6 (a) Given that $e^{-0.5x} = 5$, find the exact value of x.

[2 marks]

(b) Use integration by parts to find $\int x e^{-0.5x} \, dx$.

[4 marks]

6 Use the substitution $u = 2 + \ln x$ to show that

$$\int_1^e \frac{\ln x}{x(2+\ln x)^2} \, dx = p + \ln q$$

where p and q are rational numbers.

[7 marks]

Q6	Solution	Mark	Total	Comment
(a)	$-\frac{1}{2}x = \ln 5$	M1		
	$x = -2\ln 5$	A1	2	ACF
(b)	$u = x \quad \frac{dv}{(dx)} = e^{-0.5x}$	M1		All 4 terms in this form with $\frac{du}{dx}$ and $\int dx$ attempted
	$\frac{du}{(dx)} = 1 \quad v = -2e^{-0.5x}$	A1		All correct
	$\int = -2xe^{-0.5x} - \int -2e^{-0.5x}\ (dx)$	dM1		Correct substitution of their terms into the parts formula
	$\int = -2xe^{-0.5x} - 4e^{-0.5x}\ [+c]$ oe	A1	4	

Q6	Solution	Mark	Total	Comment
	$u = 2 + \ln x \Rightarrow du = \frac{1}{x}dx$ **OE**	B1		may have $x = e^{u-2}$ etc
		M1		clear attempt to get integral all in terms of u including dx in terms of du
	$\int \frac{u-2}{u^2}\,du$	A1		correct – condone omission of du if seen on later line
	$\int\left(\frac{A}{u}+\frac{B}{u^2}\right)du = A\ln u + \frac{C}{u}$	dM1		
	$\ln u + \frac{2}{u}$	A1		Integration by parts gives $\int = \ln u + \frac{2}{u} - 1$
	$\left[\ln 3 + \frac{2}{3}\right] - [\ln 2 + 1]$	dM1		correct substitution of correct limits but must have earned previous **dM1**
	$= -\frac{1}{3} + \ln\left(\frac{3}{2}\right)$	A1	7	
	Total		7	

8 For this question assume $0 < x < 0.5$

(a) Using the **product rule**, show that if $y = (\sin x)(\cos x)^{-1}$ then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x$$

[3 marks]

(b) Given that $x = \frac{1}{2}\sin u$ show that

$$\frac{\sqrt{(1 - 4x^2)}}{2x} = \cot u$$

[2 marks]

(c) Use the substitution $x = \frac{1}{2}\sin u$ to find

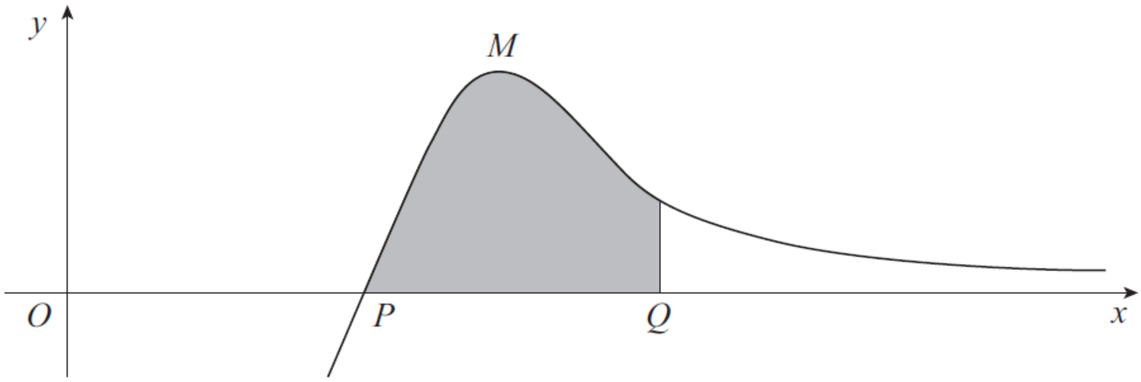
$$\int \frac{1}{(1 - 4x^2)^{\frac{3}{2}}}\,\mathrm{d}x$$

giving your answer in terms of x.

[7 marks]

Q8	Solution	Mark	Total	Comment
(a)	$(\frac{dy}{dx})$			
	$= \sin x \times -1 \times (\cos x)^{-2} \times -\sin x - \cos x(\cos x)^{-1}$	M1		Attempt at product rule
		A1		
	$= \frac{\sin^2 x}{\cos^2 x} + 1$			
	$= \tan^2 x + 1$			
	$= \sec^2 x$	A1		Must see 'a middle line' **AG**, all correct and no errors seen
			3	
(b)	$\frac{\sqrt{(1-4x^2)}}{2x} = \frac{\sqrt{1-\sin^2 u}}{\sin u}$	M1		
	$= \frac{\cos u}{\sin u}$			
	$= \cot u$	A1		Must see a middle line
			2	
(c)	$\frac{dx}{du} = 0.5\cos u$ oe	B1		Correct expression for $\frac{du}{dx}$ or du or dx
	$\int \frac{1}{(1-4(0.5\sin u)^2)^{1.5}} \times 0.5\cos u[du]$	oe M1		Replacing all terms in x to all in terms of u, including replacing dx, but condone omission of du
		A1		All correct, must see du here or on later line
	$=0.5\int \frac{1}{(\cos^2 u)^{1.5}} \times \cos u[du]$	dM1		Correct use of trig identity
	$= 0.5\int \sec^2 u[du]$	dM1		Correct simplification of powers
	$= 0.5\tan u$	A1		Using part (a)
	$[\tan u = \frac{2x}{\sqrt{1-4x^2}}]$			
	$\int = \frac{x}{\sqrt{1-4x^2}}$	A1		

7 Part of a curve is sketched below.



The curve has equation $y = (x-1)\,e^{-3x}$.

(a) The curve has a stationary point M. Show that the x-coordinate of M is $\frac{4}{3}$.

[3 marks]

(b) (i) By using integration by parts twice, find

$$\int (x-1)^2 e^{-6x}\,dx$$

[6 marks]

3 Use the substitution $u = \cos 2x$ to find

$$\int \cos^2 2x \sin^3 2x\,dx$$

[5 marks]

Q7	Solution	Mark	Total	Comment
(a)	$\frac{dy}{dx} = e^{-3x} - 3(x-1)e^{-3x}$	M1 A1		$e^{-3x} + A(x-1)e^{-3x}$ **OE** $A = -3$
	Equate gradient to 0, hence $x = \frac{4}{3}$	A1cso	3	**AG** be convinced, must see a middle line
(b) (i)	$u = (x-1)^2 \quad \frac{dv}{dx} = e^{-6x}$	M1		u and $\frac{dv}{dx}$ correct and 4 terms in this form
	$\frac{du}{dx} = 2(x-1) \quad v = -\frac{1}{6}e^{-6x}$	A1		all correct
	$I = -\frac{1}{6}(x-1)^2 e^{-6x} + \frac{1}{3}\int (x-1)e^{-6x}\, dx$	dM1		correct sub of their terms into parts formula
	$\int (x-1)e^{-6x}dx = -\frac{(x-1)}{6}e^{-6x} + \int \frac{1}{6}e^{-6x}dx$	M1		must be correct but may be seen anywhere in solution
	$= -\frac{(x-1)}{6}e^{-6x} - \frac{1}{36}e^{-6x}$	A1		
	$\int = -\frac{1}{6}(x-1)^2 e^{-6x} - \frac{(x-1)}{18}e^{-6x} - \frac{1}{108}e^{-6x}$	A1	6	**OE** condone omission of +c

(a) Withhold final **A1** for verification

(b)(i) Condone omission of dx throughout

Alt: If a candidate expands $(x-1)^2 = x^2 - 2x + 1$, then

$u = x^2 \quad \frac{dv}{dx} = e^{-6x} \quad \frac{du}{dx} = 2x \quad v = -\frac{1}{6}e^{-6x}$ **M1A1** u and $\frac{dv}{dx}$ correct and 4 terms in this form

$\int x^2 e^{-6x}dx = -\frac{1}{6}x^2 e^{-6x} + \frac{1}{6}\int 2xe^{-6x}\, dx$ **dM1** correct sub of their terms into parts formula

$\int xe^{-6x}dx = -\frac{x}{6}e^{-6x} + \int \frac{1}{6}e^{-6x}dx$ **M1** must be correct but may be seen anywhere in solution

$= -\frac{x}{6}e^{-6x} - \frac{1}{36}e^{-6x}$ **A1**

$\int = e^{-6x}(-\frac{1}{6}x^2 + \frac{5}{18}x - \frac{13}{108})$ **OE** **A1**

Q3	Solution	Mark	Total	Comment
	$\frac{du}{dx} = -2\sin 2x$ OE	B1		
	$k\int u^2 \times (1-u^2)\,du$	M1		Condone omission of du
	$= m\int \left(u^2 - u^4\right)du$	dM1		Condone omission of brackets and du
	$= -\frac{1}{2}\left(\frac{u^3}{3} - \frac{u^5}{5}\right) \quad (+c)$ OE	A1		Must have seen du on an earlier line where all terms are in 'u' only
	$= \frac{\cos^5 2x}{10} - \frac{\cos^3 2x}{6} \quad (+c)$ OE	A1	5	Condone omission of $+c$

6 Use integration by parts to find the value of $\int_1^5 \frac{3x}{\sqrt{2x-1}}\,dx$.

[6 marks]

6 (a) Use integration by parts to find $\int \frac{\ln(3x)}{x^2}\,dx$.

[4 marks]

Q6	Solution	Mark	Total	Comment
	$\int 3x(2x-1)^{-0.5}\ \mathrm{d}x$			
	$u = 3x \qquad \dfrac{\mathrm{d}v}{(\mathrm{d}x)} = (2x-1)^{-0.5}$	M1		u and $\dfrac{\mathrm{d}v}{(\mathrm{d}x)}$ correct, with $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ and $\int \mathrm{d}v$ attempted
	$\dfrac{\mathrm{d}u}{(\mathrm{d}x)} = 3 \qquad v = \dfrac{2}{2}(2x-1)^{0.5}$	A1		All correct
	$\int = 3x(2x-1)^{0.5} - \int (2x-1)^{0.5} \times 3\ (\mathrm{d}x)$	dM1		Correct substitution of their terms into the parts formula
	$= 3x(2x-1)^{0.5} - (2x-1)^{1.5}$ OE	A1		
	$[\int_1^5\ ] = (15\times 3 - 27) - (3-1)$	dM1		F(5) – F(1), correct from $Ax(2x-1)^{0.5} - B(2x-1)^{1.5}$
	$=16$	A1		
			6	

Q6	Solution	Mark	Total	Comment
a	$u = \ln 3x \qquad \dfrac{\mathrm{d}u}{(\mathrm{d}x)} = \dfrac{1}{x}$ oe	B1		PI by further work
	$\dfrac{\mathrm{d}v}{(\mathrm{d}x)} = \dfrac{1}{x^2} \qquad v = -x^{-1}$	B1		PI by further work
	$\int = -\dfrac{1}{x}\ln 3x - \int -x^{-1} \times \dfrac{1}{x}\ (\mathrm{d}x)$ oe	M1		Correct substitution of **their terms** into the parts formula
	$= -\dfrac{1}{x}\ln 3x - \dfrac{1}{x}\ (+c)$ oe	A1	4	

8 Use the substitution $u = 4x - 1$ to find the exact value of

$$\int_{\frac{1}{4}}^{\frac{1}{2}} (5 - 2x)(4x - 1)^{\frac{1}{3}}\, dx$$

[7 marks]

Q8	Solution	Mark	Total	Comment
	$\frac{du}{dx} = 4$ oe	B1		Correct expression for $\frac{du}{dx}$ or du or dx
	$[u = 4x - 1]$ $4x = u + 1$ oe	B1		Correct term in kx, where k = 1, 2, 4
	$\int \frac{9-u}{2} \times \sqrt[3]{u} \times \frac{(du)}{4}$ oe	M1		Replacing all terms in x to all in terms of u, including replacing dx, but condone omission of du
		A1		All correct, must see du here or on next line
	$\left(= \frac{1}{8} \int 9u^{\frac{1}{3}} - u^{\frac{4}{3}} (du) \right)$ oe			
	$= \frac{1}{8} (\frac{3}{4} \times 9u^{\frac{4}{3}} - \frac{3}{7} u^{\frac{7}{3}})$ oe	m1		Correct integration from an expression of the form $au^{\frac{1}{3}} + bu^{\frac{4}{3}}$ or $au^{\frac{1}{3}} + bu^{\frac{4}{3}} + cu^{\frac{1}{3}}$
	Limits $[x]_{0.25}^{0.5} = [u]_0^1$ may be seen earlier	B1		Or, correctly changing variable back into x
	$\left(= \frac{1}{8} [(\frac{27}{4} - \frac{3}{7}) - 0] \right)$			
	$= \frac{177}{224}$ oe	A1		allow equivalent fraction
	Total		7	

5 (a) By writing $\tan x$ as $\dfrac{\sin x}{\cos x}$, use the quotient rule to show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x$.

[2 marks]

(b) Use integration by parts to find $\displaystyle\int x \sec^2 x \, \mathrm{d}x$.

[4 marks]

7 Use the substitution $u = 6 - x^2$ to find the value of $\displaystyle\int_1^2 \frac{x^3}{\sqrt{6 - x^2}} \, \mathrm{d}x$, giving your answer in the form $p\sqrt{5} + q\sqrt{2}$, where p and q are rational numbers.

[7 marks]

a	$(\frac{dy}{dx}=)\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$	M1		$\frac{\pm\cos^2 x\pm\sin^2 x}{\cos^2 x}$
	$=\frac{1}{\cos^2 x}$ or $1+\tan^2 x$			Must see this line
	$=\sec^2 x$	A1	2	AG; no errors seen and all notation correct
b	$\int x\sec^2 x\,dx$ $u=x$ $\frac{dv}{(dx)}=\sec^2 x$			
	$\frac{du}{(dx)}=1$ $v=\tan x$	M1		All 4 terms in this form with $\frac{du}{dx}$ correct and $\int\frac{dv}{dx}$ attempted
	$v=\tan x$	B1		
	$x\tan x-\int\tan x\,(dx)$	A1		
	$=x\tan x-\ln\sec x+c$	A1		OE (e.g. $x\tan x+\ln\cos x$); must have constant of integration

Q7	Solution	Mark	Total	Comment
	$\frac{du}{dx}=-2x$ or $du=-2x\,dx$	M1		Condone $\frac{du}{dx}=2x$ or $du=2x\,dx$
	$\int\frac{6-u}{u^{0.5}}\times\frac{du}{-2}$	A1		OE correct unsimplified integral in terms of u only, with du seen on this line or later
	$=-\frac{1}{2}\int(6u^{-0.5}-u^{0.5})du$	m1		Terms in the form $\int(a\,u^{-0.5}+bu^{0.5})du$
	$=-\frac{1}{2}(6\frac{u^{0.5}}{0.5}-\frac{2u^{1.5}}{3})$	A1F		Ft must be in the form $cu^{0.5}+du^{1.5}$ Oe (eg allow $c\sqrt{u}$)
	$(=-6u^{0.5}+\frac{1}{3}u^{1.5})$			
	(Limits $[x]_1^2=[u]_5^2$)			
	$\int_5^2=\left[-6u^{0.5}+\frac{1}{3}u^{1.5}\right]_5^2$			
	$=(-6\times2^{0.5}+\frac{1}{3}\times2^{1.5})-(-6\times5^{0.5}+\frac{1}{3}\times5^{1.5})$	m1		Correct substitution into expression of the form $eu^{0.5}+fu^{1.5}$ and F(2) – F(5), or if using x, F(2) – F(1)
	$=\frac{13}{3}\sqrt{5}-\frac{16}{3}\sqrt{2}$	A1A1	7	oe any correct exact form

6. (i) Find, in its simplest form,

$$\int \frac{5}{6e^{3x}}\,dx$$

(2)

(ii) (a) Express $\dfrac{4y^2 + 3y - 4}{y(2y-1)}$ in partial fractions.

(4)

(b) Hence find

$$\int \frac{4y^2 + 3y - 4}{y(2y-1)}\,dy \qquad y > \frac{1}{2}$$

(3)

(iii) Use integration by parts to find

$$\int_1^4 \frac{1}{\sqrt{x}}\ln(2x)\,dx$$

giving your answer in the form $a + b\ln 2$, where a and b are constants to be found.

(5)

6. (i)	$\int \frac{5}{6e^{3x}}\,dx = \int \frac{5}{6}e^{-3x}\,dx = -\frac{5}{18}e^{-3x}\ \{+c\}$	Integrates to give $\pm\alpha e^{-3x}$, $\alpha \neq 0, \frac{5}{6}, 30$	M1
		$-\frac{5}{18}e^{-3x}$ or $-\frac{5}{18e^{3x}}$ with or without $+c$	A1
			(2)
(ii)(a)	$\frac{4y^2+3y-4}{y(2y-1)} \equiv A + \frac{B}{y} + \frac{C}{(2y-1)}$		
	$\{y^2 : 4 = 2A \Rightarrow\}\ A = 2$	Their constant term = 2	B1
	$4y^2+3y-4 \equiv Ay(2y-1) + B(2y-1) + Cy$	Forming a correct identity	B1
	Either • constant: $-4 = -B \Rightarrow B = 4$ • $y: -A + 2B + C \Rightarrow 3 = -2 + 8 + C \Rightarrow C = -3$ • $y = 0 \Rightarrow -4 = -B \Rightarrow B = 4$ • $y = \frac{1}{2} \Rightarrow 1 + \frac{3}{2} - 4 = \frac{1}{2}C \Rightarrow C = -3$	Uses their identity in an attempt to find the value of at least one of either their B or their C	M1
	$\left\{\frac{4y^2+3y-4}{y(2y-1)} \equiv\right\}\ 2 + \frac{4}{y} - \frac{3}{(2y-1)}$	Correct partial fractions. Can be seen anywhere in part (ii)	A1
			(4)
(b) **Way 1**	$\left\{\int \frac{4y^2+3y-4}{y(2y-1)}\,dy\right\}$ $= \int \left(2 + \frac{4}{y} - \frac{3}{(2y-1)}\right)dy$ $= 2y + 4\ln y - \frac{3}{2}\ln(2y-1)\ \{+c\}$	Integrates to give at least one of either $\frac{B}{y} \to \pm\lambda \ln y$ or $\frac{C}{2y-1} \to \pm\mu\ln(2y-1)$ or $\gamma\ln(y-\frac{1}{2})$, $B \neq 0$, $C \neq 0$	M1
		Correct follow through integration for at least two terms from their $A \neq 0$ or from their B or from their C	A1 ft
		$2y + 4\ln y - \frac{3}{2}\ln(2y-1)$ **or** $2y + 4\ln y - \frac{3}{2}\ln(y-\frac{1}{2})$ can apply isw	A1
	Final A1: Correct bracketing required. Can be simplified or un-simplified, with/without $+c$		**(3)**
(iii)	$\left\{\int \frac{1}{\sqrt{x}}\ln(2x)\,dx\right\}$, $\left\{u = \ln(2x) \Rightarrow \frac{du}{dx} = \frac{1}{x};\ \frac{dv}{dx} = x^{-\frac{1}{2}} \Rightarrow v = 2x^{\frac{1}{2}}\right\}$		
	$= 2x^{\frac{1}{2}}\ln(2x) - \int 2x^{\frac{1}{2}}\left(\frac{1}{x}\right)\{dx\}$	Either $\frac{1}{\sqrt{x}}\ln(2x) \to \pm\lambda x^{\frac{1}{2}}\ln(kx) \pm \int \mu x^{\frac{1}{2}}\left(\frac{\alpha}{\beta x}\right)\{dx\}$ or $\pm\lambda x^{\frac{1}{2}}\ln(kx) \pm \int \mu x^{-\frac{1}{2}}\{dx\}$; $\lambda, \mu, k \neq 0$	M1
	$= 2x^{\frac{1}{2}}\ln(2x) - 4x^{\frac{1}{2}}$	**dependent on the previous M mark** Integrates the second term to give $Ax^{\frac{1}{2}}$; $A \neq 0$	dM1 A1 on epen
		$2x^{\frac{1}{2}}\ln(2x) - 4x^{\frac{1}{2}}$, simplified or un-simplified	A1
	$\left\{\left[2\sqrt{x}\ln(2x) - 4\sqrt{x}\right]_1^4\right\}$ $= (2\sqrt{4}\ln(8) - 4\sqrt{4}) - (2\sqrt{1}\ln(2) - 4\sqrt{1})$	**dependent on the first M mark** Some evidence of applying limits of 4 and 1 and subtracts the correct way round	dM1
	$\{= 4\ln 8 - 8 - 2\ln 2 + 4\} = -4 + 10\ln 2$	$-4 + 10\ln 2$	A1 **cso**
			(5)

3. (i) Given that

$$\frac{13-4x}{(2x+1)^2(x+3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)}$$

(a) find the values of the constants A, B and C.

(4)

(b) Hence find

$$\int \frac{13-4x}{(2x+1)^2(x+3)}\,\mathrm{d}x, \quad x > -\frac{1}{2}$$

(3)

(ii) Find

$$\int (\mathrm{e}^x + 1)^3\,\mathrm{d}x$$

(3)

(iii) Using the substitution $u^3 = x$, or otherwise, find

$$\int \frac{1}{4x + 5x^{\frac{1}{3}}}\,\mathrm{d}x, \quad x > 0$$

(4)

(a)	$B=6,\ C=1$	At least one of $B=6$ or $C=1$	B1
		Both $B=6$ and $C=1$	B1
	$13-4x \equiv A(2x+1)(x+3) + B(x+3) + C(2x+1)^2$ $x=-3 \Rightarrow 25 = 25C \Rightarrow C=1$ $x=-\frac{1}{2} \Rightarrow 13--2 = \frac{5}{2}B \Rightarrow 15 = 2.5B \Rightarrow B=6$	Writes down a correct identity and attempts to find the value of either one of A or B or C	M1
	Either $x^2: 0 = 2A+4C,$ constant $: 13 = 3A+3B+C,$ $x: -4 = 7A+B+4C$ or $x=0 \Rightarrow 13 = 3A+3B+C$ leading to $A=-2$	Using a correct identity to find $A=-2$	A1
			[4]
(b)	$\int \frac{13-4x}{(2x+1)^2(x+3)}\,dx = \int \frac{-2}{(2x+1)} + \frac{6}{(2x+1)^2} + \frac{1}{(x+3)}\,dx$		
	$= \frac{(-2)}{2}\ln(2x+1) + \frac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3)\ \{+c\}$ o.e. $\{= -\ln(2x+1) - 3(2x+1)^{-1} + \ln(x+3)\ \{+c\}\}$	See notes	M1
		At least two terms correctly integrated	A1ft
		Correct answer, o.e. Simplified or un-simplified. The correct answer must be stated on one line Ignore the absence of '$+c$'	A1
			[3]
(ii)	$\{(e^x+1)^3 =\}\ e^{3x} + 3e^{2x} + 3e^x + 1$	$e^{3x}+3e^{2x}+3e^x+1$, simplified or un-simplified	B1
	$\left\{\int (e^x+1)^3\,dx\right\} = \frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x\ \{+c\}$	At least 3 examples (see notes) of correct ft integration	M1
		$\frac{1}{3}e^{3x}+\frac{3}{2}e^{2x}+3e^x+x$, simplified or un-simplified with or without $+c$	A1
			[3]
(iii)	$\int \frac{1}{4x+5x^{\frac{1}{3}}}\,dx,\ x>0;\ u^3 = x$		
	$3u^2\frac{du}{dx} = 1$	$3u^2\frac{du}{dx} = 1$ or $\frac{dx}{du} = 3u^2$ or $\frac{du}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$ or $3u^2 du = dx$ o.e.	B1
	$= \int \frac{1}{4u^3+5u}.3u^2\,du\ \left\{= \int \frac{3u}{4u^2+5}\,du\right\}$	Expression of the form $\int \frac{\pm ku^2}{4u^3 \pm 5u}\{du\}$, $k \neq 0$ Does not have to include integral sign or du Can be implied by later working	M1
	$= \frac{3}{8}\ln(4u^2+5)\ \{+c\}$	**dependent on the previous M mark** $\pm\lambda\ln(4u^2+5)$; λ is a constant; $\lambda \neq 0$	dM1
	$= \frac{3}{8}\ln\left(4x^{\frac{2}{3}}+5\right)\ \{+c\}$	Correct answer in x with or without $+c$	A1
			[4]

6. (i) Given that $y > 0$, find

$$\int \frac{3y - 4}{y(3y + 2)}\, \mathrm{d}y$$

(6)

(ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4 - x}\right)}\, \mathrm{d}x = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta\ \mathrm{d}\theta$$

where λ is a constant to be determined.

(5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4 - x}\right)}\, \mathrm{d}x$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

(i) **Way 1**	$\dfrac{3y-4}{y(3y+2)} \equiv \dfrac{A}{y} + \dfrac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow -4 = 2A \Rightarrow A = -2$	At least one of their $A=-2$ **or** their $B=9$	A1
	$y = -\frac{2}{3} \Rightarrow -6 = -\frac{2}{3}B \Rightarrow B = 9$	Both their $A=-2$ **and** their $B=9$	A1
	$\displaystyle\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{-2}{y} + \frac{9}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{A}{y} \to \pm\lambda \ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu \ln(3y+2)$, $A \neq 0$, $B \neq 0$	M1
		At least one term correctly followed through from their A or from their B	A1 ft
	$= -2\ln y + 3\ln(3y+2)\ \{+c\}$	$-2\ln y + 3\ln(3y+2)$ **or** $-2\ln y + 3\ln(y+\frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 **cao**
			[6]

3.

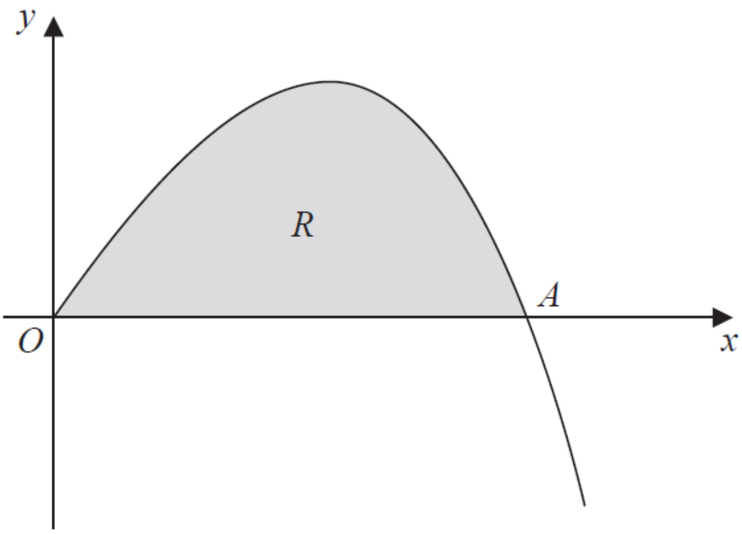


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of $\ln 2$, the x coordinate of the point A.

(2)

(b) Find

$$\int xe^{\frac{1}{2}x}\,dx$$

(3)

The finite region R, shown shaded in Figure 1, is bounded by the x-axis and the curve with equation

$$y = 4x - xe^{\frac{1}{2}x}, \quad x \geqslant 0$$

(c) Find, by integration, the exact value for the area of R.
Give your answer in terms of $\ln 2$

(3)

3.	$y = 4x - x\mathrm{e}^{\frac{1}{2}x},\ x \geqslant 0$		
(a)	$\left\{ y = 0 \Rightarrow 4x - x\mathrm{e}^{\frac{1}{2}x} = 0 \Rightarrow x(4 - \mathrm{e}^{\frac{1}{2}x}) = 0 \Rightarrow \right\}$		
	$\mathrm{e}^{\frac{1}{2}x} = 4 \Rightarrow x_A = 4\ln 2$	Attempts to solve $\mathrm{e}^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda \ln \mu$ where $\mu > 0$	M1
		$4\ln 2$ **cao** (Ignore $x = 0$)	A1
			[2]
(b)	$\left\{ \int x\mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x \right\} = 2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$	$\alpha x\mathrm{e}^{\frac{1}{2}x} - \beta\int \mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\},\ \alpha > 0, \beta > 0$	M1
		$2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$, with or without $\mathrm{d}x$	A1 **(M1 on ePEN)**
	$= 2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}\ \{+c\}$	$2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}$ o.e. with or without $+c$	A1
			[3]
(c)	$\left\{ \int 4x\,\mathrm{d}x \right\} = 2x^2$	$4x \to 2x^2$ or $\frac{4x^2}{2}$ o.e.	B1
	$\left\{ \int_0^{4\ln 2} (4x - x\mathrm{e}^{\frac{1}{2}x})\,\mathrm{d}x \right\} = \left[2x^2 - \left(2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x} \right) \right]_0^{4\ln 2 \text{ or } \ln 16 \text{ or their limits}}$		
	$= \left(2(4\ln 2)^2 - 2(4\ln 2)\mathrm{e}^{\frac{1}{2}(4\ln 2)} + 4\mathrm{e}^{\frac{1}{2}(4\ln 2)} \right) - \left(2(0)^2 - 2(0)\mathrm{e}^{\frac{1}{2}(0)} + 4\mathrm{e}^{\frac{1}{2}(0)} \right)$	**See notes**	M1
	$= \left(32(\ln 2)^2 - 32(\ln 2) + 16 \right) - (4)$ $= 32(\ln 2)^2 - 32(\ln 2) + 12$	$32(\ln 2)^2 - 32(\ln 2) + 12$, **see notes**	A1

6.

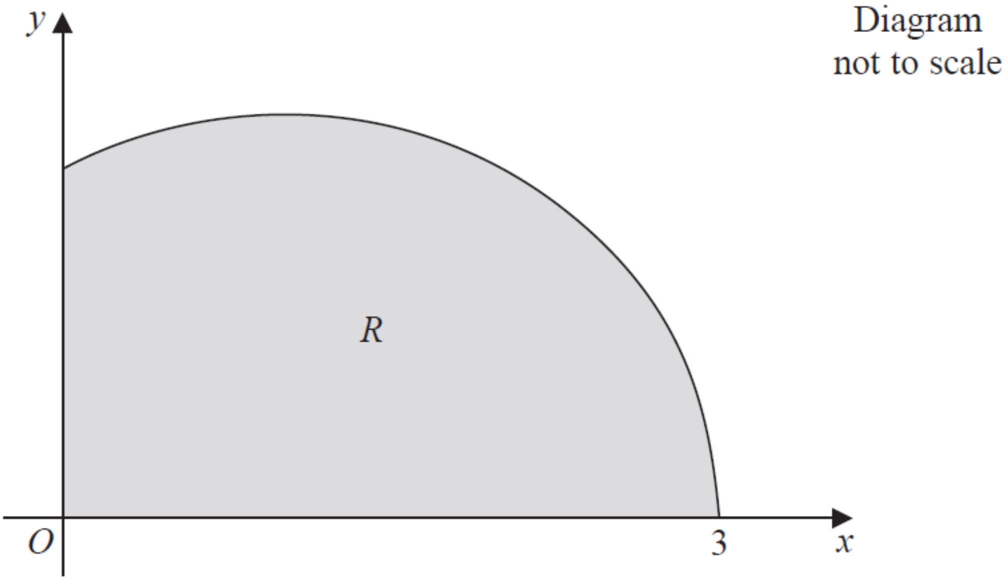


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \leqslant x \leqslant 3$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\,\mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,\mathrm{d}\theta$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R.

(3)

6. (a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x\ ,\ \ x = 1 + 2\sin\theta$		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ or $2\cos\theta$ used correctly in their working. Can be implied.	B1
	$\left\{\int \sqrt{(3-x)(x+1)}\ \mathrm{d}x \text{ or } \int \sqrt{(3+2x-x^2)}\ \mathrm{d}x\right\}$		
	$= \int \sqrt{(3-(1+2\sin\theta))((1+2\sin\theta)+1)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$	Substitutes for both x and $\mathrm{d}x$, where $\mathrm{d}x \neq \lambda \mathrm{d}\theta$. Ignore $\mathrm{d}\theta$	M1
	$= \int \sqrt{(2-2\sin\theta)(2+2\sin\theta)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$ $= \int \sqrt{(4-4\sin^2\theta)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$		
	$= \int \sqrt{(4-4(1-\cos^2\theta))}\ 2\cos\theta\ \{\mathrm{d}\theta\}$ or $\int \sqrt{4\cos^2\theta}\ 2\cos\theta\ \{\mathrm{d}\theta\}$	Applies $\cos^2\theta = 1 - \sin^2\theta$ **see notes**	M1
	$= 4\int \cos^2\theta\, \mathrm{d}\theta,\ \{k = 4\}$	$4\int \cos^2\theta \mathrm{d}\theta$ or $\int 4\cos^2\theta \mathrm{d}\theta$ **Note:** $\mathrm{d}\theta$ is required here.	A1
	$0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2} \Rightarrow \theta = -\frac{\pi}{6}$ **and** $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$	See notes	B1
			[5]
(b)	$\left\{k\int \cos^2\theta\ \{\mathrm{d}\theta\}\right\} = \{k\}\int \left(\frac{1+\cos 2\theta}{2}\right)\{\mathrm{d}\theta\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \{k\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$	Integrates to give $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha \neq 0, \beta \neq 0$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$	M1 (A1 on ePEN)
	$\left\{\text{So } 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta \mathrm{d}\theta = \left[2\theta + \sin 2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\right\}$ $= \left(2\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right)\right) - \left(2\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right)\right)$		
	$\left\{= (\pi) - \left(-\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right\} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$	$\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ **or** $\frac{1}{6}\left(8\pi + 3\sqrt{3}\right)$	A1 **cao cso**
			[3]