7 (a) Given that $y = \frac{2x-5}{x^2-4}$, use the quotient rule to show that

$$\frac{dy}{dx} = \frac{ax^2+bx+c}{(x^2-4)^2}$$

where a, b and c are integers.

[3 marks]

(b) A curve has equation $y = \frac{2x-5}{x^2-4}$.

(i) Find the values of the coordinates of the stationary points of the curve.

[3 marks]

(ii) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of these stationary points.

[4 marks]

1. Given $y = 2x(3x-1)^5$,

(a) find $\frac{dy}{dx}$, giving your answer as a single fully factorised expression.

(4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} \leqslant 0$

(2)

Q7	Solution	Mark	Total	Comment
(a)	$[\frac{dy}{dx}=]\ \frac{(x^2-4)\times 2-(2x-5)\times 2x}{(x^2-4)^2}$	M1 A1		$\frac{(x^2-4)\times k-(2x-5)\times lx}{(x^2-4)^2}$
	$[=\frac{2x^2-8-4x^2+10x}{(x^2-4)^2}]$			
	$=\frac{-2x^2+10x-8}{(x^2-4)^2}$	A1	3	
(b)(i)	$\frac{-2x^2+10x-8}{(x^2-4)^2}=0$			
	$x=1,\ 4$	M1		Solving their quadratic equ
	$x=1,\ y=1$	A1		
	$x=4,\ y=0.25$	A1	3	
(b)(ii)	$\frac{d^2y}{dx^2}=$ $\frac{(x^2-4)^2(-4x+10)-(-2x^2+10x-8)(x^2-4)\times(4x)}{(x^2-4)^4}$	M1 A1		Condone slips in $(-4x+10)$ and $(4x)$ for **M** mark
	$y'=0,\ y''=\frac{-4x+10}{(x^2-4)^2}$			
	$x=1,\ y''=\frac{2}{3}>0$ Min pt	M1		Allow 6 / (+ve) > 0
	$x=4,\ y''=-\frac{1}{24}<0$ Max pt	A1	4	Allow –6 / (+ve) < 0
	Total		10	

1.(a)	$y=2x(3x-1)^5 \Rightarrow \frac{dy}{dx}=2(3x-1)^5+30x(3x-1)^4$	M1A1
	$\Rightarrow \left(\frac{dy}{dx}\right)=2(3x-1)^4\{(3x-1)+15x\}=2(3x-1)^4(18x-1)$	M1A1
		(4)
(b)	$\frac{dy}{dx}\leqslant 0 \Rightarrow 2(3x-1)^4(18x-1)\leqslant 0 \Rightarrow x\leqslant\frac{1}{18}\quad x=\frac{1}{3}$	B1ft, B1
		(2) **(6 marks)**

2 (a) By writing $\cot x$ as $\dfrac{\cos x}{\sin x}$, use the quotient rule to show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$.

[2 marks]

(b) The curve with equation $x = \dfrac{\pi}{12} - y + \cot 3y$ is defined for $0 < y < \dfrac{\pi}{3}$.

(i) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

[2 marks]

(ii) Hence find the exact equation of the tangent to the curve at the point $\left(1, \dfrac{\pi}{12}\right)$, giving your answer in the form $y = mx + c$, where m is a rational number.

[4 marks]

(b) A curve has equation $y = \mathrm{e}^{3x - x^3}$. Find the exact values of the coordinates of the stationary points of the curve and determine the nature of these stationary points.

[7 marks]

Q2	Solution	Mark	Total	Comment
(a)	$\dfrac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}$	M1		Condone 'poor' use of brackets
	Use of $\sin^2 x+\cos^2 x=1$ **OE** $\left(\dfrac{\mathrm{d}(\cot x)}{\mathrm{d}x}=\right)\quad \dfrac{-1}{\sin^2 x}=-\operatorname{cosec}^2 x$	A1	2	**AG** be convinced
(b)(i)	$\left(\dfrac{\mathrm{d}x}{\mathrm{d}y}=\right)-1-3\operatorname{cosec}^2 3y$ **OE**	M1 A1	2	$-1+k\operatorname{cosec}^2(3p)\quad k\neq 0,\ p=x$ or y correct
(ii)	$y=\dfrac{\pi}{12}\Rightarrow\left(\dfrac{\mathrm{d}x}{\mathrm{d}y}=\right)-1-3\operatorname{cosec}^2\left(\dfrac{3\pi}{12}\right)$	M1		Correct substitution of $\pi/4$ into expression of form $-1+k\operatorname{cosec}^2 3y\quad k\neq 0$ **PI**
	Grad of tangent = $1/(\mathrm{d}x/\mathrm{d}y)$	M1		**FT** reciprocal of "their" $\mathrm{d}x/\mathrm{d}y$
	Grad of tangent = $-\dfrac{1}{7}$	A1		Must have scored **M1M1**
	$y=-\dfrac{1}{7}x+\dfrac{\pi}{12}+\dfrac{1}{7}$	A1cso	4	
	Total		8	

(b)	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)\ e^{3x-x^3}(3-3x^2)$ OE	B1		Do not condone poor use of brackets for this mark, unless written correctly later
	Equating their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	M1		**FT** 'their' $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, PI by further working
	$x=1,\ y=e^2$	A1		From $3-3x^2=0$ oe
	$x=-1,\ y=e^{-2}$	A1		And no 'extra' answers, coming from 'exponential terms'
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=e^{3x-x^3}(-6x)+(3-3x^2)^2e^{3x-x^3}$ OE	B1		Do not condone poor use of brackets for this mark, unless written correctly later
	$x=1,\ y''(1)=e^2(-6)$; $x=-1,\ y''(-1)=e^{-2}(6)$	M1		Sub **both** correct x values into their $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$
	$-6e^2<0$ so **maximum** at $x=1$ $6e^{-2}>0$ so **minimum** at $x=-1$	A1		Including inequalities (symbol or wording), and both conclusions. Must have scored 2nd **B1** Final **A** mark can be earned even if **A0A0** earlier
			7	

1 (a) Given that $y = (4x+1)^3 \sin 2x$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

(b) Given that $y = \frac{2x^2+3}{3x^2+4}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{px}{(3x^2+4)^2}$, where p is a constant.

[2 marks]

(c) Given that $y = \ln\left(\frac{2x^2+3}{3x^2+4}\right)$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

7. The curve C has equation $y = \frac{\ln(x^2+1)}{x^2+1}, \quad x \in \mathbb{R}$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a single fraction, simplifying your answer.

(3)

(b) Hence find the exact coordinates of the stationary points of C.

(6)

Q1	Solution	Mark	Total	Comment
(a)	$\left[\frac{dy}{dx}=\right]m(4x+1)^3\cos 2x+n(4x+1)^2\sin 2x$	M1		$m,n\neq 0$
	$m=2$ and $n=4\times 3[=12]$ isw	A1		
			2	
(b)	$\left[\frac{dy}{dx}=\right]\frac{(3x^2+4)4x-(2x^2+3)6x}{(3x^2+4)^2}$ oe	M1		Or $(2x^2+3)(-1)(3x^2+4)^{-2}6x+(3x^2+4)^{-1}4x$
	$=\frac{-2x}{(3x^2+4)^2}$	A1		
			2	
(c)	$\left[\frac{dy}{dx}=\right]\frac{1}{\frac{2x^2+3}{3x^2+4}}\times\text{their } b(i)$ PI	M1		'their b(i)' must be in the correct form $\frac{kx}{(3x^2+4)^2}$
	$\left[\frac{dy}{dx}=\right]\frac{(3x^2+4)}{(2x^2+3)}\times\frac{-2x}{(3x^2+4)^2}$ isw	A1		Or (using rules of logs) $y=\ln(2x^2+3)-\ln(3x^2+4)$
	$\left(=\frac{-2x}{(2x^2+3)(3x^2+4)}\right)$			$\frac{dy}{dx}=\frac{ax}{2x^2+3}-\frac{bx}{3x^2+4}$ M1 $a>0, b>0$ $a=4,\ b=6$ A1

7.(a)	Applies $\frac{vu'-uv'}{v^2}$ to $y=\frac{\ln\left(x^2+1\right)}{x^2+1}$ with $u=\ln\left(x^2+1\right)$ and $v=x^2+1$		
	$\frac{dy}{dx}=\frac{\left(x^2+1\right)\times\frac{2x}{x^2+1}-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$	M1 A1	
	$\frac{dy}{dx}=\frac{2x-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$	A1	
			(3)
(b)	Sets $2x-2x\ln\left(x^2+1\right)=0$	M1	
	$2x\left(1-\ln\left(x^2+1\right)\right)=0\Rightarrow x=\pm\sqrt{e-1},$	M1,A1	
	Sub $x=\pm\sqrt{e-1}, 0$ into $f(x)=\frac{\ln\left(x^2+1\right)}{x^2+1}$	dM1	
	Stationary points $\left(\sqrt{e-1},\frac{1}{e}\right),\left(-\sqrt{e-1},\frac{1}{e}\right),\underline{\underline{(0,0)}}$	A1 B1	
			(6)

7 (a) By writing $\sec x = (\cos x)^{-1}$, use the chain rule to show that, if $y = \sec x$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sec x \tan x$$

[2 marks]

(b) The function f is defined by

$$\mathrm{f}(x) = 2\tan x - 3\sec x, \quad \text{for} \quad 0 < x < \frac{\pi}{2}$$

Find the value of the y-coordinate of the stationary point of the graph of $y = \mathrm{f}(x)$, giving your answer in the form $p\sqrt{q}$, where p and q are integers.

[6 marks]

Q7	Solution	Mark	Total	Comment
(a)	$(\frac{dy}{dx}) = -1\times(\cos x)^{-2}\times -\sin x$	M1		
	$= \frac{\sin x}{\cos^2 x}$ oe			
	$= \frac{\sin x}{\cos x}\times\frac{1}{\cos x}$ oe			Must see 'a middle line'
	$= \tan x \sec x$	A1		AG, all correct and no errors seen
(b)	$(\frac{dy}{dx} =)\ 2\sec^2 x - 3\sec x\tan x$	M1		$m\sec^2 x + n\sec x\tan x$
	$(\frac{dy}{dx} = 0)$			
	$[\sec x](2\sec x - 3\tan x)[=0]$ oe	m1		$[\sec x](m\sec x + n\tan x)[=0]$
	$\sin x = \frac{2}{3}$	A1		Finding **any** correct exact trig ratio
	$\cos x = \frac{\sqrt{5}}{3}$ $\tan x = \frac{2}{\sqrt{5}}$ $\sec x = \frac{3}{\sqrt{5}}$	A1		Finding a second correct exact trig ratio
	$y = 2\times\frac{2}{\sqrt{5}} - 3\times\frac{3}{\sqrt{5}}$	M1		For subst their exact values correctly into 'y' (PI by correct final answer following previous 4 marks earned)
	$y = -\sqrt{5}$	A1 **CSO**	6	Must have used correct exact values throughout If second M mark is not earned, then **SC1** for AWRT –2.24 or $-\sqrt{5}$

8. (a) By writing $\sec\theta = \dfrac{1}{\cos\theta}$, show that $\dfrac{d}{d\theta}(\sec\theta) = \sec\theta\tan\theta$

(2)

(b) Given that

$$x = e^{\sec y} \qquad x > e, \quad 0 < y < \frac{\pi}{2}$$

show that

$$\frac{dy}{dx} = \frac{1}{x\sqrt{g(x)}}, \qquad x > e$$

where $g(x)$ is a function of $\ln x$.

(5)

7. (i) Given $y = 2x(x^2 - 1)^5$, show that

(a) $\dfrac{dy}{dx} = g(x)(x^2 - 1)^4$ where $g(x)$ is a function to be determined.

(4)

(b) Hence find the set of values of x for which $\dfrac{dy}{dx} \geqslant 0$

(2)

(ii) Given

$$x = \ln(\sec 2y), \qquad 0 < y < \frac{\pi}{4}$$

find $\dfrac{dy}{dx}$ as a function of x in its simplest form.

(4)

8(a)	$\dfrac{d}{d\theta}(\sec\theta)=\dfrac{d}{d\theta}(\cos\theta)^{-1}=-1\times(\cos\theta)^{-2}\times-\sin\theta$	M1
	$=\dfrac{1}{\cos\theta}\times\dfrac{\sin\theta}{\cos\theta}$	
	$=\sec\theta\tan\theta$	A1*
		(2)
(b)	$x=e^{\sec y}\Rightarrow\dfrac{dx}{dy}=e^{\sec y}\times\sec y\tan y$ oe	M1A1
	$\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{e^{\sec y}\times\sec y\tan y}$	M1
	Uses $1+\tan^2 y=\sec^2 y$ with $\sec y=\ln x\Rightarrow\tan y=\sqrt{(\ln x)^2-1}$	M1
	$\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{x\times\ln x\times\sqrt{(\ln x)^2-1}}=\dfrac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ oe	A1
		(5)

7(i) (a)	$y=2x(x^2-1)^5\Rightarrow\dfrac{dy}{dx}=(x^2-1)^5\times2+2x\times10x(x^2-1)^4$	M1A1
	$\Rightarrow\dfrac{dy}{dx}=(x^2-1)^4(2x^2-2+20x^2)=(x^2-1)^4(22x^2-2)$	M1 A1
		(4)
(b)	$\dfrac{dy}{dx}\ldots0\Rightarrow(22x^2-2)\ldots0\Rightarrow$ critical values of $\pm\dfrac{1}{\sqrt{11}}$	M1
	$x\ldots\dfrac{1}{\sqrt{11}}$ $x,,\ -\dfrac{1}{\sqrt{11}}$	A1
		(2)
(ii)	$x=\ln(\sec2y)\Rightarrow\dfrac{dx}{dy}=\dfrac{1}{\sec2y}\times2\sec2y\tan2y$	B1
	$\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{2\tan2y}=\dfrac{1}{2\sqrt{\sec^2 2y-1}}=\dfrac{1}{2\sqrt{e^{2x}-1}}$	M1 M1 A1
		(4)

2.

$$y = \frac{4x}{x^2 + 5}$$

(a) Find $\frac{dy}{dx}$, writing your answer as a single fraction in its simplest form.

(4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} < 0$

(3)

5. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

2(a)	$y=\dfrac{4x}{(x^2+5)}\Rightarrow\left(\dfrac{dy}{dx}\right)=\dfrac{4(x^2+5)-4x\times 2x}{(x^2+5)^2}$	M1A1
	$\Rightarrow\left(\dfrac{dy}{dx}\right)=\dfrac{20-4x^2}{(x^2+5)^2}$	M1A1
		(4)
(b)	$\dfrac{20-4x^2}{(x^2+5)^2}<0\Rightarrow x^2>\dfrac{20}{4}$ Critical values of $\pm\sqrt{5}$	M1
	$x<-\sqrt{5},\ x>\sqrt{5}$ or equivalent	dM1A1
		(3)
		7 marks

5.(a)	$p=4\pi^2$ or $(2\pi)^2$	B1
		(1)
(b)	$x=(4y-\sin 2y)^2\Rightarrow\dfrac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	M1A1
	Sub $y=\dfrac{\pi}{2}$ into $\dfrac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	
	$\Rightarrow\dfrac{dx}{dy}=24\pi\quad(=75.4)\ /\ \dfrac{dy}{dx}=\dfrac{1}{24\pi}(=0.013)$	M1
	Equation of tangent $y-\dfrac{\pi}{2}=\dfrac{1}{24\pi}\left(x-4\pi^2\right)$	M1
	Using $y-\dfrac{\pi}{2}=\dfrac{1}{24\pi}\left(x-4\pi^2\right)$ with $x=0\Rightarrow y=\dfrac{\pi}{3}$ cso	M1, A1
		(6)

9. Given that k is a **negative** constant and that the function $f(x)$ is defined by

$$f(x) = 2 - \frac{(x - 5k)(x - k)}{x^2 - 3kx + 2k^2}, \quad x \geqslant 0$$

(a) show that $f(x) = \dfrac{x + k}{x - 2k}$

(3)

(b) Hence find $f'(x)$, giving your answer in its simplest form.

(3)

(c) State, with a reason, whether $f(x)$ is an increasing or a decreasing function.

Justify your answer.

(2)

9.(a)	$x^2-3kx+2k^2=(x-2k)(x-k)$	B1
	$2-\dfrac{(x-5k)(x-k)}{(x-2k)(x-k)}=2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$	M1
	$=\dfrac{x+k}{(x-2k)}$	A1*
		(3)
(b)	Applies $\dfrac{vu'-uv'}{v^2}$ to $y=\dfrac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$	
	$\Rightarrow \mathrm{f}'(x)=\dfrac{(x-2k)\times 1-(x+k)\times 1}{(x-2k)^2}$	M1, A1
	$\Rightarrow \mathrm{f}'(x)=\dfrac{-3k}{(x-2k)^2}$	A1
		(3)
(c)	If $\mathrm{f}'(x)=\dfrac{-Ck}{(x-2k)^2}\Rightarrow \mathrm{f}(x)$ is an increasing function as $\mathrm{f}'(x)>0$,	M1
	$\mathrm{f}'(x)=\dfrac{-3k}{(x-2k)^2}>0$ for all values of x as $\dfrac{\text{negative}\times\text{negative}}{\text{positive}}=\text{positive}$	A1