Z < **0.75**)	M1		CAO –0.75 and +0.75
	= (p – (1 – p)) or (2p – 1)	M1		OE; 0 < p < 1 Independent of previous M1
	= 0.77337 – (1 – 0.77337)			
	= **0.546 to 0.547**	A1	(3)	AWFW (0.54674)
			8	
(b) (i)	5% (0.05) ⇒ z = **1.64 to 1.65**	B1		AWFW (1.64485) Seen; ignore sign
	$\frac{\pm((150 \text{ or } 170)-160)}{\sigma / s} = \pm(1.64 \text{ to } 1.65)$	M1		Standardising 150 with 160 and σ / s; allow $(160-(150 \text{ or } 170))$ **and** equating to ±(1.64 to 1.65) **and** with **consistent** signs
	σ / s = **6.06 or 6.08 or 6.10**	A1	3	CAO (6.07957) Seen incorrect rounding ⇒ A0
(b)(ii) (A)	P(Y > 150) = 1 – 0.05 = 0.95			
	P(Y_1 & Y_2 & Y_3 > 150) = 0.95^3			
	= **0.857 to 0.858**	B1	(1)	AWFW (0.857375)
Note	1 A calculation of P(Y > 150) = p followed by p^3 ⇒ B1 only if result falls within above range			

The random variable $Z \sim N(0, 1)$

A is the event $Z > 1.1$
B is the event $Z > -1.9$
C is the event $-1.5 < Z < 1.5$

(a) Find

(i) $P(A)$

(ii) $P(B)$

(iii) $P(C)$

(iv) $P(A \cup C)$

(6)

The random variable X has a normal distribution with mean 21 and standard deviation 5

(b) Find the value of w such that $P(X > w \mid X > 28) = 0.625$

(6)

The heights of adult females are normally distributed with mean 160 cm and standard deviation 8 cm.

(a) Find the probability that a randomly selected adult female has a height greater than 170 cm.

(3)

Any adult female whose height is greater than 170 cm is defined as tall.

An adult female is chosen at random. Given that she is tall,

(b) find the probability that she has a height greater than 180 cm.

(4)

Half of tall adult females have a height greater than h cm.

(c) Find the value of h.

(5)

6.	**(a)(i)**	$P(A) = P(Z > 1.1) = 1 - 0.8643 = \underline{\mathbf{0.1357}}$ (accept awrt 0.136)	B1
	(ii)	$P(B) = P(Z > -1.9) = \underline{\mathbf{0.9713}}$ (accept awrt 0.971)	B1
	(iii)	$P(C) = [P(-1.5 < Z < 1.5)] = 0.9332 - (1 - 0.9332)$ or $(0.9332 - 0.5) \times 2$ $= \underline{\mathbf{0.8664}}$ (accept awrt 0.866)	M1 A1
	(iv)	$P(A \cup C) = P(Z > -1.5)$ or $P(Z < 1.5)$ or $= P(A) + P(C) - P(A \cap C) = "0.1357" + "0.8664" - (0.9332 - 0.8643)$ $= \underline{\mathbf{0.9332}}$ (accept awrt 0.933)	M1 A1 (6)
	(b)	$[P(X > w \mid X > 28) =] \frac{P(X > w)}{P(X > 28)} = [0.625]$	M1
		$P(X > 28) = P\left(Z > \frac{28-21}{5}\right) = P(Z > 1.4) = [0.0808 \text{ calc: } 0.80756..]$	M1
		$P(X > w) = 0.0808 \times 0.625 \ (= 0.0505)$ or $(P(X < w) = 0.9495)$	A1
		$\frac{w-21}{5} = 1.64$	M1 B1
		$w =$ awrt $\underline{\mathbf{29.2}}$	A1 (6) (12 marks)

7	(a)	The random variable $H \sim$ height of females $P(H > 170) = P\left(Z > \frac{170-160}{8}\right)$ $[= P(Z > 1.25)]$	M1
		$= 1 - 0.8944$	M1
		$= 0.1056$ (calc 0.1056498…) **awrt 0.106** (accept 10.6%)	A1 **(3)**
	(b)	$P(H > 180) = P\left(Z > \frac{180-160}{8}\right)$ $[= 1 - 0.9938]$	M1
		$= 0.0062$ (calc 0.006209…) awrt 0.0062 or $\frac{31}{5000}$	A1
		$[P(H > 180 \mid H > 170)] = \frac{0.0062}{0.1056}$	M1
		$= 0.0587$ (calc 0.0587760…) **awrt 0.0587 or 0.0588**	A1 **(4)**
	(c)	$P(H > h \mid H > 170)\ (= 0.5)$ or $\frac{P(H > h)}{P(H > 170)} (= 0.5)$	M1
		$[P(H > h)] = 0.5 \times "0.1056" = 0.0528$ (calc 0.0528249…) or $[P(H < h)] = 0.9472$	A1ft
		$\frac{h-160}{8} = 1.62$ (calc 1.6180592…)	M1 B1
		$h =$ awrt 173 cm **awrt 173**	A1 (5)
			Total 12

The time taken, in minutes, by children to complete a mathematical puzzle is assumed to be normally distributed with mean μ and standard deviation σ. The puzzle can be completed in less than 24 minutes by 80% of the children. For 5% of the children it takes more than 28 minutes to complete the puzzle.

(a) Show this information on the Normal curve below.

(2)

(b) Write down the percentage of children who take between 24 minutes and 28 minutes to complete the puzzle.

(1)

(c) (i) Find two equations in μ and σ.

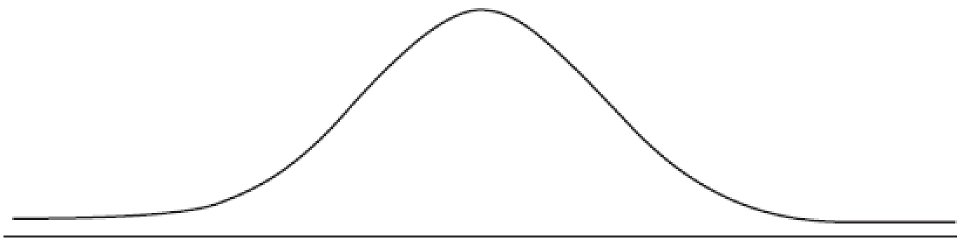
(ii) Hence find, to 3 significant figures, the value of μ and the value of σ.

(7)

A child is selected at random.

(d) Find the probability that the child takes less than 12 minutes to complete the puzzle.

(3)



The time, in minutes, taken to fly from London to Malaga has a normal distribution with mean 150 minutes and standard deviation 10 minutes.

(a) Find the probability that the next flight from London to Malaga takes less than 145 minutes.

(3)

The time taken to fly from London to Berlin has a normal distribution with mean 100 minutes and standard deviation d minutes.

Given that 15% of the flights from London to Berlin take longer than 115 minutes,

(b) find the value of the standard deviation d.

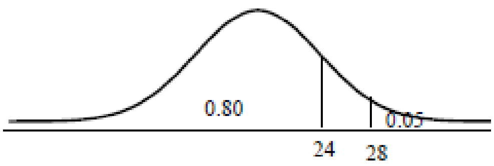
(4)

The time, X minutes, taken to fly from London to another city has a normal distribution with mean μ minutes.

Given that $P(X < \mu - 15) = 0.35$

(c) find $P(X > \mu + 15 \mid X > \mu - 15)$.

(3)

Question	Scheme		Marks
6. (a)		24 and 28 (above the mean) For 0.80 and 0.05 (clearly indicated)	B1 B1 **(2)**
(b)	15%		B1 **(1)**
(c)(i)	$\frac{(28-\mu)}{\sigma}=1.64(49)$ or $\frac{(24-\mu)}{\sigma}=0.84(16)$		M1
	0.8416 and 1.6449 seen		B1
	$\mu=28-1.64(49)\sigma$, $\mu=24-0.84(16)\sigma$		A1,A1
(ii)	$24-0.8416\sigma=28-1.6449\sigma$	eliminating μ or σ	M1
	$\sigma=4.9794597...$	**awrt** 4.98	A1
	$\mu=19.809286...$	**awrt** 19.8	A1 **(7)**
(d)	$z=\frac{(12-'19.8...')}{'4.97...'}$		M1
	$\text{P}(Z<-1.57)=1-\text{P}(Z<1.57)$		dM1
	$1-0.9418=0.0582$	**awrt** 0.06	A1 **(3)**
			[Total 13]

Question	Scheme		Marks
4. (a)	$[\text{P}(M<145)=]\ \text{P}\left(Z<\frac{145-150}{10}\right)$		M1
	$=\text{P}(Z<-0.5)$ or $\text{P}(Z>0.5)$		A1
	= **awrt** <u>**0.309**</u>		A1 (3)
(b)	$[\text{P}(B>115)=0.15\Rightarrow]\ \frac{115-100}{d}=1.0364$	(Calc gives 1.036433...)	M1B1A1
	<u>***d* = 14.5**</u>	(Calc gives 14.4727...)	A1 (4)
(c)	$[\text{P}(X>\mu+15 \mid X>\mu-15)=]\ \frac{\text{P}(X>\mu+15)}{\text{P}(X>\mu-15)}$		M1
	$=\frac{0.35}{1-0.35}$		A1
	$=\frac{7}{13}$ or <u>**awrt 0.538**</u>		A1 (3)
			[10]

The weight, in grams, of beans in a tin is normally distributed with mean μ and standard deviation 7.8

Given that 10% of tins contain less than 200 g, find

(a) the value of μ

(3)

(b) the percentage of tins that contain more than 225 g of beans.

(3)

The machine settings are adjusted so that the weight, in grams, of beans in a tin is normally distributed with mean 205 and standard deviation σ.

(c) Given that 98% of tins contain between 200 g and 210 g find the value of σ.

(4)

The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(a) Find $P(L > 127)$.

(3)

(b) Find the value of d such that $P(L < d) = 0.10$

(3)

Alice is about to go on a 6 hour journey.
Given that it is 127 hours since Alice last charged her phone,

(c) find the probability that her phone will not need charging before her journey is completed.

(4)

1 The random variable T is normally distributed with mean μ and standard deviation σ. It is given that $P(T > 80) = 0.05$ and $P(T > 50) = 0.75$. Find the values of μ and σ. [6]

6.	**(a)**	[Let X be the amount of beans in a tin. $P(X < 200) = 0.1$]	
		$\frac{200-\mu}{7.8} = -1.2816$ [calc gives 1.28155156…]	M1 B1
		$\mu = 209.996….$ awrt 210	A1
			(3)
	(b)	$P(X > 225) = P\left(Z > \frac{225-"210"}{7.8}\right)$	M1
		$= P(Z > 1.92)$ or $1 - P(Z < 1.92)$ (allow 1.93)	A1
		$= 1 - 0.9726$ $= 0.0274$ (or better) [calc gives 0.0272037…]	
		$= 0.0274$	
		= awrt **2.7%** allow **0.027**	A1
			(3)
	(c)	[Let Y be the new amount of beans in a tin]	
		$\frac{210-205}{\sigma} = 2.3263$ or $\frac{200-205}{\sigma} = -2.3263$ [calc gives 2.3263478…]	M1 B1
		$\sigma = \frac{5}{2.3263}$	dM1
		$\sigma = 2.15$ (2.14933…)	A1
			(4)
			(10 marks)

4.			
	(a)	$\frac{127-100}{15}$	M1
		So $P(L > 127) = P(Z > 1.8)$ or $1 - P(Z < 1.8)$ o.e.	A1
		$= 1 - 0.9641 = \underline{0.0359}$ (awrt $\underline{0.0359}$)	A1
			(3)
	(b)	$\frac{d-100}{15} = -1.2816$ (Calculator gives $-1.2815515…$)	M1, B1
		$d = 80.776$ (awrt $\underline{80.8}$)	A1
			(3)
	(c)	Require $P(L > 133 \mid L > 127)$	M1
		$= \left[\frac{P(L > 133)}{P(L > 127)}\right] = \frac{P(Z > 2.2)}{P(L > 127)}$	dM1
		$= \left[\frac{1-0.9861}{1-0.9641}\right] = \frac{0.0139}{[0.0359]}$	A1
		$= 0.3871...$ = awrt $\underline{0.39}$	A1
			(4)
	S.C.	An attempt at $P(L < 133 \mid L > 127)$ that leads to awrt 0.61 (M0M1A0A0)	
			10

1	$\frac{80-\mu}{\sigma} = \Phi^{-1}(0.95) = 1.645$ $\frac{\mu-50}{\sigma} = \Phi^{-1}(0.75) = 0.674(5)$ Solve simultaneously $\mu = 58.7$, $\sigma = 12.9$	M1 B1 A1 M1 A1 A1	 **6**	Standardise once with Φ^{-1}, allow σ^2, cc Both 1.645 (1.64, 1.65) and [0.674, 0.675], ignore signs Both equations correct apart from wrong z, *not* 1–1.645 Solve two standardised equations μ, a.r.t 58.7 σ, a.r.t. 12.9 [*not* σ^2] [σ^2: M1B1A0M1A1A0]

The heights of an adult female population are normally distributed with mean 162 cm and standard deviation 7.5 cm.

(a) Find the probability that a randomly chosen adult female is taller than 150 cm.

(3)

Sarah is a young girl. She visits her doctor and is told that she is at the 60th percentile for height.

(b) Assuming that Sarah remains at the 60th percentile, estimate her height as an adult.

(3)

The heights of an adult male population are normally distributed with standard deviation 9.0 cm.

Given that 90% of adult males are taller than the mean height of adult females,

(c) find the mean height of an adult male.

(4)

A manufacturer fills jars with coffee. The weight of coffee, W grams, in a jar can be modelled by a normal distribution with mean 232 grams and standard deviation 5 grams.

(a) Find $\mathrm{P}(W < 224)$.

(3)

(b) Find the value of w such that $\mathrm{P}(232 < W < w) = 0.20$

(4)

Two jars of coffee are selected at random.

(c) Find the probability that only one of the jars contains between 232 grams and w grams of coffee.

(3)

6. (a)	$[z=]\pm\left(\frac{150-162}{7.5}\right)$		M1	
	$[z=]-1.6$		A1	
	$[P(F>150)=P(Z>-1.6)=]=0.9452(0071\ldots)$	**awrt <u>0.945</u>**	A1	(3)
(b)		$z=\pm 0.2533$ (or better seen)	B1	
	$(\pm)\frac{s-162}{7.5}=0.2533(47\ldots)$		M1	
	$s=163.9$	**awrt <u>164</u>**	A1	(3)
(c)		$z=\pm 1.2816$ (or better seen)	B1	
	$\frac{162-\mu}{9}=-1.2815515\ldots$		M1 A1	
	$\mu=173.533\ldots$	**awrt <u>174</u>**	A1	(4)
				[10]

7 (a)	$P(W<224)\quad = P\left(z<\frac{224-232}{5}\right)$		M1	
	$= P(z<-1.6)$			
	$= 1 - 0.9452$		M1	
	$= 0.0548$	awrt 0.0548	A1	
				(3)
(b)	$0.5-0.2=0.3$	0.3 or 0.7 seen	M1	
	$\frac{w-232}{5}=0.5244$	0.5244 seen	B1; M1	
	$w=234.622$	awrt 235	A1	
				(4)
(c)	$0.2\times(1-0.2)$		M1	
	$2\times 0.8\times(1-0.8)=0.32$		M1 A1	
				(3)
				Total 10

The random variable $X \sim N(\mu, 5^2)$ and $P(X < 23) = 0.9192$

(a) Find the value of μ.

(4)

(b) Write down the value of $P(\mu < X < 23)$.

(1)

Past records show that the times, in seconds, taken to run 100 m by children at a school can be modelled by a normal distribution with a mean of 16.12 and a standard deviation of 1.60

A child from the school is selected at random.

(a) Find the probability that this child runs 100 m in less than 15 s.

(3)

On sports day the school awards certificates to the fastest 30% of the children in the 100 m race.

(b) Estimate, to 2 decimal places, the slowest time taken to run 100 m for which a child will be awarded a certificate.

(4)

The weight, X grams, of soup put in a tin by machine A is normally distributed with a mean of 160 g and a standard deviation of 5 g.
A tin is selected at random.

(a) Find the probability that this tin contains more than 168 g.

(3)

The weight stated on the tin is w grams.

(b) Find w such that $P(X < w) = 0.01$

(3)

The weight, Y grams, of soup put into a carton by machine B is normally distributed with mean μ grams and standard deviation σ grams.

(c) Given that $P(Y < 160) = 0.99$ and $P(Y > 152) = 0.90$ find the value of μ and the value of σ.

(6)

2. (a)		awrt ± 1.40	B1
	$\frac{23-\mu}{5} = "1.40"$ (o.e)		M1A1ft
	$\mu = 16$	(or awrt 16.0)	A1
			(4)
(b)	<u>0.4192</u>		B1
			(1)
			5

4. (a)	$(z=\pm)\frac{15-16.12}{1.6}(=-0.70)$		M1
	$P(Z < -0.70) = 1 - 0.7580$		M1
	$= \underline{0.2420}$	**(awrt 0.242)**	A1
			(3)
(b)	[P($T < t$)=0.30 implies] $z = \frac{t-16.12}{1.6} = -0.5244$		M1 A1
	$\frac{t-16.12}{1.6} = -0.5244 \Rightarrow t = 16.12 - 1.6 \times "0.5244"$		M1
	$t =$ awrt <u>15.28</u> (allow awrt 15.28/9)		A1
			(4)
			7

8. (a)	$P(X > 168) = P\left(Z > \frac{168-160}{5}\right)$		M1
	$= P(Z > 1.6)$		A1
	$= 0.0548$	**awrt 0.0548**	A1
			(3)
(b)	$P(X < w) = P\left(Z < \frac{w-160}{5}\right)$		
	$\frac{w-160}{5} = -2.3263$		M1 B1
	$w = 148.37$	**awrt 148**	A1
			(3)
(c)	$\frac{160-\mu}{\sigma} = 2.3263$		M1 B1
	$\frac{152-\mu}{\sigma} = -1.2816$		B1
	$160 - \mu = 2.3263\sigma$		
	$152 - \mu = -1.2816\sigma$		
	$8 = 3.6079\sigma$		M1
	$\sigma = 2.21\ldots.$	**awrt 2.22**	A1
	$\mu = 154.84\ldots$	**awrt 155**	A1 (6)
			[12]

The distances travelled to work, D km, by the employees at a large company are normally distributed with $D \sim \mathrm{N}(30, 8^2)$.

(a) Find the probability that a randomly selected employee has a journey to work of more than 20 km.

(3)

(b) Find the upper quartile, Q_3, of D.

(3)

(c) Write down the lower quartile, Q_1, of D.

(1)

An outlier is defined as any value of D such that $D < h$ or $D > k$ where

$$h = Q_1 - 1.5 \times (Q_3 - Q_1) \quad \text{and} \quad k = Q_3 + 1.5 \times (Q_3 - Q_1)$$

(d) Find the value of h and the value of k.

(2)

An employee is selected at random.

(e) Find the probability that the distance travelled to work by this employee is an outlier.

(3)

The heights of a population of women are normally distributed with mean μ cm and standard deviation σ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information.

(3)

(b) Show that $\mu = 154 + 1.6449\sigma$.

(3)

(c) Obtain a second equation and hence find the value of μ and the value of σ.

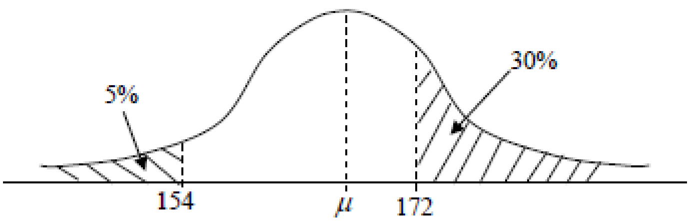
(4)

A woman is chosen at random from the population.

(d) Find the probability that she is taller than 160 cm.

(3)

Q7	(a)	$P(D > 20) = P\left(Z > \frac{20-30}{8}\right)$	M1	
		$= P(Z > -1.25)$	A1	
		$= \underline{0.8944}$ **awrt 0.894**	A1	(3)
	(b)	$P(D < Q_3) = 0.75$ so $\frac{Q_3 - 30}{8} = 0.67$	M1 B1	
		$Q_3 =$ **awrt** $\underline{\mathbf{35.4}}$	A1	(3)
	(c)	$35.4 - 30 = 5.4$ so $Q_1 = 30 - 5.4 =$ **awrt** $\underline{\mathbf{24.6}}$	B1ft	(1)
	(d)	$Q_3 - Q_1 = 10.8$ so $1.5(Q_3 - Q_1) = 16.2$ so $Q_1 - 16.2 = h$ or $Q_3 + 16.2 = k$	M1	
		$h=$**8.4 to 8.6** and $k=$ **51.4 to 51.6** both	A1	(2)
	(e)	$2P(D > 51.6) = 2P(Z > 2.7)$	M1	
		$= 2[1 - 0.9965] =$ awrt $\underline{\mathbf{0.007}}$	M1 A1	(3)

Q7	(a)	 bell shaped, must have inflexions	B1	
		154,172 on axis	B1	
		5% and 30%	B1	(3)
	(b)	$P(X < 154) = 0.05$		
		$\frac{154-\mu}{\sigma} = -1.6449$ or $\frac{\mu - 154}{\sigma} = 1.6449$	M1 B1	
		$\mu = 154 + 1.6449\sigma$ **given**	A1 cso	(3)
	(c)	$172 - \mu = 0.5244\sigma$ or $\frac{172-\mu}{\sigma} = 0.5244$ (allow $z = 0.52$ or better here but must be in an equation)	B1	
		Solving gives $\sigma = 8.2976075$ (**awrt 8.30**) and $\mu = 167.64873$ (**awrt 168**)	M1 A1 A1	(4)
	(d)	P(Taller than 160cm) $= P\left(Z > \frac{160-\mu}{\sigma}\right)$	M1	
		$= P(Z < 0.9217994)$	B1	
		$= 0.8212$ **awrt 0.82**	A1	(3)

The lifetimes of bulbs used in a lamp are normally distributed.
A company X sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

(a) Find the probability of a bulb, from company X, having a lifetime of less than 830 hours.

(3)

(b) In a box of 500 bulbs, from company X, find the expected number having a lifetime of less than 830 hours.

(2)

A rival company Y sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

(c) Find the standard deviation of the lifetimes of bulbs from company Y.

(4)

Both companies sell the bulbs for the same price.

(d) State which company you would recommend. Give reasons for your answer.

(2)

The random variable X has a normal distribution with mean 30 and standard deviation 5.

(a) Find $P(X < 39)$.

(2)

(b) Find the value of d such that $P(X < d) = 0.1151$

(4)

(c) Find the value of e such that $P(X > e) = 0.1151$

(2)

(d) Find $P(d < X < e)$.

(2)

8. (a)	Let the random variable X be the lifetime in hours of bulb			
	$P(X < 830) = P(Z < \frac{\pm(830-850)}{50})$	Standardising with 850 and 50	M1	
	$= P(Z < -0.4)$			
	$= 1 - P(Z < 0.4)$	Using 1-(probability>0.5)	M1	
	$= 1 - 0.6554$			
	$= 0.3446$ or 0.344578 by calculator	awrt 0.345	A1	(3)
(b)	0.3446×500	Their (a) x 500	M1	
	$= 172.3$	Accept 172.3 or 172 or 173	A1	(2)
(c)	Standardise with 860 and σ and equate to z value $\frac{\pm(818-860)}{\sigma} = z$ value		M1	
	$\frac{818-860}{\sigma} = -0.84(16)$ or $\frac{860-818}{\sigma} = 0.84(16)$ or $\frac{902-860}{\sigma} = 0.84(16)$ or equiv.		A1	
		$\pm 0.8416(2)$	B1	
	$\sigma = 49.9$	50 or awrt 49.9	A1	(4)
(d)	Company Y as the ***mean*** is greater for Y.	both	B1	
	They have (approximately) the same ***standard deviation*** or ***sd***		B1	(2)
			(11 marks)	

6 (a)	$P(X < 39) = P\left(Z < \frac{39-30}{5}\right)$		M1	
	$= P(Z < 1.8) = \underline{0.9641}$	(allow awrt 0.964)	A1	(2)
(b)	$P(X < d) = P\left(Z < \frac{d-30}{5}\right) = 0.1151$			
	$1 - 0.1151 = 0.8849$		M1	
	$\Rightarrow \quad z = -1.2$	(allow $\pm$ 1.2)	B1	
	$\therefore \frac{d-30}{5} = -1.2$	$\underline{d = 24}$	M1A1	(4)
(c)	$P(X > e) = 0.1151$ so $e = \mu + (\mu$ - their $d)$ or $\frac{e-30}{5} = 1.2$ or $-$ their z		M1	
		$\underline{e = 36}$	A1	(2)
(d)	$P(d < X < e) = 1 - 2 \times 0.1151$		M1	
	$= 0.7698$	AWRT $\underline{0.770}$	A1	(2)
				[10]

2 The annual salaries of employees in a company have mean £30 000 and standard deviation £12 000.

(i) Assuming a normal distribution, calculate the probability that the salary of one randomly chosen employee lies between £20 000 and £24 000. [4]

(ii) The salary structure of the company is such that a small number of employees earn much higher salaries than the others. Explain what this suggests about the use of a normal distribution to model the data. [2]

1 The random variable H has the distribution $N(\mu, \sigma^2)$. It is given that $P(H < 105.0) = 0.2420$ and $P(H > 110.0) = 0.6915$. Find the values of μ and σ, giving your answers to a suitable degree of accuracy. **[6]**

6 The continuous random variable X has the distribution $N(\mu, \sigma^2)$.

(i) Each of the three following sets of probabilities is impossible. Give a reason in each case why the probabilities cannot both be correct. (You should not attempt to find μ or σ.)

(a) $P(X > 50) = 0.7$ and $P(X < 50) = 0.2$ **[1]**

(b) $P(X > 50) = 0.7$ and $P(X > 70) = 0.8$ **[1]**

(c) $P(X > 50) = 0.3$ and $P(X < 70) = 0.3$ **[1]**

(ii) Given that $P(X > 50) = 0.7$ and $P(X < 70) = 0.7$, find the values of μ and σ. **[4]**

2 The mass, in kilograms, of a packet of flour is a normally distributed random variable with mean 1.03 and variance σ^2. Given that 5% of packets have mass less than 1.00 kg, find the percentage of packets with mass greater than 1.05 kg. **[6]**

2 (i)	$\Phi\left(\frac{24-30}{12}\right)-\Phi\left(\frac{20-30}{12}\right)$	M1 A1		Standardise one, allow √12, 12^2, √n Both standardisations correct, allow cc here
	$= \Phi(-0.5) - \Phi(-0.833)$ $= (1 - 0.6915) - (1 - 0.7976)$ = **0.1061**	M1 A1	4	Correct handling of tails [0.3085 – 0.2024] Answer, a.r.t. 0.106, c.a.o.
(ii)	Not symmetrical (skewed) Therefore inappropriate	M1 A1	2	Any comment implying not symmetric Conclude "not good model" [Partial answer: B1]

1	$\frac{105.0-\mu}{\sigma} = -0.7; \frac{110.0-\mu}{\sigma} = -0.5$	M1 A1 B1		Standardise once, equate to Φ^{-1}, allow σ^2 Both correct including signs & σ, no cc (continuity correction), allow wrong z Both correct z-values. "1 –" errors: M1A0B1
	Solve: $\sigma = 25$ $\mu = 122.5$	M1 A1 A1	6	Get either μ or σ by solving simultaneously σ a.r.t. 25.0 $\mu = 122.5 \pm 0.3$ or 123 if clearly correct, allow from σ^2 but *not* from $\sigma = -25$.

6 (i)	(a) Probabilities don't total 1	B1	1	Equivalent statement
	(b) P(> 70) must be < P(> 50)	B1	1	Equivalent statement
	(c) P(> 50) = 0.3 ⇒ μ < 50 P(< 70) = 0.3 ⇒ μ > 70	B1	1	Any relevant valid statement, e.g. "P(< 50) = 0.7 but P(< 50) must be < P(< 70)"
(ii)	μ = 60 by symmetry	B1		μ = 60 obtained at any point, allow from Φ
	$\frac{10}{\sigma} = \Phi^{-1}(0.7) = 0.524(4)$	M1		One standardisation, equate to Φ^{-1}, not 0.758
	σ = 10/0.5243	B1		$\Phi^{-1} \in [0.524, 0.5245]$ seen
	= **19.084**	A1	4	σ in range [19.07, 19.1], e.g. 19.073

2	$\frac{1.03-1.00}{\sigma} = 1.645$	M1dep* A1 B1		Standardise and equate to Φ^{-1}, allow wrong sign, σ^2, 1–, cc etc All correct apart possibly from value of Φ^{-1} 1.645 seen anywhere, allow –1.645, can be implied
	[$\sigma = 0.0182\ldots \approx {}^6/_{329}$]	*M1		Solve to find σ, or eliminate σ, dependent on first M1
	$1-\Phi\left(\frac{1.05-1.03}{\sigma}\right) = 1-\Phi(1.0966)$	M1		Standardise with $\mu = 1.03$, use Φ, answer < 0.5, allow √ errors
	$= 1 - 0.8635$ = **0.1365** or 13.6(5)%	A1	6	Final answer in range [0.1355, 0.137] or [13.55%, 13.7%], must be from positive σ, not from σ^2 0.1333 from $\sigma = 0.018$ is 5+A0

(a) The mass of peanuts, X, in a large tub can be modelled by a normal random variable with a mean of 730 grams and a standard deviation of 20 grams.

Determine the probability that the mass of peanuts in a randomly selected large tub is:

(i) more than 757 grams;

[3 marks]

(ii) between 706 grams and 730 grams.

[2 marks]

(b) The mass of cashew nuts, Y, in a small tub can be modelled by a normal random variable with a mean of μ grams and a standard deviation of σ grams.

(i) Given that $\mathrm{P}(Y > 320) = 0.99$ and $\mathrm{P}(Y > 350) = 0.50$, find, to the nearest gram, values for μ and σ.

[4 marks]

(ii) Hence find the value of w such that $\mathrm{P}(\mu - w < Y < \mu + w) = 0.80$.

[3 marks]

(d) During a follow-up investigation, the following data were collected.

Plant	**N**	**O**
x	150	200
y	4.63	4.89

Give a **general reason** and a **specific reason based on numerical support** why your equation calculated in part **(b)(i)** should not be used to estimate the yield of a tomato plant when given between $150\,\mathrm{mg/l}$ and $200\,\mathrm{mg/l}$ of potassium in the liquid feed.

[2 marks]

(a)(i)	$P(X > 757) = P\left(Z > \frac{757 - 730}{20}\right) =$	M1		Standardising **757** with **730** and **20** but allow $(730 - 757)$
	$P(Z > 1.35) = 1 - P(Z < 1.35) = 1 - 0.91149$	m1		Area change; may be implied but only by the **correct** answer
	$= \underline{0.088 \text{ to } 0.089}$	A1	3	AWFW (0.08851)
(ii)	$P(706 < X < 730)$ $= P(\underline{-1.2} < Z < \underline{0})$ or $P(\underline{0} < Z < \underline{1.2})$	B1		CAO 0 and 1.2
	$= (0.5 - (1 - 0.88493))$ or $(0.88493 - 0.5)$			
	$= \underline{0.384 \text{ to } 0.386}$	B1	2	AWFW (0.38493)
(b) (i)	$\left(\frac{350 - \mu}{\sigma} = 0\right) \Rightarrow \mu = \underline{350}$	B1		CAO; this should result immediately from the two 'standard' simultaneous equations; other one is below for M1
	$0.99 \Rightarrow z = \underline{-2.32 \text{ to } -2.33}$	B1		AWFW; ignore sign (–2.3263)
	$\frac{320 - (350 \text{ or } \mu)}{\sigma} = \begin{pmatrix} \pm 2.05 \text{ to } \pm 2.06 \\ \pm 2.32 \text{ to } \pm 2.33 \\ \pm 2.57 \text{ to } \pm 2.58 \end{pmatrix}$	M1		Standardising **320** with (**350** or μ) and σ but allow ((350 or μ) – 320) **and** equating to one of 3 listed z-values but **allow inconsistent signs**
	$\Rightarrow \sigma = \underline{13}$	A1	4	CAO; (12.87 to 12.94) Penalise inconsistent signs here
(ii)	$P(Y < \mu + w) = 0.90 \Rightarrow z = \underline{1.28}$ or $P(Y > \mu - w) = 0.90 \Rightarrow z = \underline{-1.28}$	B1		AWRT; ignore sign (1.2816)
	$\frac{((350 \text{ or } \mu) + w) - (350 \text{ or } \mu)}{(12.87 \text{ to } 12.94) \text{ or } 13} = \begin{pmatrix} 0.84 \text{ AWRT} \\ 1.28 \text{ AWRT} \\ 1.64 \text{ to } 1.65 \end{pmatrix}$ or $w = (\text{listed } (+z\text{-value})) \times ((12.87 \text{ to } 12.94) \text{ or } 13)$ (OE)	M1		Standardising (w with **0**) (OE) and (**+12.8** to **+13**, even if A0 awarded in (b)(i)) **and** equating to one of 3 listed z-values **with consistent signs**
	$w = \underline{16.4 \text{ to } 16.7}$	A1	3	AWFW (16.47 or 16.66)

Still mineral water is supplied in 1.5-litre bottles. The actual volume, X millilitres, in a bottle may be modelled by a normal distribution with mean $\mu = 1525$ and standard deviation $\sigma = 9.6$.

(a) Determine the probability that the volume of water in a randomly selected bottle is:

(i) less than 1540 ml;

(ii) more than 1535 ml;

(iii) between 1515 ml and 1540 ml;

(iv) not 1500 ml.

[7 marks]

(b) The supplier requires that only 10 per cent of bottles should contain more than 1535 ml of water.

Assuming that there has been no change in the value of σ, calculate the reduction in the value of μ in order to satisfy this requirement. Give your answer to one decimal place.

[4 marks]

(c) Sparkling spring water is supplied in packs of six 0.5-litre bottles. The actual volume in a bottle may be modelled by a normal distribution with mean 508.5 ml and standard deviation 3.5 ml.

Stating a necessary assumption, determine the probability that:

(i) the volume of water in **each** of the 6 bottles from a randomly selected pack is more than 505 ml;

Q	Solution	Marks	Total	Comments
5(a)	**Accept the equivalent percentage answers with %-sign (see GN5)**			
(i)	$P(X < 1540) = P\left(Z < \frac{1540-1525}{9.6}\right)$	M1		Standardising 1540 with 1525 and 9.6 but allow (1525 – 1540)
	$= P(Z < 1.56(25)) =$ **0.94 to 0.942**	A1	(2)	AWFW (0.94091)
(ii)	$P(X > 1535) =$ $P(Z > 1.04(17)) = 1 - P(Z < 1.04)$	M1		Area change; can be implied by **any final answer** < 0.5
	$= 1 - 0.85122 =$ **0.148 to 0.15**	A1	(2)	AWFW (0.14878)
(iii)	$P(1515 < X < 1540) = P(-1.04 < Z < 1.56)$ $= 0.94091 - (1 - 0.85122)$ $=$ **0.79 to 0.793**	B2	(2)	AWFW (0.79213)
(iv)	$P(X \neq 1500) =$ **1 or one or unity or 100%**	B1	(1)	CAO; accept nothing else but ignore zeros after decimal point (eg 1.00) Ignore additional words providing that they are not contradictory (eg certain so = 1)
(b)	10% (0.1) $\Rightarrow z =$ **1.28**	B1		AWRT (1.2816) Seen; ignore sign
	$\frac{\pm(1535-(\mu \text{ or } \bar{x} \text{ or } x))}{9.6} = \pm(1.28 \text{ to } 1.29)$	M1		Standardising 1535 with $\mu / \bar{x}$ and 9.6; allow $((\mu \text{ or } \bar{x}) - 1535)$ and equating to $\pm(1.28 \text{ to } 1.29)$
	$\mu =$ **1522 to 1523**	A1		AWFW (1522.697)
	Reduction $= 1525 - 1522.7 =$ **2.3**	Adep1	4	**CAO (1 dp only)**; dependent on A1
(c)	Each **pack** contains a **random sample** of bottles **or** **Packs** contain **random samples** of bottles	B1	(1)	Must contain at least the 3 emboldened words and clearly infer that 'bottles in a pack are a random sample' **This mark can be scored anywhere in (c)**
(i)	$p = P(\text{bottle} > 505) = P\left(Z > \frac{505-508.5}{3.5}\right) =$			
	$P(Z > -1) = P(Z < 1) =$ **0.84**	B1		AWRT (0.84134) Can be implied by a **correct** answer
	$P(6 \text{ bottles} > 505) = p^6$	M1		Providing $0 < p < 1$
	$=$ **0.35 to 0.356**	A1	(3)	AWFW (0.35469)
Notes	1 Calculation of $(1 - 0.84134) = 0.15866 \Rightarrow$ B0			

(a) Wooden lawn edging is supplied in $1.8\,\text{m}$ length rolls. The actual length, X metres, of a roll may be modelled by a normal distribution with mean 1.81 and standard deviation 0.08.

Determine the probability that a randomly selected roll has length:

(i) less than $1.90\,\text{m}$;

(ii) greater than $1.85\,\text{m}$;

(iii) between $1.81\,\text{m}$ and $1.85\,\text{m}$.

[6 marks]

(b) Plastic lawn edging is supplied in $9\,\text{m}$ length rolls. The actual length, Y metres, of a roll may be modelled by a normal distribution with mean μ and standard deviation σ.

An analysis of a batch of rolls, selected at random, showed that

$$\text{P}(Y < 9.25) = 0.88$$

(i) Use this probability to find the value of z such that

$$9.25 - \mu = z \times \sigma$$

where z is a value of $Z \sim \text{N}(0, 1)$.

[2 marks]

(ii) Given also that

$$\text{P}(Y > 8.75) = 0.975$$

find values for μ and σ.

[4 marks]

5				**Accept percentage equivalent answers in (a) but see GN4**
(a)(i)	$P(X < 1.9) = P\left(Z < \frac{1.9 - 1.81}{0.08}\right)$	M1		Standardising 1.9 with 1.81 and 0.08 but allow (1.81 – 1.9)
	$= P(Z < 1.125) = \underline{0.87}$	A1	(2)	AWRT (0.86971)
(ii)	$P(X > 1.85) = P(Z > 0.5) = 1 - P(Z < 0.5)$	M1		Area change; can be implied by **any final answer < 0.5**
	$= 1 - 0.69146 = \underline{0.31}$	A1	(2)	AWRT (0.30854)
(iii)	$P(1.81 < X < 1.85)$ $= (0.691 \text{ to } 0.692) - 0.5$ **or** $= 0.5 - (0.308 \text{ to } 0.309)$	B1		Can be implied by a correct answer
	$= \underline{0.19}$	B1	(2)	AWRT (0.19146)
			6	
(b)(i)	$z = \text{or} < \frac{9.25 - \mu}{\sigma}$ **or** $9.25 = \mu + z\sigma$	M1		Either expression **or** with z replaced by 1.17 to 1.18 (AWFW)
	$0.88 \Rightarrow z = \underline{1.17 \text{ to } 1.18}$	B1	2	AWFW (ignore sign) (1.175)
(ii)	$P(Y > 8.75) = 0.975 \Rightarrow z = \underline{1.96}$	B1		AWRT (ignore sign)
	Thus $9.25 - \mu = +1.175\sigma$ $8.75 - \mu = -1.96\sigma$			(1.17 to 1.18) AWFW (ignore sign) (1.96) AWRT (ignore sign)
	giving $0.5 = 3.135\sigma$	M1		A valid method for solution of two equations that are correct except for signs of z-values (see Note **1**)
	$\sigma = \underline{0.16}$	Adep1		AWRT (0.15949) Dependent on **two fully correct equations** including signs of z-values
	$\mu = \underline{9 \text{ to } 9.1}$	Adep1	4	AWFW (9.06260)

A garden centre sells bamboo canes of nominal length 1.8 metres. The length, X metres, of the canes can be modelled by a normal distribution with mean 1.86 and standard deviation σ.

(a) Assuming that $\sigma = 0.04$, determine:

(i) $\mathrm{P}(X < 1.90)$;

(ii) $\mathrm{P}(X > 1.80)$;

(iii) $\mathrm{P}(1.80 < X < 1.90)$;

(iv) $\mathrm{P}(X \neq 1.86)$.

[7 marks]

(b) It is subsequently found that $\mathrm{P}(X > 1.80) = 0.98$.

Determine the value of σ.

[3 marks]

2	No MR or MC in this question			Accept %age equivalents in (a)(i) to (iii)
(a)	<u>Length, $X \sim \text{N}(1.86,\ 0.04^2)$</u>			
(i)	$\text{P}(X < 1.90) = \text{P}\left(Z < \dfrac{1.90-1.86}{0.04}\right)$	M1		Standardising 1.90 with 1.86 and 0.04 but allow $(1.86-1.90)$
	$= \text{P}(Z < 1) = \underline{0.841}$	A1	(2)	AWRT (0.84134)
(ii)	$\text{P}(X > 1.80) = \text{P}(Z > -1.5) = \text{P}(Z < 1.5)$	M1		Correct area change; **neither** 1.5 or correct standardising are required Can be implied by **final answer > 0.5**
	$= \underline{0.933}$	A1	(2)	AWRT (0.93319)
(iii)	$\text{P}(1.80 < X < 1.90) = \text{P}(Z<1) - \text{P}(Z<-1.5) =$ (i) − [1 − (ii)] **or** (ii) − [1 − (i)] **or** (i) + (ii) − 1	M1		OE; any **correct** difference in areas that results in answer **> 0** Can be implied by **correct answer** but see Notes
	$= \underline{0.774 \text{ to } 0.775}$	A1	(2)	AWFW (0.77453)
(iv)	$\text{P}(X \neq 1.86) = \underline{1 \text{ or one or unity or } 100\%}$	B1	(1)	CAO; accept nothing else but ignore zeros after decimal place (eg 1.00) Ignore additional words providing that they are not contradictory (eg certain so = 1)
Note	1 $\text{P}(X \neq 1.86) = \text{P}(Z \neq 0) \Rightarrow$ B0 unless followed by 1 OE			
			7	
(b)	$0.98 \Rightarrow z = \underline{2.05 \text{ to } 2.06}$	B1		AWFW; seen anywhere, ignore sign (2.0537)
	$\left(\dfrac{1.80-1.86}{\sigma}\right)$ </=/> $\left(\begin{matrix}-2.05 \text{ to } -2.06 \\ \textbf{or} \\ -2.32 \text{ to } -2.33\end{matrix}\right)$	M1		Standardising 1.80 with 1.86 and σ or s but allow $(1.86-1.80)$; **and** equating to a **z-value** in either range (*ignore sign*)
	σ </=/> $\underline{0.029 \text{ to } 0.03}$	A1	3	AWFW (0.02922) If working is shown, then there must be consistent signs throughout so, for example, $(1.80-1.86)/\sigma = +2.0537$ $\Rightarrow$ B1 M1 A0

The weight, X grams, of the contents of a tin of baked beans can be modelled by a normal random variable with a mean of 421 and a standard deviation of 2.5.

(a) Find:

(i) $P(X = 421)$;

(ii) $P(X < 425)$;

(iii) $P(418 < X < 424)$. *(6 marks)*

(b) Determine the value of x such that $P(X < x) = 0.98$. *(3 marks)*

(c) The weight, Y grams, of the contents of a tin of ravioli can be modelled by a normal random variable with a mean of μ and a standard deviation of 3.0.

Find the value of μ such that $P(Y < 410) = 0.01$. *(4 marks)*

	Weight, $X \sim N(421, 2.5^2)$			
2(a)(i)	$P(X = 421) =$ **0 or zero or nought or 0%**	B1		CAO; accept nothing else but ignore additional words providing that they are not contradictory (eg impossible so = 0)
(ii)	$P(X < 425) = P\left(Z < \frac{425 - 421}{2.5}\right)$	M1		Standardising 425 with 421 and 2.5 but allow (421 – 425)
	$= P(Z < 1.6) =$ **0.945 to 0.946**	A1		AWRT (0.94520)
(iii)	$P(418 < X < 424) = P(-a < Z < a) =$ $P(Z < a) - (1 - P(Z < a))$ **or** $2 \times P(Z < a) - 1$	M1		OE; $a = 1.2$ or correct standardising are **not required** May be implied by **0.885** (AWRT) **seen anywhere** **or by a correct answer**
	$= 0.885 - (1 - 0.885) = 0.885 - 0.115$ **or** $= 2 \times 0.885 - 1$	A1		AWRT (0.88493/0.11507) Implied by a **correct** answer
	$=$ **0.769 to 0.77**	A1	6	AWFW (0.76986)
(b)	$0.98 \Rightarrow z =$ **2.05 to 2.06**	B1		AWFW (2.0537)
	$\left(\frac{x - 421}{2.5}\right) = 2(.0)$ to 2.4	M1		Standardising x with 421 and 2.5 but allow $(421 - x)$; **and** equating to a **z-value** (*ignore sign*) May be implied by a **correct** answer
	$x =$ **426 to 426.3**	A1	3	AWFW (426.13) *Must be consistent signs throughout*
(c)	$0.01 \Rightarrow z =$ **–2.33 to –2.32**	B1		AWFW; (*ignore sign*) (–2.3263)
	$z = \left(\frac{410 - \mu}{3.0 \text{ or } 2.5}\right)$	M1		Standardising 410 with μ and (3.0 or 2.5) but allow $(\mu - 410)$
	$\left(\frac{410 - \mu}{3.0}\right) = -2.6$ to -2.3	A1		Equating to a **z-value** (*ignore sign*) May be implied by a **correct** answer
	$\mu =$ **417**	Adep1	4	AWRT (416.98) Dependent on previous A1 *Must be consistent signs throughout*

The volume of *Everwhite* toothpaste in a pump-action dispenser may be modelled by a normal distribution with a mean of 106 ml and a standard deviation of 2.5 ml.

Determine the probability that the volume of *Everwhite* in a randomly selected dispenser is:

(a) less than 110 ml; *(3 marks)*

(b) more than 100 ml; *(2 marks)*

(c) between 104 ml and 108 ml; *(3 marks)*

(d) **not** exactly 106 ml. *(1 mark)*

2				**In (a), ignore the inclusion of a lower limit of 0; it has no effect on the answer**
	Volume, $V \sim N(106,\ 2.5^2)$			
(a)	$P(V < 110) = P\left(Z < \frac{110-106}{2.5}\right)$	M1		Standardising 110 with 106 and 2.5; allow (106 – 110)
	$= P(Z < \underline{1.6})$	A1		CAO; ignore inequality and sign May be implied by a correct answer
	$= \underline{0.945}$	A1	3	AWRT (0.94520)
(b)	$P(V > 100) = P(Z > -2.4) = P(Z < +2.4)$	M1		Correct area change May be implied by a correct answer or by an answer > **0.5**
	$= \underline{0.991 \text{ to } 0.992}$	A1	2	AWFW (0.99180)
(c)	$P(104 < V < 108) = P(-a < Z < a) =$ $P(Z < a) - (1 - P(Z < a))$ **or** $2 \times P(Z < a) - 1$	M1		OE; $a = 0.8$ is not a requirement May be implied by **0.788** seen or by a correct answer
	$= 0.788 - (1 - 0.788) = 0.788 - 0.212$ **or** $= 2 \times 0.788 - 1$	A1		AWRT (0.78814/0.21186) Condone 0.211 May be implied by a correct answer
	$= \underline{0.576}$	A1	3	AWRT (0.57628)
(d)	$P(V \neq 106) = \underline{1 \text{ or one or unity or } 100\%}$	B1	1	CAO; accept nothing else but ignore additional words providing they are not contradictory (eg certain so = 1)

A machine, which cuts bread dough for loaves, can be adjusted to cut dough to any specified set weight. For any set weight, μ grams, the actual weights of cut dough are known to be approximately normally distributed with a mean of μ grams and a fixed standard deviation of σ grams.

It is also known that the machine cuts dough to within 10 grams of any set weight.

(a) Estimate, with justification, a value for σ. *(2 marks)*

(b) The machine is set to cut dough to a weight of 415 grams.

As a training exercise, Sunita, the quality control manager, asked Dev, a recently employed trainee, to record the weight of each of a random sample of 15 such pieces of dough selected from the machine's output. She then asked him to calculate the mean and the standard deviation of his 15 recorded weights.

Dev subsequently reported to Sunita that, for his sample, the mean was 391 grams and the standard deviation was 95.5 grams.

Advise Sunita on whether or not **each** of Dev's values is likely to be correct. Give numerical support for your answers. *(3 marks)*

(c) Maria, an experienced quality control officer, recorded the weight, y grams, of each of a random sample of 10 pieces of dough selected from the machine's output when it was set to cut dough to a weight of 820 grams. Her summarised results were as follows.

$$\sum y = 8210.0 \quad \text{and} \quad \sum (y - \bar{y})^2 = 110.00$$

Explain, with numerical justifications, why **both** of these values are likely to be correct. *(4 marks)*

7 (a)	$\sigma \approx \frac{10}{a}$ or $\frac{20}{b}$ or $\frac{\text{range}}{b}$ or $10c$ or $20d$	M1		OE; with $2 \le a \le 4$ $4 \le b \le 8$ or with c or d in equiv percentages Cannot be implied from a correct answer (justification required)
	<u>2.5 or 3.3(OE) or 5</u>	A1	2	
SC	Award B1 for only **2.5 or 3.3(OE) or 5** with no justification Award B0 for any other answer with no justification or with incorrect justification (eg $\sqrt{10}$ = 3.16)			
(b)	**Valid statement** involving: 391 and 405 **OR** 401 and 415 **OR** 24 and 10 **OR** 391 and 415 and 10/24 with linking statement	B1		Allow 'set weight' to imply 415 and/or 'mean' to imply 391 B0 for 10 linked to σ
	95.5 > (value of σ of **2.5** or **3.3(OE)** or **5**)	B1		Accept ≠ rather than > **Clear correct numerical** comparison
	Neither (likely to be) **correct**	Bdep1	3	Dependent on B1 B1
(c)	Mean or $\overline{y} = \frac{8210.0}{10}$ = <u>**821**</u> **OR** $\sum y$ = <u>**8200**</u>	B1		CAO;
	Variance $\frac{110.00}{9}$ = <u>**12.2**</u> or $\frac{110.00}{10}$ = <u>**11**</u> **OR** SD <u>**3.5**</u> or <u>**3.3**</u>	B1		AWRT CAO Award on value; ignore notation AWRT
	821 **is similar to/within 10 of** 820 **OR** 8210 **is within 100 of** 8200	B1		OE; **clear correct numerical** comparison of **821** with **820** Allow 'set weight' to imply 820 **Or** OE; **clear correct numerical** comparison of **8210** with **8200** but **do not** accept 'within 10' here
	3.5 or 3.3 is **similar to** **a value** of σ of **3.3(OE)** or **2.5**	B1	4	**Clear correct numerical** comparison

A general store sells lawn fertiliser in 2.5 kg bags, 5 kg bags and 10 kg bags.

(a) The actual weight, W kilograms, of fertiliser in a 2.5 kg bag may be modelled by a normal random variable with mean 2.75 and standard deviation 0.15.

Determine the probability that the weight of fertiliser in a 2.5 kg bag is:

(i) less than 2.8 kg;

(ii) more than 2.5 kg. *(5 marks)*

(b) The actual weight, X kilograms, of fertiliser in a 5 kg bag may be modelled by a normal random variable with mean 5.25 and standard deviation 0.20.

(i) Show that $\mathrm{P}(5.1 < X < 5.3) = 0.372$, correct to three decimal places. *(2 marks)*

(ii) A random sample of **four** 5 kg bags is selected. Calculate the probability that none of the four bags contains between 5.1 kg and 5.3 kg of fertiliser. *(2 marks)*

				limit of 0; it has no effect on either answer
(a)	<u>Weight, $W \sim N(2.75, 0.15^2)$</u>			
(i)	$P(W < 2.8) = P\left(Z < \frac{2.8 - 2.75}{0.15}\right)$	M1		Standardising 2.8 with 2.75 and 0.15; allow $(2.75 - 2.8)$
	$= P(Z <$ **$\underline{0.33 \text{ or } 1/3}$**$)$	A1		AWRT/CAO; ignore inequality and sign May be implied by a correct answer
	$=$ **$\underline{0.629 \text{ to } 0.633}$**	A1		AWFW (0.63056)
(ii)	$P(W > 2.5) = P(Z > -1.67) = P(Z < +1.67)$	M1		Correct area change May be implied by a correct answer or an answer > **0.5**
	$=$ **$\underline{0.951 \text{ to } 0.953}$**	A1	5	AWFW (0.95221)
(b)	<u>Weight, $X \sim N(5.25, 0.20^2)$</u>			
(i)	$P(5.1 < X < 5.3) = P(Z < 0.25) - P(Z < -0.75)$ **= 0.59871** **MINUS [(1 – 0.77337) or 0.22663]** = **<u>0.372(08)</u>**	 B1 B1	 2	Must have diff of 2 probs for each B1 Accept 0.599 Accept 0.773 or 0.227 AG; do **not** mark simply on answer
(ii)	$P(0 \text{ in } 4) = [1 - 0.372]^4$	M1		Accept $[1 - \text{c's (b)(i)}]^4$
	$= 0.628^4 =$ **$\underline{0.155 \text{ to } 0.156}$**	A1	2	AWFW (0.15554)

During June 2011, the volume, X litres, of unleaded petrol purchased per visit at a supermarket's filling station by private-car customers could be modelled by a normal distribution with a mean of 32 and a standard deviation of 10.

(a) Determine:

(i) $\mathrm{P}(X < 40)$;

(ii) $\mathrm{P}(X > 25)$;

(iii) $\mathrm{P}(25 < X < 40)$. *(7 marks)*

(b) Given that during June 2011 unleaded petrol cost £1.34 per litre, calculate the probability that the unleaded petrol bill for a visit during June 2011 by a private-car customer exceeded £65. *(3 marks)*

(c) Give **two** reasons, in context, why the model $\mathrm{N}(32, 10^2)$ is unlikely to be valid for a visit by **any** customer purchasing fuel at this filling station during June 2011. *(2 marks)*

3(a) (i)	Volume, $X \sim N(32,\ 10^2)$			
	$P(X < 40) = P\left(Z < \frac{40-32}{10}\right)$	M1		Standardising 40 with 32 and 10; allow (32 – 40)
	$= P(Z < 0.8)$	A1		CAO; ignore inequality and sign May be implied by a correct answer
	= 0.788	A1	3	AWRT (0.78814)
(ii)	$P(X > 25) = P(Z > -0.7)$ $= P(Z < +0.7)$	M1		Area change May be implied by a correct answer or an answer > 0.5
	= 0.758	A1	2	AWRT (0.75804)
(iii)	$P(25 < X < 40) =$ (i) – (1 – (ii)	M1		OE; allow new start ignoring (i) & (ii) Allow even if incorrect standardising providing 0 < answer < 1 May be implied by a correct answer
	= 0.78814 – (1 – 0.75804) = 0.546 **Note:** If (ii) is 0.242, then (0.788 – 0 242) = 0.546 ⇒ M0 A0	A1	2	AWRT (0.54618)
(b)	$P(B > £65) =$ $P\left(Z > \frac{48.5-32}{10}\right)$ or $P\left(Z > \frac{65-42.88}{13.4}\right)$	M1		Attempt to change from B to X using (48 to 49), 32 and 10 or Attempt to work with distribution of B using 65, (42.8 to 42.9) and 13.4
	$= P(Z > 1.65) = 1 - P(Z < 1.65)$	m1		Area change May be implied by a correct answer or an answer < 0.5
	= 1 – 0.95053 = 0.049 to 0.05(0)	A1	3	AWFW (0.04947)
(c)	Other fuels Other vehicles with an example (not other cars) Other types of customer Minimum purchase (policy) Purchases in integer/fixed £s Customers filling fuel cans	B2,1	2	Size of car/engine/fuel tank ⇒ B0 Price of fuel ⇒ B0 Customer paying capacity ⇒ B0 Must be two clearly different valid reasons for award of B2 Drivers and vehicles related ⇒ B1 eg lorry drivers & lorries