Mechanics Sector 2: Forces

Introduction

A force is a push or a pull.
Forces can:

- Change the **motion** of the particle
- Change the **shape** of the particle

Force is a **vector quantity**. This means that it has a magnitude (size) and a direction.
Forces are measured in **Newtons (N)**.

Types of forces

- **Weight**
  - Force due to gravity
  - Weight = mass(kg) x gravitational field strength ($\mathrm{Nkg^{-1}}$)
  - This can be written as

$$W = mg$$

  - On Earth, **g = 9.8 $\mathbf{Nkg^{-1}}$**
  - Weight is an example of a non-contact force (the object does not have to be in contact with the Earth for weight to act on it)
  - Weight always acts **vertically downwards**
  - On a force diagram, we represent weight with an arrow pointing vertically downwards from the particle (or from the centre of mass).

- **Tension**
  - Force that acts through taut strings, cables, rods and springs
  - We assume that the tension is the **same throughout the string**
  - If the string is **slack** then there is **no tension**
  - On a force diagram, tension acts **along the string** pointing **away from** the object

- **Thrust**
  - Forward force that drives an object forward
  - Used in situations such as the force generated via the engine that drives a car forward

- **Normal reaction force**
  - This is the force that acts on an object from a surface due to the object pushing on the surface
  - Always acts in a **perpendicular direction to the surface** that the object is on

- **Friction**
  - Force that acts between two surfaces
  - Always acts **against the desired motion**
  - Depends on the **coefficient of friction μ** and the **normal reaction force**

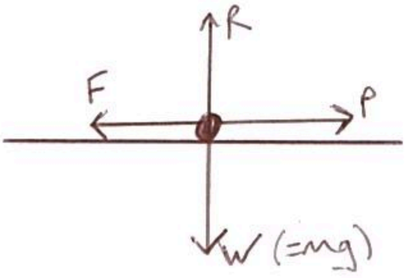
$$F \leq \mu R$$

- **Drag**
  - Force that acts on an object due to the object **moving through a fluid**
  - When object is moving through air, it is called **air resistance**
  - Drag increases as the speed of the object increases
  - When drag = thrust the object will move at terminal velocity
  - Drag acts against the direction of motion
  - If the object is **stationary** then there is **no drag/air resistance**

Example 1

Draw force diagrams for the following situations. State any assumptions that you make.

a) An object at rest on a rough horizontal table with a horizontal force P applied to the object.



Object is a particle

b) A ball bearing freefalling through air.

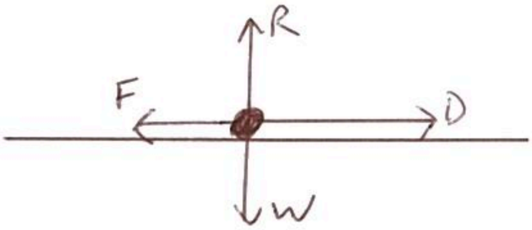
object is a particle

This diagram is assuming that air resistance is negligible

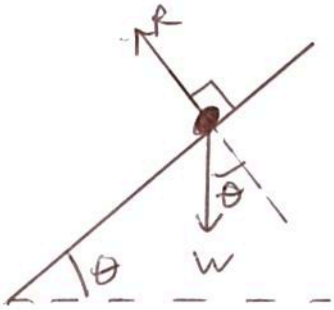
NB
If air resistance is not negligible

c) A toy car accelerating on a rough horizontal surface

Object is a particle



d) A block sliding down a smooth inclined plane (a slope at an angle to the horizontal direction).



Vectors Recap

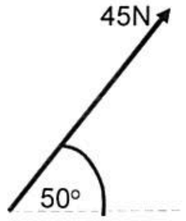
Example 2

NB You must include units
i.e. forces → N

Solve the following situations:

a) Resolve the following vectors either horizontally and vertically or parallel and perpendicular to the plane (whichever is most appropriate)

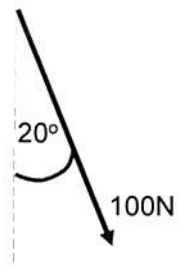
i)



→ $45\cos 50 = 28.9\text{N}$

↑ $45\sin 50 = 34.5\text{N}$ (upwards)

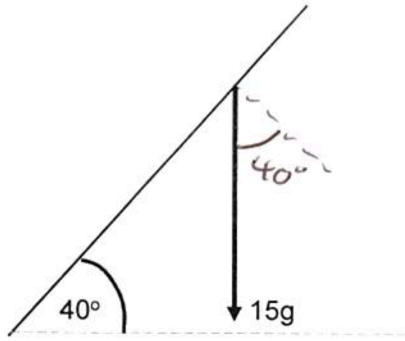
ii)



→ $100\sin 20 = 34.2\text{N}$

↑ $100\cos 20 = 94.0\text{N}$ (downwards)

iii)

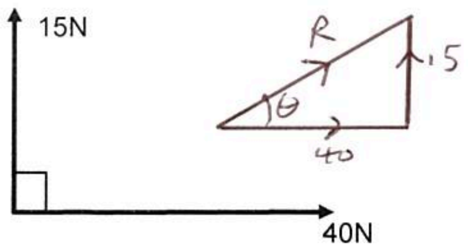


// $15g\sin 40 = 94.5\text{N}$ (down slope)

⊥ $15g\cos 40 = 112.6\text{N}$

b) Find the resultant force of the following set of forces

i)



$R = \sqrt{40^2 + 15^2} = 42.7\text{N}$

$\theta = \tan^{-1}\left(\frac{15}{40}\right) = 20.6°$

ii)

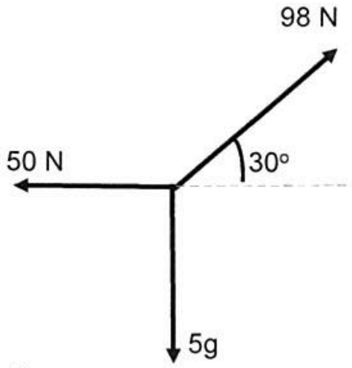
$\underset{\sim}{F_1} = \binom{2}{7}$, $\underset{\sim}{F_2} = \binom{-1}{4}$, $\underset{\sim}{F_3} = \binom{-6}{-3}$

$\underset{\sim}{F_R} = \underline{F_1} + \underline{F_2} + \underline{F_3}$

$= \binom{2}{7} + \binom{-1}{4} + \binom{-6}{-3}$

$= \binom{-5}{8}$

iii)

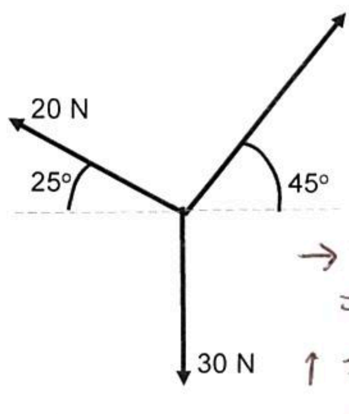


→ $98\cos 30 - 50 = 34.9$

↑ $98\sin 30 - 5g = 0$

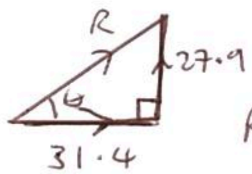
∴ resultant force $= 34.9\text{N}$ to the right

iv)



→ $70\cos 45 - 20\cos 25$
$= 31.4..$

↑ $70\sin 45 + 20\sin 25 - 30$
$= 27.9..$



$R = \sqrt{(31.4..)^2 + (27.9..)^2}$
$= 42.0\text{N}$

$\theta = \tan^{-1}\left(\frac{27.9}{31.4}\right) = 41.7°$

Equilibrium

An object is said to be in equilibrium if

- There is **no resultant force** acting on an object
- There is **no resultant torque** acting on an object (see notes on moments)

This is due to **Newton's 1st Law**

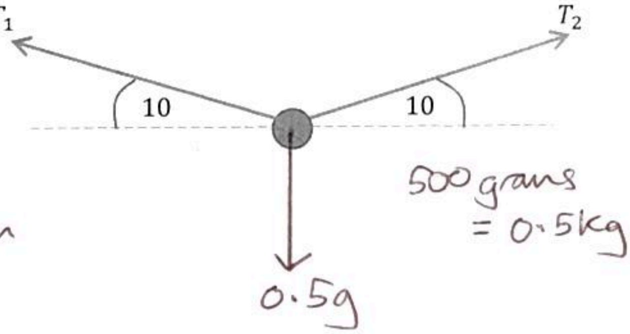
"A particle will remain stationary or, if already moving, will continue to move with a constant velocity in a straight line unless acted on by a resultant force."

When an object is in equilibrium then the components of the forces acting on the object will balance horizontally and vertically

- The sum of the components (of the forces) that are acting horizontally to the left will be equal to the sum of the components that are acting horizontally to the right.
- The sum of the components (of the forces) that are acting vertically upwards will be equal to the sum of the components that are acting vertically downwards

Example 3

A peg bag of mass 500 grams hanging from a washing line may be modelled as a particle supported in equilibrium by two strings, both of which are inclined at an angle $10°$ to the horizontal, as shown in the diagram below.
Find the tension in each string.

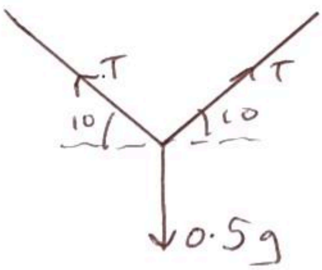


horizontal

$T_2 \cos 10 = T_1 \cos 10$

$\therefore T_1 = T_2$ ← change tension to T

vertical



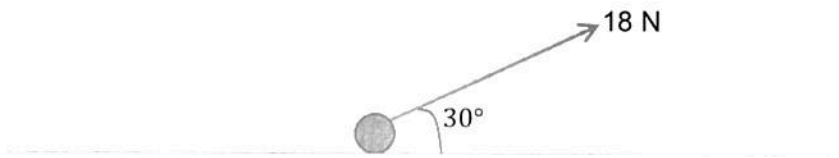
$T \sin 10 + T \sin 10 = 0.5g$

$2T \sin 10 = 0.5g$

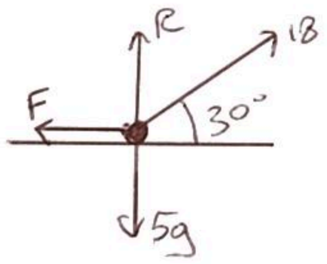
$T = \dfrac{0.5g}{2 \sin 10} = 14.1\text{N}$

Example 4

A particle of mass 5 kg rests in equilibrium on a rough horizontal surface as shown below. A string is attached to the particle. The string makes an angle of 30° to the horizontal and the tension in the string is 18 N.



Find the magnitude of the normal reaction force and the frictional force acting on the particle.



Friction opposes the desired motion

$\rightarrow \quad 18\cos 30 = F$

$F = 15.6\text{N}$

$\uparrow \quad R + 18\sin 30 = 5g$

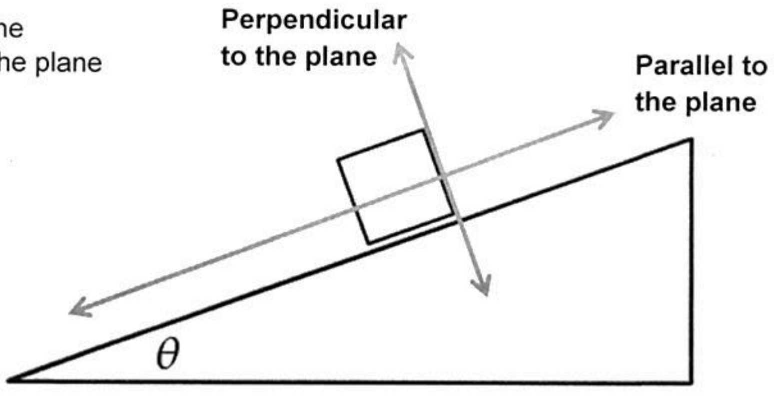
$R = 5g - 18\sin 30$

$= 40\text{N}$

Inclined planes

Sometimes an object may be in equilibrium on an inclined plane. We therefore need to consider the following directions:

- Parallel to the plane
- Perpendicular to the plane

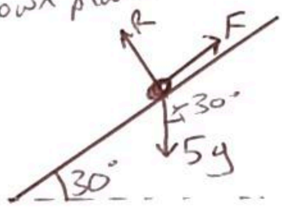


The components of the forces acting on the object will also **balance in these directions**.

Example 5

A particle of mass 5 kg is at rest on a rough plane inclined at an angle of 30° above the horizontal. Find the normal reaction force and the frictional force exerted by the surface of the plane on the particle

desired motion due to weight is down plane
→ F acts up the plane

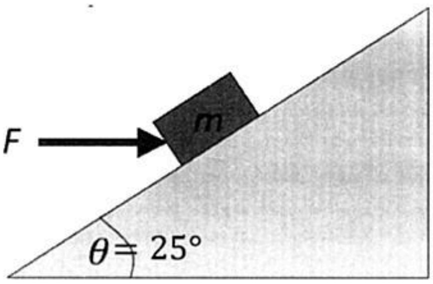


$\perp$ $R = 5g\cos 30 = 42.4N$

// $F = 5g\sin 30 = 24.5N$

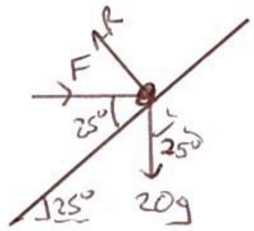
Example 6

An object of mass 20 kg is held in equilibrium on a smooth plane inclined at 25° by a horizontal force F as shown below.



smooth plane
→ no friction force

Find the magnitude of the force F and the normal reaction force acting on the object.



R does not act // to plane but F acts // and $\perp$
→ consider // first to find F

// $F\cos 25 = 20g\sin 25$

$F = 91.4N$

F acts into the plane

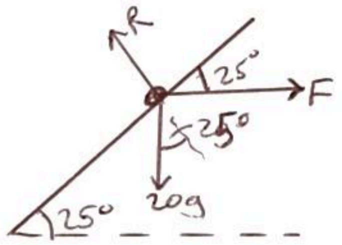
$\perp$ $R = F\sin 25 + 20g\cos 25$

$= 91.4\sin 25 + 20g\cos 25$

$= 216.3N$

$= 216N$ (3sf)

NB
could use this diagram



Forces in Vector Form

As mentioned, force is a vector quantity. Forces can therefore be expressed in vector notation.

If the object is in equilibrium, the forces will balance (there is no resultant force). This means that the sum of each of the components of the vectors (the $\mathbf{i}$ component, $\mathbf{j}$ component and $\mathbf{k}$ component) will be zero.

Example 7

Three forces, $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ are applied to an object. The forces are:

$$\mathbf{F_1} = 2\mathbf{i} - \mathbf{j} \leftarrow \begin{pmatrix}2\\-1\end{pmatrix}$$
$$\mathbf{F_2} = 3\mathbf{i} + 4\mathbf{j} \leftarrow \begin{pmatrix}3\\4\end{pmatrix}$$
$$\mathbf{F_3} = c\mathbf{i} + d\mathbf{j} \leftarrow \begin{pmatrix}c\\d\end{pmatrix}$$

Given that the object remains in equilibrium, find the values of c and d.

equilibrium $\quad \underline{F}_1 + \underline{F}_2 + \underline{F}_3 = 0$

$$\begin{pmatrix}2\\-1\end{pmatrix} + \begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}c\\d\end{pmatrix} = 0$$

$$\begin{pmatrix}5\\3\end{pmatrix} + \begin{pmatrix}c\\d\end{pmatrix} = 0 \quad \therefore \quad c = -5, \quad d = -3$$

or $\underline{F}_3 = \begin{pmatrix}-5\\-3\end{pmatrix}$

Example 8

A particle is in equilibrium, subject to forces $(5\mathbf{i} + \mathbf{j})$ N, $(2\mathbf{i} - 7\mathbf{j})$ N and $\mathbf{P}$ N.

a) Find $\mathbf{P}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.
b) Find the magnitude of $\mathbf{P}$ and the direction of $\mathbf{P}$ as a bearing.

a) $\begin{pmatrix}5\\1\end{pmatrix} + \begin{pmatrix}2\\-7\end{pmatrix} + \underline{P} = 0$

$\begin{pmatrix}7\\-6\end{pmatrix} + \underline{P} = 0 \qquad \underline{P} = \begin{pmatrix}-7\\6\end{pmatrix} = -7\underline{i} + 6\underline{j}$

b) magnitude $= \sqrt{7^2 + 6^2} = \sqrt{85} = 9.22\text{N}$

to left

$\begin{pmatrix}-7\\6\end{pmatrix}$

up

$\alpha = \tan^{-1}\left(\frac{6}{7}\right) = 40.6°$

$\therefore$ bearing $= 270 + 40.6$

$= 310.6°$

$= 311°$ (nearest degree)

Newton's Laws of Motion

Newton's 1st Law

"A particle will remain stationary or, if already moving, will continue to move with a constant velocity in a straight line unless acted on by a resultant force."

We have been obeying this law when dealing with forces that are in **equilibrium**.

Newton's 2nd Law

"The resultant force acting on an object is proportional to the rate of change of momentum of the object."

For an object with a **constant mass** (which is always the case in M1) then this can be simplified to:

"The resultant force F applied to a particle is proportional to the mass of the particle and the acceleration produced."

$$\boxed{F_R = ma}$$

F_R = resultant force (N)
m = mass (kg)
a = acceleration (m/s^2) – note that this acceleration is constant, so you can use it in the **suvat equations** too.

Basically, if there is a resultant force acting on an object, then the object will accelerate in that direction, with the resultant force, mass and acceleration all being linked by the equation above.

Take the direction that the object is moving in as the positive direction.

If there is **no acceleration** in a particular direction then the components of the forces in that direction will **balance** (as they are in equilibrium).

E.g. If an object is **accelerating horizontally** on a **horizontal plane**

- The resultant horizontal component will obey $F_R = ma$ and can be used to find the acceleration (and vice-versa)

- As there is no vertical acceleration, the sum of the components acting **vertically upwards** will **equal** the sum of the forces acting **vertically downwards**

This also applies **parallel and perpendicular** to an inclined plane when an object is accelerating either up or down the plane

- **Parallel** to the plane, the resultant component will obey $F_R = ma$ and can be used to find the acceleration (and vice-versa)

- As there is no acceleration **perpendicular** to the plane, the sum of the components acting **out of the plane** will **equal** the sum of the forces acting **into the plane**

Newton's 3rd Law

"If an object A is applied to an object B then object B applies an equal and opposite force to object A"

This also applies when an object is in contact with a surface (or between two bodies that are not in contact but experience non-contact forces such as gravitational forces).

Examples

1. If an object is at rest on a surface, the object will apply a force on the surface (due to its weight), so the surface applies an equal and opposite force on the object.

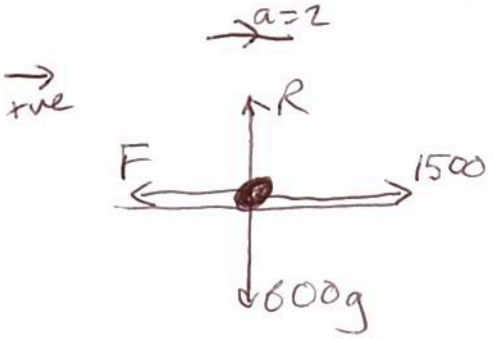
2. The Earth will apply a force on a falling object, and so the object applies an equal and opposite force on the Earth

3. If you punch a wall, you apply a force on the wall, so the wall applies an equal and opposite force on your fist

4. If a positive ion applies a force of repulsion on a second positive ion, the second ion applies an equal and opposite force of repulsion on the first ion.

Example 9

A car of mass 600 kg is travelling along a straight horizontal road with a constant acceleration of 2 ms^{-2}. The engine is exerting a forward force of magnitude 1500N. By modelling the car as a particle, find the magnitude of the resistance it is experiencing.



$\uparrow$ forces balance

$R = 600g$ N

$\rightarrow$ $F_R = ma$

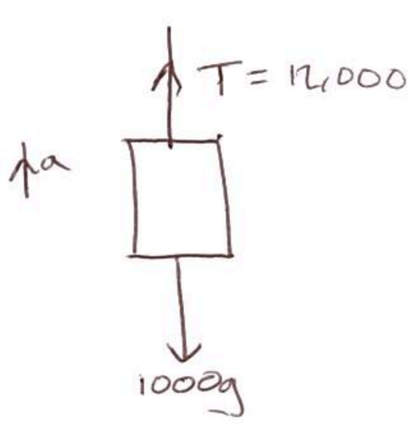
$1500 - F = 600 \times 2$

$F = 1500 - 1200$

$= 300\text{N}$

Example 10

The total mass of a lift and its passengers is 1000 kg. The lift is accelerating upwards. The tension in the cable pulling the lift up is 12000N. Find the acceleration of the lift.



$\uparrow$ +ve

$F_R = ma$

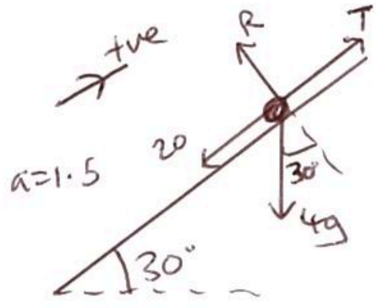
$12{,}000 - 1000g = 1000a$

$a = \dfrac{12{,}000 - 1000g}{1000}$

$= 2.2\ \mathrm{ms}^{-2}$

Example 11

A particle of mass 4kg is being pulled up a rough plane inclined at $30°$ to the horizontal via a light, inextensible string. The frictional force acting against the particle is assumed to be constant with a magnitude of 20N. Given that the particle is accelerating at a uniform rate of 1.5 ms^{-2}, find the tension in the string.



$\perp$ $R = 4g\cos 30$ (equilibrium)

$//$ $F_R = ma$

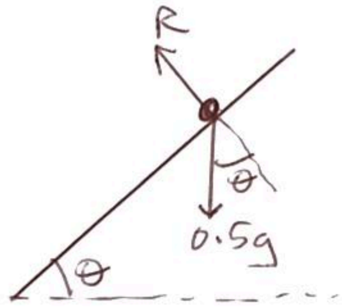
$T - 20 - 4g\sin 30 = 4 \times 1.5$

$T = 45.6\,\mathrm{N}$

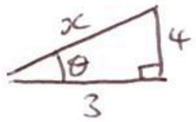
You must resolve parallel and perpendicular to the plane in this case. If you tried to resolve horizontally and vertically, there'd be a component of the acceleration (which is acting parallel to the plane) in both of these directions. You'd therefore have to apply $F_R = ma$ in both dirsctions

Example 12

A small brick of mass 0.5 kg is placed on a smooth plane which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{4}{3}$, and released from rest. Find the acceleration of the brick.



$\tan\theta = 4/3$



$x = \sqrt{3^2 + 4^2} = 5$

$\therefore \cos\theta = 3/5 \quad \sin\theta = 4/5$

$\perp$ $R = 0.5g\cos\theta = 0.5g \times 3/5 = \frac{3}{10}g\ \mathrm{N}$

$//$ $F_R = ma$

$0.5g\sin\theta = 0.5a$

$a = g\sin\theta = 9.8 \times 4/5$

$= 7.84\,\mathrm{ms}^{-2}$

Example 13

Two forces of $3\mathbf{i} + 2\mathbf{j}$ and $5\mathbf{i} - 3\mathbf{j}$ act on a particle of mass 10 kg.

a) What is the acceleration of the particle?

b) What additional force must act on the particle to give it an acceleration of $2\mathbf{i} + \mathbf{j}$?

a) $\underline{F}_R = \underline{F}_1 + \underline{F}_2 = \begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}5\\-3\end{pmatrix} = \begin{pmatrix}8\\-1\end{pmatrix}$ N

$\therefore \underline{a} = \frac{\underline{F}}{10} = \begin{pmatrix}0.8\\-0.1\end{pmatrix}\ \mathrm{ms}^{-2}$ or $0.8\underline{i} - 0.1\underline{j}\ \mathrm{ms}^{-2}$

b) $\underline{a} = \begin{pmatrix}2\\1\end{pmatrix} \quad \therefore \underline{F}_R = \begin{pmatrix}20\\10\end{pmatrix}$

$\underline{F}_R = m\underline{a} = 10\begin{pmatrix}2\\1\end{pmatrix}$

$\underline{F}_1 + \underline{F}_2 + \underline{F}_3 = \underline{F}_R$

$\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}5\\-3\end{pmatrix} + \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}20\\10\end{pmatrix}$

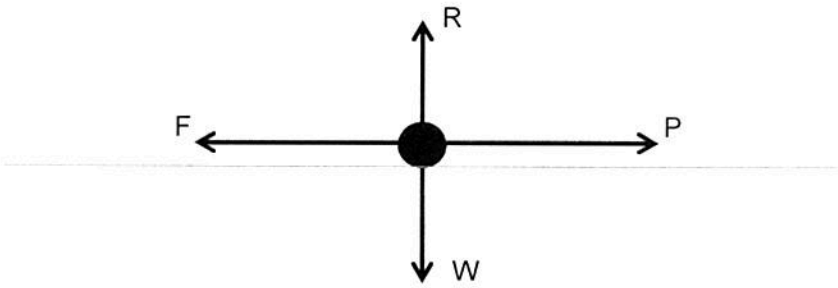
$\therefore \underline{F}_3 = \begin{pmatrix}12\\11\end{pmatrix}$

Friction

Frictional forces act when two surfaces rub together. Friction always acts to oppose the desired direction of motion.

i.e. **the direction of the frictional force is always in the opposite direction to the desired motion.**

Consider the situation below:



If the object is in equilibrium, F = P.

If P = 0, then F = 0 i.e. there is no friction acting on the object (theoretically).

As P increases, F will also increase. The magnitude of friction will be just enough to prevent motion from occurring, up to a maximum value.
So, if P is steadily increased and the body remains at rest, F will also increase so that **F = P remains true**.

However, F can only increase up to a certain limit F_{max}.

At this maximum value, the object is just about to move and is said to be in **a state of limiting equilibrium.** If P increases beyond this maximum value, the object will move (accelerate) and the **friction force will remain at this maximum value**.

The maximum value can be calculated using:

$$\boxed{F_{max} = \mu R}$$

Where

F_{max} = maximum frictional force (N)
R = normal reaction force (N)
μ = coefficient of friction

In general

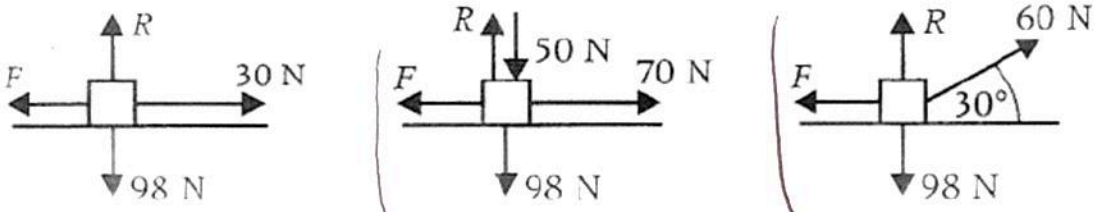
$$\boxed{F \leq \mu R}$$

NB

- If the surface is smooth then F = 0 (as μ = 0)
- $0 \leq \mu \leq 1$ for any two surfaces

Example 14

In the following situations a body of mass 10 kg is placed on a rough horizontal plane. If $\mu = \frac{1}{2}$ in each case determine whether motion will occur.



(a) (b) (c)

a)

$\uparrow \; R = 98N$

$\mu = 1/2$

$\therefore F_{max} = \frac{1}{2} \times 98 = 49N$

Required F of equilibrium = 30N

∴ body remains stationary in equilibrium

b)

$\uparrow \; R = 98 + 50 = 148N$

$F_{max} = \frac{1}{2} \times 148 = 74N$

F required for equilibrium = 70N

∴ body remains stationary in equilibrium

c)

$\uparrow \; R + 60\sin 30 = 98$

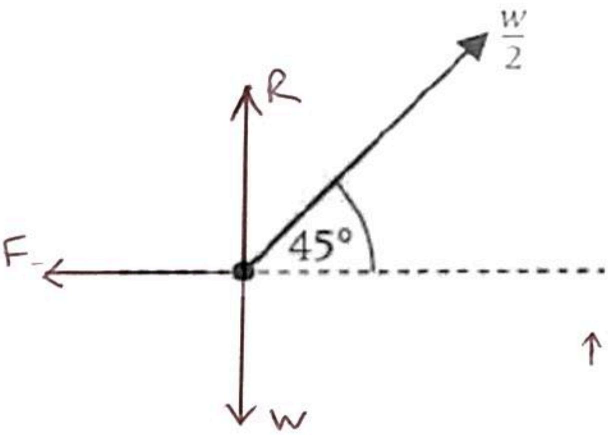
$\therefore R = 68N$

$\therefore F_{max} = 68 \times 1/2 = 34N$

F required for equilibrium = $60\cos 30 = 52.0N$

∴ object will accelerate

Example 15



A particle of weight W is at rest on a rough horizontal surface. A force of magnitude $\frac{W}{2}$ is applied to the particle as shown. Find the coefficient of friction, μ, between the particle and the surface, if the particle is on the point of slipping.

$\rightarrow \; F = \frac{W}{2}\cos 45 = \frac{\sqrt{2}W}{4}$ $\qquad \cos 45 = \frac{\sqrt{2}}{2}$

$\uparrow \; R + \frac{W}{2}\sin 45 = W$

$R = W - \frac{W}{2} \times \frac{\sqrt{2}}{2}$

$= W\left(1 - \frac{\sqrt{2}}{4}\right) = \frac{W(4-\sqrt{2})}{4}$

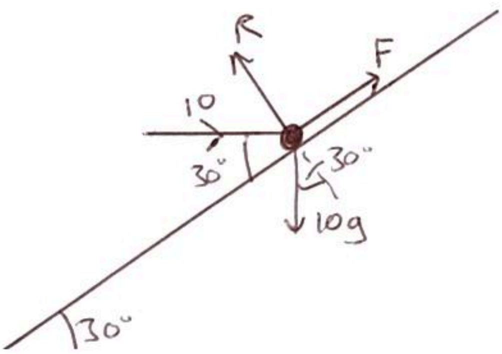
$F = \mu R \quad \therefore \mu = \frac{F}{R} = \frac{\frac{\sqrt{2}}{4}\cancel{W}}{\frac{\cancel{W}(4-\sqrt{2})}{4}} = \frac{\sqrt{2}}{4} \times \frac{4}{4-\sqrt{2}} = \frac{\sqrt{2}}{4-\sqrt{2}}$ or $\frac{1+2\sqrt{2}}{7}$

or 0.547 (3sf)

Example 16

A particle of mass 10 kg is at rest on a rough plane inclined at 30° to the horizontal. A horizontal force of magnitude 10N acts on the particle.

a) Find the magnitude of the force of friction on the particle.
b) The coefficient of friction between the particle and the slope is μ. Find an inequality that μ must satisfy.



$10g\sin 30 = 49N$ $\qquad 49 > 8.66$

$10\cos 30 = 8.66N$

∴ F must act up the plane for object to remain in equilibrium

// $\quad 10g\sin 30 = F + 10\cos 30$

$F = 10g\sin 30 - 10\cos 30$

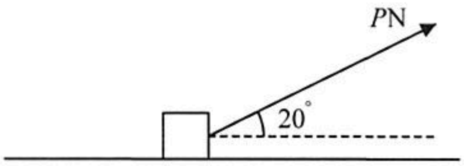
$= 49 - 5\sqrt{3} \quad (= 40.33\ldots)$

⊥ $R = 10\sin 30 + 10g\cos 30$
$= 5 + 49\sqrt{3}$

$F \leq \mu R$ $\quad \frac{F}{R} \leq \mu$

$\mu \geqslant \frac{F}{R}$ $\quad \mu \geqslant \frac{49 - 5\sqrt{3}}{5 + 49\sqrt{3}}$

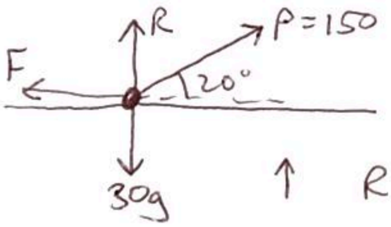
Example 17



A box of mass 30 kg is being pulled from rest along rough horizontal ground using a rope. The rope makes an angle of 20° with the ground, as shown in the diagram above. The tension in the rope is 150 N.

The coefficient of friction between the box and the ground is 0.4. The box is modelled as a particle and the rope as a light, inextensible string.

Find the acceleration of the box. ← if accelerating, $F = F_{max} = \mu R$



↑ $R + 150\sin 20 = 30g$ $\quad \therefore R = 30g - 150\sin 20$
$= 242.7\text{N}$

$\xrightarrow{+ve}$

$\rightarrow$ $F_R = ma$

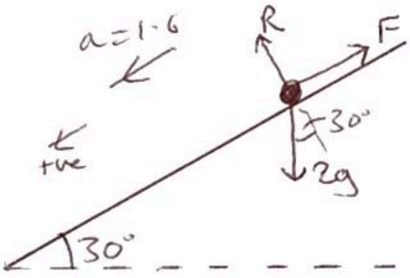
$150\cos 20 - F = 30a$ $\quad F = \mu R = 0.4 \times 242.7..$

$150\cos 20 - 0.4 \times 242.7.. = 30a$

$a = 1.46\ \text{ms}^{-2}$

Example 18

A particle of mass 2 kg slides down a rough plane inclined at 30° to the horizontal. Given that the acceleration is 1.6ms^{-2} find the coefficient of friction between the particle and the plane.



⊥ forces balance

$2g\cos 30 = R$
$R = 16.97..$

// $F_R = ma$

$2g\sin 30 - F = 2 \times 1.6$
$F = 2g\sin 30 - 2 \times 1.6$
$= 6.6$

$F = \mu R$

$\mu = \frac{F}{R} = \frac{6.6}{16.97..} = 0.389$

Exam Questions

AQA M1 January 2012

6 A cyclist freewheels, with a constant acceleration, in a straight line down a slope. As the cyclist moves 50 metres, his speed increases from $4\,\text{m}\,\text{s}^{-1}$ to $10\,\text{m}\,\text{s}^{-1}$.

(a) (i) Find the acceleration of the cyclist. *(3 marks)*

(ii) Find the time that it takes the cyclist to travel this distance. *(3 marks)*

(b) The cyclist has a mass of 70 kg. Calculate the magnitude of the resultant force acting on the cyclist. *(2 marks)*

(c) The slope is inclined at an angle α to the horizontal.

(i) Find α if it is assumed that there is no resistance force acting on the cyclist. *(3 marks)*

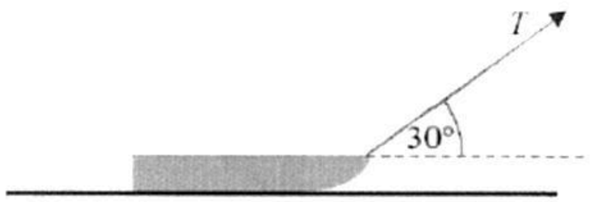
(3 marks)

(ii) Find α if it is assumed that there is a constant resistance force of magnitude 30 newtons acting on the cyclist. *(3 marks)*

(d) Make a criticism of the assumption described in part **(c)(ii)**. *(1 mark)*

AQA M1 June 2012

6 A child pulls a sledge, of mass 8 kg, along a rough horizontal surface, using a light rope. The coefficient of friction between the sledge and the surface is 0.3. The tension in the rope is T newtons. The rope is kept at an angle of $30°$ to the horizontal, as shown in the diagram.



Model the sledge as a particle.

(a) Draw a diagram to show all the forces acting on the sledge. *(1 mark)*

(b) Find the magnitude of the normal reaction force acting on the sledge, in terms of T. *(3 marks)*

(c) Given that the sledge accelerates at $0.05\,\text{m}\,\text{s}^{-2}$, find T. *(6 marks)*

Edexcel M1 January 2013

3. A particle P of mass 2 kg is attached to one end of a light string, the other end of which is attached to a fixed point O. The particle is held in equilibrium, with OP at 30° to the downward vertical, by a force of magnitude F newtons. The force acts in the same vertical plane as the string and acts at an angle of 30° to the horizontal, as shown in Figure 3.

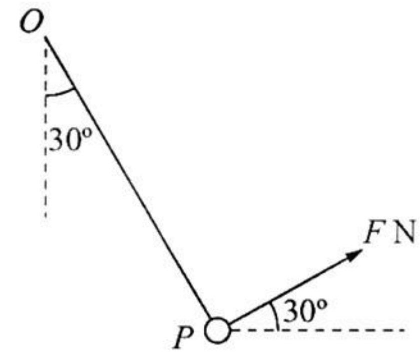


Figure 3

Find

(i) the value of F,

(ii) the tension in the string.

(8)

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7.

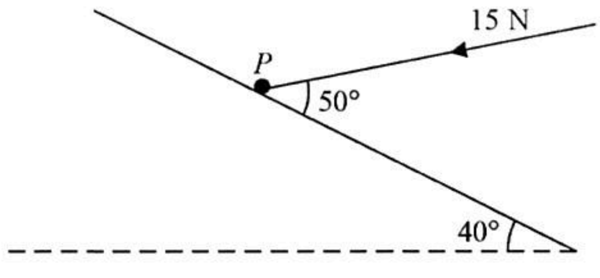

Figure 4

A particle P of mass 2.7 kg lies on a rough plane inclined at 40° to the horizontal. The particle is held in equilibrium by a force of magnitude 15 N acting at an angle of 50° to the plane, as shown in Figure 4. The force acts in a vertical plane containing a line of greatest slope of the plane. The particle is in equilibrium and is on the point of sliding down the plane.

Find

(a) the magnitude of the normal reaction of the plane on P,

(4)

(b) the coefficient of friction between P and the plane.

(5)

The force of magnitude 15 N is removed.

(c) Determine whether P moves, justifying your answer.

(4)

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7.

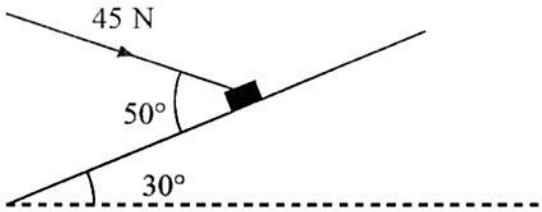

Figure 3

A package of mass 4 kg lies on a rough plane inclined at 30° to the horizontal. The package is held in equilibrium by a force of magnitude 45 N acting at an angle of 50° to the plane, as shown in Figure 3. The force is acting in a vertical plane through a line of greatest slope of the plane. The package is in equilibrium on the point of moving up the plane. The package is modelled as a particle. Find

(a) the magnitude of the normal reaction of the plane on the package,

(5)

(b) the coefficient of friction between the plane and the package.

(6)

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6 a) i) $s = 50$ $\quad v^2 = u^2 + 2as$

$u = 4$ $\quad 10^2 = 4^2 + 2 \times a \times 50$

$v = 10$ $\quad a = \dfrac{10^2 - 4^2}{2 \times 50} = 0.84\,ms^{-2}$

$a = ?$

ii) $t = ?$ $\quad s = \frac{1}{2}(u+v)t$ $\quad \therefore t = \dfrac{50}{\frac{1}{2}(4+10)} = 7.14s$

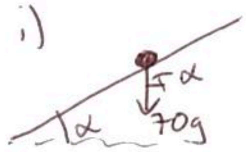
$50 = \frac{1}{2}(4+10)t$

NB could use 'a' answer and one of the other SUVAT eqns

b) $F_R = ma$

$= 70 \times 0.84 = 58.8N$

c) i)

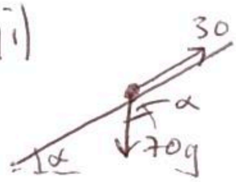


$F_R = 70g \sin\alpha$

$\sin\alpha = \dfrac{58.8}{70g}$ $\quad \therefore \alpha = \sin^{-1}\left(\dfrac{58.8}{70g}\right)$

$= 4.92°$

ii)



$F_R = 70g\sin\alpha - 30 = 58.8$

$\therefore \sin\alpha = \dfrac{58.8 + 30}{70g} = 0.129...$

$\alpha = \sin^{-1}(0.129...)$

$= 7.44°$

d) The air resistance force will increase with speed rather than remaining constant as assumed in c)ii)

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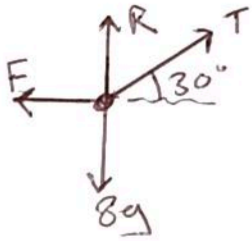
a)



b) $\uparrow$ $R + T\sin 30 = 8g$

$R = 8g - T\sin 30$ $\quad \sin 30 = \frac{1}{2}$

$R = 8g - \frac{1}{2}T$

c) $\rightarrow$ $F_R = ma$

$T\cos 30 - F = 8a$ $\quad F = \mu R$

$= 0.3(8g - \frac{1}{2}T)$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2}T - 0.3(8g - \frac{1}{2}T) = 8 \times 0.05$

$\frac{\sqrt{3}}{2}T + 0.15T - 2.4g = 0.4$

$T\left(\frac{\sqrt{3}}{2} + 0.15\right) = 0.4 + 2.4g$

$T = \dfrac{0.4 + 2.4g}{\frac{\sqrt{3}}{2} + 0.15} = 23.5\,N$

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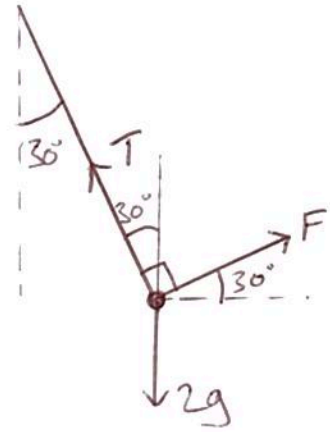

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

i) $\rightarrow$ $F\cos30 = T\sin30$

$F = T\tan30$

$T = \frac{F}{\tan30}$

$\uparrow$

$T\cos30 + F\sin30 = 2g$

$\frac{F\cos30}{\tan30} + F\sin30 = 2g$

$F\left(\frac{\cos30}{\tan30} + \sin30\right) = 2g$

$F = \frac{2g}{\left(\frac{\cos30}{\tan30} + \sin30\right)}$

$= \frac{4g}{5}\text{ N} = 9.8\text{ N}$

ii)

$T = \frac{\frac{4g}{5}}{\tan30} = \frac{4g\sqrt{3}}{5} = 17.0\text{N}$

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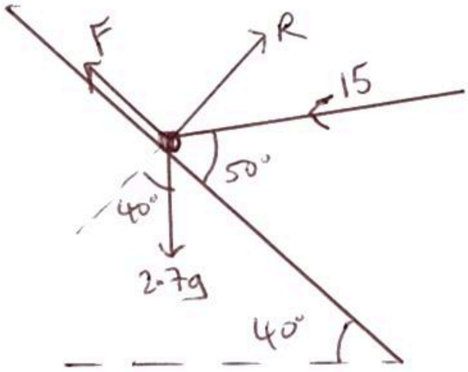


point of sliding down plane

$\rightarrow$ friction acts up the plane

a) $\perp$ $R = 15\sin 50 + 2.7g\cos 40$

$= 31.8N$ (3sf)

b) // $F + 15\cos 50 = 2.7g\sin 40$

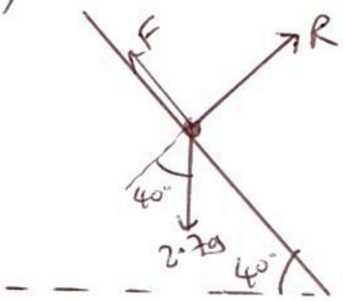
$F = 2.7g\sin 40 - 15\cos 50$

$= 7.37N$

$F = \mu R$ $\therefore \mu = \frac{F}{R} = \frac{7.37\ldots}{31.8\ldots}$

$= 0.232$ (3sf)

c)



15N force affects R $\rightarrow$ need to find new R

$R = 2.7g\cos 40$

$= 20.3$

μ is same

$\rightarrow \mu = 0.232$

$\therefore F_{max} = 0.232\ldots \times 20.3\ldots$

$= 4.70N$

F required for equilibrium $\Rightarrow$ consider // to plane

// $F = 2.7g\sin 40 = 17.0$

$17.0 > 4.70$

i.e. $F_{\substack{\text{required} \\ \text{for equilibrium}}} > F_{max}$

$\therefore$ object will move (accelerate) down the slope

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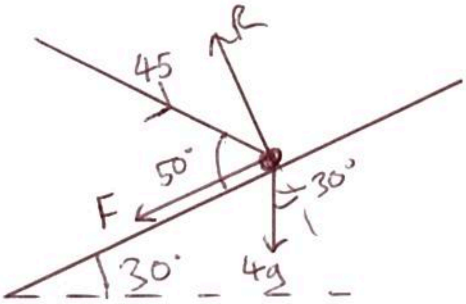

on point of moving up the plane

$\Rightarrow$ F acts down the plane

a) $\perp$ $R = 4g\cos 30 + 45\sin 50$

$= 68.4N$

b) $//$ $F + 4g\sin 30 = 45\cos 50$

$F = 45\cos 50 - 4g\sin 30$

$= 9.33N$

$F = \mu R$

$\therefore \mu = \frac{F}{R} = \frac{9.33...}{68.4...} = 0.136$