6 (a) The curve C has equation $y = \dfrac{1}{4^x}$. The line $y = \dfrac{3}{4}$ intersects C at the point A.

(i) Sketch the curve C, indicating the value of the intercept on the y-axis.

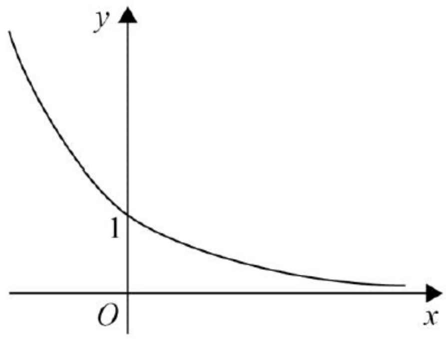
[2 marks]

(ii) Use logarithms to find the x-coordinate of A, giving your answer to three significant figures.

[3 marks]

(b) Given that $\log_4 c = m$ and $\log_{64} d = n$, express $\dfrac{c^2}{\sqrt{d^3}}$ in the form 4^p, where p is an expression in terms of m and n.

[4 marks]

Q	Solution	Mark	Total	Comment
6 (a)(i)		B1 B1	2	Correct shaped graph in 1st two quadrants only and indication of correct behaviour of curve for large positive and negative values of x. Ignore any scaling on axes. y-intercept indicated as 1 on diagram or stated as intercept =1 or as coords (0, 1).
(ii)	$\frac{1}{4^x}=\frac{3}{4} \Rightarrow 4^{-x}=\frac{3}{4}$ (or $4^x=\frac{4}{3}$ or $4^{1-x}=3$ or $4^{x-1}=\frac{1}{3}$)	M1		Correct 'rearrangement' to eg $4^x=\frac{4}{3}$ or $4^{-x}=\frac{3}{4}$ or $0.25^x=0.75$ PI or $\log 1-\log 4^x=\log(3/4)$ or better
	$\log 4^{-x}=\log 0.75 \Rightarrow -x\log 4=\log 0.75$ [$\log 4^x=\log(4/3) \Rightarrow x\log 4=\log(4/3)$] [$\log 4^{1-x}=\log 3 \Rightarrow (1-x)\log 4=\log 3$] [$\log 4^{x-1}=\log\frac{1}{3} \Rightarrow (x-1)\log 4=\log\frac{1}{3}$] [$4^x=4/3$, $x=\log_4(4/3)$]; [$0.25^x=0.75$, $x=\log_{0.25}0.75$]	M1		Takes logs of both sides of eqn of form either $4^x=k$ or $4^{-x}=k$ OE and uses 3rd law of logs or log to base 4 (or base 1/4) correctly
	$x=0.2075187\ldots$ so $x=0.208$ (to 3sf)	A1	3	Condone >3sf [Logs must be seen to be used otherwise **max** of M1M0A0]
(b)	$c=4^m,\quad d=64^n$ $c^2=4^{2m}$ $\sqrt{d^3}=(4^{3n})^{1.5}=4^{4.5n}$ $\frac{c^2}{\sqrt{d^3}}=4^{2m-4.5n}$	M1 A1 A1 A1	 4	Either $c=4^m$ or $d=64^n$ $c^2=4^{2m}$ $\sqrt{d^3}=4^{4.5n}$ seen or used $\frac{c^2}{\sqrt{d^3}}=4^{2m-4.5n}$ OE expression for p in terms of m and n.
	Total		9	

7 (a) (i) Express $\log_b(6x) - \log_b 18$ as a single term.

[1 mark]

(ii) Solve the equation

$$\log_b(x+4) = \log_b(6x) - \log_b 18 + \log_b(x-1)$$

where b is a positive constant.

[4 marks]

(b) (i) Given that $\log_m n = k$, express n in terms of m and k.

[1 mark]

(ii) Given that $p \log_8 x^2 = \log_2(x^2\sqrt{x})$, find the value of the constant p.

[4 marks]

Q	Solution	Mark	Total	Comment
7(a) (i)	$\log_b \frac{6x}{18}$	B1	1	$\log \frac{6x}{18}$ OE Condone base b missing
(ii)	$\log_b \frac{6x}{18} + \log_b(x-1) = \log_b \frac{6x(x-1)}{18}$	M1		Eg $\log D + \log(x-1) = \log D(x-1)$ or $\log(x+4) - \log(x-1) = \log\left(\frac{x+4}{x-1}\right)$ OE results so as to have no more than two log terms remaining in the given equation. Condone base b missing PI by a correct eqn. with no log terms provided no errors seen in (ii) in determining such an eqn.
	$\log_b(x+4) = \log_b \frac{6x(x-1)}{18}$ $\Rightarrow x+4 = \frac{6x(x-1)}{18}$	A1		OE A correct eqn after all logarithms eliminated in a correct manner. Condone $\log(x+4) = \log \frac{6x(x-1)}{18}$ with 'log' on each side crossed out.
	$x^2 - 4x - 12 = 0 \Rightarrow x = 6, \quad x = -2$	A1		$x = 6, \quad x = -2$; if -2 is missing we must see either $(x-6)(x+2)$ or a valid statement for ignoring it
	$x = 6$	A1	4	6 as the only solution.
(b)(i)	$n = m^k$	B1	1	
(ii)	$(\log_2)x^2\sqrt{x} = (\log_2)x^{2.5}$	B1		$x^2\sqrt{x} = x^{2.5}$ seen or used at any stage
	$p\log_8 x^2 = \log_8(x^2)^p$	M1		Use of log law $a\log b = \log b^a$ at any stage in **(b)(ii)**, or $8^T = x^{2p}$ OE seen
	Let $T = p\log_8 x^2 = \log_2 x^2\sqrt{x}$			
	eg $2^T = x^{2.5}$, $8^T = x^{2p} = 2^{3T}$; $\underline{x^{2p} = x^{7.5}}$ eg $\log_2 x^{2.5} = \log_8 x^{2p} = \frac{\log_2 x^{2p}}{\log_2 8}$ $= \log_2 x^{\frac{2p}{3}}$; $\underline{x^{2.5} = x^{\frac{2p}{3}}}$	M1		Correctly converting to the same base OE and eliminating in a correct manner all logarithmic terms. Can also be awarded after B0 if cand has $x^2\sqrt{x} = x^q$, where q is non-integer
	$2p = 7.5$; $p = 3.75$ or eg $\frac{2p}{3} = 2.5$; $p = 3.75$	A1	4	Correct value for p, obtained convincingly for a general x. NMS scores 0/4
	Total		**10**	

9 (a) (i) Describe the geometrical transformation that maps the graph of $y = 2^x$ onto the graph of $y = 2^{2x}$.

[2 marks]

(ii) Describe the single geometrical transformation that maps the graph of $y = 2^x - 15$ onto the graph of $y = 2^{x+3} - 15$.

[2 marks]

(b) The curve C_1 has equation $y = 2^{2x}$. The curve C_2 has equation $y = 2^{x+3} - 15$.

The curves C_1 and C_2 intersect at the points A and B.

(i) Given that $u = 2^x$, express 2^{x+3} in terms of u.

[1 mark]

(ii) Find the gradient of the line AB, giving your answer in the form $\dfrac{p}{\log_2 q}$.

[6 marks]

Q	Solution	Mark	Total	Comment
9(a)(i)	Stretch (**I**) in x(-direction) OE (**II**) (scale factor) 0.5 OE (**III**)	M1 A1	 2	Need (**I**) **and** either (**II**) or (**III**) Need (**I**) and (**II**) and (**III**) More than one transformation scores 0/2
(a)(ii)	Translation $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$	E2,1	2	**E2**: 'translat…' and $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$. If not **E2** award **E1** for either 'translat…' or for $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$. More than one transformation scores 0/2
(b)(i)	$2^{x+3} = 2^3 2^x = 8u$	B1	1	$8u$ Accept $2^3 u$ if in later work it is simplified to $8u$
(ii)	$u^2 - 8u + 15 = 0$ $y = \left(\dfrac{y+15}{2^3}\right)^2 \quad (y^2 - 34y + 225 = 0)$	M1		Eliminating y with $2^{2x} = u^2$ or $(2^x)^2$ to form a quadratic eqn in u or in 2^x (terms in any order) **OR** eliminating x with $2^{2x} = y = \left(\dfrac{y+15}{2^3}\right)^2$ condoning one sign or one numerical error
	$u = 3,\ u = 5;\ y = 9,\ y = 25;$	A2,1		Correct values for u (or 2^x) and correct values for y. If not **A2** award **A1** for any two correct values. If y-values not simplified look for later evidence; eg p as '16' in the final answer is sufficient evidence.
	$u = 3 \Rightarrow x = \log_2 3$; $u = 5 \Rightarrow x = \log_2 5$	M1		From a quadratic eqn, use of $2^x = k \Rightarrow x = \log_2 k$ OE, for $k>0$ **M0** if c's quadratic eqn would give non-real roots or no positive root when solved correctly
	Gradient of $AB = \dfrac{25-9}{\log_2 5 - \log_2 3}$	dM1		Dep on both previous M1. $\dfrac{y_A - y_B}{x_A - x_B}$ used with c's x and y values
	$= \dfrac{16}{\log_2\left(\dfrac{5}{3}\right)}$	A1	6	OE in the requested form eg $\dfrac{-16}{\log_2(0.6)}$
	Total		**11**	

9 Given that $3\log_2(c+2)-\log_2\left(\frac{c^3}{2}+k\right)=1$, express $(c+1)^2$ in terms of k.

[7 marks]

2 (a) Sketch the graph of $y=(0.2)^x$, indicating the value of the intercept on the y-axis.

[2 marks]

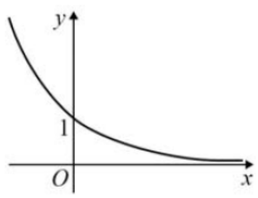
(b) Use logarithms to solve the equation $(0.2)^x=4$, giving your answer to three significant figures.

[2 marks]

(c) Describe the geometrical transformation that maps the graph of $y=(0.2)^x$ onto the graph of $y=5^x$.

[1 mark]

Q9	Solution	Mark	Total	Comment
	$\log_2(c+2)^3 - \log_2\left(\frac{c^3}{2}+k\right) = 1$	M1		$3\log(c+2) = \log(c+2)^3$
	$\log_2\left(\frac{(c+2)^3}{\left(\frac{c^3}{2}+k\right)}\right) = 1$	M1		Either $\log A - \log B = \log\frac{A}{B}$ or $1+\log_2 B = \log_2 2B$ used with correct A and B; if cand is using their expansion for $(c+2)^3$ in place of $(c+2)^3$, ignore any errors in the expansion in awarding this M1 mark
	$\log_2\left(\frac{(c+2)^3}{\left(\frac{c^3}{2}+k\right)}\right) = \log_2 2$	B1		$1 = \log_2 2$ stated or used **at any stage**. This also includes the step $\log_2 \mathrm{f}(c,k) = p \Rightarrow \mathrm{f}(c,k) = 2^p$.
	$(c+2)^3 = 2\left(\frac{c^3}{2}+k\right)$ $c^3+6c^2+12c+8$ $= 2\left(\frac{c^3}{2}+k\right)$	B2,1,0		(*) see below $(c+2)^3 = c^3+6c^2+12c+8$ seen **or used** at any stage; **B1** if 3 of the 4 terms are correct. May have to check correct collecting of like terms at a later stage in soln. [See below for altn for these two B marks]
	$\Rightarrow 6c^2+12c+8 = 2k$	A1		OE Correct equation with no logs and no c^3 term.
	$\Rightarrow 6(c^2+2c+1) = 2k-2$ $\Rightarrow (c+1)^2 = \frac{2k-2}{6} = \frac{k-1}{3}$	A1	7	ACF for the expression in k.
	Total		7	

Q2	Solution	Mark	Total	Comment
(a)		B1 B1	2	Only one y- intercept, marked as 1 or coordinates (0,1) stated or 'y = 1 when x = 0' Correct graph having no other 'crossing point' on either axis.
(b)	$x\log 0.2 = \log 4$	M1		OE eg $(x =)\log_{0.2} 4$
	$(x =) -0.861(35\ldots) = -0.861$ (to 3sf)	A1	2	Condone > 3sf, rounded or truncated. If use of logarithms not explicitly seen then score 0/2
(c)	Reflection in the y-axis.	E1	1	OE **E0** if more than one transformation
	Total		5	

9 (a) Given that $\log_3 c = m$ and $\log_{27} d = n$, express $\frac{\sqrt{c}}{d^2}$ in the form 3^y, where y is an expression in terms of m and n.

[4 marks]

(b) Show that the equation

$$\log_4(2x+3) + \log_4(2x+15) = 1 + \log_4(14x+5)$$

has only one solution and state its value.

[4 marks]

2. Find the exact solutions, in their simplest form, to the equations

(a) $e^{3x-9} = 8$

(3)

(b) $\ln(2y+5) = 2 + \ln(4-y)$

(4)

Q9	Solution	Mark	Total	Comment
(a)	$c=3^m$, $d=27^n$	M1		Either $c=3^m$ or $d=27^n$ seen or used
	$d=3^{3n}$, $d^2=3^{6n}$	A1		Either $d=3^{3n}$ or $d^2=3^{6n}$ seen or used
	$\sqrt{c}=3^{0.5m}$	A1		$\sqrt{c}=3^{0.5m}$ seen or used
	$\frac{\sqrt{c}}{d^2}=3^{\frac{m}{2}-6n}$	A1	4	$\frac{\sqrt{c}}{d^2}=3^{\frac{m}{2}-6n}$ OE expression for y in terms of m and n.
Altn	$\frac{1}{2}\log_3 c-2\log_3 d=y\log_3 3$	(M1)		A correct expression in y in terms of logs to base 3 or base 27 where no further log laws are required
	$\log_3 d=\frac{\log_{27} d}{\log_{27} 3}$	(A1)		$\log_3 d=\frac{\log_{27} d}{\log_{27} 3}$ or $\log_{27} c=\frac{\log_3 c}{\log_3 27}$ seen or used
	$\log_{27} 3=\frac{1}{3}$	(A1)		$\log_{27} 3=\frac{1}{3}$ or $\log_3 27=3$ seen or used
	$y=\frac{1}{2}m-6n$	(A1)	(4)	Correct expression for y in terms of m and n OE eg $\frac{\sqrt{c}}{d^2}=3^{\frac{m}{2}-6n}$
(b)	$1=\log_4 4$	B1		$1=\log_4 4$ seen or used at any stage.
	$\log_4(2x+3)(2x+15)=1+\log_4(14x+5)$ $\log_4(2x+3)(2x+15)=\log_4 4(14x+5)$	M1		Applying a log law correctly to two correct log terms. [Condone missing base] NB: Lots of other possibilities after correct rearrangements! PI by $(2x+3)(2x+15)=4(14x+5)$ OE with no errors seen
	$(2x+3)(2x+15)=4(14x+5)$	A1		OE eqn with logs eliminated in a correct manner
	$4x^2+36x+45=56x+20$ $4x^2-20x+25=0$; $(2x-5)^2=0$ Only one solution 2.5	A1	4	Must include statement and correct value
	Total		8	

2.(a)	$e^{3x-9}=8\Rightarrow 3x-9=\ln 8$	M1
	$\Rightarrow x=\frac{\ln 8+9}{3}, = \ln 2+3$	A1, A1
		(3)
(b)	$\ln(2y+5)=2+\ln(4-y)$	
	$\ln\left(\frac{2y+5}{4-y}\right)=2$	M1
	$\left(\frac{2y+5}{4-y}\right)=e^2$	M1
	$2y+5=e^2(4-y)\Rightarrow 2y+e^2y=4e^2-5\Rightarrow y=\frac{4e^2-5}{2+e^2}$	dM1, A1

9 (a) Use logarithms to solve the equation $2^{3x} = 5$, giving your value of x to three significant figures.

[2 marks]

(b) Given that $\log_a k - \log_a 2 = \frac{2}{3}$, express a in terms of k.

[4 marks]

(c) (i) By using the binomial expansion, or otherwise, express $(1+2x)^3$ in ascending powers of x.

[3 marks]

(ii) It is given that

$$\log_2[(1+2n)^3 - 8n] = \log_2(1+2n) + \log_2[4(1+n^2)]$$

By forming and solving a suitable quadratic equation, find the possible values of n.

[5 marks]

Q9	Solution	Mark	Total	Comment
(a)	$3x\log 2 = \log 5$	M1		OE eg $3x = \log_2 5$ or eg $x\log 8 = \log 5$
	$x = 0.773(976...)$ $= 0.774$ (to 3sf)	A1	2	Condone > 3sf. If use of logarithms not explicitly seen then score 0/2
(b)	$\log_a \frac{k}{2} = \frac{2}{3}$	M1		Either $\log k - \log 2 = \log\frac{k}{2}$ or $\frac{2}{3} = \log a^{\frac{2}{3}}$ seen at any stage
	$\frac{k}{2} = a^{\frac{2}{3}}$	A1		OE eqn with logs eliminated with no incorrect work
	$a^{\frac{2}{3}} = \frac{k}{2} \Rightarrow a = \left(\frac{k}{2}\right)^{\frac{3}{2}}$	m1		$a^{\frac{m}{n}} = C \Rightarrow a = C^{\frac{n}{m}}$
		A1	4	$a = \left(\frac{k}{2}\right)^{\frac{3}{2}}$ OE exact form with no obvious incorrect working
(c)(i)	$(1+2x)^3 = 1 + 3(2x) + 3(2x)^2 + (2x)^3$ $= 1 + 6x + 12x^2 + 8x^3$	B3,2,1	3	B3: expansion correct and simplified B2: 3 of the 4 terms correct and simplified B2; 4 terms correct but not all simplified B1 2 of the 4 terms correct and simplified (ignore the ordering of the terms)
(c)(ii)	$[(1+2n)^3 - 8n] = 1 - 2n + 12n^2 + 8n^3$	B1F		Ft at most two incorrect coefficients in **(c)(i)**
	$\log(1+2n) + \log 4(1+n^2) = \log 4(1+n^2)(1+2n)$	M1		Log law 1 applied correctly to RHS of given eqn., ignore base. Those who rearrange the terms first before applying log law 2 correctly must also attempt to deal with the resulting fraction in a correct manner.
	Given equation becomes $1 - 2n + 12n^2 + 8n^3 = 8n^3 + 4n^2 + 8n + 4$ $8n^2 - 10n - 3$ $(=0)$	A1		Correct three term quadratic
	$(4n+1)(2n-3)$ $(=0)$	A1		PI by correct two roots from a correct quadratic equation
	$n = -\frac{1}{4}$, $n = \frac{3}{2}$	A1	5	Need both as the final two values of n with no extras
	Total		**14**	

6. (a) Given $y = 2^x$, show that

$$2^{2x+1} - 17(2^x) + 8 = 0$$

can be written in the form

$$2y^2 - 17y + 8 = 0$$

(2)

(b) Hence solve

$$2^{2x+1} - 17(2^x) + 8 = 0$$

(4)

6.(a)	Replaces 2^{2x+1} with $2^{2x}\times 2$ or states $2^{2x+1}=2^{2x}\times 2$ or states $\left(2^x\right)^2=2^{2x}$	Uses the addition **or** power law of indices on 2^{2x} or 2^{2x+1}. E.g. $2^x\times 2^x=2^{2x}$ or $\left(2^x\right)^2=2^{2x}$ or $2^{2x+1}=2\times 2^{2x}$ or $2^{x+0.5}=2^x\times\sqrt{2}$ or $2^{2x+1}=\left(2^{x+0.5}\right)^2$.	M1
	$2^{2x+1}-17\times 2^x+8=0$ $\Rightarrow 2y^2-17y+8=0$*	Cso. Complete proof that includes explicit statements for the addition **and** power law of indices on 2^{2x+1} with no errors. The equation needs to be as printed including the "= 0". If they work backwards, they do not need to write down the printed answer first but must end with the version in 2^x including '= 0'.	A1*
(b)	$2y^2-17y+8=0\Rightarrow(2y-1)(y-8)(=0)\Rightarrow y=...$ or $2\left(2^x\right)^2-17\left(2^x\right)+8=0\Rightarrow\left(2\left(2^x\right)-1\right)\left(\left(2^x\right)-8\right)(=0)\Rightarrow 2^x=...$ Solves the **given** quadratic either in terms of y or in terms of 2^x See General Principles for solving a 3 term quadratic Note that completing the square on e.g. $y^2-\frac{17}{2}y+4=0$ requires $\left(y\pm\frac{17}{4}\right)^2\pm q\pm 4=0\Rightarrow y=...$		M1
	$(y=)\frac{1}{2},8$ or $\left(2^x=\right)\frac{1}{2},8$	Correct values	A1
	$\Rightarrow 2^x=\frac{1}{2},8\Rightarrow x=-1,3$	M1: Either finds one correct value of x for their 2^x or obtains a correct numerical expression in terms of logs e.g. for $k>0$ $2^x=k\Rightarrow x=\log_2 k$ or $\frac{\log k}{\log 2}$ A1: $x=-1,3$ only. Must be values of x.	M1 A1
			(4)

7. Given that $y = 2^x$,

(a) express 4^x in terms of y.

(1)

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0$$

(4)

7.(a)	$\left(4^x =\right) y^2$	Allow y^2 or $y \times y$ or "y squared" "$4^x =$" not required	B1
	Must be seen in part (a)		
			(1)
(b)	$8y^2 - 9y + 1 = (8y-1)(y-1) = 0 \Rightarrow y = ...$ or $(8(2^x)-1)((2^x)-1) = 0 \Rightarrow 2^x = ...$	For attempting to solve the given equation as a **3 term quadratic** in y or as a **3 term quadratic** in 2^x leading to a value of y or 2^x (Apply usual rules for solving the quadratic – see general guidance) Allow x (or any other letter) instead of y for this mark e.g. an attempt to solve $8x^2 - 9x + 1 = 0$	M1
	$2^x (\text{or } y) = \frac{1}{8},\ 1$	Both correct answers of $\frac{1}{8}$ (oe) and 1 for 2^x or y or their letter but **<u>not *x*</u>** unless $2^x (\text{or } y)$ is implied later	A1
	$x = -3 \quad x = 0$	M1: A correct attempt to find one **numerical value** of x from their $2^x (\text{or } y)$ **which must have come from a 3 term quadratic equation**. If logs are used then they must be evaluated. A1: Both $x = -3$ and/or $x = 0$ May be implied by e.g. $2^{-3} = \frac{1}{8}$ and $2^0 = 1$ **and no extra values.**	M1A1
			(4)

8. (i) Find the real value of x such that

$$\log_x 600 = 3$$

Give your answer to 2 decimal places.

(2)

(ii) Solve the equation

$$\log_9(3x) + \log_9\left(\frac{x^4}{81}\right) = 2$$

giving the exact answer in the form $x = 3^k$, where k is a rational number.

(5)

7. (i) Find the value of y for which

$$1.01^{y-1} = 500$$

Give your answer to 2 decimal places.

(2)

(ii) Given that

$$2\log_4(3x + 5) = \log_4(3x + 8) + 1, \qquad x > -\frac{5}{3}$$

(a) show that

$$9x^2 + 18x - 7 = 0$$

(4)

(b) Hence solve the equation

$$2\log_4(3x + 5) = \log_4(3x + 8) + 1, \qquad x > -\frac{5}{3}$$

(2)

8. (i)	$\log_x 600 = 3$ means $x^3 = 600$	M1
	$x = \sqrt[3]{600} = 8.43$	A1 **(2)**
(ii) Way 1	$\log_9 3x + \log_9 \frac{x^4}{81} = \log_9 \frac{3x^k}{81}$ where $k = 4$ or 5	M1
	Uses $\log_9 81 = 2$ or $9^2 = 81$	M1
	So $3x^5 = 81 \times 81$	A1
	So $x^5 = \frac{81\times 81}{3} = 3^7$ so $x =$ (must be power of 3)	dM1
	$x = 3^{1.4}$ (accept $k = 1.4$)	A1 **(5)**
Way 2	$\log_9 3 + \log_9 x + 4\log_9 x - \log_9 81 = 2$	M1
	$5\log_9 x = \log_9 81 + \underline{\log_9 81} - \log_9 3$	M1
	$5\log_9 x = 3.5$	A1
	$\log_3 x = \frac{7}{5}$ so $x =$ (must be power of 3)	dM1
	$x = 3^{1.4}$ (accept $k = 1.4$)	A1 **(5)**

7. (i)	Use of power rule so $(y-1)\log 1.01 = \log 500$ or $(y-1) = \log_{1.01} 500$	M1
	625.56	A1 **(2)**
(ii) (a)	Ignore labels (a) and (b) in part ii and mark work as seen	
	$\log_4 (3x+5)^2 =$ Applies power law of logarithms	M1
	Uses $\log_4 4 = 1$ or $4^1 = 4$	M1
	Uses quotient or product rule so e.g. $\log(3x+5)^2 = \log 4(3x+8)$ or $\log \frac{(3x+5)^2}{(3x+8)} = 1$	M1
	Obtains with no errors $9x^2 + 18x - 7 = 0$ *	A1* cso **(4)**
(b)	Solves given or "their" quadratic equation by any of the standard methods	M1
	Obtains $x = \frac{1}{3}$ and $-\frac{7}{3}$ and rejects $-\frac{7}{3}$ to give just $\frac{1}{3}$	A1 **(2)**
		[8]

7. (i) $$2\log(x + a) = \log(16a^6), \text{ where } a \text{ is a positive constant}$$

Find x in terms of a, giving your answer in its simplest form.

(3)

(ii) $$\log_3(9y + b) - \log_3(2y - b) = 2, \text{ where } b \text{ is a positive constant}$$

Find y in terms of b, giving your answer in its simplest form.

(4)

8. (i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2$$

express b in terms of a.

(3)

(ii) Solve the equation

$$2^{2x+5} - 7(2^x) = 0$$

giving your answer to 2 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

7. (i)	Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$	M1
	Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$ Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$	M1
	$(x =)$ $4a^3 - a$ (depends on previous M's and must be this expression or equivalent)	A1cao **(3)**
(ii) Way 1	$\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ — Applies quotient law of logarithms	M1
	$\dfrac{(9y+b)}{(2y-b)} = 3^2$ — Uses $\log_3 3^2 = 2$	M1
	$(9y+b) = 9(2y-b) \Rightarrow y =$ — Multiplies across and makes y the subject	M1
	$y = \frac{10}{9}b$	A1cso **(4)**
Way 2	Or : $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ — 2nd M mark	M1
	$\log_3(9y+b) = \log_3 9(2y-b)$ — 1st M mark	M1
	$(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ — Multiplies across and makes y the subject	M1 A1cso **(4)** **[7]**

8(i)	Two Ways of answering the question are given in part (i)	
Way 1	$\log_3\left(\dfrac{3b+1}{a-2}\right) = -1$ or $\log_3\left(\dfrac{a-2}{3b+1}\right) = 1$ — Applying the subtraction law of logarithms	M1
	$\dfrac{3b+1}{a-2} = 3^{-1}\left\{=\dfrac{1}{3}\right\}$ or $\left(\dfrac{a-2}{3b+1}\right) = 3$ — Making a correct connection between log base 3 and 3 to a power.	M1
	$\{9b+3 = a-2 \Rightarrow\}$ $b = \dfrac{1}{9}a - \dfrac{5}{9}$ — $b = \dfrac{1}{9}a - \dfrac{5}{9}$ or $b = \dfrac{a-5}{9}$	A1 oe
		[3]
	In **Way 2** a correct connection between log base 3 and "3 to a power" is used before applying the subtraction or addition law of logs	
(i) **Way 2**	Either $\log_3(3b+1) - \log_3(a-2) = -\log_3 3$ or $\log_3(3b+1) + \log_3 3 = \log_3(a-2)$	2nd M1
	$\log_3(3b+1) = \log_3(a-2) - \log_3 3 = \log_3\left(\dfrac{a-2}{3}\right)$ or $\log_3 3(3b+1) = \log_3(a-2)$	1st M1
	$\left\{3b+1 = \dfrac{a-2}{3}\right\}$ $b = \dfrac{1}{9}a - \dfrac{5}{9}$	A1
		[3]
(ii) **Way 1** **See also common approach below in notes**	$32(2^{2x}) - 7(2^x) = 0$ — Deals with power 5 correctly giving $\times 32$	M1
	So, $2^x = \dfrac{7}{32}$ — $2^x = \dfrac{7}{32}$ or $y = \dfrac{7}{32}$ or awrt 0.219	A1 oe
	$x\log 2 = \log\left(\dfrac{7}{32}\right)$ or $x = \dfrac{\log\left(\frac{7}{32}\right)}{\log 2}$ or $x = \log_2\left(\dfrac{7}{32}\right)$ — A valid method for solving $2^x = \frac{7}{32}$ Or $2^x = k$ to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]

7. (i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places.

(3)

(ii) Find the values of y such that

$$\log_2(11y - 3) - \log_2 3 - 2\log_2 y = 1, \qquad y > \frac{3}{11}$$

(6)

7. (i)	$8^{2x+1} = 24$	
	$(2x+1)\log 8 = \log 24$ or $(2x+1) = \log_8 24$ — or $8^{2x} = 3$ and so $(2x)\log 8 = \log 3$ or $(2x) = \log_8 3$	M1
	$x = \frac{1}{2}\left(\frac{\log 24}{\log 8} - 1\right)$ or $x = \frac{1}{2}(\log_8 24 - 1)$ — $x = \frac{1}{2}\left(\frac{\log 3}{\log 8}\right)$ or $x = \frac{1}{2}(\log_8 3)$ o.e.	dM1
	$=0.264$	A1
		(3)
(ii)	$\log_2(11y-3) - \log_2 3 - 2\log_2 y = 1$	
	$\log_2(11y-3) - \log_2 3 - \log_2 y^2 = 1$	M1
	$\log_2 \frac{(11y-3)}{3y^2} = 1$ or $\log_2 \frac{(11y-3)}{y^2} = 1 + \log_2 3 = 2.58496501$	dM1
	$\log_2 \frac{(11y-3)}{3y^2} = \log_2 2$ or $\log_2 \frac{(11y-3)}{y^2} = \log_2 6$ (allow awrt 6 if replaced by 6 later)	B1
	Obtains $6y^2 - 11y + 3 = 0$ o.e. i.e. $6y^2 = 11y - 3$ for example	A1
	Solves quadratic to give $y =$	ddM1
	$y = \frac{1}{3}$ and $\frac{3}{2}$ (need both- one should not be rejected)	A1
		(6)
		[9]