Mechanics Sector 2: Moment of a force

If we consider a body that is not a particle, then the point through which a force acts is very important.

Particle: an object that has mass but whose dimensions are negligible

Rigid body: an object that has mass and size/shape but does not deform when a force is applied

When considering an object that is assumed to be a rigid body:

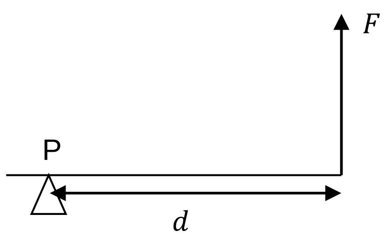
- If the force acts through the centre of mass of the object, then the force will cause the object to accelerate as explained in the Forces section.

- If the force **does not act through the centre of mass** then the force will cause the body to experience a **rotational motion**.
  - The body will now turn as well as or instead of moving in a straight line.
  - This force is called a **torque.**

When considering situations where a body is turning, we must consider **moments**.

A moment of a force is defined as the following:

Moment about a point P = Force x perpendicular distance from P to the line of action of the force

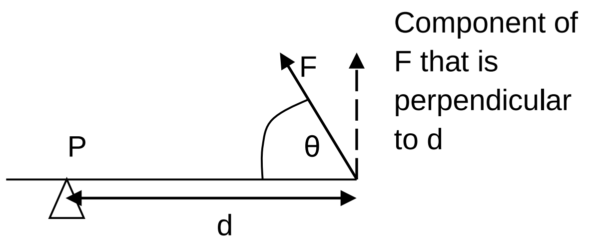


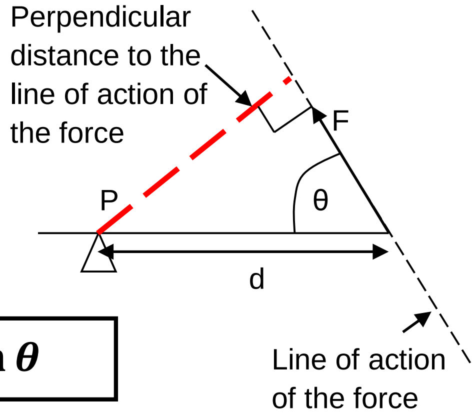
$$\boxed{\boldsymbol{Moment = Fd}}$$

The unit for a moment is Nm (as the force is measured in N and the distance is measured in m)

If the unit of distance is in cm then the unit for the calculated moment would be in Ncm

NB
If the force is not perpendicular to the distance, then you must find the component of the force that is perpendicular to the distance or the perpendicular distance to the line of action of the force.

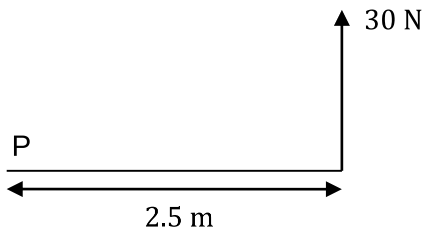




$$\boxed{\boldsymbol{Moment = Fd\sin\theta}}$$

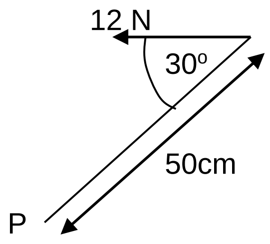
E.g.
Calculate the moments caused by these forces relative to P

1.



$$Moment = 30 \times 2.5$$
$$= 75 \text{ Nm}$$

2.



The component of the force perpendicular to the rod is

$$F_{\perp} = 12 \sin 30$$

Therefore

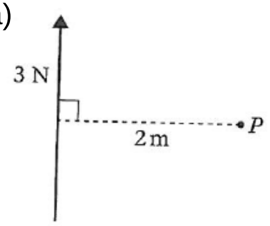
$$Moment = 12 \sin 30 \times 0.5$$
$$= 3 \text{ Nm}$$

I've changed the distance to metres

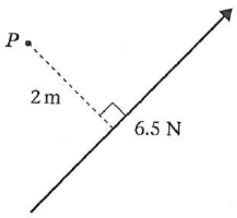
<u>Example 1</u>

Find the moment of each of these forces about the point P.

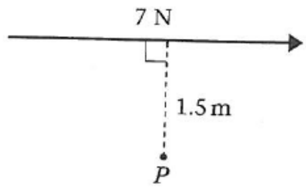
a)



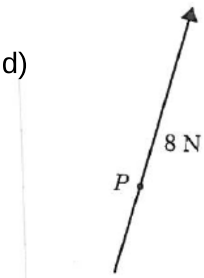
b)



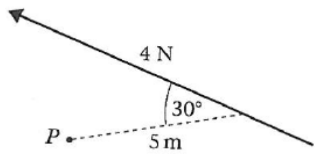
c)



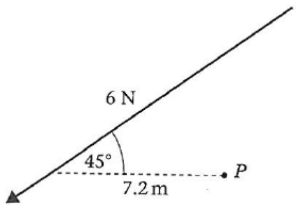
d)



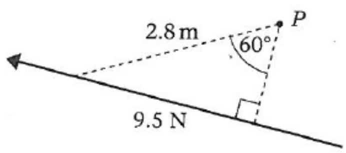
e)



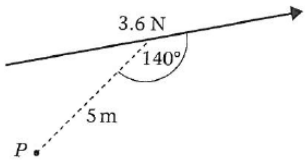
f)



g)

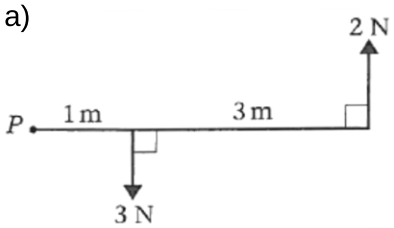


h)

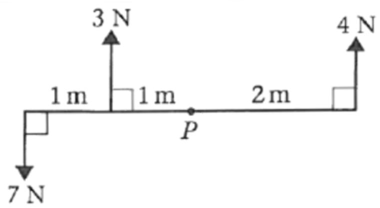


<u>Example 2</u>
Calculate the resultant moment about the point P

a)



b)



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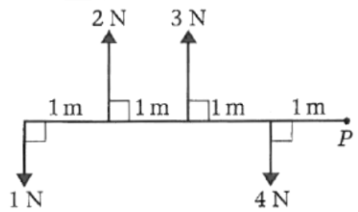


Equilibrium Situations

For a body in equilibrium we can say that:

- There is no resultant force in any direction
  - The sum of the components acting vertically upwards will equal the sum of the components acting vertically downwards

- There is no resultant torque acting on the body
  - The algebraic sum of the moments around any point must be zero
    Or in easier terms
  - The total anticlockwise moment will equal the total clockwise moment around any point on the body
  - This is the **principle of moments**

Therefore, when considering any object assumed to be a rigid body, we can apply the following methods:

1. **Resolve horizontally and vertically** (or **parallel and perpendicular to the rod**). The forces will **balance**, so that:

   > Sum of components vertically upwards = Sum of components vertically downwards
   > Sum of components to the left = Sum of components to the right

   or similarly for parallel and perpendicular directions

2. Take moments about a particular point on the body:

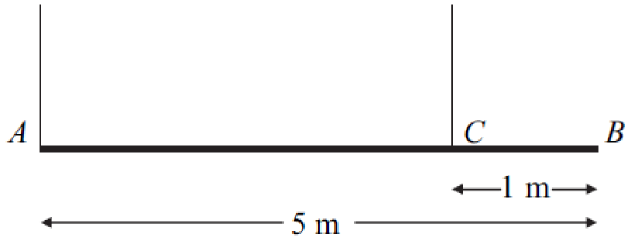
   > The total anticlockwise moment = The total clockwise moment

   If there is more than one unknown force, choose a point where one of the unknown forces acts from, as the moment caused by this force will equal **zero**.

Extra notes

- Always draw a diagram with all the known forces and distances/angles considered
- A **light** rod will have **negligible weight** (the weight is so small compared to the other forces that we can ignore it)
- If the rod is **uniform** then the weight will act through the centre of mass **at the centre of the rod**
- If the rod is **non-uniform** then the centre of mass **will not be at the centre of the rod**.
- Other words used instead of rod are plank, ladder and beam
- Any force that acts through the point about which you are considering moments has a moment of zero
- If a rod is balanced on a support or pivot then there will be a **normal reaction force** acting perpendicular to the rod.
- If the point of contact is rough then there will also be friction acting parallel to the rod at the point of contact. This can be combined with the reaction force to give a **single resultant force at that point**

Example 3



A beam AB has length 5m and mass 12kg. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached at the point A, the other to the point C where BC = 1m, as shown in the diagram above. The beam is modelled as a uniform rod and the strings are assumed to be light.

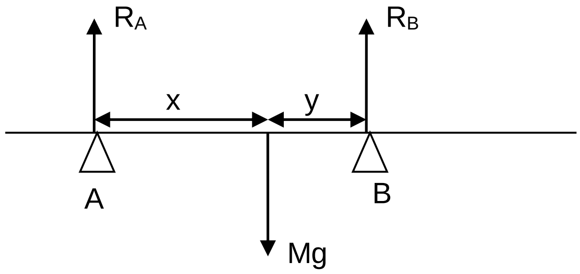
Calculate the tension in each of the ropes.

Example 4

A uniform plank AB of length 1.6m and mass 4kg rests on two supports which are 0.3m from each end of the plank. A mass, m kg, is attached to end B of the plank. If the normal reaction force acting at the support closest to B is twice the normal reaction force acting at the other support, determine the mass attached.

Tilting

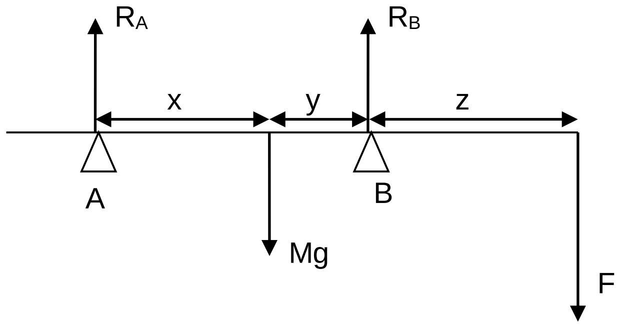
Consider a heavy rod of mass M that has been placed on two supports at positions A and B. The beam is horizontal when placed on the supports and is in equilibrium.



From the diagram, taking moments around B gives

$R_A(x + y) = Mgy$ (Weight = Mg)

If an extra vertical force is applied to the rod at some point other than between A and B the rod may or may not remain in equilibrium, depending on the magnitude of the extra force and the point of application (where the force is applied).



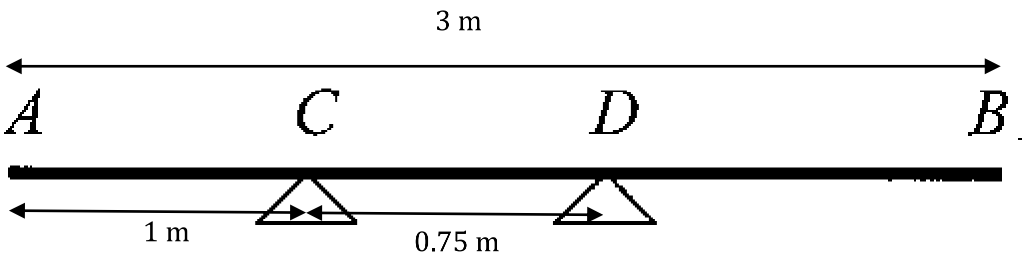
With the extra force (using the second diagram) and assuming that the rod is still in equilibrium, taking moments around B gives

$$R_A(x + y) + Fz = Mgy$$

The right hand side of each equation are identical, so we see that the **reaction at A (R_A) must have decreased as a result of applying the extra force**. R_A will decrease further if the magnitude of the extra force increases.

If the magnitude of the extra force is such that the body is in equilibrium but **R_A has decreased to zero** then the rod is said to be **on the point of tilting about B.**

<u>Example 5</u>

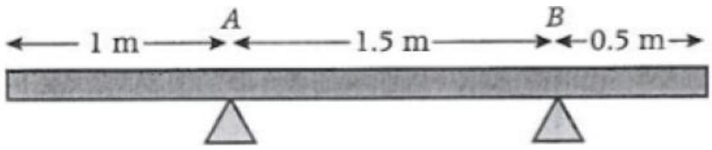


A uniform rod AB has a weight of 120N and length 3m. The plank rests horizontally in equilibrium on two smooth supports C and D, where AC = 1m and CD = 0.75m, as shown in the diagram above.

A rock is placed at B and the plank is at the point of tilting about D. Modelling the rock as a particle, find:

a) the weight of the rock
b) the magnitude of the reaction force of the support on the plank at D

Example 6



A uniform beam of mass 20kg and length 3m rests on two supports as shown below.

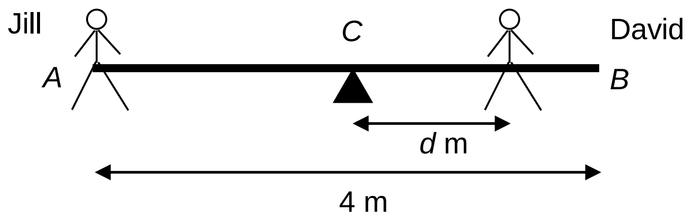
a) Find the reaction force exerted by each support.
b) Find the greatest mass that can be placed at the left-hand end of the beam, if it is to remain in equilibrium.

Non-uniform situations

If the rod is non-uniform then the centre of mass of said rod cannot be assumed to be at the centre of the rod. This means that the weight of the rod does not act at the centre of the rod. Instead, we must position the weight at a different point along the rod. This point will generally be at an unknown distance from either end. However, we can represent this distance with a suitable letter (such as x or d) if this is the case.

Example 7

Figure 1



A seesaw in a playground consists of a beam *AB* of length 4 m which is supported by a smooth pivot at its centre *C*. Jill has mass 25 kg and sits on the end *A*. David has mass 40 kg and sits at a distance *d* metres from *C*, as shown in Figure 1. The beam is initially modelled as a uniform rod. Using this model,

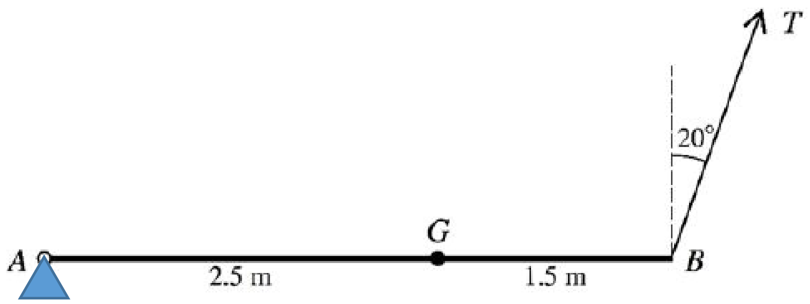
(*a*) find the value of *d* for which the seesaw can rest in equilibrium in a horizontal position.

David realises that the beam is not uniform as he finds he must sit at a distance 1.4 m from *C* for the seesaw to rest horizontally in equilibrium. The beam is now modelled as a non-uniform rod of mass 15 kg. Using this model,

(*b*) find the distance of the centre of mass of the beam from *C*.

(*c*) find the magnitude of the reaction force caused by the pivot at C

Example 8
A non-uniform beam AB of length 4m and mass 5kg has its centre of mass at the point T of the beam, where AG = 2.5m. The beam rests on a pivot at A and is held in a horizontal position by means of a wire attached to the end B. The wire makes an angle of 20^o to the vertical and the tension acting through the wire is T N, as shown in the diagram below.



Find
a) The value of T
b) The magnitude of the reaction force acting at the point A.

This is slightly beyond the specification, as not all the forces are parallel or perpendicular to each other, but the same method applies.

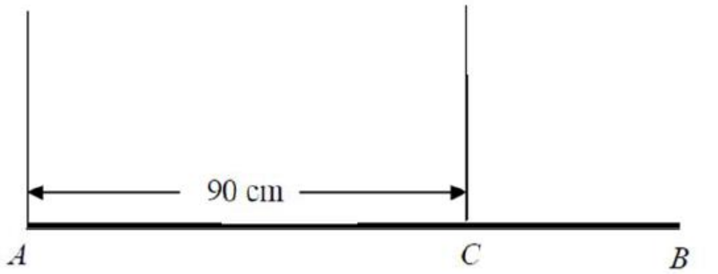
If you can do this then you should be able to handle anything that the examiners give you!

Exam Questions

Edexcel M1 June 2006

5.

Figure 3



A steel girder AB has weight 210 N. It is held in equilibrium in a horizontal position by two vertical cables. One cable is attached to the end A. The other able is attached to the point C on the girder, where $AC = 90$ cm, as shown in Figure 3. The girder is modelled as a uniform rod, and the cables as light inextensible strings.

Given that the tension in the cable at C is twice the tension in the cable at A, find

(*a*) the tension in the cable at A,

(2)

(*b*) show that $AB = 120$ cm.

(4)

A small load of weight W newtons is attached to the girder at B. The load is modelled as a particle. The girder remains in equilibrium in a horizontal position. The tension in the cable at C is now three times the tension in the cable at A.

(*c*) Find the value of W.

(7)

Edexcel M1 June 2011

3.

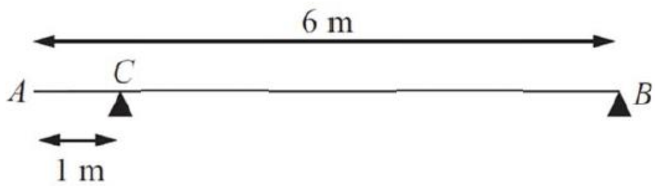


Figure 1

A uniform beam AB has mass 20 kg and length 6 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at C, where $AC = 1$ m, and the other is at the end B, as shown in Figure 1. The beam is modelled as a rod.

(a) Find the magnitudes of the reactions on the beam at B and at C.

(5)

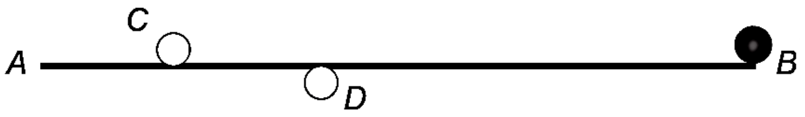
A boy of mass 30 kg stands on the beam at the point D. The beam remains in equilibrium. The magnitudes of the reactions on the beam at B and at C are now equal. The boy is modelled as a particle.

(b) Find the distance AD.

(5)

WJEC M1 June 2014

4. The diagram shows a uniform rod AB, of length $1{\cdot}8$ m and mass 3 kg, held in horizontal equilibrium by two small fixed cylinders C and D. An object of mass 12 kg rests on the rod at B. The length AC is $0{\cdot}3$ m and CD, the distance between the cylinders, is $0{\cdot}4$ m. The force exerted on the rod by each of the cylinders is vertical.



Find the magnitude of each of the forces exerted on the rod by the cylinders. [7]

WJEC M1 June 2012

8. A light uniform rod AB has length $1{\cdot}4$ m. A particle of mass 5 kg is attached to end A, and a particle of mass 2 kg is attached to end B. The rod rests horizontally in equilibrium on a smooth support at C.

(a) Calculate the reaction of the support at C. [2]

(b) Find the distance AC. [4]

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4.

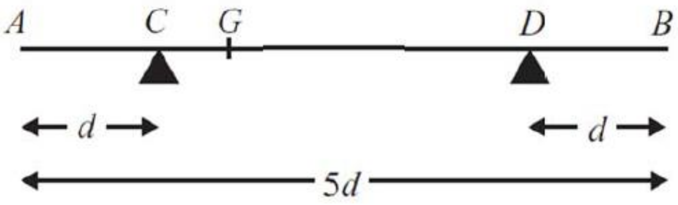


Figure 1

A non-uniform rod AB, of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D, where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D.

(a) Show that $GD = \frac{5}{2}d$.

(4)

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

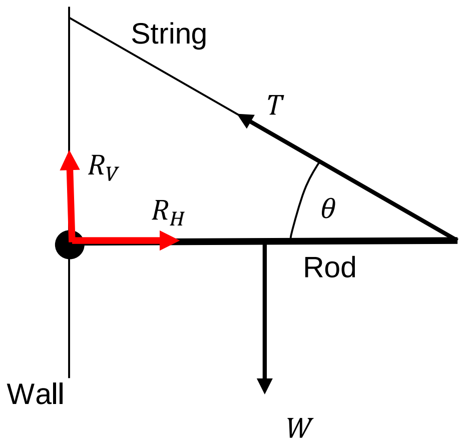
(b) Find the magnitude of the normal reaction between the support at D and the rod.

(5)

Harder Examples

Hinges

Consider the following diagram of a rod hinged to a wall.



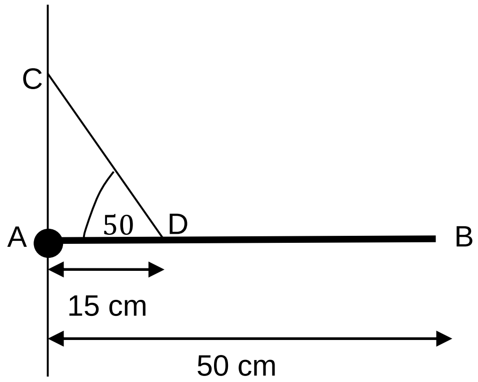
The magnitude and direction of the resultant force acting on the rigid body at the hinge is unknown. However, we can represent this force as two components, one horizontal (R_H) and the other vertical (R_V).
These will balance the other forces acting on the rigid body.

The best approach here (depending on the question) is to:

1. Take moments about the hinge (which will help you find any unknown forces or lengths, such as the tension).
2. Resolve the forces both horizontally and vertically to find R_H and R_V

<u>Example 9</u>
A uniform horizontal shelf AB of length 50cm and mass 5kg is freely hinged to a vertical wall and is supported by a chain CD as shown in the diagram. The chain is attached to the shelf at D such that the length AD is 15cm and the angle CDA between the shelf and the chain is 50°.
Find

a) The tension in the chain
b) The magnitude and direction of the reaction force acting on the shelf at the hinge



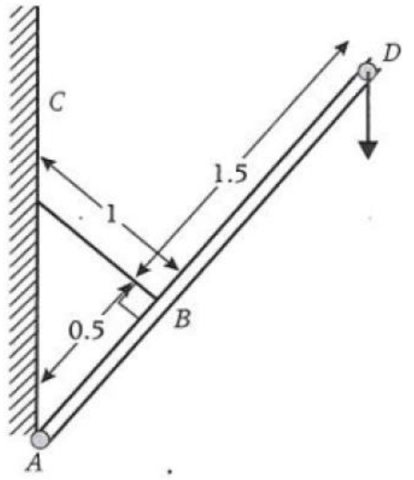
Example 10

A boom on a yacht is held in a horizontal position by a rope attached to the top of the mast. The boom is freely pivoted where it is attached to the mast. The length of the boom is 4m and its mass is 15kg. Assume that the boom is uniform. The rope makes an angle of 70^o with the boom.

a) Find the tension in the rope
b) Find the magnitude of the force that the mast exerts on the boom.

Example 11 (Harder)

A uniform rod of length 2m and mass 5kg is connected to a vertical wall by a smooth hinge at A and wire CB as shown.



If a 10kg mass is attached to D, find:

a) The tension in the wire
b) The magnitude of reaction at the hinge A.

This is slightly beyond the specification, as not all the forces are parallel or perpendicular to each other, but the same method applies.

If you can do this then you should be able to handle anything that the examiners give you!

Ladders

Situations involving ladders resting against a wall at an angle to a horizontal surface can be challenging, as there are multiple forces which are not perpendicular to the ladder.

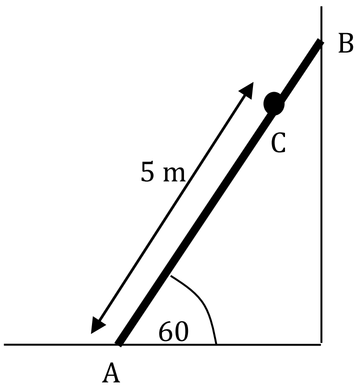
However, we can still apply the same methods as before.

Example 12

A uniform ladder of length 12.5m and mass 48kg rests with its top against a smooth wall and its foot on rough ground, 3.5m from the base of the wall.

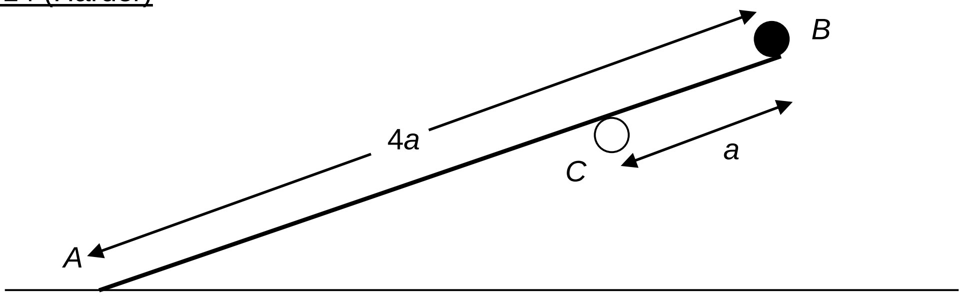
Find the frictional and reaction forces at the base of the ladder.

Example 13



A uniform ladder of length 6m and mass 25 kg rests with its foot, A, on a rough, horizontal surface and its top, B, against a smooth, vertical wall. The ladder makes an angle of 60^{o} with the horizontal surface.
A man of mass 80 kg stands on the ladder at position 5m from A, as shown in the diagram above. The ladder is at the point of slipping.
Find the coefficient of friction between the ladder and the horizontal surface.

Example 14 (Harder)



A wooden plank AB has mass $4m$ and length $4a$. The end A of the plank lies on rough horizontal ground. A small stone of mass m is attached to the plank at B. The plank is resting on a small smooth horizontal peg C, where $BC = a$, as shown in Figure 2. The plank is in equilibrium making an angle α with the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the plank and the ground is μ. The plank is modelled as a uniform rod lying in a vertical plane perpendicular to the peg, and the stone as a particle. Show that

(*a*) the reaction of the peg on the plank has magnitude $\frac{16}{5}\,mg$,

(*b*) $\mu \geq \frac{48}{61}$.

(*c*) State how you have used the information that the peg is smooth.

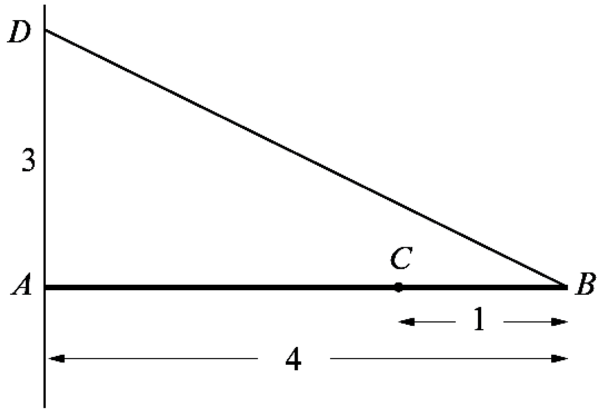
This is slightly beyond the specification, as not all the forces are parallel or perpendicular to each other, but the same method applies.

If you can do this then you should be able to handle anything that the examiners give you!

Exam Questions

WJEC Jun 13 M3

6. The diagram shows a uniform rod AB, of mass 6 kg and length 4 m, held in a horizontal position by means of a light inextensible string BD, where D is a point 3 m vertically above A. The end A of the rod rests against a rough vertical wall. A particle of mass 3 kg is attached to the rod at C, where $BC = 1$ m. The rod is in limiting equilibrium in a vertical plane perpendicular to the wall.



(a) Calculate the tension in the string. [4]

(b) Find the vertical component and the horizontal component of the force exerted by the wall on the rod.
Hence find

(i) the magnitude of the resultant force exerted by the wall on the rod,

(ii) the coefficient of friction between the rod and the wall. [9]

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6.

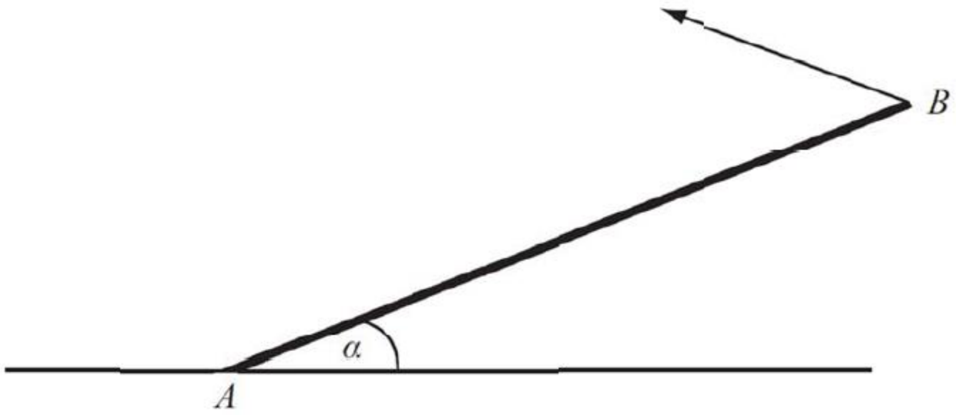


Figure 2

A uniform rod AB, of mass 20 kg and length 4 m, rests with one end A on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$, by a force acting at B, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5. Find the magnitude of the normal reaction of the ground on the rod at A.

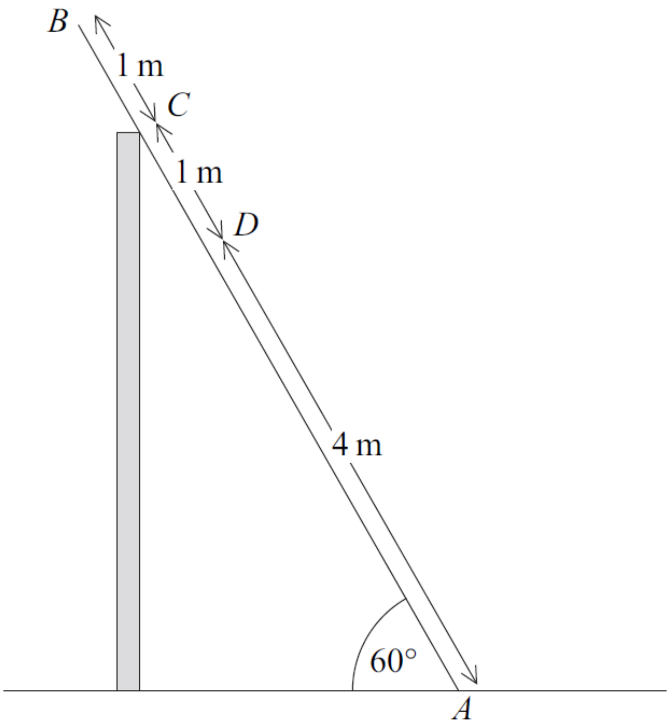
(7)

AQA June 2014 M2

7 A uniform ladder AB, of length 6 metres and mass $22\,\text{kg}$, rests with its foot, A, on rough horizontal ground. The ladder rests against the top of a smooth vertical wall at the point C, where the length AC is 5 metres. The vertical plane containing the ladder is perpendicular to the wall, and the angle between the ladder and the ground is $60°$. A man, of mass $88\,\text{kg}$, is standing on the ladder.

The man may be modelled as a particle at the point D, where the length of AD is 4 metres.

The ladder is on the point of slipping.



(a) Draw a diagram to show the forces acting on the ladder.

[2 marks]

(b) Find the coefficient of friction between the ladder and the horizontal ground.

[6 marks]

AQA June 2016 M2

7 A uniform ladder, of weight W, rests with its top against a rough vertical wall and its base on rough horizontal ground.

The coefficient of friction between the wall and the ladder is μ and the coefficient of friction between the ground and the ladder is 2μ .

When the ladder is on the point of slipping, the ladder is inclined at an angle of θ to the horizontal.

(a) Draw a diagram to show the forces acting on the ladder.

[2 marks]

(b) Find $\tan\theta$ in terms of μ.

[7 marks]