3 (a) (i) Sketch the graph of $y = \ln(2x)$, stating the coordinates of any point where the curve crosses the coordinate axes.

[2 marks]

(ii) Describe a sequence of two geometrical transformations that maps the graph of $y = \ln(2x)$ onto the graph of $y = \ln(3 + 4x)$.

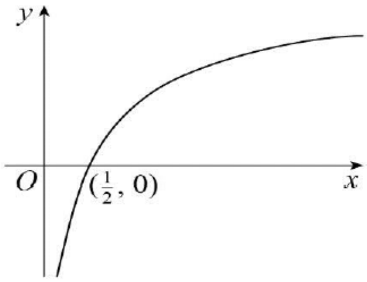
[4 marks]

7 You are given that k is a positive constant.

By sketching the graphs of $y = |5x - 3k|$ and $y = 3|x + 4k|$ on the same axes, solve the inequality

$$|5x - 3k| \geqslant 3|x + 4k|$$

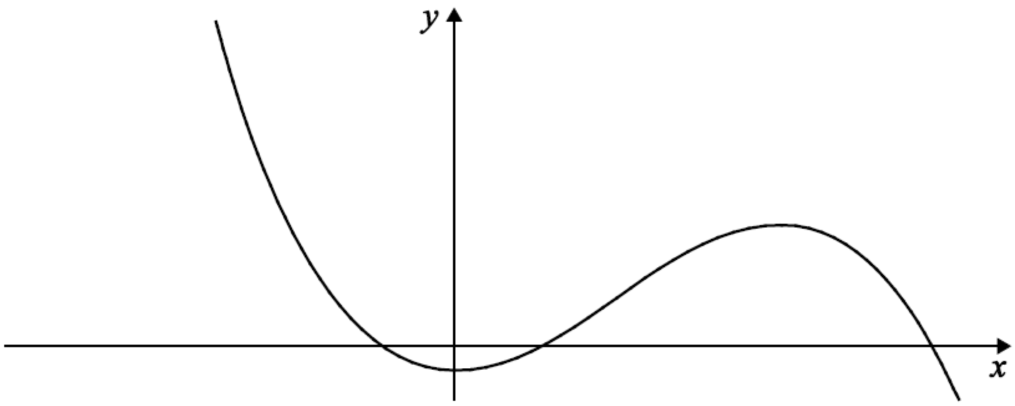
[5 marks]

Q3	Solution	Mark	Total	Comment
(a)(i)		**M1**		"log" graph correct shape in 1st and 4th quadrants Graph not touching y-axis
		A1	**2**	cuts x-axis at $\left(\frac{1}{2},0\right)$ stated or 0.5 marked on x-axis and no other intersections with coordinate axes
(ii)	Stretch + either I or II Parallel to x-axis I	**M1**		**Alt 1:** Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$ **M1**
	SF $\frac{1}{2}$ II	**A1**		$k=-\dfrac{3}{2}$ **A1**
	Followed by Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	**M1**		Followed by Stretch in x-direction **M1** SF $\frac{1}{2}$ **A1**
	$k=-\dfrac{3}{4}$	**A1**	**4**	

Q7	Solution	Mark	Total	Comment
	2 V-shaped mod graphs, one with vertex on positive x-axis and other with vertex on negative x-axis	**B1**		
	Critical values $\dfrac{15k}{2}$	**B1**		
	$5x-3k=-3(x+4k)$ OE	**M1**		PI
	$[x=]-\dfrac{9k}{8}$	**A1**		And no other values
	$x\le -\dfrac{9k}{8}$, $x\ge \dfrac{15k}{2}$	**A1**	**5**	May have **OR** between two inequalities but not **AND**

9 **Figure 2** shows part of the curve with equation $y = \mathrm{f}(x)$.

Figure 2



(c) The curve with equation $y = \mathrm{f}(x)$ has a minimum point at $(0, b-2)$ and a maximum point at $(a, 9b)$, where $0 < b < 2$.

(i) Find the coordinates of the minimum point of the curve with equation $y = \mathrm{f}(x+a) + 2b$.

[2 marks]

(ii) Find the coordinates of the maximum point of the curve with equation $y = 3\mathrm{f}(2x)$.

[2 marks]

(c)(i)	$x = -a$ $y = 3b - 2$	**B1** **B1**		 Each value may be stated or given as coordinates
			2	
(ii)	$x = 0.5a$ $y = 27b$	**B1** **B1**		 Each value may be stated or given as coordinates
			2	

2 (a) Sketch, on the axes below, the curve with equation $y = 4 - |2x + 1|$, indicating the coordinates where the curve crosses the axes.

[4 marks]

(b) Solve the equation $x = 4 - |2x + 1|$.

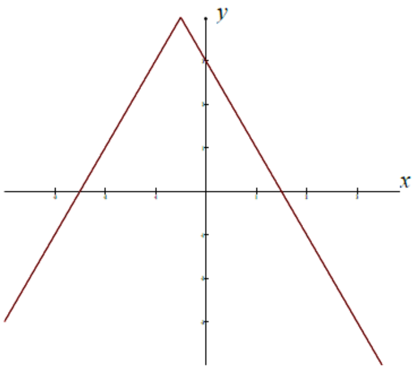
[3 marks]

(c) Solve the inequality $x < 4 - |2x + 1|$.

[2 marks]

(d) Describe a sequence of two geometrical transformations that maps the graph of $y = |2x + 1|$ onto the graph of $y = 4 - |2x + 1|$.

[4 marks]

Q2	Solution	Mark	Total	Comment
a		M1		Correct shape, inverted V, roughly symmetrical, with vertex in the 2nd quadrant
		A1		In all 4 quadrants
	(1.5, 0) **and** (-2.5, 0)	B1		Shown on sketch or coordinates stated
	(0, 3)	B1	4	Shown on sketch or coordinates stated (diagram takes precedence)
b	$(x =)1$	B1		OE
	$x = 4 + (2x + 1)$	M1		
	$(x =) - 5$	A1	3	
c	$-5 < x < 1$	B2	2	Or for $x >$ -5 **AND** $x < 1$
d	Reflection in $y = k$	M1		Translation $\begin{bmatrix} 0 \\ p \end{bmatrix}$ (M1)
	x-axis (or line $y = 0$)	A1		$p = 4$ (A1)
	(followed by)			(followed by) (PI)
	Translation $\begin{bmatrix} 0 \\ p \end{bmatrix}$	M1		Reflection in $y = k$ (M1)
	$p = 4$	A1	4	$k = 4$ (A1) oe
	Total		13	

(a) For **M1** must be attempt at straight lines. Condone correct values on axes for **B1, B1**

(b) **NMS**: $x = -5$ scores **SC1**
If squaring: $x^2 - 8x + 16 = 4x^2 + 4x + 1$ therefore $3x^2 + 12x - 15 = 0$ scores **M1**, then **A1, B1** as above

(c) $x >$ -5, $x < 1$ scores **SC1** $x >$ -5 or $x < 1$ scores **SC1**
SC1 for $-5 \le x \le 1$ or $-5 \le x \le 1$ or $-5 \le x \le 1$

(d) There are other correct possible transformations, but for full marks the order of the two transformations must produce the correct answer.

10.

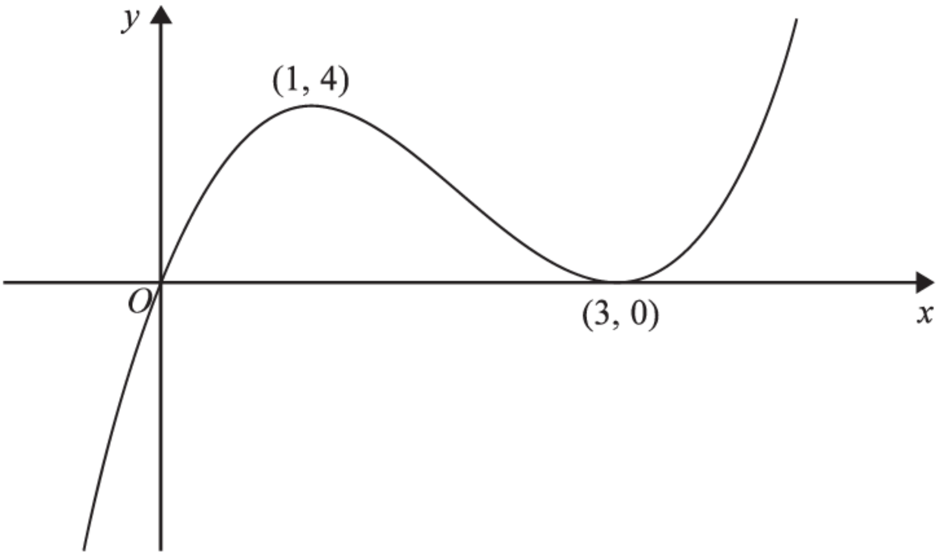


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x(3 - x)^2 \qquad x \in \mathbb{R}$$

The curve passes through the origin and touches the x-axis at the point (3, 0).
There is a maximum point at (1, 4) and a minimum point at (3, 0).

(a) On separate diagrams, sketch the curve with equation

(i) $y = \mathrm{f}(\frac{1}{2}x)$,

(ii) $y = \mathrm{f}(x + 2)$.

On each sketch indicate clearly the coordinates of

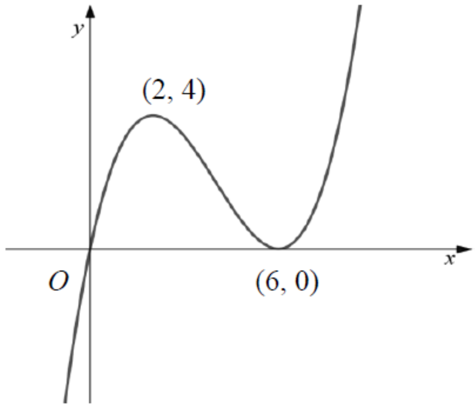
- any points where the curve crosses or touches the x-axis,
- the point where the curve crosses the y-axis,
- any maximum or minimum points.

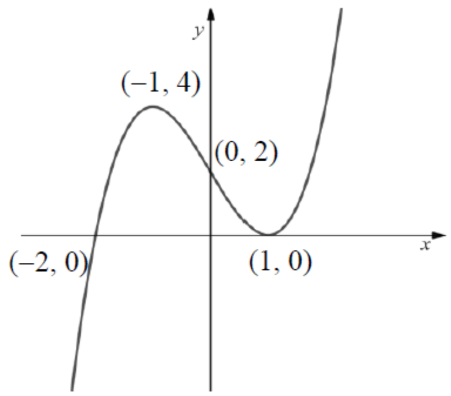
(6)

The curve with equation $y = \mathrm{f}(x) + k$, where k is a non-zero constant, has a maximum point at $(a, 0)$.

(b) Write down the values of a and k.

(2)

<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td rowspan="3">10.(a)(i)</td><td rowspan="3">


</td><td>Similar shape to the given figure passing through O (be generous if it just misses O but the intention is clear) and with evidence of a horizontal stretch taken from the x coordinates of the max/min point(s) but with no contradiction if both points are given. There should be no change in the y coordinates. The origin does not need to be labelled.</td><td>B1</td></tr>
<tr><td>Maximum at (2, 4)</td><td>B1</td></tr>
<tr><td>Minimum at (6, 0)</td><td>B1</td></tr>
<tr><td></td><td colspan="2">The coordinates may appear on the sketch, or separately in the text. If a point on an axis appears on the sketch it is not necessary to give both coordinates.
So, for example, 6 or (0, 6) on the x - axis would get credit, but if the answer is given in the text (6, 0) is needed. If there is any ambiguity, the sketch has precedence.</td><td></td></tr>
<tr><td rowspan="4">(a)(ii)</td><td rowspan="3">


</td><td>Similar shape translated horizontally. Ignore any coordinates given.</td><td>M1</td></tr>
<tr><td>Minimum at (1, 0) and crosses or at least reaches x-axis at (−2, 0)</td><td>A1</td></tr>
<tr><td>Maximum at (−1, 4) – must correspond to a maximum in the 2nd quadrant and crosses the y-axis at (0, 2)</td><td>A1</td></tr>
<tr><td></td><td></td><td>[6]</td></tr>
<tr><td></td><td colspan="2">The coordinates may appear on the sketch, or separately in the text. If a point on an axis appears on the sketch it is not necessary to give both coordinates.
So, for example, 2 or (2, 0) on the y-axis would get credit but if the answer is given in the text (0, 2) is needed. If there is any ambiguity, the sketch has precedence.</td><td></td></tr>
<tr><td rowspan="5">(b)</td><td>$a = 1$ or $k = -4$</td><td>One correct value</td><td>B1</td></tr>
<tr><td>$a = 1$ and $k = -4$</td><td>Both correct</td><td>B1</td></tr>
<tr><td colspan="2">Note that these marks may be implied by sight of e.g. “f(x) – 4” and/or “(1, 0)”</td><td></td></tr>
<tr><td colspan="2">Note that the answer to (b) often appears at the bottom of page 1</td><td></td></tr>
<tr><td colspan="2"></td><td>[2]</td></tr>
<tr><td></td><td colspan="2"></td><td>8 marks</td></tr>
</table>

5.

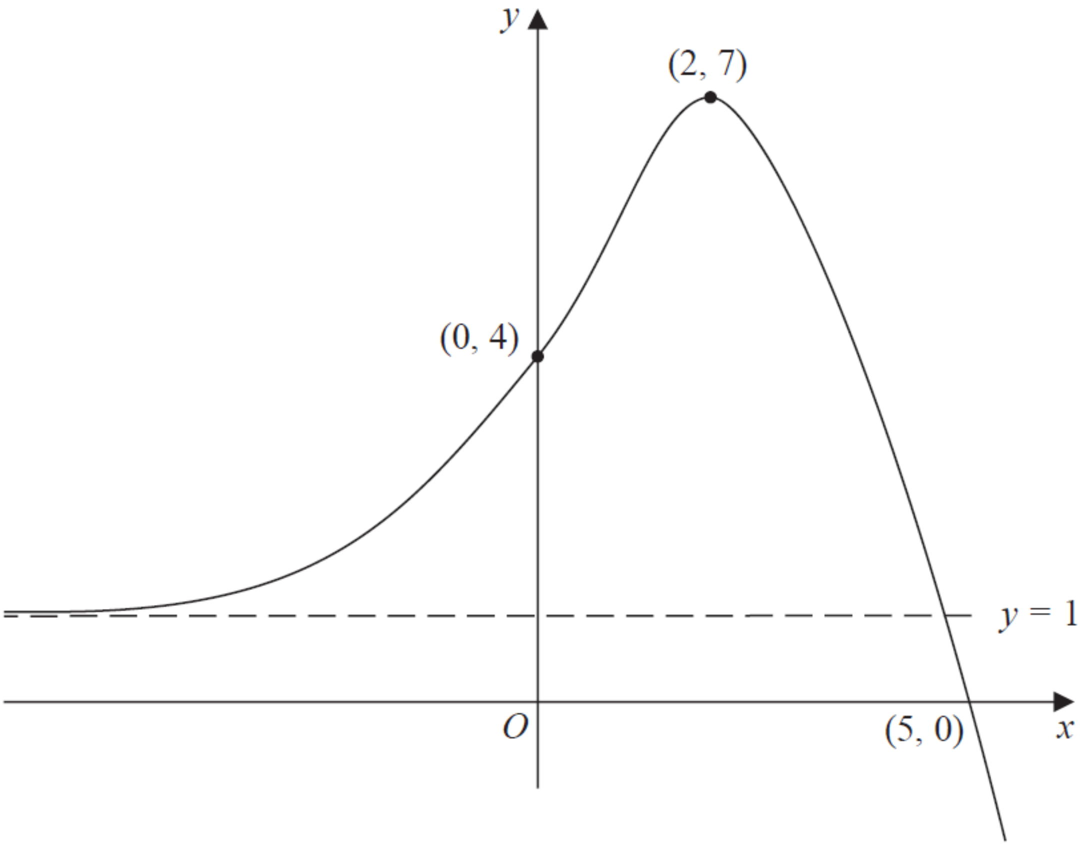


Figure 1

Figure 1 shows the sketch of a curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve crosses the y-axis at (0, 4) and crosses the x-axis at (5, 0).

The curve has a single turning point, a maximum, at (2, 7).

The line with equation $y = 1$ is the only asymptote to the curve.

(a) State the coordinates of the turning point on the curve with equation $y = \mathrm{f}(x - 2)$.

(1)

(b) State the solution of the equation $\mathrm{f}(2x) = 0$

(1)

(c) State the equation of the asymptote to the curve with equation $y = \mathrm{f}(-x)$.

(1)

Given that the line with equation $y = k$, where k is a constant, meets the curve $y = \mathrm{f}(x)$ at only one point,

(d) state the set of possible values for k.

(2)

5(a)	(4, 7)	Accept (4, 7) or $x = 4$, $y = 7$ or a sketch of $y = \mathrm{f}(x-2)$ with a maximum point marked at (4, 7). (Condone missing brackets) There should be no other coordinates.	B1
			(1)
(b)	$(x =)$ 2.5	Allow (2.5, 0) (condone missing brackets) but no other values or points. Allow a sketch of f(2x) with the only x-intercept marked at $x = 2.5$ (Allow (0, 2.5) marked in the correct place.	B1
			(1)
(c)	$y = 1$ (oe e.g. $y - 1 = 0$)	Must be an equation and not just '1' and no other asymptotes stated.	B1
			(1)
(d)	$k \leq 1$ **or** $k = 7$	Either of $k \leq 1$ or $k = 7$ Accept either of $y \leq 1$ or $y = 7$ Note that $k = 7$ may sometimes be seen embedded in e.g. $k = 0, 1, 7$ and can score B1 here.	B1
	$k \leq 1 \quad k = 7$	Both correct and in terms of k with no other solutions.	B1

4.

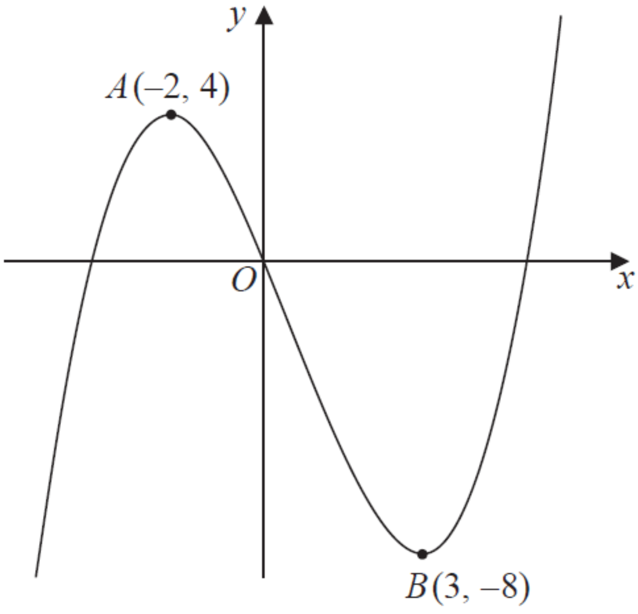


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$. The curve has a maximum point A at (–2, 4) and a minimum point B at (3, –8) and passes through the origin O.

On separate diagrams, sketch the curve with equation

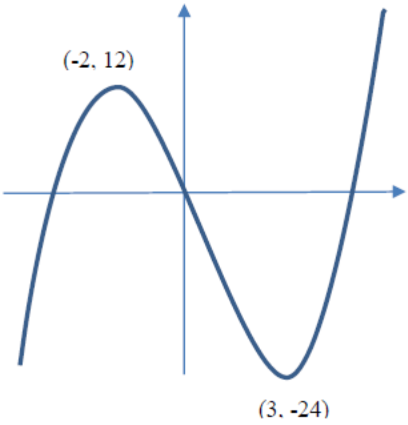
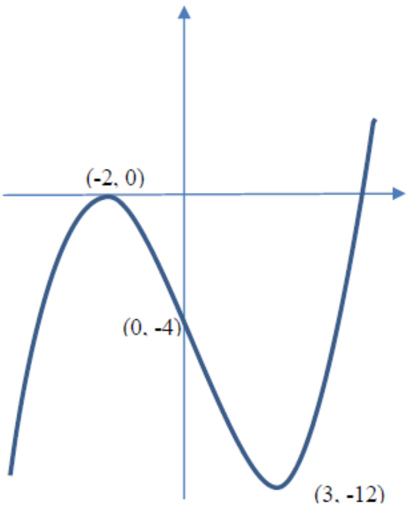
(a) $y = 3\mathrm{f}(x)$,

(2)

(b) $y = \mathrm{f}(x) - 4$

(3)

On each diagram, show clearly the coordinates of the maximum and the minimum points and the coordinates of the point where the curve crosses the y-axis.

4.(a)		Similar shape to given figure passing through the origin. A cubic shape with a maximum in the second quadrant and a minimum in the 4th quadrant. There must be evidence of a change in at least one of the *y*-coordinates (inconsistent changes in the y-coordinates are acceptable) but **not the *x*-coordinates**.	B1
		Maximum at (-2, 12) and minimum at (3, -24) with coordinates written the right way round. Condone missing brackets. The coordinates may appear on the sketch, or separately in the text (not necessarily referenced as *A* and *B*). If they are on the sketch, the *x* and *y* coordinates can be positioned correctly on the axes rather than given as coordinate pairs. In cases of ambiguity, the sketch has precedence. The origin does not need to be labelled. Nor do the *x* and *y* axes.	B1
			[2]
(b)		A positive cubic which does not pass through the origin with a maximum to the left of the *y*-axis and a minimum to the right of the *y*-axis.	M1
		Maximum at (-2, 0) and minimum at (3, -12). Condone missing brackets. For the max allow just -2 or (0, -2) if marked in the correct place. If the coordinates are in the text, they must appear as (-2, 0) and must not contradict the sketch. The curve must **touch** the *x*-axis at (-2, 0). For the min allow coordinates as shown or 3 and -12 to be marked in the correct places on the axes. In cases of ambiguity, the sketch has precedence.	A1
		Crosses *y*-axis at (0, -4). Allow just -4 (not +4) and allow (-4, 0) if marked in the correct place. If the coordinates are in the text, they must appear as (0, -4) and must not contradict the sketch. In cases of ambiguity, the sketch has precedence.	A1
			[3]

5. Given that a is a positive constant and

$$f(x) = |3x - a|, \quad x \in \mathbb{R}$$

(a) sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the coordinate axes.

(2)

Given that $x = 4$ is a solution to the equation $|3x - a| = \frac{1}{2}x + 2$

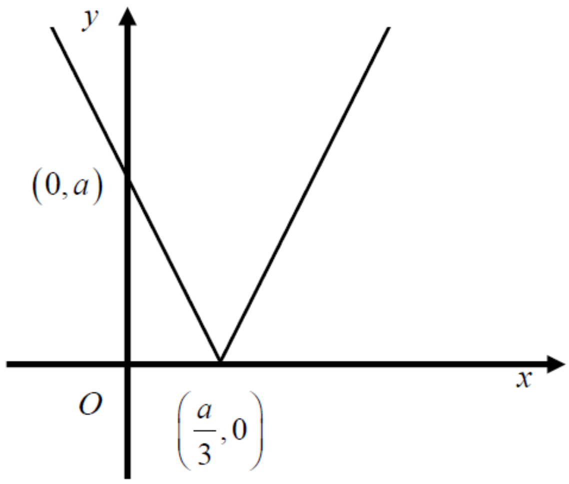
(b) find the two possible values of a.

(3)

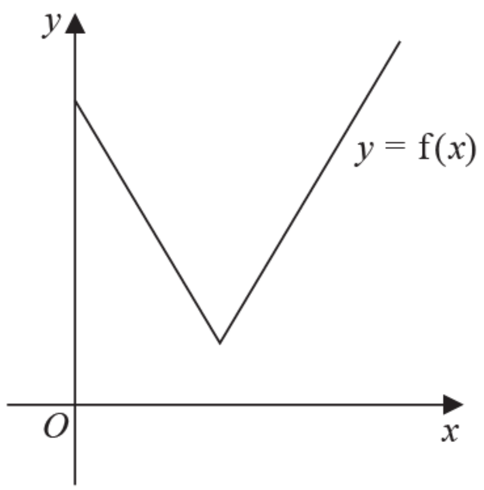
For one of the values of a, $x = 4$ is the smaller of the two solutions. For this value of a,

(c) find the value of the larger solution.

(2)

5(a)		
	V shape on the +ve x axis	B1
	$(0,a)$ and $\left(\frac{a}{3},0\right)$	B1
		(2)
(b) Way 1	Substitutes $x=4$ into $\lvert 3x-a\rvert=\frac{1}{2}x+2 \Rightarrow \lvert 3\times 4-a\rvert=\frac{1}{2}\times 2+2$	M1
	Solves $12-a=\pm 4 \Rightarrow a=8,16$	dM1 A1
		(3)
(c)	Chooses larger value of 'a' solves $3x-a=\frac{1}{2}x+2 \Rightarrow x=...$	M1
	$x=\frac{36}{5}$ or 7.2	A1
		(7 marks)

5.



Not to scale

Figure 2

Figure 2 shows part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|5 - x| + 3, \qquad x \geqslant 0$$

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly one root,

(a) state the set of possible values of k.

(2)

(b) Solve the equation $\mathrm{f}(x) = \frac{1}{2}x + 10$

(4)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = 4\mathrm{f}(x - 1)$. The vertex on the graph with equation $y = 4\mathrm{f}(x - 1)$ has coordinates (p, q).

(c) State the value of p and the value of q.

(2)

5(a)	Either $k > 13$ or $k = 3$	B1
	Both $k > 13$ $k = 3$	B1
		(2)
(b)	Smaller solution: $2(5-x)+3 = \frac{1}{2}x + 10 \Rightarrow x = \frac{6}{5}$	M1 A1
	Larger solution: $-2(5-x)+3 = \frac{1}{2}x + 10 \Rightarrow x = \frac{34}{3}$	M1 A1
		(4)
(c)	$(6,12)$	B1B1
		(2)
		(8 marks)

6. Given that a and b are positive constants,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$

(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

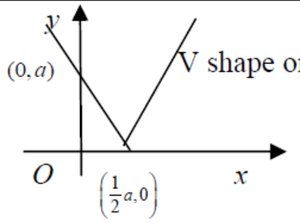
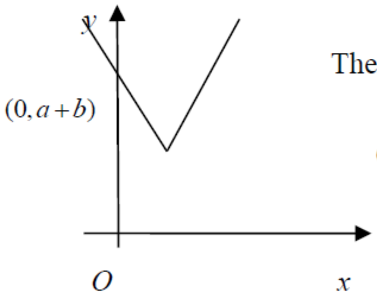
Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

(b) find c in terms of a.

(4)

6.(a)(i)	 V shape on x - axis **or** coordinates $\left(\frac{1}{2}a, 0\right)$ **and** $(0, a)$	B1
	Correct shape, position and coordinates	B1
(ii)	 Their "V" shape translated up or $(0, a+b)$	B1ft
	Correct shape, position and $(0, a+b)$	B1
		(4)
(b)	States or uses $a+b=8$	B1
	Attempts to solve $\lvert 2x-a\rvert+b=\frac{3}{2}x+8$ in either x or with $x=c$	
	$2c-a+b=\frac{3}{2}c+8 \Rightarrow kc=\mathrm{f}(a,b)$	M1
	Combines $kc=\mathrm{f}(a,b)$ with $a+b=8 \quad \Rightarrow c=4a$	dM1 A1
		(4)