Mechanics Sector 1: Motion in One Dimension

Modelling

When we consider a real life situation, we will often ignore certain conditions, and therefore **model** the situation. Exam questions often use specific terminology to model with.

Example 1
Complete the table

Terminology	Modelling meaning	Result
Rigid		Object (e.g. rod) can be modelled in 1 dimension as a straight line
Light	Negligible mass (and therefore weight) relative to other masses	
Inextensible		The acceleration of the connected particles are the same in magnitude
Smooth	The friction between the object and the surface is negligible	
Particle	The object's dimensions are negligible	
Uniform		The centre of mass of the object is positioned at the centre of the object

Vectors and scalars

A scalar is something that only has a magnitude (size), a vector quantity is something that has a magnitude and a direction.

With vector quantities in one dimension, one direction is specified as positive. If you travel in the direction **opposite** to that it counts as **negative**.

Choosing the **initial direction of motion** as the positive direction is usually a good approach to take.

Scalar	Vector	Unit
Distance	Displacement	m
Speed	Velocity	m/s or ms^{-1}
Acceleration	Acceleration	m/s^2 or ms^{-2}

Definitions

Displacement – "**The length in a straight line between two points**"

Velocity – "**The change of displacement per unit time**" or
"The rate of change of displacement"

i.e. how far in displacement an object moves each second

Average speed and velocity

Average speed and average velocity can be found using the following equations

$$\textbf{\textit{Average speed}} = \frac{\textbf{\textit{Total distance}}}{\textbf{\textit{Total time}}} \qquad \textbf{\textit{Average velocity}} = \frac{\textbf{\textit{Total displacement}}}{\textbf{\textit{Total time}}}$$

Acceleration

Acceleration – "**The change in velocity per unit time**" or
"The rate of change of velocity"

i.e. how much the velocity of an object changes each second

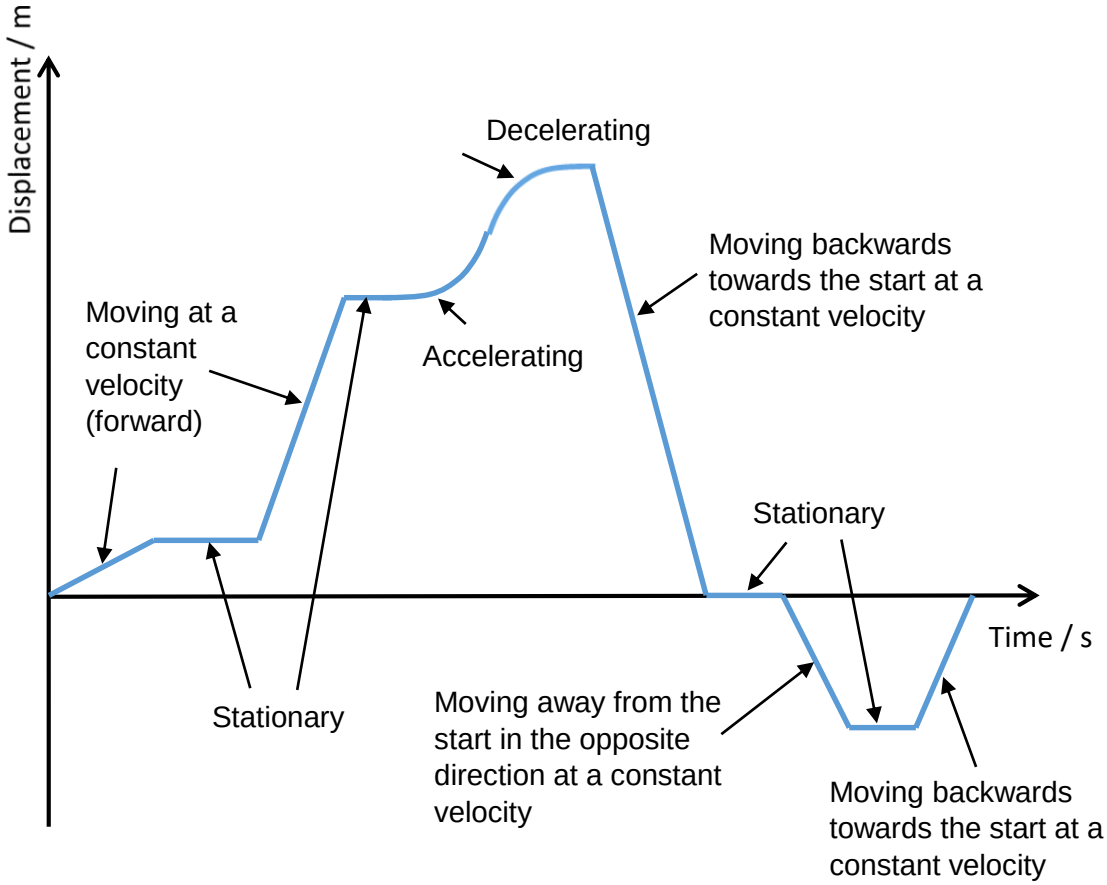
Example 2

A runner moves along a straight horizontal road at a constant speed of 6 ms^{-1} for 12s. She then runs back towards the start at a constant speed and covers a distance of 30m in 8s.

Find:

a) The speed that she is moving at in the second part of her journey
b) The total distance that she travelled in 20s.
c) Her displacement after 20s.
d) Her average speed
e) Her average velocity

Displacement – time graphs



A straight line that is not horizontal means that the object is travelling at a **constant velocity**.
The velocity is equal to the **gradient of the line**.

Therefore

- The steeper the line, the larger the magnitude of the velocity, as the gradient is larger.
- If the line is sloping downwards from left to right, the gradient is negative. This means that the object is travelling in the opposite direction to the direction that was initially considered as the positive direction.

A line that **curves upwards** with a positive gradient means that the object is **accelerating** (the gradient of the line is increasing, meaning that the velocity is increasing).

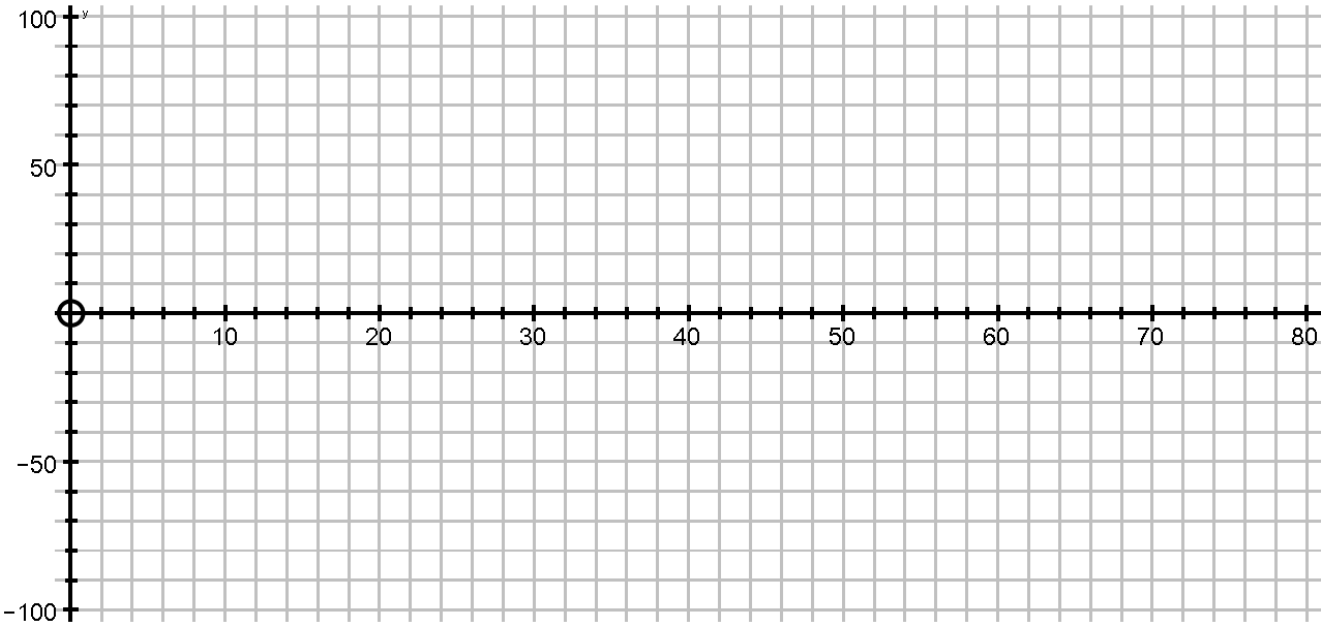
A line that **curves downwards** with a positive gradient (so the line is becoming more horizontal) means that the object is **decelerating** (the gradient of the line is decreasing, meaning that the velocity is decreasing).

A negative displacement means that the object is in a position that is in the opposite direction from the start point to the direction that was initially considered positive.

NB
If the graph is a distance time graph then the y-axis will not have a negative part to it

- ⇨ Any section of the graph below the time axis (i.e. the x-axis) will be reflected in that axis so that the section is now above the time axis.

Example 3



Bob lives on a straight road. He walks 100m down the road to the shop at a constant speed, taking 20 seconds to get there.
He spends a 30 seconds at the shop before walking home at the same speed. He then runs 80m in the other direction, away from home, to get to the bus stop to catch a bus. He runs at a constant speed of 8 ms^{-1}.

Plot this as a displacement time graph.

What distance did Bob travel?

What is Bob's displacement:

i) After 30 seconds

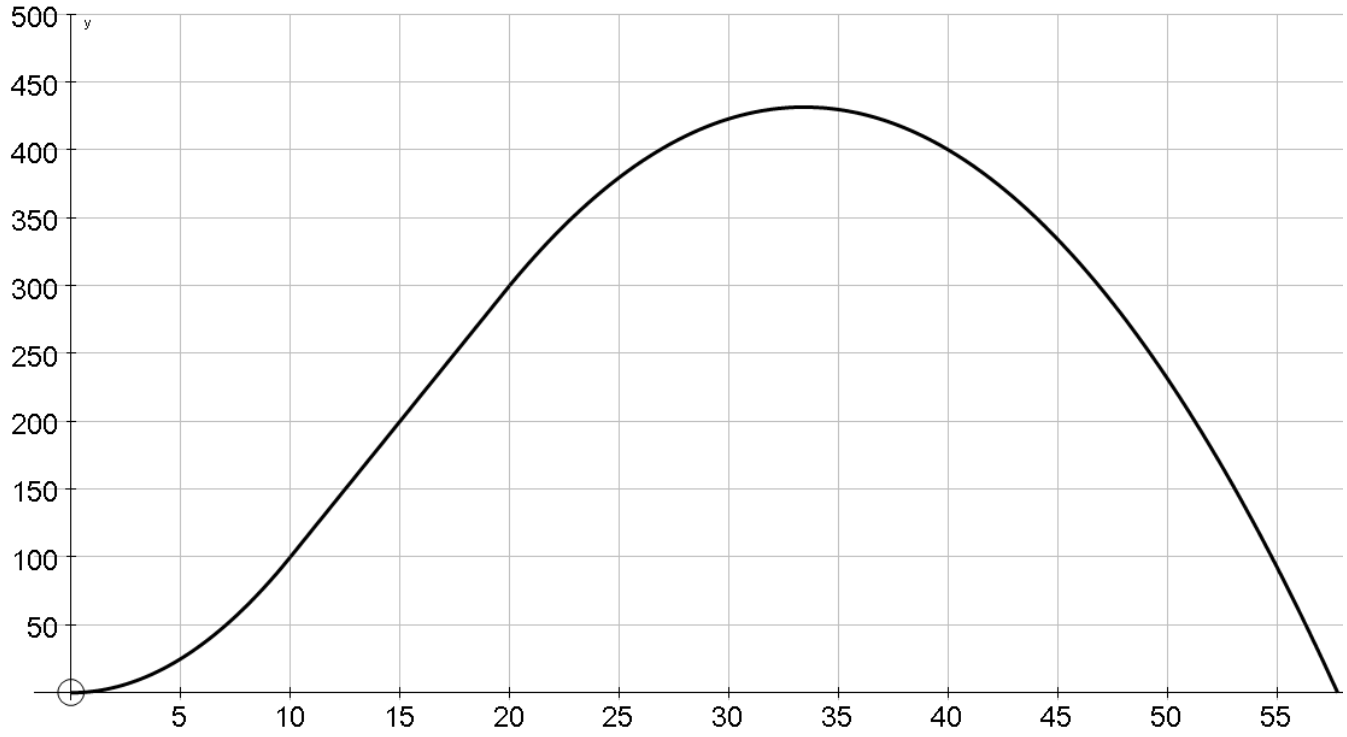
ii) After 75 seconds

What was Bob's speed for the first 20 seconds?

What was Bob's average speed?

What was Bob's average velocity?

Example 4



A rocket is fired from ground level. Its displacement time graph is shown.

a) What is happening from 0-10 seconds?

b) Describe the motion from 40-50 seconds.

c) How long was the rocket more than 200m above the ground?

d) When did the rocket achieve its greatest height?

Velocity – time graphs

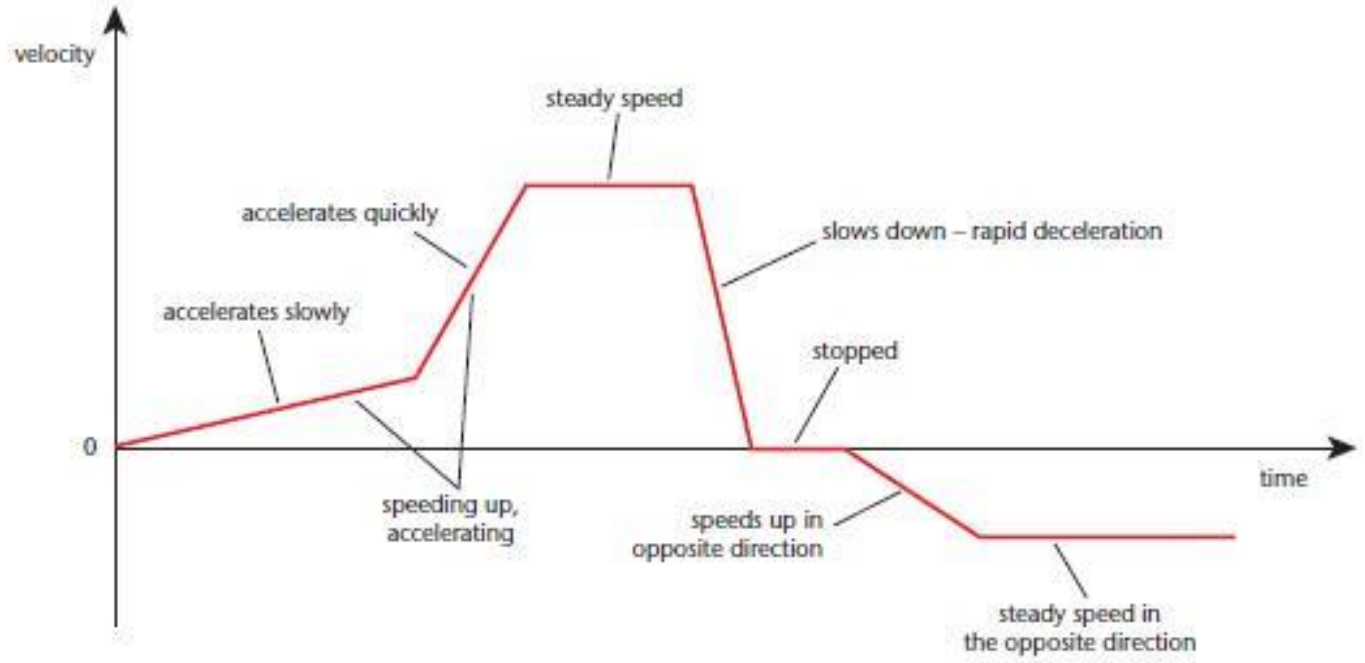


Fig. 9.2 A velocity–time graph.

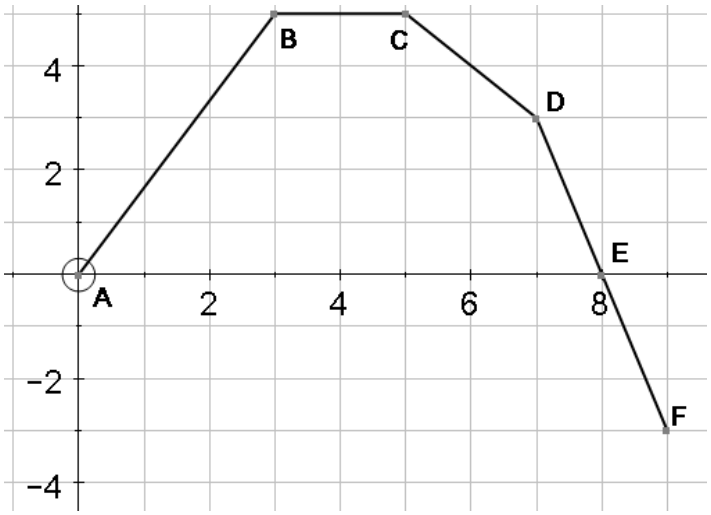
Extra notes

- Gradient of the line = acceleration
- Negative gradient (when velocity is positive) = deceleration
- Negative gradient (when velocity is negative) = acceleration in the opposite direction
- Area under graph of a particular section = displacement travelled in that section
- Area below the x-axis = negative displacement
    - The magnitude of any area below the x-axis should be added to any area above the x-axis to find the total distance travelled
    - The magnitude of any area below the x-axis should be subtracted from any area above the x-axis to find the total displacement

If the acceleration is constant for a particular section, you can **use SUVAT (to be covered later) for that section**.

However, if there are any changes to the type of motion e.g. if the object then started to move at a constant speed, you would not be able to use SUVAT when considering the second section or the entire journey.

Example 5



Consider this velocity-time graph of a different journey

What is happening from:
a) A to B?
b) B to C?
c) C to D?
d) E to F?

Find the acceleration of the object
e) In the section A to B
f) In the section D to E

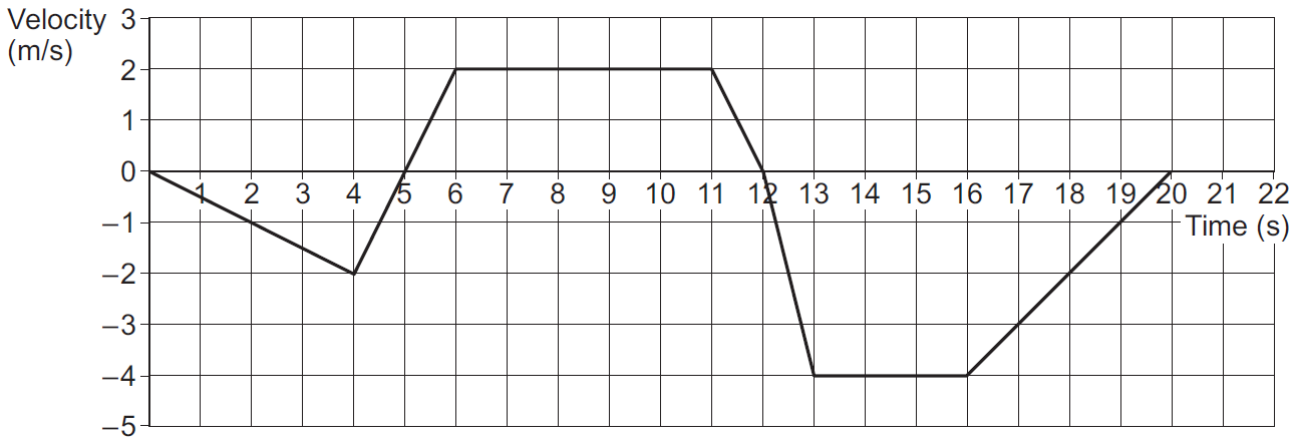
What is the total **distance** of this journey?

What is the displacement of the object from the start point at the end of the journey.

What happens at E?

Example 6

The graph below shows the velocity of an object moving in a straight line over a 20 second journey.



a) Find the maximum magnitude of the acceleration of the object
b) The the object is at its starting position at the times 0, t_1 and t_2 seconds. Find t_1 and t_2.

Example 7

A car accelerates from rest to a speed of 20 ms^{-1} in 10 s. It then travels at a constant speed for a further 20s before decelerating at a constant rate of $4ms^{-2}$ to rest.

a) Draw a speed-time graph to represent this information

Find

b) The initial acceleration of the car
c) The total distance travelled by the car in the journey.

Example 8 (Harder)

Two cars A and B are travelling in the same direction along a motorway. They pass a warning sign at the same instant, and subsequently arrive at a toll booth after the same amount of time.

Car A passes the warning sign at a speed of 24 ms^{-1}, continues at this speed for one minute, then decelerates uniformly, coming to rest at the toll booth.

Car B passes the warning sign at a speed of 30 ms^{-1}, continues at this speed for T seconds, then decelerates uniformly, coming to rest at the toll booth.

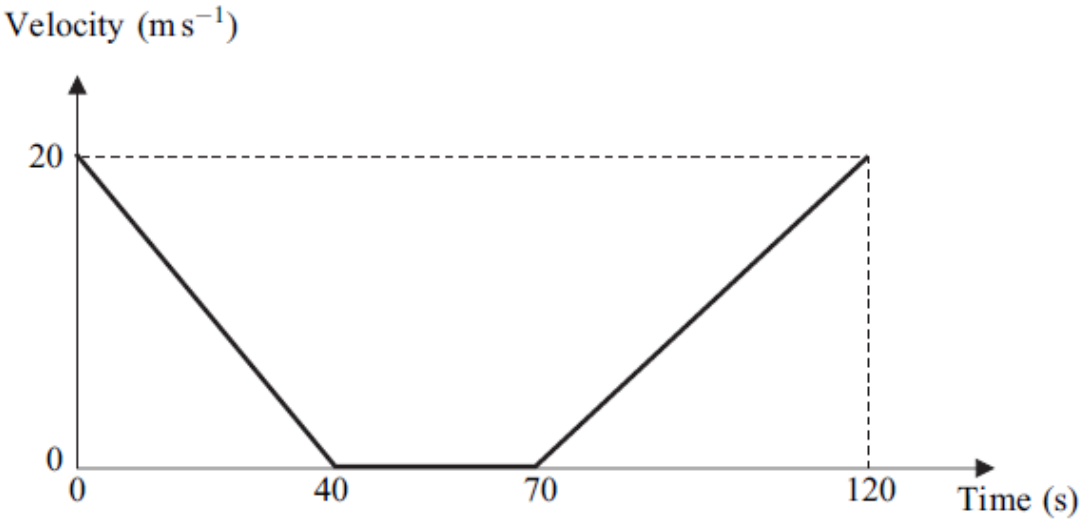
The distance from the warning sign to the toll booth is 1.56km.

a) On the same diagram, sketch the speed-time graph of each car.
b) Calculate the length of time, in seconds, for which A is decelerating.
c) Find the value of T.

Exam questions: Graphs

AQA June 2010

1 A bus slows down as it approaches a bus stop. It stops at the bus stop and remains at rest for a short time as the passengers get on. It then accelerates away from the bus stop. The graph shows how the velocity of the bus varies.



Assume that the bus travels in a straight line during the motion described by the graph.

(a) State the length of time for which the bus is at rest. *(1 mark)*

(b) Find the distance travelled by the bus in the first 40 seconds. *(2 marks)*

(c) Find the total distance travelled by the bus in the 120-second period. *(2 marks)*

(d) Find the average speed of the bus in the 120-second period. *(2 marks)*

(e) If the bus had not stopped but had travelled at a constant 20 m s^{-1} for the 120-second period, how much further would it have travelled? *(2 marks)*

AQA Jan 2007

2 A lift rises vertically from rest with a constant acceleration.

After 4 seconds, it is moving upwards with a velocity of 2 m s^{-1}.

It then moves with a constant velocity for 5 seconds.

The lift then slows down uniformly, coming to rest after it has been moving for a total of 12 seconds.

(a) Sketch a velocity–time graph for the motion of the lift. *(4 marks)*

(b) Calculate the total distance travelled by the lift. *(2 marks)*

Motion in a straight line – SUVAT equations

Any motion with a **constant acceleration** can be described using **equations of motion (suvat equations).**

These are:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

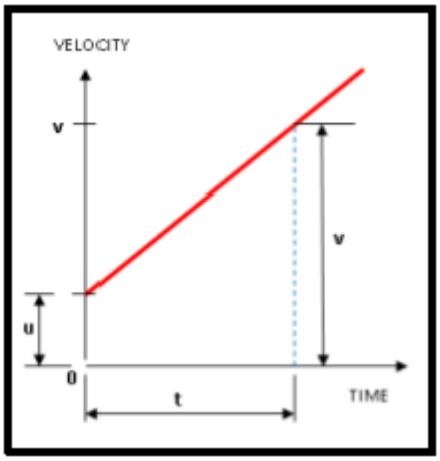
$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

u = initial velocity	(ms^{-1})
v = final velocity	(ms^{-1})
t = time taken	(s)
s = displacement	(m)
a = acceleration	(ms^{-2})

Example 9 – Derivation

Consider the following graph which describes the motion of a particle.



The particle moves in a straight line with a constant acceleration $\boldsymbol{a}$ ms^{-2}.

The initial velocity of the particle is $\boldsymbol{u}$ ms^{-1} and the velocity after $\boldsymbol{t}$ seconds is $\boldsymbol{v}$ ms^{-1}.

a) Use the graph to show that

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

b) Hence, or otherwise, show that $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$

Extra notes

- Always write down the **initial values** that you have been given in the question.
- **Direction is important**
    - Define which direction is positive (usually **the initial direction** is the positive direction but technically you can pick any direction to be positive, as long as you stick with this throughout the solution).
    - **Negative displacement** means that the object has moved to a position in the **opposite direction from the starting point:**
        - If horizontal and to the right is positive, the object's end position is to the left of the starting point.
        - If vertical and upwards is positive, the object's end position is below the starting position.
    - **Negative velocity** means that the object is **moving in the opposite direction.**
    - **Negative acceleration** means that the object is **decelerating (slowing down)** if the object has a positive velocity
    - If both the velocity and acceleration are negative, **the object is accelerating back towards the starting point.**
- If a = 0 then the object is not accelerating – it will be moving at a constant velocity.
- If an object is moving at a **constant velocity** then we can use $v = \frac{s}{t}$. This is the only situation where you can use this formula.

<u>Example 10</u>

A particle is accelerating at $3ms^{-2}$. Its initial velocity is $4ms^{-1}$.

a) What is its velocity after 5 seconds?
b) What is its displacement after 5 seconds?
c) How long does the particle take to get to $10ms^{-1}$?

Example 11

A bus leaving a bus stop accelerates from rest at 0.8 ms^{-2} for 5s. The bus then travels at a constant speed for 2 minutes before decelerating uniformly at 0.4ms^{-2} to come to rest at the next bus stop.
Find

a) The constant speed
b) The distance travelled while the bus is accelerating
c) The total distance travelled

Example 12 (Harder)

A sports car is being driven along a straight test track. It passes the point O at time $t = 0$ at which time it begins to decelerate uniformly. The car passes the points L and M at times $t = 1$ and $t = 4$ respectively.

Given that OL is 54 m and LM is 90m

a) Find the deceleration of the car.

The car subsequently comes to rest at N.

b) Find the distance MN.

Exam Question: SUVAT

AQA Jan 10

2 A sprinter accelerates from rest at a constant rate for the first 10 metres of a 100-metre race. He takes 2.5 seconds to run the first 10 metres.

(a) Find the acceleration of the sprinter during the first 2.5 seconds of the race. *(3 marks)*

(b) Show that the speed of the sprinter at the end of the first 2.5 seconds of the race is $8\,\mathrm{m\,s^{-1}}$. *(2 marks)*

(c) The sprinter completes the 100-metre race, travelling the remaining 90 metres at a constant speed of $8\,\mathrm{m\,s^{-1}}$. Find the total time taken for the sprinter to travel the 100 metres. *(3 marks)*

(d) Calculate the average speed of the sprinter during the 100-metre race. *(2 marks)*

AQA June 2011

3 A pair of cameras records the time that it takes a car on a motorway to travel a distance of 2000 metres. A car passes the first camera whilst travelling at $32\,\mathrm{m\,s^{-1}}$. The car continues at this speed for 12.5 seconds and then decelerates uniformly until it passes the second camera when its speed has decreased to $18\,\mathrm{m\,s^{-1}}$.

(a) Calculate the distance travelled by the car in the first 12.5 seconds. *(1 mark)*

(b) Find the time for which the car is decelerating. *(3 marks)*

(c) Sketch a speed–time graph for the car on this 2000-metre stretch of motorway. *(3 marks)*

(d) Find the average speed of the car on this 2000-metre stretch of motorway. *(2 marks)*

Vertical Motion Under Gravity - Freefall

For a freefalling object we can make the following assumptions:

- Air resistance is **negligible**

- The object is a **particle**
  - It has no dimensions or shape that will affect it's progression
  - It will not turn or spin as it falls

Because of these assumptions, we can say that the particle will fall at a **constant acceleration**.
This is the acceleration due to gravity, g, which has a **magnitude** equal to **9.8** $\mathbf{ms^{-2}}$.

$$\mathbf{a = 9.8\ ms^{-2}}$$

NB

- g can be given in an exam question to a different level of accuracy.
- If g is not given, use g = 9.8 ms^{-2}.
- You must use the value of g given in the question and make sure that your final answer matches the number of significant figures as that used for g
  - If g = 9.8 ms^{-2} – answer is 2sf
  - If g = 9.81 ms^{-2} – answer is 3sf
  - If g = 10 ms^{-2} – answer is 1sf

As the acceleration is constant, we can use the **suvat equations** to calculate the displacement, velocity and time at any point on the object's path.

It is important to:

- State **which direction** (up or down) is **positive**.
- Remember that the acceleration due to gravity always acts in a **downwards** direction

<u>Example 13</u>
A ball is projected upwards at a speed of 25 ms^{-1}.
Find the maximum height that the ball reaches.

In this question, use g = $\mathbf{10\ ms^{-2}}$

If the entire motion of the particle is **upwards** then:

- Say that upwards is positive
- Note that acceleration will be negative

Example 14

A coin is dropped from rest at the top of a building of height 12m and travels in a straight line to the ground.

Find the time taken for the coin to reach the ground and the speed of impact.

In this question, use g = 9.81 $\mathbf{ms^{-2}}$

If the entire motion is **downwards** then:

- Say that downwards is positive
- Note that acceleration will be positive

Example 15

A juggler throws a ball up in the air with an initial speed of 5 ms^{-1} from a height of 1.2 m above a horizontal ground.

Find the maximum height of the ball above the ground and the time taken for the ball to reach the ground.

In this question, use g = 9.8 ms^{-2}

If the motion involves **moving up and down** then you can either solve each part of the motion (up and down) **separately**, carrying any values through, or you can solve the **entire motion in one go**.

Exam Question: Vertical Motion

Edexcel Jan 03 M1

7.

A ball is projected vertically upwards with a speed u ms^{-1} from a point A which is 1.5 m above the ground. The ball moves freely under gravity until it reaches the ground. The greatest height attained by the ball is 25.6 m above A.

(a) Show that u = 22.4. (3)

The ball reaches the ground T seconds after it has been projected from A.

(b) Find, to 2 decimal places, the value of T. (4)

Edexcel Jan 06 M1

1. A stone is thrown vertically upwards with speed 16 m s^{-1} from a point h metres above the ground. The stone hits the ground 4 s later. Find

(a) the value of h,

(3)

(b) the speed of the stone as it hits the ground.

(3)

7. Whilst looking over the edge of a vertical cliff, 122.5 metres in height, Jim dislodges a stone. The stone falls freely from rest towards the sea below.

Ignoring the effect of air resistance,

(a) calculate the time it would take for the stone to reach the sea, **(3 marks)**

(b) find the speed with which the stone would hit the water. **(2 marks)**

Two seconds after the stone begins to fall, Jim throws a tennis ball downwards at the stone. The tennis ball's initial speed is u m s^{-1} and it hits the stone before they both reach the water.

(c) Find the minimum value of u. **(5 marks)**

(d) If you had taken air resistance into account in your calculations, what effect would this have had on your answer to part *(c)*? Explain your answer.

(2 marks)